Three Essays on Social Polarization and Conflict

A thesis presented

by

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Abstract

Theoretically, polarization is associated with a higher probability of social conflict. In the first chapter, I present the main theoretical issues underlying the economic theory of polarization and applied them to analyze polarization in Bogotá for the period 2000-2003. In this period the Estaban-Gradín-Ray index of polarization has been reduced by 10 % in the case of bipolar representation of the original distribution and by 8% in the case of multipolar representation. The reduction, in the bipolar case, is mainly explained by a reduction in the income distance between the two groups. In fact, there is an increase in the average income of the lower income group and by a decrease in the average income of the high income group. In the case of multipolar representation the reduction is explained by a reduction in simple polarization due to the reduction in the income distance between the groups and the fact that they become more uneven.

Additionally, when looking at Group and Explained Polarization idexes education seems to be the most relevant socioeconomic characteristic affecting economic polarization.

In the second chapter, I use the concept of Stability Sets in order to analyze the relative probability of the two possible equilibria of a political-economy model char-

acterizing a polarized society. In the model, society is composed of two social groups and I focus on how changes in the economic parameters of the model, underlying the political prferences of the groups, alter the probabilty of collective action taking place in order to pass from one equilibrium to the other.

Finally, in the third chapter, I propose a microeconomic model based on the theory of social networks for analyzing how changes in the network's structure affect the level of some basic parameters associated with the concept of polarization and through it to a higher probability of conflict.

Contents

Introduction

To some it may seem strange that a doctoral thesis in economics should be dedicated to the study of social problems such as civil wars. I must explain my personal reasons for this.

I grew up in a country in upheaval where deaths, massacres, bombs and, without exaggeration, terror were a part of daily life. Such are the horrors of a war; a war that seems so complex that apparently the only possible response is to learn to live with it.

My earliest memories of this reality are of the daily television newscasts. In them I heard of bloodshed in remote regions and of the conditions of extreme poverty that many of my countrymen bore: an extreme and patent inequality. As a child, my perception of our conflict was somewhat blurred but my feelings were sharp – a deep sense of pain. I never understood why one human being would kill another, and much less with such cruelty. But these events happened in places far from Bogotá, the city where I was born and lived, and where I felt safe. I understood that this war was not fought in cities and that for many "citizens" (that is, residents of cities) it went unnoticed.

But then, new factors – illicit trade in drugs – and new actors – paramilitary groups and mafias at the service of druglords – arose. They made conflict more intense and led to its transformation in unforeseeable ways.

Violence reached the cities, Bogotá amongst them, with new faces. Some of its aspects were indiscriminate attacks against all sorts of persons and institutions, bombs in buildings and passenger airplanes and a heyday of contract killing.

I remember, for example, the armed assault on the Palace of Justice in Bogotá by the M19 guerrilla. This event, which resulted in the tragic death of at least 100 people and the disappearance of at least 11 more, put and end to the sense of security that living in the city had given me. An imposing edifice which had housed the judicial power of the State was reduced to ashes in a few hours. I understood the State was weak and vulnerable. I remember that on that afternoon I hid, with my tears and fright, under my mother's bed, while I wondered whether she would return alive – she worked a few blocks from the Palace. I cannot forget my happiness and relief when I heard her come home at nightfall.

I also remember how more than 2000 members of a left-wing party created in the mid-eighties, the "Union Patriotica", were systematically killed under the blind gaze of the State. A State that has never been strong, that has been unwilling to remedy social exclusion and that has been shaken time and time again by the blows of corruption, insurgence, paramilitaries and drug trafficking. I cannot forget that those brave enough to speak these truths out loud have also been systematically murdered with impunity.

My consciousness of the grave situation of my country and its somber prospects made me feel that I could not allow anguish to paralyze me or to make light of that reality. I did not want to be trapped by the distraction of a beauty pageant or a "vallenato" music concert, and to act as if nothing was happening (which is an almost infallible prescription in Colombia). What was to be done? This is the origin of my thesis. It is a childhood promise, made to myself: to never be indifferent towards war, poverty, death and pain. This is my modest tribute to those who have died in the midst of the war that my country lives and to those that cannot do anything about it, aside from mourning them. For the author, any merit this thesis may have is not merely academic. It is a promise kept

I am conscious that my contribution is minor. During the last years, I have done the best I of which am capable, with total honesty and with the courage to face my own fears and limitations. This is my best reward for the efforts I have made during my doctorate.

The process that led to this work began with my first idea about the relation between inequality and social conflict. It sprang from the reality of my country, and found a theoretical structure in Amartya Sen's work on inequality. In the course of my doctoral studies my research led me to the concept of polarization. The idea was new to me but after having assimilated it I decided it was more appropriate for the study of social conflict than inequality. The initial step from inequality to polarization constitutes the first chapter of this thesis. In it, I present both concepts and their similarities and differences, with emphasis on the detailed analysis of the components that underlie the notion of polarization. In order to see these at work, this chapter concludes with an empirical exercise that measures levels of polarization in Bogotá in the first years of this century.

With a better understanding of polarization's relevance for the analysis of social conflict, I anticipated that my work would require a further approximation to other social sciences and especially political science. I began to work on the idea of a political economy model with polarization which would permit the analysis of changes in certain objective parameters could affect the probability of a social conflict, such as a civil war, occurring. The result of this effort is a model in which a society is seen as a combination of two groups which are polarized regarding economic policy decisions and with different levels of political power. The model allows the analysis of changes in the probability of a group's members being able to undertake collective action – such as a civil war – when faced with changes in the objective parameters of the model, especially economic inequality, variations in production factor prices or the distribution of the latter.

Finally, in the third paper, my purpose is to translate some of the concepts that underlie polarization into a different language, that of social networks. This has the object of evaluating, a priori, what kinds of social configurations can be described as more or less polarized. In this particular context a given network is like a picture of a society. If with such a picture we can identify the risk of a high degree of polarization, then when we have more detailed knowledge of the society (its individuals, institutions or groups and the way they are linked) we will be better placed to prevent polarization from deepening.

Chapter 1
Analyzing Polarization: Evidence from Bogotá.

In a nutshell, the driving force of my whole dissertation is my belief that polarized societies have a higher probability to suffer from situations of social unrest.

A theory of polarization has been just recently developed in economics, therefore, and aware of the importance of this concept in my work I devote this first chapter to a detailed presentation of the main issues lying at the core of the theory.

In economics, It was probably Marx the first one dealing with the very notion of polarization and its relation with social conflict. He describes two well defined groups engaged in social conflict, workers and capitalists. However, the lack of a theory of polarization postponed a systematic analysis of the matter until very recently, in fact the concept was commonly used in social sciences with some vagueness and there was no clear understanding of the channels through which polarization may foster social conflict. For example, also in economics, the celebrated work on economic inequality by Amartya Sen opens the first chapter claiming that "The relation between inequality and rebellion is indeed a close one, and it runs both ways" Sen[12]. However, the black box containing the connections allowing for such a relation is still uncover in his work. Furthermore, are we sure that inequality is the relevant concept to relate with rebellion instead of polarization, for example?

In fact, it appears to be that polarization has a closer relation than inequality with the generation of conflict. Therefore, the distinction between the two concepts becomes crucial. Additionally, clarification on the theoretical differences between these two notions shares light when matters come to taking decisions on economic policy.

It is fair to say that the idea that a society split up into few groups increases the probability of potential social conflict is not exclusive to economics, sociologists such and political scientists such as Simmel[14], Gurr[9] and Tilly[15] also noticed this causal relationship. However, to the best of my knowledge there is no a developed theory of polarization in these sciences; so the present work relies entirely on the theory of polarization developed in economics.

The rest of the chapter proceeds as follows: Part I is divided into two sections. Section 1 provides a glance at the concept of inequality and the basic axioms for any inequality measure to be accurate, then and with the aim of posterior comparison with some polarization measures, I just mention the most widely used measure of income inequality in economics, the Gini coefficient.

Similarly, but in a more detailed manner, Section 2 presents the notion of economic polarization, the basic axioms for any measure of polarization to be accurate and the measure proposed by Esteban and Ray [4], which is the most commonly measure of polarization being used. Further I analyze some of its extensions.

Part II is an application of the concept, in particular, I measure income polarization in Bogotá at the beginning of the 2000s. After describing both, the data and the income variable used in the calculations, I analyze the results of the Esteban-Gradín-Ray Index of polarization by looking at the behavior of its main components and some extensions of the index, in particular those of group and explained polarization.

1.1 Glancing at Inequality in Economics.

A natural way of introducing the concept of economic polarization is by analyzing the more familiar concept of economic inequality. This section provides by no means an exhaustive study of inequality nor of the way it is measured. A detailed analysis of the concept of inequality can be found in Sen and Foster [12], for its measurement see Silber [13]. Here, I limited myself to presenting inequality in such a way that the reader can more easily incorporate the more intuitive concept of polarization and grasp differences between the two notions. The notion of polarization is presented in the next section.

Next, I present a definition of the inequality concept in economics and the minimal desirable requirements for having an accurate inequality measure of it. Considering that these requirements become more palatable in the so called Lorenz Criteria, I briefly present the Lorenz contribution to the analysis of inequality, the Lorenz curve, and the Lorenz Criteria. Finally the most popular index of inequality in economics is presented, the Gini coefficient.

1.1.1 Defining Economic Inequality.

When talking about inequality one could easily inquire about the very meaning of inequality and, as a natural step further, for the meaning of justice in general and, that of economic justice in particular. Not only philosophers but also sociologists, political theorists, statisticians and economics have devoted time and effort to these matters however, the present section narrows its scope to the definition of economic inequality in terms of income no because inequality can be fully understood through income (or wealth) but because it is believed that income (or wealth) is a fundamental component of the rest of economic inequalities.

In a simple manner *economic inequality* can be defined as the fundamental disparity that permits one individual certain material choices while denying another individual those very same choices (Ray[11] Ch. 6 pp170).

This notion of economic inequality is quantified by means of an inequality measure. An *inequality measure* is a rule that assigns a degree of inequality to each possible distribution of income or wealth. This rule should satisfy some minimal requirements such as anonymity, independence with respect to both, size of the population and absolute levels of income and the “Dalton” criteria, in order to provide an appropriate quantification of economic inequality. A higher value of the measure means the presence of greater inequality.

To be more precise let $I = I(y_1, y_2, \ldots, y_n)$ be a function defined over all conceivable income distributions $\mathbf{y} = (y_1, y_2, \ldots, y_n)$. with y_i equal to the income of individual $i = 1, ..., n$ for any positive integer n. The function I can be interpreted as an inequality measure if it satisfies, at least, the requirements stated above and formalized as follows:

1 (Anonymity) *given an income distribution* $\mathbf{y}$*, permutations of incomes among individuals should no change the inequality measure.*

2 (Population) *Any population, independently of their size, with the same proportions of people earning the same different levels of income should have the same inequality measure.*

$$I(y_1, y_2, \ldots, y_n) = I(y_1, y_2, \ldots, y_n; y_1, y_2, \ldots, y_n).$$

3 (Relative Income) *Any population, independently of the absolute levels of income, with the same relative values of income should have the same inequality measure.*

$$I(y_1, y_2, \ldots, y_n) = I(\rho y_1, \rho y_2, \ldots, \rho y_n) \text{ for any } \rho > 0.$$

4 (Dalton) *The Dalton principle, due to Dalton 1920, states that if one income distribution can be achieved from another by constructing a sequence of regressive* *transfers, then the former distribution must be deemed more unequal than the latter. In other words, any transfer of income from y_i towards y_j, with $y_i \leq y_j$,must increase inequality.*

The inequality measure I satisfies the Dalton Principle if for every income distribution $\mathbf{y}$ and every transfer $\delta > 0$

$$I(y_1, y_2, \ldots, y_i, \ldots, y_j, \ldots, y_n) \ < \ I(y_1, y_2, \ldots, y_i \ - \ \delta, \ldots, y_j \ + \ \delta, \ldots, y_n),$$

whenever $y_i \leq y_j$

In economics, several measures of inequality have been proposed going from very simple ones as the Range to more sophisticated as the Theil's Entropy measure, passing through the well known Gini coefficient[1].

Before explaining the Gini coefficient, the most popular inequality measure in economics and the one that will be used in the empirical part of the present chapter, I briefly introduce the Lorenz curve. The Lorenz curve, in my opinion, allows for an intuitive introduction of the formula for the Gini coefficient and, more importantly, as stated above it makes more palatable those four criteria by grouping them in a single criteria, the Lorenz criteria.

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[1] The range, perhaps the simplest measure of economic inequality, results from the comparison between the two extreme values of an income distribution as a ratio of mean income: $R = (Max_i y_i - Min_i y_i) / \mu$.

Th Theil's entropy meausre of economic inequality: $T = \log(n) - H(x)$, where $H(x) = \sum_{i=1}^{n} x_i \log(1/x_i)$.

results from the comparison between the maximun value of $H(x)$ and the actual value of it for a given income distribution. If the income distribution is such that each person gets the same share of income, $x_i = 1/n$, then $H(x)$ achives its maximum value $H(X) = log(n)$ and then $T = 0$.By the contrary, in the case of complete inequality $H(X) = 0$ and then $T = \log(n)$.

A complete revision of different measures of inequality can be found in Sen (1997, Ch.2).

1.1.2 The Lorenz Contribution.

The Lorenz curve is a graphical representation of the distribution of income in any society. In obtaining the curve population is arranged in cumulative percentages from poorest to richest on the horizontal axis and the percentages of national income accruing to any fraction of the population are measured on the vertical axis.

The Lorenz Curve

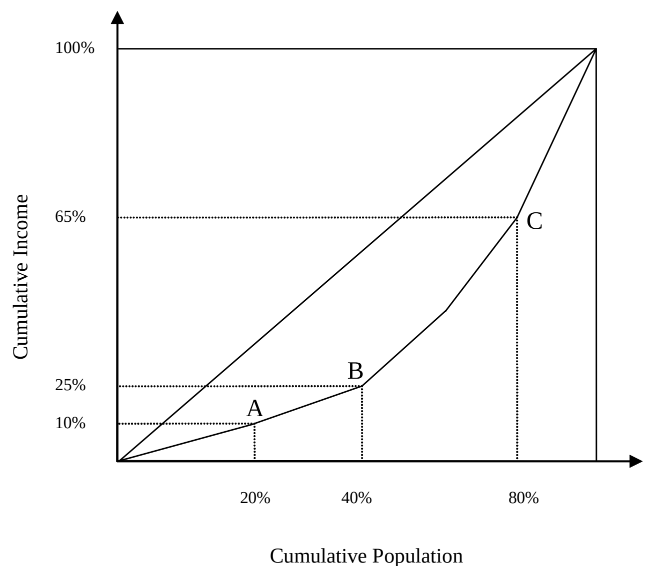

Figure 1

Thus, the point A in Figure 1. tell us that the cumulative 20% of the poorest population receives the cumulative 10% of the income; the point B tell us that the cumulative 40% of the poorest population receives the cumulative 25% of the income and so on.

Three important properties of the Lorenz curve should be noticed. First, it begins and ends on the 45° line, it means that the poorest 0% earns 0% of national income and that the poorest 100% earn 100% of national income i.e. total income is earned by the whole population. Second, the Lorenz curve can never get flatter as we move from left to right. This is so because the slope of the curve at any point represents the marginal contribution of a particular individual to the cumulative share of national income and because we have ordered individuals from poorest to richest that contribution cannot ever fall. Third, if everybody earns the same amount of income then the Lorenz curve must coincide with the 45° line. Any point on the 45° diagonal indicates the relationship $y = x$, meaning, for instance, that the poorest 10% earn 10% of national income and so on. In other words if national income is equally distributed among individuals in the population then the Lorenz curve coincides with the 45° line. In fact the 45° line is also called line of absolute equality.

It is easy to have an intuitive idea of the amount of inequality present in any society by glancing at the Lorenz curve diagram; the greater the area between the line of absolute equality and the Lorenz curve the grater the amount of inequality in the society.

As stated at the beginning of the section, the importance of the Lorenz curve dwells on summarizing, in a single criteria, the four previous criteria for an accurate inequality measure.

5 (Lorenz) *The inequality measure I satisfies the Lorenz criteria if, for every pair of income distributions* $\mathbf{y}$ *and* $\mathbf{z}$, $I(y_1, y_2, \ldots, y_n) \geq I(z_1, z_2, \ldots, z_n)$ *whenever the Lorenz curve of* $\mathbf{y}$ *lies, at all points, to the right of* $\mathbf{z}$.

In a sense, the Lorenz criteria embodies all the four previous criteria because, *an inequality measure is consistent with the Lorenz criterion if and only if it is simultaneously consistent with the anonymity, population, relative income and Dalton criteria* (See[12]).

1.1.3 An Index of Economic Inequality: The Gini Coefficient.

The Lorenz curve is an indicative of the amount of inequality in any society however the curve by itself does not provide any specific measure of it, the Gini coefficient does.

The Gini coefficient, due to Gini [7], measures the difference between the line of absolute equality and the Lorenz curve and express it as a ratio to the triangular region underneath the diagonal. Formally, it can be defined as:

$$G = \frac{1}{2n^2\mu} \sum_{i=1}^{n} \sum_{j=1}^{n} n_i n_j \left| yi - y_j \right| \tag{1.1}$$

The formulation of the coefficient can be seen as the normalization of the sum of the absolute values of differences between all pairs of income. More precisely the sum is normalized by dividing it by 2 and by population squared as well as mean income. Notice than when taking all pairwise income comparisons each $y_i - y_j$ is counted twice (again as $y_j - y_i$), that is why the sum is normalized by dividing by 2; the second normalizer correspond to population considerations, the number of all possible pairs of incomes is n^2, finally, the mean income, μ, is used as income normalizer. For our purposes, it is important to notice that the measure uses individual comparisons (no groups) and that individual n_i has the same weight than any other n_j in the population, they both have power equal to one.

1.2 Measuring Polarization in Economics.

The main purpose of this section is to present the Esteban-Gradín-Ray index of polarization, the one that will be used for measuring polarization in Bogotá at the beginning of the 2000s in the next section. Here, I formally present the concept of polarization developed by Esteban and Ray (1994), and present some of the main theoretical refinements of the index, in particular those by Esteban, Gradín and Ray[6] , Gradín [8] and Duclos, Esteban and Ray[3] . Finally, I suggest my own contribution to the way of measuring polarization.

In economics, the proposal of an axiomatic definition and measure for polarization was started, independently, by the work of Esteban and Ray [4] and Wolfson [16]. The former work formally defines the concept of polarization as a way to overcome the weakness of classic inequality measures when dealing with a wider interpretation of social distance in economic terms. This interpretation, though not new, relates societies divided in few, homogeneous, but distant groups with the increase in the probability of conflictive situations. In other words they arrive to the polarization index by studying the inequality measures itself. With a different approach, the latter work, that of Wolfson, provides a formal definition of the concept of polarization as a necessity for differentiating the phenomena of “disappearing middle class” from the more general of income inequality. In fact he identifies the phenomena of “disappearing middle class”, present in the studies of changes in income distribution in America at the beginning of the 80’s, with the concept of polarization, different from just income inequality.

The reader might wonder why to concentrate here in analyzing the concept and measurement of polarization within the theoretical framework developed by Esteban and Ray? There are two reasons that justify this choice: first, as I said above, the empirical part of this chapter handles precisely the index developed by Esteban and Ray as well as further developments of the index using the same theoretical framework and, second, it can be shown that the Wolfson’s polarization index can be seen as a particular case of the Esteban and Ray’s polarization index so there is no loss of generality in this selection.

1.2.1 Defining Polarization.

Suppose we are able to observe the distribution of a population with respect to the set of attributes Y. If the distribution describes a population grouped around few but distant poles then we can say that the population is polarized in terms of the set Y. In general, the set of attributes might include several variables however, in practice, it usually contains only one characteristic i.e. political preferences, religious preferences, income, etc. If the set of attributes reduces to income or wealth, then we say that population exhibits *economic polarization*.

To facilitate the understanding of the concept consider the following intuitive example: Suppose that the second column in Table 1. reports levels of income for each of the ten closed economies of the countries labeled with capital letters in the first column. Further, suppose that all the ten countries engage in a Free Trade Agreement with the aim of reducing economic inequality. Column 3 in Table 1. reports countries' income levels for all countries after five years of free trade in their new open economies scheme.

Table 1

Country	Income 1	Income 2
	(No Trade)	(Trade)
A	1	3
B	2	3
C	3	3
D	4	3
E	5	3
F	6	8
G	7	8
H	8	8
I	9	8
J	10	8

The two income distributions, before and after the Free Trade Agreement, are plotted in Figures 2 and 3 respectively.

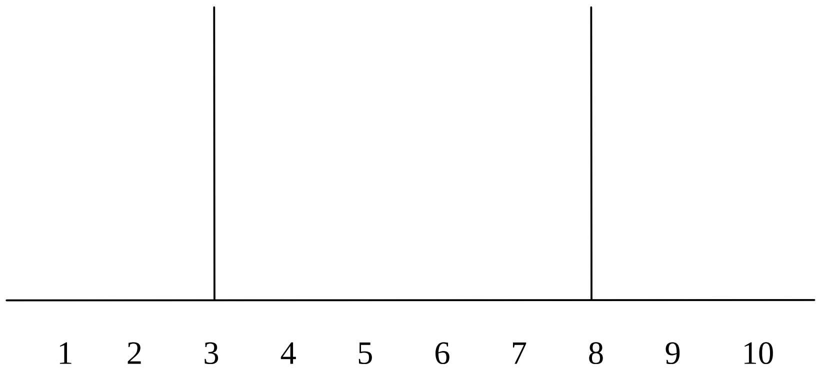

Figure 2

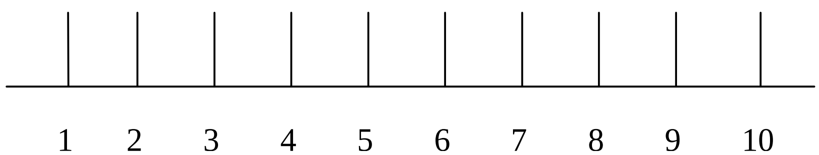

Figure 3

In the case of the first income distribution the mean income is equal to $\mu = 5.5$. Next, after five years of the free trade agreement the second income distribution can be seen as a redistributive process of transfers. Countries above the mean of the distribution converge to their local mean $\mu_a = 8$, while countries below the mean converge to their local mean $\mu_b = 3$. Comparing the two income distributions it is always the case that inequality, under any definition based on the Dalton principle, would decrease between the first and the second distribution but polarization would certainly increase. (See section 1 for the definition of the concept of inequality and the Dalton Principle)

This example help us to realize that the phenomenon of polarization differs from the one of inequality, in fact a decrease in inequality is consistent with an increase in polarization[2]. Observe that in the latter distribution, the one after transfers, individuals gather in two groups, one around μ_a and the other around μ_b and that the distance between these two groups has increased. Therefore, we can characterize the latter distribution by the existence of two homogenous groups which at the same time exhibit an important amount of distance or heterogeneity between them. These two characteristics, within-group homogeneity and between-group heterogeneity are the key elements when talking about polarization and lie at the core of the alienation-identification theoretical framework proposed by Esteban and Ray: If a distribution of a population with respect to a particular set of characteristics is grouped into few clusters, such that each cluster is very similar in terms of the attributes of its members, but different clusters have members with very dissimilar attributes, then it could be said that the population is polarized. It is usually said in the literature that members within each group feel identification but, at the same time, alienated from members of other groups.

1.2.2 A Polarization Index.

How to measure economic polarization? By now, it should be clear that any measure of polarization must embody three basic elements:

1. There must be a small number of significantly sized groups[3].

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[2] Notice that polarization is different from inequality, then it does not mean that both concepts are necessarily opposite, they are just different. In fact it is possible to have an increase in polarization together with an increase in inequality (See Esteban and Ray 1994).

[3] As the authors state it is possible to think about non-significantly sized groups or even isolated

2. An individual belonging to any of the groups must feel a sense of *identification* with the rest of individuals belonging to the same group, and

3. Individuals belonging to the same group must feel some degree of *alienation* from individuals belonging to other groups.

Formally, let the couple of vectors $(\boldsymbol{\pi}, \mathbf{y})$ represent an income distribution. $\pi_i \in \boldsymbol{\pi}$ is the fraction of the population located at the $y_i \in \mathbf{y}$ level of income. The vector $\mathbf{y}$ is ordered in such a way that $y_i \leq y_{i+1}$ for all $i = 1, ..., n$ and the total population associated with $(\boldsymbol{\pi}, \mathbf{y})$ is given by $\sum_{i=1}^{n} \pi_i$ and $\boldsymbol{\pi} > 0$. Then, a polarization measure (P) is a mapping $P : (\boldsymbol{\pi}, \mathbf{y}) \rightarrow R_+$.

The within group identification feeling stated above is formally characterized by a continuous and increasing *identification function* $I(\pi_i) : R_+ \rightarrow R_+$, for all i and with $\pi_i > 0$ as the measure of people in the same income class of individual y_i. The alienation feeling of individual y_i with respect to individual y_j is formally described by a continuous and non-decreasing *alienation function* $a(\delta(y_i, y_j)) : R_+ \rightarrow R_+$, with $\delta(y_i, y_j) = \mid y_i - y_j \mid$.

Both alienation and identification concepts are assumed to be symmetric, in other words, y_i's alienation feeling from y_j is assumed to be exactly the same alienation that individual y_j feels from individual y_i; the same is assumed to be true true in the case of identification feelings.

individuals, however, the probability for social conflict to arise in such a cases is irrelevant.

Finally, the total polarization in a society is defined as the sum of all the *effective antagonisms* $T(I, a)$, it is to say the sum of all alienation feelings between individuals but simultaneously accounting for the feelings of individual identification.

$$P(\pi, y) = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i \pi_j T(I(\pi_i),\, a(\delta(y_i, y_j))), \tag{1.2}$$

with $T(I, a)$ being an strictly increasing function in a and $T(I, 0) = 0$ giving the effective antagonism felt by y_i towards y_j.

Expression (1) above can be seen a as a general functional form for a polarization measure. A particular measure of polarization will depend on the choice of the functions $I, a,$and T.

Similar to the case of an income inequality measure, there are certain basic desirable axioms for any polarization measure to hold:

Axiom 1 *Fix population masses $p > 0$, $q > 0$ with $p > q$. Locate p at level y_o and two q masses one at level y_x and the other at y_z with $y_o < y_x < y_z$. There exists $\varepsilon > 0$ and $\mu > 0$ (possibly depending on p and y_x) such that if $\delta(y_x, y_z) < \varepsilon$ and $q < \mu p$, then the joining of the two q masses at their mid-point, $(x + y)/2$, increases polarization.*

The intuition behind this axiom is the following: if we join the close two small right hand side masses while keeping unchanged their average distance from the third mass then, polarization should increase.

Axiom 2 *Fix population masses $p > 0$, $q > 0$ and $r > 0$ with $p > r$. Locate p at level y_o, q at level y_x and r at level y_z with $y_o < y_x < y_z$ and $y_x >\mid y_x - y_z \mid$. There is $\varepsilon > 0$ such that if the population mass q is moved to the right (towards r) by an amount not exceeding ε, polarization increases.*

The intuition behind this axiom is the following: The p-mass is larger than the r-mass. Additionally, the q-mass located between the p-mass and the r-mass is at least as close to the latter as it is to the former. Then, small positional changes in the q-mass toward the nearer and smaller mass should rise polarization.

Axiom 3 *Fix population masses $q > 0$. Locate q at level y_x. Any new distribution formed by shifting population mass from the central mass q equally to two lateral masses, p and r, located each at d units of distance away from y_x, must increase polarization.*

The intuition behind this axiom is the following: If the middle population mass divides into to two equally spread away population masses, then polarization should increase.

The polarization measure proposed by Esteban and Ray, satisfying the axioms above, is given by:

$$P_{ER}(\pi, y) = K \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i^{1+\alpha} \pi_j \mid y_i - y_j \mid, \tag{1.3}$$

for any constant $K > 0$ and $\alpha \in (0, 1.6]$. Here $a =\mid y_i - y_j \mid$, $I = \pi_i^\alpha$ and $T = aI$. Henceforth, I refer to this measure as the Esteban-Ray (ER) measure of polarization.

At this point, I must stress a second important difference between inequality and polarization: polarization deals with the formation of groups and its relative importance, attaching weights to them when being measured, instead of dealing with comparisons among individuals as the Gini index does.

1.2.3 Two Main Caveats Regarding the ER Index of Polarization.

So far, I have presented the theoretical framework where polarization is defined and a way for measuring it, the ER index. The ER index of polarization is based on a discrete, finite, number of income groups and this translates into two difficulties. First, given any distribution, it is assumed that population is already grouped in such a way that the identification-alienation framework is captured. The second drawback is that the ER measure is, as a consequence, discontinuous.

The Optimal Grouping Problem, Gradin´s Contribution

The intuition behind the first problem is that the ER index treats several individuals belonging to the different clusters as single groups and, therefore, information about the dispersion of those individuals in each cluster, present in the original distribution, is lost. The work of Esteban, Gradín and Ray [6] overcomes this problem, that of *optimal grouping* [4]. The proposed solution is to correct the ER index by a factor capturing precisely that lose of information. Specifically, the authors define a measure of *extended polarization* (Henceforth EGR) in the following way:

$$P_{EGR} = P_{ER} - \beta \left[G(F) - G(\rho^*) \right] \tag{1.4}$$

where f is the density function of the original data, ρ^* represents the optimal n-group representation of the original distribution, $G(\cdot)$ assigns the Gini coefficient to the distribution in its argument and β is the weight we assign to the error term $[G(F) - G(\rho^*)]$.

Notice that the number of groups, $n,$ continuous to be an exogenous decision but the location of the groups has been endogenized through the use of the optimal representation (partition), ρ^*, of the original income distribution.

The optimal parition of the income distribution is the one solving the so called average condition (See more on this in Aghevli and Mehran 1981[1]). Given an income distribution function F with continuous and differentiable density function f and finite mean μ. The problem of optimal grouping can be seen as finding group limits $a_0, a_1, ..., a_k$, where k corresponds to the number of income intervals one wishes the data to be grouped, which minimizes the sum:

$\Phi = \sum_{i=1}^{k} \int_{a_{i-1}}^{a_i} \int_{a_{i-1}}^{a_i} \mid x - y \mid dF(x) dF(y),$

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[4] The problem of optimal grouping can be stated in the following terms: given an income distribution group the data into specified number of groups in such a way that income differences are minimized within the groups and maximized between the groups (Aghevli and Mehran 1981[1]).

where x and y represent the incomes of members of the population with income distribution F. It can be shown that the solution to this problem is given by:

$ai = \int_{a_{i-1}}^{a_{i+1}} xdF(x) \big/ \int_{a_{i-1}}^{a_{i+1}} dF(x) = E\left(X \mid a_{i-1} \leq X < a_{i+1}\right).$

This Average condition says that eah group limit must be equal to the average income of the population in its two adjacent groups.

Diagramatically, the optimal representation of the original distribution is equivalent to transforming the original Lorenz curve into a n-piecewise linear Lorenz curve. Hence, the process of finding the optimal location of the groups is equivalent to minimizing the area between the original Lorenz curve and the piecewise linear representation (See Esteban, Gradín and Ray 1999). That is the reason why they can write the lack of identification or error term as: $\varepsilon\left(f, \rho^*\right) = \left[G(f) - G(\rho^*)\right].$

The Discontinuity Problem, Duclos' Contribution.

The work of Duclos, Esteban and Ray[3] (Henceforth DER) overcomes the second theoretical drawback, that of discontinuity, by introducing an axiomatically derived measurement theory of polarization for the case of continuous income distributions. Additionally and as a different solution for the optimal grouping problem, they use kernel density procedures in order to estimate nonparametrically the optimal size of the groups.

The measure proposed by DER in such a case is given by:

$$P_{DER} : \iint f(x)^{1+\alpha} f(y) \mid y - x \mid dydx, \tag{1.5}$$

where $\alpha \in [0.25, 1]$ for technical reasons. See Duclos, Esteban and Ray (2004).

I do not dig deeply into this considering that the empirical part of the present chapter is based on the ER and EGR indexes of polarization.

1.3 Understanding Economic Polarization.

Up to this point I have focused on the theoretical issues behind polarization and how they can be measured within the identification-alienation framework. Now I turn into the socio-economic interpretation of the index and its components.

Undoubtedly variables such as age, education or innate ability are related to income levels and through it to economic polarization. So a natural step forward in the analysis of economic polarization concerns understanding possible causal effect relationships between different socio-economic variables and their effect into economic polarization.

To the best of my knowledge there are only two theoretical tools trying to disentangle causal relationships between socio-economic variables and economic polarization, both of them, group polarization and explained polarization, proposed by Gradín (2000)[8]. Here I present the existing theoretical tools sharing light on these matters and propose my own theoretical suggestions for developing additional instruments.

1.3.1 Group Polarization. (GP)

A first approach inquiring how socio-economic variables may affect economic polarization is the so called group polarization. The basic idea is not difficult. We can consider any characteristic different from income e.g. age, occupation, religion, race, education, etc. yielding an exhaustive partition of the whole population into k groups. The most relevant characteristics will be those that at the same time show much polarization between groups and within group homogeneity.

In order to implement this group polarization we choose a characteristic c yielding $\rho^c(\mathbf{q}, \mathbf{x}) = (q_1, q_2, ..., q_k; x_1, x_2, ..., x_k)$, an exhaustive partition of the whole population into k groups, where q_i indicates the population share in group i and $x_1 \leq x_2 \leq ... \leq x_k$ indicate groups' average incomes respectively. Next, we define the level of *group polarization* (P_{GP}) as the level of polarization when using the exogenous partition ρ^c to represent F. In other words:

$$P_{GP}(F; \alpha, \beta, \rho^c) = P_{ER}(\alpha, \rho^c) - \beta [G(F) - G(\rho^c)] \tag{1.6}$$

This time groups are not necessarily income intervals instead of, groups are exogenously conformed according to whether their members share the same category for a given characteristic, c. Hence, it is possible to obtain negative values for P_G. In order to make interpretation of the results easier, the index is normalized to have no negative values in the following way:

$$P_{GP}\left(F;\alpha,\beta,\rho^{c}\right) \equiv P_{ER}\left(\alpha,\rho^{c}\right)-\left(-\beta\right)\left[G(F)-G(\rho^{c})\right] \simeq P_{ER}\left(\alpha,\rho^{c}\right)-\beta\left[\left(G(F)-G(\rho^{c})\right)-1\right] \tag{1.7}$$

1.3.2 Explained Polarization (EP).

A second approach is that of *Explained Polarization*. The intuition behind the method is the following: there is no reason to assume that we know, in advance, what are the most relevant characteristics at explaining polarization, as we implicitly did under group polarization. In fact, there are different socio-economic variables affecting income levels at the same time and, therefore, the probability of appertaining to each of the relevant income groups when measuring income polarization. The key issue here is that we can still using income as a proxy of all those variables which are relevant and we focus on finding out those characteristics that better explain social antagonism or alienation in terms of income, given that individuals appertain to each income group precisely because they share that characteristic.

Formally, let $\rho^{*}\left(\boldsymbol{\pi},\mathbf{y},\mathbf{z}\right)=\left(\pi_{1},\pi_{2},...,\pi_{n};y_{1},y_{2},...,y_{n};z_{0},z_{1},...,z_{n}\right)$ be the optimal partition we use to estimate income polarization on $\left(\pi,y\right)$, the original income distribution. π_{i} is the fraction of population in group i, located between the interval $[z_{j-1},z_{j}]$ and y_{i} is the mean income of group i. Next, suppose we identify characteristic c as a relevant socio-economic variable at explaining economic polarization. Then, the method suggests to make a new partition of the original distribution but this time based on c. Let $\rho^c(\mathbf{q},\mathbf{x}) = (q_1, q_2, ..., q_k; x_1, x_2, ..., x_k)$ be such a partition. This time, q_j is the fraction of the population in group j and $x_1 \leq x_2 \leq ... \leq x_k$ are the mean incomes of the groups. Notice that we do not have income intervals for the location of the groups precisely because groups are determined according to c instead of income.

Now it is possible to define a new partition ρ^+ of (π, y) in the following way: individuals in the first group are all individuals in ρ^c whose mean income level is in the interval $[z_0, z_1]$;individuals in the second group are all individuals in ρ^c whose mean income level is in the interval $[z_1, z_2]$ and so on. This partition is given by: $\rho^+(\mathbf{r},\mathbf{w},\mathbf{z}) = (r_1, r_2, ..., r_n; w_1, w_2, ..., w_n; z_0, z_1, ..., z_n)$. Here, r_i is the fraction of population in group i, located between the interval $[z_{j-1}, z_j]$ and w_i is the mean income of group i[5].

The following example would clarify the idea. Consider $\rho^*(\boldsymbol{\pi},\mathbf{y},\mathbf{z})$ be the optimal partition for 2 income groups i.e. one group will be formed by all individuals below the income mean whereas the second group will be formed by individuals above the income mean. Next, suppose $\rho^c(\mathbf{q},\mathbf{x})$ be a new 2-group partition of the original income distribution based on geographical location (i.e. rural and urban areas). We calculate the income mean for each of the groups in ρ^c and observe if they are under or below the income mean in ρ^*. For the sake of the argument imagine that income mean in rural areas is below that of the original distribution while that in

[5] We can express r_i and w_i as: $r_j = \sum\limits_{i\in\phi_j} q_i$ and $g_j = \frac{1}{r_j}\sum\limits_{i\in\phi_j} q_i m_i$ with $\Phi_i \equiv \{i \mid x_i \in [z_{i-1}, z_i]\}$.

urban areas is above. Now, we arrange the new partition $\rho^{+}(\mathbf{r}, \mathbf{w}, \mathbf{z})$. This partition will contain in the first group r_1 all individuals belonging to q_1 in ρ^c i.e. individuals living in rural areas and in the second group all individuals belonging to q_2 in ρ^c i.e. individuals living in urban areas. The important issue to notice is that people with income above the original mean but living in rural areas will be this time in the first income group under partition ρ^{+}.

If the polarization measure calculated on ρ^{+}, based on characteristic c, equals the polarization measure calculated on ρ^{*}, then we say that characteristic c totally accounts for the level of income polarization in ρ^{*}. In general, the higher the EP the larger the share a given characteristic explains. EP will equal 1 if $\rho^{+} = \rho$ and it will equal 0 when there is no polarization between the groups by the characteristic c.

In practice the EP Index can be computed as:

$$P_{EP}(z^{*} = \mu, \alpha = 1) = \frac{G(\rho^{+})}{G(\rho^{*})} \tag{1.8}$$

1.3.3 Further Contributions.

Gradin's contribution to the analysis of economic polarization can be seen as an attempt to uncover socio-economic characteristics of individuals, different from income, that help at explaining why individuals might be located in particular income clusters. In other words people is in a particular income group because they have in common some relevant socio-economic characteristics.

In my view and for the sake of understanding polarization it might be also important to find out if people have some relevant socio-economic characteristics precisely because they appertain to the same income group, my question behind the argument is how is the distribution of socio-economic characteristics over the different income groups?

1.4 Income Polarization in Bogotá: 2000-2003.

In this section I present the results of the EGR index of polarization and its components (i.e. the ER index and the error term) as well as the results for group and explained polarization for Bogotá at the beginning of the 2000s.

Bogotá has renewed itself during the last 10 years. Indeed, the city recovers not only his role as the leading city inside the country but also stands up as an example for other Latinamerican cities in terms of cycle paths, public parks, public libraries, public transport (Transmilenio) and fiscal policies, for example. In terms of population the city growths by 2% annually between 2000 and 2003 meaning an increase of 108 thousand people every year, in average. More of the 70% of the population appartain to the economically active population, they are older than 15[6].

Bogotá is the biggest labor market in the country. Just to give an idea, Bogota employed in 2000 near 2.6 million in average of people representing 45% of total urban employment and, between 2001 and 2003 the number of workers raised by

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6 Source: Departamento Nacional de Planeación.

10% meaning a total number of 2.9 million of workers[7]. This study uses an individual monthly salary as a proxy of household income as the dimension for measuring economic polarization in Bogotá[8].

1.4.1 The Data.

To estimate household income polarization in Bogotá, I use data from the Fundación para la Educación Superior y el Desarrollo's Social Survey (FEDESARROLLO) , one of the most complete social surveys for the main Colombian cities. The survey is available from 1999 until 2003, with semi-annual periodicity.

I use salary as a proxy of Household Income to estimate income polarization in the cases of two and three groups representations of the original income distribution.

In order to analyze possible causal effect relationships between different socioeconomic variables and their effect into economic polarization through the GP and EP index I generate the following categories based on the householders characteristics:

7 Source: Departamento Nacional de Estadística.

8 I generate household income under diferent specifications yielding always the same polarization ranking, the same I obtain using just salary so I decided to work, for simplicity, with salary as a proxy of household income.

gender, twelve categories according to age and gender[9], maximum educational level attained[10], migrational status[11], work category[12], economic activity[13]

I agree that economic polarization is, in principle, a long term concern for any society; however as an economist, I get used to find out second best solutions because those optimal conditions allowing first best solutions rarely come. In this case, it would be ideal to have a long detailed series of income data for analyzing polarization in Bogotá unfortunately this first best solution was not available.

1.4.2 Income Polarization and its Components, the Esteban-Gradín-Ray (EGR) Index.

The EGR index of polarization for Bogotá is reported in Table 2. The index is reported for the cases of 2 and 3 groups representations as well as for two different values of α, the sensitivity parameter, $\alpha = 1$ and $\alpha = 1.6$; keeping for all cases the same weight for the error term i.e. $\beta = 1$.

Importantly, it can be seen that independently of either the number of groups or the weight on the group importance (the alpha value), the index decreased between

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[9] The categories gather people each ten years starting at people older than 15 and ending at 65, plus a final group of people older than 65. Each of these six groups was made either for male and female.

[10] Broken down in 4 categories: illiterate, primary school, high school and university.

[11] Subdivided into 4 categories: people who have lived always in the city, people who have arrived to the city between the last 0-6 months, people who arrived to the city between 7 months and 5 years ago and people who arrived to the city since more than 5 years ago.

[12] Broken down in 6 categories: government employee, private employee, domestic worker, self-employed, manager and family worker.

[13] Broken down in 17 categories: agriculture; fishing; miner; manufacture; electricity, gas and water; construction; trade; transport and communication; tourism; financial services; house market; public administration and national defense; education; social service and health; home services; local administration; no reported.

October 2000 and October 2003. For example in the case of bipolar representation the index decreased in 10 and 15% for $\alpha = 1$ and $\alpha = 1.6$ respectively.

As explained above, polarization can be seen as the result of two main features, few but significantly homogeneous groups which are distant one from another. Therefore, in order to understand changes in polarization we need to look carefully at the behavior of the population shares of the groups as well as the intra-group dispersion and the difference/distance between the mean incomes of each of them. To do so I present separately the cases of bipolarization and multi-polar representations.

In the case of two-groups (bi-polar) representation there is a reduction in the proportion of people belonging to the first group, that of lower income between October 2000 and October 2003, it reduced in 4% while the proportion of people belonging to the second group, that of higher income, increased in 13% (Table 3.). Therefore and considering only the significance of the relative size of the groups bipolarization should increase in Bogotá, these population movements drive society to a more equally sized 2 groups representation. However, and looking at the behavior of the mean income for each group, reported in Table 4. , it can be seen that mean income for people in the lower income group increased in 5.5% while that for the group of higher income decreased in 21.6%. In other words the income distance between the 2 groups reduces, so mean income movements alone should reduce bipolarization.

So far I have analyzed changes in the size and mean income of the groups in the case of bipolarization, which are the main components of the ER index but the reader should remember that The EGR index accounts for the error resulting from a simplified representation of the original distribution. In other words I am still to analyze the intra-group dispersion.

Table 5 reports the intra-group dispersion for the 2 and 3 groups representations. In the case of bipolarization the level of intra-group dispersion has decreased period by period, with the only one exception of the last period. Therefore, on average, income groups have become more homogeneous. From the same Table it is possible to infer that the bi-polar representation of the income distribution accounts for more than 73% of the Gini coefficient of the original distribution, the rest being of course within group dispersion.

Summing up, in the case of bipolarization, the general decrease in bipolarization has been driven by the decrease in simple bipolarization driven in turn by the decrease in the mean income distance between the two groups that overwhelms the fact of the two groups becoming more even sized. It should be noticed at this point that the evolution of the simplified polarization has a similar pattern independently of the two values of the sensitivity parameter.

When putting together the simplified bipolarization decrease and the steadily decrease in the error term or intra-group dispersion we have that in general the effect of simple polarization outweights that of the error term. So it can be concluded that the reduction in the income distance drives the reduction in the extended bipolarization.

Now, in the case of multi-polar representation i.e. the three groups representation, It can be seen, looking at Table 2., that there is also a reduction in the polarization index; the index decreased in 8 and 7% for $\alpha = 1$ and $\alpha = 1.6$ respectively. Notice, from Table 3., that the increase in the fraction of the first group between October 2000 and October 2003 made group sizes more uneven, reducing *ceteris paribus* simple polarization. Regarding the mean income terms; while the lower and middle income groups exhibit an increase in 18 and 10% respectively, the highest income group decreased its mean income in 13%. Therefore, income distances have diminished. These two forces, more uneven groups and less income distances, work in the same direction, they reduce simple polarization.

Finally, Looking the behavior of the intra-group dispersion in Table 5., it can be seen that there is not really a big change so we can conclude that the reduction in simple polarization accounts almost entirely for the reduction in extended polarization.

1.4.3 Group Polarization: Bogotá, 2001-2003.

For each characteristic reported in Table 6. I computed the group polarization index as defined in 1.7 for values of the parameters $\alpha = \beta = 1$. According to the results polarization has increased for almost all characteristics between the first semester in

2001 and the second semester in 2003. The most significative increases occurring when groups of age and gender and migrational status are the relevant characteristic, 27 and 12% respectively. Only in the case of gender there is a decrease in group polarization, it decreases by 7%.

The results show that in the first semester 2001 education appears to be the element which generates the highest polarized salary distribution, followed by work category, age-gender, migrational status and gender. While for the second semester 2003 education was still appearing to be the element which generates the highest polarized salary distribution but this time followed by age-gender, economic activity, work category, migrational status and gender.

So, even if we deter from making straightforward comparisons between the two rankings, because the final period has on category more, looking each period independently, it appears that the educational level is the most relevant characteristic generating the highest polarized salary distribution.

In general, looking at the decomposition of the index, reported in Table 6., groups became internally more identified independently of the level of simple polarization (the error term diminishes). Indeed, the increase in within group identification overcomes the decrease in polarization in the cases of education and age-gender.

1.4.4 Explained Polarization: Bogotá, 2001-2003.

Here, I use the same group of socio-economic characteristics in order to identify which of them might be accounting for changes in polarization between 2001 and 2003. I focus on the case of bipolarization for values of the parameters $\alpha = \beta = 1$, and compute the EP index as defined in (1.8). Results are reported in Table 7.

According to the results, we can conclude that in 2001 the educational level achieved was the main element explaining observed bipolarization, followed by work category, age-gender, migrational status and gender. Similarly, for the second semester 2003 the educational level achieved was still being the main element explaining observed bipolarization, but this time followed by age-gender, economic activity, work category, migrational status and gender.

It means that education is the most important characteristic explaining the division of the salary distribution in Bogotá into two homogeneous but distant groups in both periods. By the other hand characteristics such as gender and migrational status seem to be less relevant.

Table 8. reports the degenerated bimodal distribution underlying each characteristic. Focused on two of the most relevant characteristics, education and age-gender, it can be seen that the increasing role played by education was due to changes in group shares which generated a more even distribution while, for the case of age-gender the rise was due to the reduction in the distance between the two groups.

1.4.5 Concluding Remarks

It is important to distinguish between polarization and inequality because of the closer relation of the former to the generation of social conflict instead of the latter.

Suppose we are able to observe the distribution of a population with respect to the set of attributes Y. If the distribution describes a population grouped around few but distant poles then we can say that the population is polarized in terms of the set Y. If we focus on income or wealth as the attribute, Y, characterizing the distribution then we talk about economic polarization. In very few words one may say that polarization differs from inequality mainly because of the weight that the former gives to the size of the groups.

Here I use the alienation-identification framework to calculate the Esteban-Gradín-Ray index of economic polarization. Further I calculate the group and explained polarization indexes of polarization seeking to identify socio-economic characteristics causing economic polarization.

The data show a decrease in the EGR index of polarization for Bogotá between the second semester 2000 and the second semester 2003 not only in the case of bipolar representation but also that of multipolar representation of the original salary distribution; It decreases by 10 and 8% respectively with $\alpha = 1$. In the case of bipolarization the decrease in the index is explained by a reduction in income distance between the two groups in the simple polarization component of the index while for the multipolar case the reduction is explained by the reduction in income distances between the groups as well as the fact that the groups become more uneven.

The results for the GP and EP indexes of polarization show that the educational leve attained is the most relevant socio-economic characteristics at explaining salary polarization. In the case of group polarization education appears to be the element which generates the highest polarized salary distribution. In the case of explained polarization education is the most important characteristic explaining the division of the salary distribution in Bogotá into two homogeneous but distant groups in both periods.

1.4.6 Annexes

Table 2
Polarization Index

	$\alpha = 1$		$\alpha = 1.6$		Gini
Number of Groups	2	3	2	3	
October 2000	0.262	0.251	0.161	0.119	0.55
April 2001	0.273	0.261	0.170	0.133	0.56
October 2001	0.257	0.237	0.151	0.109	0.51
April 2002	0.207	0.202	0.111	0.087	0.46
October 2002	0.226	0.215	0.126	0.099	0.47
October 2003	0.237	0.231	0.136	0.110	0.51

Source: Own construction using FEDESARROLLO ´s data

Table 3
Group Polpulation Shares

Number of Groups	2		3		
	1	2	1	2	3
October 2000	0.77	0.23	0.51	0.37	0.11
April 2001	0.76	0.24	0.55	0.35	0.10
October 2001	0.73	0.27	0.51	0.34	0.15
April 2002	0.70	0.30	0.47	0.38	0.15
October 2002	0.70	0.30	0.51	0.34	0.14
October 2003	0.74	0.26	0.54	0.34	0.12

Source: Own construction using FEDESARROLLO ´s data

Table 4
Mean Income by Groups

Number of Groups	2		3		
	1	2	1	2	3
October 2000	0.469	2.750	0.322	0.948	4.262
April 2001	0.452	2.748	0.334	0.982	4.705
October 2001	0.470	2.409	0.348	0.955	3.270
April 2002	0.526	2.107	0.396	0.958	2.940
October 2002	0.507	2.157	0.404	1.000	3.100
October 2003	0.495	2.408	0.380	1.035	3.705

Source: Own construction using FEDESARROLLO ´s data

Table 5
Intra-Group Dispersion

	2 Groups		3 Groups	
	Absolute Level	Relative (% Gini)	Absolute Level	Relative (% Gini)
October 2000	0.146	26.4	0.066	12.0
April 2001	0.143	25.6	0.065	11.7
October 2001	0.129	25.1	0.062	12.0
April 2002	0.125	27.3	0.058	12.7
October 2002	0.120	25.8	0.058	12.4
October 2003	0.135	26.7	0.065	12.8

Source: Own construction using FEDESARROLLO ´s data

Table 6

Group Polarization			Decomposition			
			ER Polarization		Error Term	
Sub-Group	2001	2003	2001	2003	2001	2003
Gender	0.57	0.53	0.040	0.007	0.466	0.482
Education	0.96	0.97	0.137	0.122	0.176	0.154
Age-Gender	0.63	0.80	0.026	0.017	0.392	0.219
Migration	0.57	0.64	0.019	0.031	0.445	0.387
Work Category	0.75	0.75	0.025	0.029	0.275	0.275
Economic Activity	N.A.	0.79	N.A.	0.026	N.A.	0.238

Source: Own construction using FEDESARROLLO ´s data

Table 7
Explained Polarization

Sub-Group	April 2001	October 2003
Gender	0.18	0.03
Education	0.88	0.92
Age-Gender	0.36	0.74
Migration	0.23	0.29
Work Category	0.64	0.59
Economic Activity	N.A.	0.69

Source: Own construction using FEDESARROLLO ´s data

Table 8

Bimodal Partitions by Characteristic	Mean Ratio		Population Shares (%) Poor		Rich	
	2001	2003	2001	2003	2001	2003
Gender	1.35	1.05	24.5	73.8	75.5	26.2
Education	4.47	3.52	79.2	73.1	20.8	26.9
Age-Gender	1.57	2.06	61.8	60.4	38.2	39.6
Migration	1.28	1.37	43	75.7	57	24.3
Work Category	2.35	2.02	89.7	41.5	10.3	58.5

Source: Own construction using FEDESARROLLO ´s data

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Chapter 2
APolitical Economy Model of Social Polarization and Civil War

Economic structures and perturbances play an important role at explaining different political outcomes. This economic influence on political outcomes has received considerable attention among scholars and policy makers. For instance, recent evidence suggests that increasing income plays a prominent role in promoting political stability: countries experiencing positive growth shocks appear to face a lower risk of civil war (Collier and Hoeffler 1998; Fearon and Laitin, 2003; and Miguel et al, 2004). Also, economic shocks may affect the political power structure among different groups in society and, as a consequence, opportunities to institutional changes arise (Acemoglu and Robinson 2006; Roemer 2001).

The relationship between changes in prices and civil wars has been theoretically studied, for example, by Dal Bó and Dal Bó. Dal Bó and Dal Bó (2008) with the canonical 2x2 international economics model of trade, study how economic shocks affect the intensity of conflict and conclude that positive shocks to labor intensive industries diminish conflict, while positive shocks to capital intensive industries increase it. This conclusion is based on the assumption that conflict activities are labor intensive so that an increase in the relative remuneration to labor into the legal activities makes it less attractive for people to join illegal conflict activities, which are assumed to be labor intensive. However, most of the analysis relating price shocks and social conflict remains silent about how the groups involved in conflict are able, precisely, to act as a group. In other words the collective action problem that each group faces before deciding to engage into conflict is ignored.

The importance of collective action problems underlying revolutions and civil wars has been longly recognized. There are several explanations about the mecanisms that help insurgent movements to overcome the collective action problem they face when runing a civil war or a revolution. In a nutshell, the puzzle behind insurgent collective action is how to explain participation of people in insurgent movements and activities given that the costs of such risky activities may include their own lives?

Pioneer work such as those of Tullock (1971) and Popkin (1979), based on Olson's approach to the collective action problem, states that individual material incentives to peasants contingent on their participation constitute a possible solution to the problem. In contrast, Skocpol (1982), Goodwin and Skocpol (1989) and Goodwin (2001) state that instead of selective, individual, incentives guerrillas offer collective incentives to peasants e.g. they offer certain public goods including for instance security, as a state might do, in areas under they control to those who join the insurgent movements.

The role of social networks has also been analyzed as a way to overcome the problem, Taylor (1988). The main idea is that communities where strong ties among members exist have a high capacity for collective action given that repeated interaction among its members generates high costs for nonparticipants, because of the high degree of cultural homogeneity and the existence of social norms such as strong reciprocity among its members.

Another approach looks at the change in political opportunities as way to overcome the collective action problem. As political opportunity varies then relative benefits and costs of collective action also change. Political oppurtunity changes occur whe elite alliances weaken or relevant legal positions change; then, guerrillas may seize such opportunities to increase the insurgent level of participation. As pointed out by Goodwin and Jasper (1999) it remains challenging to specify changes in political opportunities. More recently, Wood (2003) uses a Schellingean model with multiple equilibria to analyze civil war in El Salvador.

The main contribution of this paper is to provide a rational-choice model of collective action that links changes in prices with changes in the probability of solving collective action problems in civil wars. Hence, the model allows for comparative statics on the probability for citizens to solve their collective action problem. In particular, I analyze how changes in international prices of intermediate goods, an objective parameter, affect, through the distribution and levels of income, the probability for citizens to engage in civil war in a politically polarized society characterized by the production of one labor intensive good in a closed economy. Therefore, the model allows for comparative statics on the probability of solving the collective action problem depending on changes on the potential benefits of coordination. A key novelty of the model is that benefits of the groups are contingent not only on participation but also in success.

By a politically polarized society I mean a society composed by *few groups*, with relative equal *political power* and having opposite (rather than different) preferences over political or institutional outcomes. This means, they face a *political conflict*. Notice the two key issues characterizing, here, political polarization: first, there is a relative equal distribution of political power between the few groups -independently of their relative size- and, second, groups have opposite preferences over political outcomes[14].

Regarding political power, and following Acemoglu and Robinson (2006), I distinguish between de jure political power and de facto political power. In short,

[14] Recently, in economics, a theory of polarization has been developed based on the works by Esteban and Ray (1994) and Wolfson (1994). In this theory, three main issues underlie the concept of polarization: the presence of few but representative amount of groups and the notions of within group homogeneity and between group heterogeneity. The former implies that people in each group have an strong sense of identification with each other while the latter implies that people belonging to each group feel distant from people in other groups. In this paper it is assumed that groups have a strong feeling of within-group identification given that they share a common political interest and, at the same time they have an strong feeling of between-group heterogeneity given that their preferred outcomes are opposite. Regarding the relative size of the groups, which might be unequal between an elite and the citizens, the important issue is the real threat a group represent for the other. It is clear than a society divided into two groups having the characteristics of within-group homogeneity and between-group heterogeneity but with very unequal sizes cannot be considered as a polarized society; a small group or a single individual represents no threat for the majority group. However in the present model, the credible threat one group has over the other is due to the fact of equal amounts of power rather than its relative size.

while the first type of power is allocated by political institutions, the second depends on the capacity of a group to obtain its favorite policies by using force. The fact that groups have opposite preferences over political outcomes defines political conflict.

This work concentrates in the extreme but not unreal case of a bi-polarized society i.e. a society composed of just two groups: an elite and the citizens, having opposite preferences over political outcomes and with relatively equal amounts of political power. Here, it is assumed that the elite enjoys the jure political power and citizens the facto political power. Further, it is assumed that the elite have little trouble exerting the jure political power they have, while citizens face what is knwon as the collective action problem in order to exert their facto political power.

The intuition behind the comparative statics I propose is the following: individuals in a given group are more likely to coordinate when the potential benefits of coordination increase (or the costs decrease). In game theoretical language, I analyze how changes in the payoffs of the players affect the probability for players of achieving different equilibrium of the game, depending on the solution, or not, of the collective action problem they face.

Following Medina (2007) I distinguish between single equilibrium models and multiple-equilibria models of collective action. In single equilibrium models, individuals have a positive cost of cooperating and no single agent's specific decision to cooperate will have a significant effect on the probability of succeeding. In this type of models, based on the Olsonian model of collective action Olson (1965), individuals only consider the value of the selective incentive and the cost of cooperation in deciding whether to cooperate. As a consequence, changes in the value of the benefits agents obtain from successful collective action become irrelevant. Only selective incentives can account for collective action. In other words, there is no possibility for comparative statics with this type of models.

On the contrary, in multiple-equilibria models, with a well specified payoff structure and with no dominant strategy for players it is possible to examine how changes in the value of the benefits agents obtain from successful collective action affect their strategies. In other words, we can perform comparative statics. A detailed exposition of the argument is in Medina (2007).

The rest of the paper proceeds as follows: section 2 provides some historical examples of polarized societies where economic shocks have proved to be crucial in generating opportunities for individuals to coordinate and enagage into civil war or revolts. In section three, I detailed present the method of stability sets, which is the game theoretical tool that allows the comparative statics I propose. Section fourth contains a political-economy model that captures the essence of the problem and presents the results of the comparative statics. Finally, I state the conclusions of the work.

2.1 Historical Examples

Here I provide a couple of historical examples of societies characterized by the existence of two main social groups engaged into conflict because of their preference over opposite economic policies or institutions and, each group having significant amounts of political power, though with different sources -de jure or de facto political power-. In other words, these are examples of, as I said in the introduction, bi-polarized societies.

Additionally, it can be seen how economic shocks, or a sequence of them, bring about oportunities for a group to solve their collective action problem and to exert their de facto political power. This situation could find parallels in other places and times. That is why I believe, systematical analysis of how changes in prices, or other objective parameters, may affect the probability of groups solving collective action problems in polarized situations is relevant.

2.1.1 The 2011 Egyptian Revolution.

On 25 January 2011 a popular violent uprising took place in Egypt. In cities like Alexandria and Cairo, among others, several demonstrations and violent clashes between antagonists and supporters of the regime of Hosni Mubarak ended on 11 February with Mubarak's resignation from office after 29 years of autocracy.

There is no doubt that Egyptians' protests focused on political issues such as the general lack of freedom, rampant corruption and a common desire for some institutional change. Additionally, the economic situation was not good either. In fact economic was featured by high unemployment rates, high inflation rates and low wages. However, all this symptoms were not new by the beginning of 2011 so a natural question comes to mind: Why is this outrage happening precisely now and not, for instance, a couple of years or let us say ten years ago? What was the starting spark of the fire?

Given that poor people spent most of his income in food; then, the connection between a severe food crises and popular outrage becomes natural. Following FAO's data on the FAO Food Price Index show that the index hit a record in January 2011 and do it again in February when the index was averaging 236 points for the eighth consecutive month with an increase of 2.2% with respect to January[15]. So there is little doubt that this increase in food prices can be seen as an important spark starting the fire in Egypt.

2.1.2 Venezuela: The Caracazo.

It can be said that the economic dependence of Venezuelan economy on oil has developed a particular economic and social structure. The existence of a very rich state, which owns the country's oil rents, intensifies the dispute among different groups for power. Once it is achieved the group in power has no intention of seeking welfare of society as a whole but rather tries to remain in power (Mejía 2009, pp124). So society is characterized by the existence of two groups, an elite that enjoys both de jure

[15] http://www.fao.org/worldfoodsituation/wfs-home/foodpricesindex/en/

political power and huge source of income that comes from oil and citizens who only own work and their de facto political power.

By the end of the 1980s Venezuela was plunged into a deep economic crisis originated by an increase in the external debt in the second half of the 70s and a drastic fall in oil prices, which decline from 28,9 dollars per barrel in 1973 to 10,9 dollars in 1986. Several economic policies that were perceived by citizens as unfair and helped to aggravate the situation. For example, the government established a preferential exchange rate to help private entrepreneurs to repay their external debt. This policy, favoring the elite, transferred significative amounts of public resources to the private sector to the detriment of social public investment. Economic inequality also increase; wages' participation on total income passed from 61,2% in 1960 to 15% at the end of the 80s.

On February 16, 1989, president Carlos Andrés Pérez, launched a new economic package including a 100% increase in the price of gasoline and a 30% increase on tarrifs in public transport. As a consequence, during February 27th and 28th a huge social protest in Caracas, know as the "Caracazo" took place. Official reports admit 246 casualties during the events. However, extra official sources mention more than 2.000 deaths.

2.1.3 South Africa: The Soweto Riot.

The Nationalist Party, which represented white Afrikaaner interests, came to power in South Africa after World War II. It implemented a number of measures to segregate the majority black, or Bantu, population (80% of the population) from the white dominant minority. In our terminology the latter, comprising about 10% of the population, held de jure political power while de facto political power was in the hands of black South Africans.

These measures were collectively known as apartheid ("apartness" in Afrikaans) and included spacial segregation for both residential and business purposes, the reservation of the most desirable job categories for whites, severe limitations on political and union activities of blacks and on their access to public facilities and land tenure regulations that specially benefited the white minority, that controlled almost 80% of the surface of the country.

One of the enclaves where blacks were permitted to live was a township in the outskirts of Johannesburg, Soweto (an acronym of South Western Township). There, inhabitants suffered the effects of another aspect of the inequality imposed by apartheid policies: segregated and inferior education. A good indicator of this discrimination was the budget allotted per student in 1975: 644 rands for each white pupil versus 42 rands for each of their black peers – a difference of over 1400%.

In 1975, in a decision widely perceived by blacks as arbitrary, educational authorities decided that Afrikaans, rather than English, would be the primary language of instruction in black schools. In June of 1976, a student boycott took place in one school. The movement quickly spread and on June 16 a peaceful protest march was called. It is estimated that some 20,000 students in uniform participated. But the reaction of South African police was brutal – the unofficial death toll was over 200. After Soweto, similar uprisings took place in the main cities of South Africa[16].

2.2 Stability Sets and the Tracing Procedure

The main purpose of this section is to formally present the concept of stability sets and the tracing procedure; the theoretical tools I will use to analyze, systematically, the relative probability of the two possible equilibria of the tipping game I present in the next section and that captures the intuition behind the historical examples above. Intuitively, and appealing to the reader's imagination, the idea behind the method of stability sets is that every single equilibrium of the game has an attraction space; therefore, if we are able to measure its size then we can provide a relative probability of the occurrence of each outcome of the game. Technically, the stability sets provide a measure of the attraction spaces by using the tracing procedure technique. This section is based on the work of Harsanyi and Selten (1988) and Medina (2007).

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[16] www.africanencyclopedia.com/apartheid/apartheid.htm
africanhistory.bout.com/library/weekly/aa060801a.htm

Before presenting a formal definition of these concepts and in order to contribute to the reader's understanding, I present a simple example where they can be seen at work.

2.2.1 Stability Sets in 2×2 Coordination Games

Consider the following coordination game:

Table 1. Coordination Game 1

		2: C	2: D
1	C	$1, 1$	$-1, 0$
	D	$0, -1$	$0, 0$

This game has three Nash equilibria, two of them in pure strategies (C, C), (D, D) and one in mixed strategies, where players choose randomizations over their pure strategies[17]

In order to obtain all possible Nash equilibria of the game let us derive the *best-response correspondences* of the players. A best-response correspondence for Player i is a mapping that assigns a set of i's optimal strategies to each strategy profile of the other players. In this example it is optimal for Player i to play C when its expected payoff is higher than that of D; she will randomize between its two pure strategies if she is indifferent between them.

Let q_1 denotes the probability of Player 1 choosing C, q_2 the probability of Player 2 choosing C, and $u_i(\phi_i)$ Player i's expected payoff from choosing strategy ϕ_i, then, following standard calculations from classic game theory, we can obtain:

[17] A mixed strategy for any Player is a probability distribution over his set of pure strategies.

$$u_1(C) = 2q_2 - 1;$$

$$u_1(D) = 0;$$

$$u_2(C) = 2q_1 - 1;$$

$$u_2(D) = 0.$$

The mixed strategy equilibrium is the pair q_1^*, q_2^* that solves:

$$u_1(C) = u_1(D);$$

$$u_2(C) = u_2(D),$$

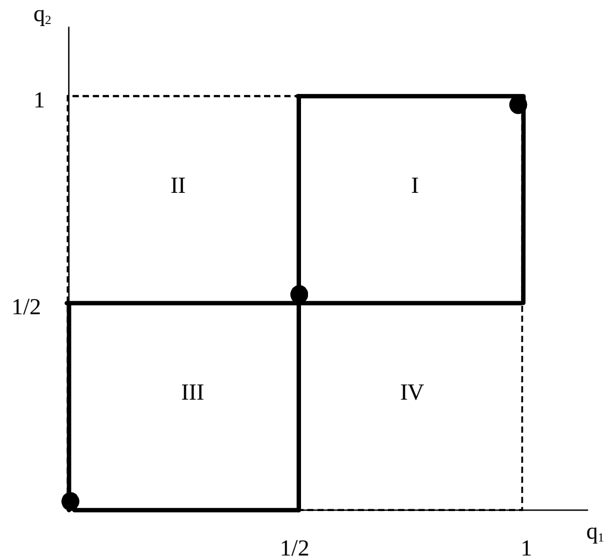
which in this case turns out to be $q_1^* = 1/2$, $q_2^* = 1/2$. Now, denoting by $q_1^*(q_2)$ Player 1's best-response correspondence and by $q_2^*(q_1)$ Player 2's best-response correspondence we obtain:

$$q_1^*(q_2) = \begin{cases} 1 & \text{if } q_2 > 1/2 \\ [0,1] & \text{if } q_2 = 1/2 \\ 0 & \text{if } q_2 < 1/2 \end{cases}$$

$$q_2^*(q_1) = \begin{cases} 1 & \text{if } q_1 > 1/2 \\ [0,1] & \text{if } q_1 = 1/2 \\ 0 & \text{if } q_1 < 1/2 \end{cases}$$

These two best-response correspondences are plotted in Figure 1. The axes are the probabilities q_1 and q_2 and the Nash equilibria of the game are represented by dots

Figure 1. Best-response Correspondences of coordination game 1.



Note how the strategy space has been partitioned into four regions, which can be thought of as possible initial conditions of the beliefs players have about each other. If each players' initial beliefs about the other's strategy are located in region I, and they become common knowledge among the players, then, the optimal thing for Player 1 and Player 2 to do is to play C. Instead, if players' initial beliefs are located in region III, then, the optimal thing for Player 1 and Player 2 to do is to play D. Regions II and IV are unstable. If initial beliefs are located in these regions and become common knowledge, then, it is optimal for the players to play against them.

It should be noted that the initial beliefs are not necessary those of equilibrium but, if it is the case, they are *attracted* towards one equilibrium of the game.

To get the idea of stability sets and the importance of the initial beliefs in the process of equilibrium selection let's modify the payoffs both players will receive for playing C. This modified coordination game is presented in Table 2:

Table 2. Coordination game 2.

		2: C	2: D
1	C	$0.5, 0.5$	$-1.5, 0$
	D	$0, -1.5$	$0, 0$

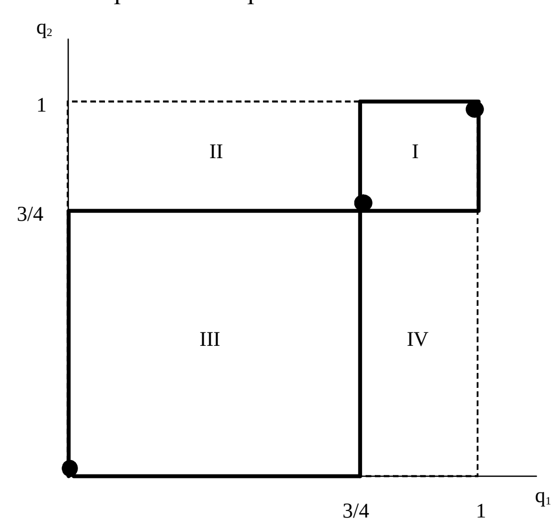
The new mixed equilibrium is $q_1 = 3/4$, $q_2 = 3/4$ and the new best-response correspondences are:

$$q_1^*(q_2) = \begin{cases} 1 & \text{if } q_2 > 3/4 \\ [0,1] & \text{if } q_2 = 3/4 \\ 0 & \text{if } q_2 < 3/4 \end{cases}$$

$$q_2^*(q_1) = \begin{cases} 1 & \text{if } q_1 > 3/4 \\ [0,1] & \text{if } q_1 = 3/4 \\ 0 & \text{if } q_1 < 3/4 \end{cases}$$

Which are plotted in Figure 2.

Figure 2. Best-response correspondences of the coordination game 2.



This game also has three equilibria, two in pure strategies ((C, C) and (D, D)) and one in mixed strategies ($q_1 = 3/4$, $q_2 = 3/4$).

However, the mixed strategy equilibrium has changed between this and the previous game as a consequence of changes in the payoffs. Moreover by focusing only on the mixed strategies of the games one would infer that the equilibrium (C, C) is less likely in the modified game. Indeed, Player 1 will choose C with probability $1/2$ in the first game and with probability $3/4$ in the second game. A similar change has occurred for Player 2 in order to choose C. But this change in the mixed strategy equilibrium implies that the stability set of the equilibrium (C, C) is smaller now; i.e., the space of those initial beliefs that are attracted towards this equilibrium is smaller in the coordination game 2 than in the coordination game 1.

Note that even if we were ignorant about the underlying probability distribution of the priors, we can still predicting the equilibrium (C, C) as less probable than the equilibrium (D, D).

Now, let's introduce a bit of notation for some of the basic concepts used in the example above before formally defining stability sets.

We denote by q_i a mixed strategy for Player i, which is nothing but a probability distribution over his set of pure strategies Φ_i. We denote by $q_i(\phi_i)$ the probability that q_i assigns to the pure strategy ϕ_i and by Q_i the set of all q_i.

An strategy profile is denoted by $q = (q_i)_{i \in L}$ and the set of all q is denoted by Q.

Finally, following Harsanyi and Selten (1988), we regard to the initial beliefs that Players have about each other as *Priors* and denote them with the vector p.

Definition 1 (Stability Sets) *The set of all $q_i \in Q_i$ such that a given pure strategy ϕ_i is a best reply to q is denoted by $S(\phi_i)$. The set $S(\phi_i)$ is called the stability set of ϕ_i.*

2.2.2 The Tracing Path Procedure in 2×2

The tracing procedure is a mathematical method developed by Harsanyi and Selten (1988) that defines rational outcomes for non-cooperative games. Intuitively, the tracing procedure records the behavior of the game's equilibria as players, starting from arbitrary initial beliefs about the other player's behavior, gradually achieve common knowledge of rationality under those initial conditions. The tracing procedure models a process by which rational agents adopt, and expect other players to adopt, one , and only one, equilibrium of a noncooperative game.

Although we will still stick to the simple 2×2 coordination game developed above to show how the tracing procedure works, we next introduce a general definition of the linear tracing procedure.

The Linear Tracing Procedure

Harsanyi and Selten propose two versions of the tracing procedure namely linear and logarithmic. Here we use only the Linear Tracing Procedure.

For explanatory purposes we will not distinguish between a pure strategy ϕ_i and a mixed strategy, q_i, that assigns probability 1 to ϕ_i and 0 to all other strategies. We denote by Q_i the set of all possible strategies q_i of player i. Note that Q_i contains all possible strategies of player i, pure and mixed.

Before defining the tracing procedure it is helpful to define some concepts:

Definition 2 (Payoff Function) *A Payoff function* $u_i(q) = \sum_{\phi \in \Phi} \left(\prod_{j \in L} q_j(\phi_j) \right) u_i(\phi)$ *is a function that assigns a payoff, for Player* i*, to each strategy profile.*

Definition 3 (Auxiliary Games) *The one-parameter family of games* Γ^μ *with* $0 \leq \mu \leq 1$*, are called auxiliary games. Each of these auxiliary games keeps,, for each player, the same set of actions but changes his payoff function with respect to that of the original game. Moreover, the payoff function* $u_i(q)$ *in the auxiliary game* Γ^μ *is given by:*

$$u_i^\mu(q_i, q_{-i}) = \mu u_i(q_i, q_{-i}) + (1 - \mu) u_i(q_i, p_{-i})$$

where u_i is the i's payoff function in the original game.

Note from the definition that when $\mu = 0$, then $u_i^0(q_i, q_{-i}) = u_i(q_i, p_{-i})$, i.e. each player's payoff function depends only on her own strategy, and on her priors about other players; it will be independent of those other players' strategies. When $\mu = 1$, then $u_i^1(q_i, q_{-i}) = u_i(q_i, q_{-i})$, i.e. each player's payoff function depends not only on her own strategy but also on the other players' strategies; it corresponds with the original game.

Definition 4 (Linear Tracing Procedure) *The linear tracing procedure computes the equilibria of all the $\mu-$auxiliary games for a given profile of initial beliefs.*

To see in action the linear tracing procedure we now return to our simple example of the 2×2 coordination game

Example (The 2×2 Coordination Game)

The linear tracing procedure starts by computing the players' payoff functions for the first auxiliary game. In the first auxiliary game, Γ^0, players' payoffs are functions of their own strategies and their initial belief conditions about others; they do not depend on other players' strategies, and importantly, as proved by Harsanyi and Selten, there is only one equilibrium even if the original game has more than one.

Consider the 2×2 coordination game represented in Figure 1. where, as I said, if initial belief conditions are located in region III such as $(1/4, 1/4)$ they belong to the stability set of equilibrium $(0, 0)$ while initial belief conditions located in region I such as $(3/4, 3/4)$ belong to the stability set of equilibrium $(1, 1)$ and let's now see how the tracing procedure formalizes this result.

Given that players choose the strategy profile $q = (q_1, q_2)$ and face initial belief conditions $p = (p_1, p_2)$, let's compute the payoff functions for each of the players in game Γ^0:

$$u_1^0(q_1, p_2) = q_1(2p_2 - 1);$$
$$u_2^0(q_2, p_1) = q_2(2p_1 - 1).$$

And in game Γ^1:

$$u_1^1(q_1, q_2) = q_1(2q_2 - 1);$$
$$u_2^1(q_2, q_1) = q_2(2q_1 - 1).$$

Next, by a linear combination of the two expressions above, one for each player, we obtain the payoff functions for any other auxiliary game (i.e. $0 < \mu < 1$):

$$u_1^\mu(q_1, q_2, p_1, p_2) = \mu q_1(2q_2 - 1) + (1-\mu)q_1(2p_2 - 1);$$
$$u_2^\mu(q_1, q_2, p_1, p_2) = \mu q_2(2q_1 - 1) + (1-\mu)q_2(2p_1 - 1).$$

Now, assuming that initial belief conditions are $p = (1/4, 1/4)$, located in region III, we obtain:

$$\begin{aligned} u_1^\mu(q_1, q_2, 1/4, 1/4) &= \mu q_1(2q_2 - 1) + (1-\mu)q_1(2(1/4) - 1) \\ &= q_1\left[\mu(2q_2 - 1) - \tfrac{1-\mu}{2}\right]; \end{aligned}$$

$$\begin{aligned} u_2^\mu(q_1, q_2, 1/4, 1/4) &= \mu q_2(2q_1 - 1) + (1-\mu)q_2(2(1/4) - 1) \\ &= q_2\left[\mu(2q_1 - 1) - \tfrac{1-\mu}{2}\right]. \end{aligned}$$

Conidering that the game is symmetric as well as the initial belief conditions then, all relevant information to see the equilibrium paths are obtained by looking at only one best-response correspondence (because a complete analogue expression holds for player 2)

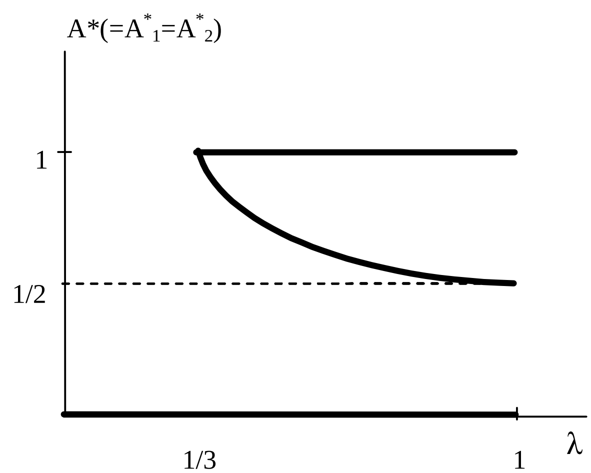
$$q_1^*(q_2) = \begin{cases} 1 & \text{if } q_2 > \frac{1+\mu}{4\mu} \\ [0,1] & \text{if } q_2 = \frac{1+\mu}{4\mu} \\ 0 & \text{if } q_2 < \frac{1+\mu}{4\mu} \end{cases}$$

We carefully must analyze three issues. First, note that for any value $\mu < 1/3$, these best-response correspondences can be satisfied *only* if $q_1^* = q_2^* = 0$. Moreover, note that the equilibrium (D, D) is also equilibrium for any other value of μ. We represent this equilibrium behavior in Figure 3. with a bold straight line on the horizontal axis meaning that whatever the value of the parameter μ the strategy $q = (0, 0)$ is an equilibrium.

Second, when $\mu = 1/3$, $\frac{(1+\mu)}{4\mu} = 1$, and the strategy $q_1^* = q_2^* = 1$ also becomes an equilibrium. We represent this new equilibrium with a dot in Figure 3.

Finally, for values of $\mu > 1/3$ the strategy $q_1^* = q_2^* = 1$ still being an equilibrium, represented in Figure 3 with a bold straight line, and a new mixed strategy equilibrium appears: $q_1^* = q_2^* = \frac{(1+\mu)}{4\mu}$, that we represent in Figure 3. with a bold curve line.

Figure 3. Tracing Paths for the coordination game 1
with $p = (p_1, p_2) = (1/4, 1/4)$

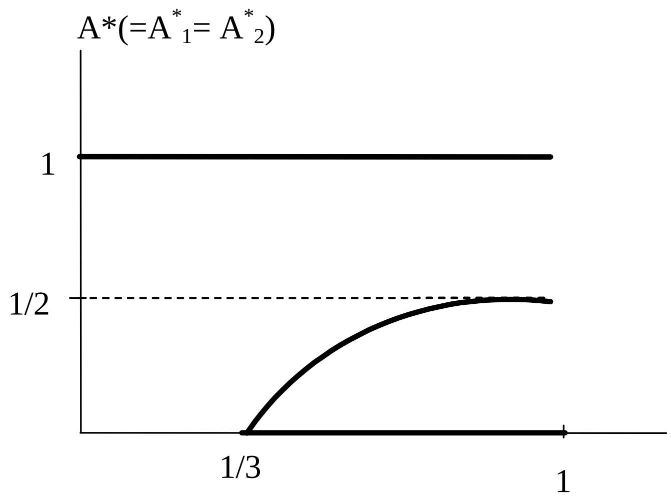


Now, if we repeat the calculations above for initial belief conditions $p = (3/4, 3/4)$, located in region I, we obtain the following best responce correspondences:

$$q_1^*(q_2) = \begin{cases} 1 & \text{if } q_2 > \frac{3\mu-1}{4\mu} \\ [0,1] & \text{if } q_2 = \frac{3\mu-1}{4\mu} \\ 0 & \text{if } q_2 < \frac{3\mu-1}{4\mu} \end{cases}$$

Now, the only equilibrium that exists for every value of μ is $q_1^* = q_2^* = 1$. The other two equilibria $q_1^* = q_2^* = 0$ and $q_1^* = q_2^* = \frac{3\mu-1}{4\mu}$ are only feasible for values $\mu \geq \frac{1}{3}$. The situation is represented in Figure 4.

Figure 4. Tracing Paths for the coordination game 1
with $p = (p_1, p_2) = (1/4, 1/4)$



Thus, we have shown that initial belief conditions such as $(1/4, 1/4)$, located in region III, belong to the stability set of equilibrium $(0, 0)$ while those such as $(3/4, 3/4)$ belong to the stability set of equilibrium $(1, 1)$.

2.2.3 Stability Sets in Large Games

While the method of stability sets and the tracing procedure are straigthforward in the case of 2×2 coordination games some difficulties arise in the case of large games, i.e. when we have a large number of players.

Briefly, in the case of tipping games, the type of game we study here, one can not say that players' actions are completely independent. Moreover, it is common in the study of collective action problems to state that the expectations of a particular group move towards a tipping point. It implicitly means that agents are not choosing their strategies independently but knowing that a certain fraction of the members of the group are more likely to cooperate. As a consequence the Nash equilibrium concept, under which players choose their strategies independently of other players' actions becomes innapropriate. Fortunately as proved by Robert Auman (1974) it is possible to generalize the framework of Nash play and solve games of this type through correlated equilibria, an equilibrium concept that does not assume strict independence among all the players' strategies. So the appropriate solution concept for games modeling collective action problems, which tipically involve a large number of players with no totally independent beliefs and actions, is the concept of correlated equilibria.

An important feature about correlated equilibria is that even if players are forming beliefs and choosing actions independently, just the fact that such beliefs and actions come from publicly available information makes them statistically correlated. So correlation does not imply, necesarily, any deliberate attempt at coordination among the players.

Medina (2007) shows that for a large game with multiple equilibria an equilibrium of any given auxiliary game along the tracing path must satisfy the following inequalities[18]:

$$\left(\lambda F\left(\gamma_{\mu}\right)+(1-\lambda) F\left(\gamma_{\eta}\right)\right) \gamma_{\mu} \geq \gamma_{\mu} W,$$

$$\left(\lambda F\left(\gamma_{\mu}\right)+(1-\lambda) F\left(\gamma_{\eta}\right)\right)\left(1-\gamma_{\mu}\right) \leq\left(1-\gamma_{\mu}\right) W$$

where F is the function that represents the probability of success, depending of the level of turnout $\gamma.\gamma_{\eta}$ is a share of expected cooperators, γ_{μ} is the equilibrium level of turnout and W is a function of the games' payoff structure:

$$W \equiv \frac{w_4-w_3}{\left(w_1-w_3\right)-\left(w_2-w_4\right)}$$

where, assuming that all players have identical payoffs (e.g., $w_{1i}=w_1$), $w_1$is the payoff a player receives if she cooperates and collective action succeeds. w_2 is the payoff a player receives if she defects and collective action succeeds. w_3 is the payoff a player receives if she cooperates and collective action fails. w_4 is the payoff a player receives if she defects and collective action fails.

The two inequalities above allow us to compute the stability sets of a collective action game. However, the most important result obtained by Medina is the following:

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18 He focuses on games having three equilibria, two of them in pure strategies: one where nobody cooperates, another one where everyone does so and a third one with intermediate level of turnout.

> If the mutual expectations players have are summarized by initial belief conditions with expected aggregate turnout $\gamma_\eta < W$, those expectations belong to the stability set of noncooperation $\left(\gamma_\mu = 0\right)$. If, instead, the initial belief conditions are such that $\gamma_\eta > W$, they belong to the stability set of cooperation $\left(\gamma_\mu = 1\right)$. Medina (2007, pp. 136).

The intuition behind the result is the following: low expected levels of turnout, those with $\gamma_\eta < W$, are not enough to make cooperation worthwhile for players. In terms of the expected payoffs for the would-be participants in the colective action the result implies that the would-be costs of a failed collective action outweigth the would-be benefits. On the contrary high expected levels of turnout, those with $\gamma_\eta > W$, players find the risk worth taking and, as these expectations are validated, they reach the cooperative equilibrium.

The importance of this result is that allows us to make comparative statics of collective action problems. The method of stability sets crucially connects beliefs and payoffs in the game. So specifying our assumptions about the distribution of initial belief conditions and, then, analyzing changes in the expected costs benefits relationship for players to participate in the collective action, captured in W, we can compute the probability of cooperation by looking at the relative size of each stability set, one for each possible equilibria of the game.

2.3 A Political Economy Model

2.3.1 The Economy.

Society is composed of two groups, the elite, E, and the citizens, C. Total population is normalized to 1; $1-\gamma$ denotes the fraction of the population belonging to group C, which is assumed to be the majority of the population i.e. $1-\gamma > 1/2$, and the remaining fraction of the population, γ, belongs to group E.

There are two possible economic scenarios for this society depending on the success, or failure, of the collective action problem faced by citizens: a closed or an open economy. Next, I describe the economic details for both scenarios and make explicit the payoffs citizens would get in each of them.

Closed Economy

In a closed economy, society uses two intermediate goods as inputs ,Y_K, and, Y_L, and produces one agricultural good, Y, by means of the Cobb-Douglas production function: $Y = Y_K^{\theta} Y_L^{1-\theta}$, where $0 < \theta < 1$. The inputs Y_K, and, Y_L are capital and labor intensive respectively and, in a closed economy with no trade, they are produced by domestic factors of production. I assume that citizens only own labor while the elite only owns capital. Furthermore, the elite is assumed to be relatively rich with fixed income y^e, and the citizens, relatively poor with income y^c so that $y^c < \bar{y} < y^e$, where $\bar{y}$ denotes the average income in this society. To make the model as much parsimonious as possible we assume that inputs are produced using only their respective factors; therefore, domestic production of each intermediate good is given by:

$$Y_K = K \tag{2.1}$$

$$Y_L = 1 - \gamma$$

Production factors are fully employed and the production function exhibits constant returns to scale implying that all revenues from production are distributed as income to the factors of production. I assume competitive markets both for intermediate goods and factors of production so that each production factor will be paid their marginal product. Prices of intermediate goods are denoted by p_K and p_L and the price of the final output by p_y. The price of the final output is normalized to 1 and used as the numeraire i.e. $p_y = 1$.

To find the relative price ratio for inputs we solve the following standard cost-minimization problem of a firm:

$$\min_{Y_K,\, Y_L} P_K Y_K + P_L Y_L \qquad s.t. \qquad Y = Y_K^{\theta} Y_L^{1-\theta} \tag{2.2}$$

To solve it we form the Lagrangean function:

$$\mathcal{L} = P_K Y_K + P_L Y_L - \lambda \left(Y_K^{\theta} Y_L^{1-\theta} - Y \right) \tag{2.3}$$

which yields the following first order conditions:

$$P_K = \theta \lambda Y_K^{\theta-1} Y_L^{1-\theta} \tag{2.4}$$

$$P_L = (1-\theta)\, \lambda Y_K^{\theta} Y_L^{-\theta} \tag{2.5}$$

From these, we obtain an expression for the relative prices

$$\frac{P_K}{P_L} = \frac{\theta}{(1-\theta)} \frac{Y_L}{Y_K} \tag{2.6}$$

Because of the competitive market assumption each production factor will be paid their marginal product, that is:

$$w = P_L, \qquad\qquad r = P_K \tag{2.7}$$

where w denotes the wage rate and r the return to capital. We can use $(1)$and $(6)$to obtain

$$p_K = \theta \left(\frac{K}{1-\gamma} \right)^{\theta-1} \tag{2.8}$$

$$p_L = (1-\theta) \left(\frac{K}{1-\gamma} \right)^{\theta} \tag{2.9}$$

Exploiting the fact that we have normalized total population to 1, then, total income, y, equals average income, $\bar{y}$, and can be expressed as: $\bar{y} = K^{\theta}(1-\gamma)^{1-\theta}$. Now, considering that citizens only own labor and the elite only owns land then we can write the income of a citizen, y^c, as

$$y^c = \frac{w}{1-\gamma} = \frac{(1-\theta)\left(\frac{K}{1-\gamma}\right)^{\theta}}{1-\gamma} = \left(\frac{1-\theta}{1-\gamma}\right)\bar{y}, \tag{2.10}$$

and the income of a member of the elite, y^e,as

$$y^e = \frac{rK}{\gamma} = \frac{\theta}{\gamma}\bar{y}. \tag{2.11}$$

It should be noted that θ and $1-\theta$ are the shares of national income accruing to capital, S_K, and labor, S_L, respectively $\left(S_K = \frac{rK}{Y} = \theta \text{ and } S_L = \frac{wL}{Y} = \ (1-\theta)\ \right)$. Therefore and because citizens only own labor and the elite only owns land higher levels of θ translate into higher levels of income inequality among the two social groups.

In this model the government face the following budget constraint:

$$T = \tau\left((1-\gamma)\,y^c + \gamma y^e\right) - C(\tau)\,\bar{y} = (\tau - C(\tau))\,\bar{y} \tag{3}$$

Where function $C(\tau)$ represents the cost of collecting taxes. The costs of collecting taxes are assumed to increase with the level of taxation, $C'(\tau) > 0$, and to increase faster as the tax rate increases, $C''(\tau) > 0$. Also it is assumed that $C'(0) = 0$ and $C'(1) = 1$. Together these properties of the cost function capture the idea of distortions existing in the economy when taxation is so high and ensure an interior solution to the optimization problem. The term $C(\tau)\,\bar{y}$ normalizes the costs of taxation so that the equilibrium tax rate does not depend in equilibrium on the size of the economy. Variations of the total income in this economy, $\bar{y}$, increase the costs of taxation.

Open Economy

In an open economy, intermediate goods are traded internationally through competitive markets. In particular we assume that countries have different endowments of factors of production so that they have incentives to trade intermediate goods, we assume that production factors cannot be traded directly. World prices for land and labor intensive intermediate goods are denoted by $\tilde{p}_K$ and $\tilde{p}_L$. Once international prices are given, factor remunerations are again given by the relevant value of the marginal products, now valuated at world prices. So $\tilde{w} = \tilde{p}_L$ and $\tilde{r} = \tilde{p}_K$ are the wage rate and and the return to capital in an open economy. International prices are determined in some world market equilibrium and are given by $\tilde{p}_K = \theta\Psi^{\theta-1}$ and $\tilde{p}_L = (1-\theta)\,\Psi^{\theta}$, where Ψ denotes the world capital to labor ratio, i.e. $\Psi = \frac{\sum_j K_j}{\sum_j L_j}$. We assume that in the world economy what matters for the determination of international prices is the capital-labor ratio, just as it was in the case of a closed economy. So we can establish the international price ratio as:

$$\frac{\tilde{P}_K}{\tilde{P}_L} = \frac{\theta}{(1-\theta)} \frac{\sum_j L_j}{\sum_j K_j} \tag{2.12}$$

Where L_j is total labor supply in country j and K_j is the stock of capital. Further, and for the sake of the argument I assume that civil wars occur in relatively poor countries, which are more abundant in labor than in capital. For the country in question we have that $\Psi > \frac{K}{(1-\gamma)}$, so that it has comparative advantage in the production of labor intensive inputs.

This assumption is crucial because it implies that passing from a closed to an open economy raises the price of the labor intensive intermediate good. Intuitivelly, after opening to trade, the price of the abundant factor increases relative to the other factor price[19]. Therefore, in this setting, international trade increases wages relative to capital returns.

Here, in an open economy, factor rewards are given by:

$$\tilde{w} = \tilde{P}_L, \qquad \tilde{r} = \tilde{P}_K or \tilde{w} = (1-\theta)\,\Psi^{\theta}, \qquad \tilde{r} = \theta\Psi^{\theta-1} \tag{2.13}$$

Remembering that citizens own only labor and the elite only capital, we can use these factor prices to obtain post-trade incomes:

$$\tilde{y}_c = (1-\theta)\,\Psi^{\theta} \tag{2.14}$$

and

[19] This proposition is the well known Stolper-Samuelson theorem in international economics: A rise in the relative price of a good will lead to a rise in the return to that factor which is used most intensively in the production of the good, and conversely, to a fall in the return to the other factor.

$$\tilde{y}_e = \frac{\theta}{\gamma}\Psi^{\theta-1}K \tag{2.15}$$

and average income:

$$\mathring{y} = \Psi^{\theta-1}\left(\left(1-\theta\right)\left(1-\gamma\right)\Psi + \theta K\right) \tag{2.16}$$

2.3.2 Conflict and Political Power

So far we have shown that in a poor country, which is assumed to be more abundant in labor than in capital, the returns to capital are higher in a closed than in an open economy. Therefore, the most preferred policy for the elite, that only owns capital, is to keep a closed economy. On the contrary, the citizens, who only own labor, would prefer an open economy because of the increase in the their incomes and utility levels through the increase of the marginal return to labor at international prices, i.e. $\tilde{w} > w$. This preferences over opposite economic institutions (closed vs open economy) characterize conflict in this society. Additionally, it is assumed that the elite has de jure political while citizens de facto political power.

Notice that the existence of two social groups having equal amounts of political power and facing conflict because of their opposite preferences over opposite economic outcomes characterizes a bi-polarized society.

For this society, the status quo is characterized by a close economy where the elite exerts their jure political power and is able to implement their most preferred policy. However, citizens may be able to modify the status quo if they can use their de facto political power. An important feature of the de facto political power in this model is that it is transitory in the sense that the amount of de facto political power citizens have today is generally different from the one they will have tomorrow. More important, economic shocks to the system might boost or cut off the amount of the political power citizens have so they become relevant for explaining the odds of moving towards an open economy.

Notice that the assumption of different types of political power affects the optimal tax rate determination process. Under Downsian competition and given that citizens are the majority (i.e. $1-\gamma > \frac{1}{2}$) the optimal tax rate will be that which maximizes the indirect utility of the citizens. In other words preferences of the elite are irrelevant for determining the tax rate. Here, instead, the optimal tax rate will depend on the political power of each group. To introduce political power into the model let us denote by X the amount of political power the elite has and by $(1-X)$ that of the citizens. The optimal policy maximizes the weighted sum of the indirect utilities of the two groups, the weights being the political power of each group[20]. To be concrete let us assume that individuals within each group have the same preferences denoted by utility functions what are linear in consumption and denoted by $V\left(y^{i} \mid t\right)$; the

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[20] Under Downsian competition the optimal tax rate, the one tha maximizes the indirect utility of the citizens, is that solving the following maximization problem:

$$\max_{\tau\in[0,1]} (1-\tau)\, y^{c} + (\tau - C(\tau)\, \bar{y}$$

with first order condition: $-y^{p} + (1 - C'(\tau^{p}))\, \bar{y} = 0$ implying $\frac{\theta-\gamma}{1-\gamma} = C'(\tau^{p})$ after replacing by y^{p} and $\bar{y}$.

utility of an individual with income level y^i when policy is given by t. It is assumed that people consume all their income so that they want to maximize their post-tax indirect utility income, given by:

$$V\left(y^i \mid \tau, T\right) = (1-\tau)\, y^i + T \qquad \text{with } i = e \text{ or } c. \tag{2.17}$$

Therefore at the equilibrium tax rate would be that which maximizes the following expression:

$$\max_{\tau\in[0,1]} (1-X)(1-\gamma)\left((1-\tau)\, y^c + T\right) + X\gamma\left((1-\tau)\, y^e + T\right) \tag{2.18}$$

Which has a first order condition that yields:

$$\frac{(1-X)(1-\theta)+X\theta}{(1-X)(1-\gamma)+X\gamma} = 1 - C'\left(\tau\left(X\right)\right) \tag{2.19}$$

where $\tau(X)$ is the equilibrium tax rate when the political power of the elite is X.

As stated above in the status quo, a closed economy, the elite has enough political power to set their most preferred policy outcome, t^e. To model this let us assume that $X = 1$. Therefore, the payoffs of the citizens under the status quo are given by:

$$V\left(y^c \mid \tau^e\right) = (1-\tau^e)\, y^c + \left(\tau^e - C\left(\tau^e\right)\right)\bar{y} = w + \tau^e\left(\bar{y} - w\right) - C\left(\tau^e\right)\bar{y} \tag{2.20}$$

However if citizens engage into civil war but fail to overthrown the elite then, they will be punished. They will be taxed but without reciving any transfer. In this case each citizen has a post-tax indirect utility income, given by:

$$V\left(y^{c} \mid \tau^{e}\right)=w\left(1-\sigma\right)^{\theta} \tag{2.21}$$

If succesful collective action takes place, citizens are able to change the status quo from a closed to an open economy. Assuming that a fraction ϕ of the income of the elites is distributed between all citizens who take part into the collective action then, the post-tax indirect utility income for a citizen who participated into a successful civil war is given by:

$$\begin{aligned} V\left(y^{c} \mid \tau^{c}\right) &= \left(1-\tau^{c}\right)\tilde{y}^{c}+\left(\tau^{c}-C\left(\tau^{c}\right)\right)\tilde{y} \text{ or,} \\ V\left(y^{c} \mid \tau^{c}\right) &= \tilde{w}+\tau^{c}\left[\left(1-\phi\right)\left(\tilde{y}-\tilde{w}\right)\right]+\phi\left(\tilde{y}-\tilde{w}\right)-C\left(\tau^{c}\right)\mathring{y} \end{aligned} \tag{2.22}$$

The payoff for those who did not participate in the civil war when it was successful are given by:

$$V\left(y^{c} \mid \tau^{c}\right)=\tilde{w}+\tau^{c}\left[\left(1-\phi\right)\left(\tilde{y}-\tilde{w}\right)\right]-C\left(\tau^{c}\right)\mathring{y} \tag{2.23}$$

Notice that with $X=0$, i.e. the elite has no political power, then the optimal tax rate becomes $\left(\frac{\theta-\gamma}{1-\gamma}\right)=C'\left(\tau^{c}\right)$, which is the tax rate preferred by the citizens.

But for any $X > 0$ i.e. the elite has political power the optimal tax rate is less than the one preferred by the citizens $\tau(X > 0) < \tau^c$. Moreover, the optimal tax rate falls as the political power of the elite increases. To see this notice that increases in X make the left hand side of equation (19) greater. Therefore, in order for the equality to hold $C'(\tau)$ must fall implying $\tau(X > 0) < \tau(X = 0)$ given that $C'(\tau) > 0$.

2.4 The Collective Action Problem.

Now we are in a position to use the stability set technique described in the previous section. We just need to identify the payoffs for citizens contingent on the success or not of the collective action i.e. w_1, w_2, w_3, and w_4 so that we can get the expression for $W = \frac{w_4 - w_3}{(w_4 - w_3) + (w_1 - w_2)}$ in this model and perform comparative statics on the probability of collective action to take place.

Just to remember $w_1 = U(C, S)$ is the payoff a player receives if she cooperates and collective action succeeds, $w_2 = U(D, S)$ is the payoff a player receives if she defects and collective action succeeds, $w_3 = U(C, F)$ is the payoff a player receives if she cooperates and collective action fails, $w_4 = U(D, F)$ is the payoff a player receives if she defects and collective action fails. Here, these payoffs are given by:

$$w_1 = \tilde{w} + \tau^c \left[(1 - \phi)(\tilde{y} - \tilde{w}) \right] + \phi(\tilde{y} - \tilde{w}) - C(\tau^c)\,\mathring{y} \qquad (2.24)$$

$$w_2 = \tilde{w} + \tau^c \left[(1-\phi) \left(\tilde{y} - \tilde{w} \right) \right] - C\left(\tau^c\right) \mathring{y} \tag{2.25}$$

$$w_3 = w\left(1-\sigma\right)^{\theta} \tag{2.26}$$

$$w_4 = w + \tau^e \left(\bar{y} - w \right) - C\left(\tau^e\right) \bar{y} \tag{2.27}$$

yielding,

$$W \equiv \frac{w - w\left(1-\sigma\right)^{\theta}}{w + \phi\left(\mathring{y} - \tilde{w}\right)} \tag{2.28}$$

which is the the size of the stability set of the noncooperative equilibrium considering that all players in each group face the same payoffs. Finally, from that expression it can be shown that:

$\frac{\partial W}{\partial w} > 0$ so increases in the level of wages in a closed economy decrease the probability of success of collective action. Stated differently, the stability set of the noncooperative equilibrium increases.

$\frac{\partial W}{\partial \tilde{w}} < 0$ so increases in the level of wages valued at international prices increase the probability of success of collective action. Stated differently, the stability set of the noncooperative equilibrium decreases.

$\frac{\partial W}{\partial \theta} < 0$ so increases in the level of inequality in the society increase the probability of success of collective action. In other words, the stability set of the noncooperative equilibrium decreases.

2.5 Concluding Remarks

The role of the economic structure in determining different social outcomes has longly been studied in economics. The works of Acemoglu and Robinson (2006) and Roemer (2001) are but a few, recent examples. More over, there already is literature on the effect of changes in prices over social conflict. The works of Collier and Hoeffler (1998) and Dal Bó and Dal Bó (2008) are notable examples. This literature, however, does not deal with a central aspect of any socioeconomic process: the collective action problem.

Though the importance of collective action problems in social conflicts such as revolts and civil wars has also been recognized (the works of Tullock 1971 and Goodwin 2001 are some examples) there is not much literature relating changes in objective parameters such as prices, rather than subjective ones, with changes in the odds for successful collective action. Therefore, meaningful questions around the theme still unanswered. Does income inequality affect the probability for citizens to engage into a collective action against the government? Are workers more likely to revolt in a closed economy with a large difference in terms of salary with respect to international wages?

In this paper, I model a politically polarized society and use the concept of stability sets in order to make explicit how changes in economic parameters of the economy may affect the probability for individuals to coordinate and solve their collective action.

The model allows for comparative statics on parameters such as prices and inequality. In particular the model shows that increases in the level of wages (in a closed economy) decrease the probability of success of collective action while increases in the level of wages (valued at international prices) or in the level of inequality in the society increase the probability of success of collective action.

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Chapter 3
Social Polarization and Conflict: A Network Approach

The opening words of Amartya Sen's famous book on economic inequality are "The relation between inequality and rebellion is indeed a close one, and it runs both ways" Sen[11.]. However, recent literature in economics has raised questions about the accuracy of the very concept of inequality and its correlation with social conflict[2.][3.]. As a result, it seems that the concept of polarization is better suited than that of inequality at explaining the probability of social conflict arising.

Not surprisingly, in his recent work on Identity and Violence Sen himself rests on the notions underlying the concept of polarization, those of within-group homogeneity and between-group heterogeneity:

"A sense of identity can be a source not merely of pride and joy but also of strength and confidence... And yet identity can also kill—and kill with abandon. A strong—and exclusive—sense of belonging to one group can in many cases carry with it the perception of distance and divergence from other groups. Within-group solidarity can help to feed between-group discord.[12.]"

Reductionist ideologies based on defining individuals or societies in terms of a unique affiliation can be easily used to foster strong feelings of within-group solidarity but also of between-group disagreement. Then, it becomes easier to promote conflict as a social outcome. The problem with these theories is that they reduce a multidimensional issue to an unidimensional one.

As an example, consider Huntington's theory of the clash of civilizations[6.] in which religion is the unique affiliation considered when talking about civilizations and conflict. Following Sen's analysis of this theory, classification of people in terms of belonging either to Western civilization or Islamic civilization is based upon two misconceptions. First, the very idea of classifying people into either one of the two civilizations is an extreme reductionist view that misses the complexity of the essence of human beings; and second, to assume that individuals in each of these civilizations must have a sense of antagonism towards those who belong to the other group ignores the multiple affiliations of individuals. In fact, every individual belongs to different social groups and as such has multiple affiliations and each of these affiliations confers to the individual a specific identity; a person's profession, employment, family role, preferences for music, sports or politics are just but a few examples.

Similar examples of reductionism, with social conflict as an outcome, can be found all over the world. In Rwanda, perception of individuals only in terms of ethnicity as either Hutu or Tutsi kept the country in continuous civil war and produced atrocities like the genocide of 1995 with almost one million causalities in just a few days. According to Sen, Bangladesh obtained its independence from Pakistan in 1971 after a bloody civil war because of the reduction of individuals affiliations to just one, that of language. In Colombia, where peace has become extremely difficult to achieve, both sides in the conflict have a reductionist view; the Fuerzas Armadas Revolucionarias de Colombia (FARC) perceive society as being the status quo they want to change while society only perceive the FARC as mere terrorists.

Recognition of the fact that individuals have multiple affiliations expands the spectrum of commonalities among people and abates their sense of antagonism.

Societies where multiple affiliations of individuals are taken into account generate more peaceful environments as social outcomes. Switzerland, for example, where Germans, French and Italians are the main three different groups forming the society and each of their languages is an official language of the country is a good example of a peaceful environment with individuals having different affiliations.

In economics most of the work relating polarization and conflict has been done considering unidimensional categories for defining groups within society. Examples of analysis of polarization in terms of only one relevant characteristic are those of Gasparini et. all.([5.]), based on income, Montalvoa and Reynal-Querol and Reynal-Querol ([8.] [10.]) based on religion or Reynal-Querol ([9.]), based on ethnicity.

The main contribution of this work is the use of the theory of social networks as a way to deal with the fact that individuals have multiple affiliations, instead of just one. Specifically, with a microeconomic model based on the theory of social networks I analyze how changes in the structure of a given network, representing a given society, affect the level of some basic parameters of the network associated with the concept of polarization and, as a consequence, its relation with social conflict.

Here, it is assumed that each individual has several affiliations, instead of having only one. Affiliations of individuals and their relationships are captured by means of a hypergraph. Each individual is endowed with a preference system over the set of all possible affiliations (i.e. the power set of the set of affiliations) so that she can choose among all of them. Both, the hypergraph and the preference system induce a communication graph.

Two specific systems of preferences are analyzed: a monotonic and a non-monotonic system. Under the monotonic system, two individuals decide to communicate with each other if they mutually consider the other's set of affiliations as being at least as good as its threshold affiliations system set i.e. the set of minimum affiliations she considers crucial for communicating with someone else.

The non-monotonic system is obtained by positing thresholds of good combinations of affiliations and ceilings of bad combinations of affiliations. In other words, each individual has an exhaustive partition over the set of all possible affiliations. Here, two individuals communicate if the affiliations of each individual are considered as good by the other and none of them are considered as a bad affiliation.

If communication among individuals occurs then, they will be linked in the associated communication graph.

The paper proceeds as follows. Some basic concepts derived from the theory of networks are introduced in section 2. In section 3 I formally define the concept of polarization and establish certain axioms concerning polarization but using networks theory concepts. A model of social communication based on the characteristics of the social structure, represented here by means of an hypergraph, and a system of social preferences is developed in section 4. In section 5, I perform some comparative statics; in particular, I analyze how changes in the affiliation sets for each individual and changes in social preferences may alter the degree of social polarization. Finally, the conclusions of the work are presented.

3.1 Preliminary Network Concepts

Here, I define the concepts of the theory of networks and groups that will be used throughout to be related with the notion of polarization and for building up the microeconomic model in order to perform comparative statics.

Definition 5 (Network) *A network* $\Gamma = (N, G)$ *consists of a set of nodes* $N = \{1, ..., n\}$ *and a set of links* $G. = \{g_1,\ g_2, ...,\ g_m\}$ *.Each link* g_i *is identified by the pair of respective nodes which are connected by* gi*. Each link in G represents a relationship between nodes* i *and* j*.*

Definition 6 (Path) *A path in a network* Γ *between nodes* i *and* j *is a sequence of links* $i_1, ..., i_K$ *such that* $i_k i_{k+1} \in \Gamma$ *for each* $k \in \{1, ..., K-1\}$*, with* $i_1 = i$ *and* $i_K = j$*.*

Definition 7 (Geodesic Distance) *The geodesic distance between nodes $i, j \in \Gamma$ is the length (number of links) of the shortest path between them. If there is not such a path then the distance between the two nodes is ∞.*

Definition 8 (Average Path Length) *Given a network Γ, its average path length is the average of geodesic distances between nodes*

Definition 9 (Component) *A (connected) component of a network $\Gamma = (N, G)$, is a nonempty subnetwork $C' = (N', G')$, such tat $\varnothing \neq N' \subset N, G' \subset G$,*

$\cdot$ if $i \in N(G')$ and $j \in N(G')$ where $j \neq i$, then there exists a path in G' between i and j, and

$\cdot$ if $i \in N(G')$ and $j \notin N(G')$ then there does not exist a path in G between i and j.

Thus, the components of a network are the distinct maximal connected subgraphs of a network. The set of components of Γ is denoted $C(\Gamma)$. Note that $\Gamma = \bigcup_{C' \in C(\Gamma)} C'$.

Definition 10 (Clique) *A clique is a maximal completely connected subnetwork Q of a given network Γ.*

It is worthy to note that a clique is a maximal completely connected subnetwork while a component is a maximal path-connected subnetwork. Neither implies the other.

Definition 11 (Relative Size) *Given a network* Γ *with more than one component the relative size of a component* $\Gamma' \in C(\Gamma)$ *is given by* $S_{\Gamma'} = \frac{\#\{N(G')\}}{\#\{N\}}$

Definition 12 (Overall Clustering) *It is a measure of the average probability that two nodes* k, j *directly connected to node* i *are also connected among themselves* $Cl(\Gamma) = \frac{\sum_i \#\{jk \in \Gamma | k \neq j,\, j \in N_i(\Gamma),\, k \in N_i(\Gamma)\}}{\sum_i \#\{jk | k \neq j,\, j \in N_i(\Gamma),\, k \in N_i(\Gamma)\}}$

Definition 13 (Hypergraph) *A hypergraph* $H = (V, E)$ *consists of a set of vertices or nodes* $V = \{v_1, ..., v_n\}$ *and a set of hyperedges* $E = \{e_1, ..., e_m\}$ *which is nothing but a collection of subsets of* V*. Each hyperedge, therefore, contains one or more vertices and we say that those vertices are linked by that particular hyperedge.*

Definition 14 (Incidence Matrix) *The incidence matrix of a hypergraph* $H = (V, E)$ *is a matrix* $I(H)$ *with* n *rows that represent the vertices and* m *columns that represent the edges of* H *such that:*

$$(i, j) - entry = \begin{cases} 1 & \text{if } v_i \in e_j, \\ 0 & \text{if } v_i \notin e_j. \end{cases}$$

Definition 15 (Dual Representation of a Hypergraph) *The dual of the hypergraph* $H = (V, E)$ *is a hypergraph* $H^*(Y, Z)$ *whose vertex set is* $Y = \{E_1, ..., E_m\}$ *(the original hyperedge set) and the edge set,* E^**, is a new set of hyperedges defined as follows:* $E^* = \{V_1, ..., V_n\}$; $V_i = \{E_j : v_i \in e_j \text{ in } H\}$

Definition 16 (Representative Graph of a Hypergraph) *Let* $H = (E;\ V_1,\ V_2,\ ..., V_n)$ *be a hypergraph with* n *edges. The representative graph of* H *is defined to be the* *simple graph of order* n *whose vertices* $v_1, v_2, ..., v_n$ *respectively represent the edges* $V_1, V_2, ..., V_n$ *of* H *and with vertices* v_i *and* v_j *joined by an edge if, and only if,* $V_i \cap V_j \neq \varnothing$.

Definition 17 (Intersecting Family) *A hypergraph* H *is called an intersecting family if all of its edges pairwise intersect.*

Definition 18 (Star) *A hypergraph* $H = (V, E)$ *is called a star if there is a vertex which belongs to all hyperedges*

Definition 19 (Bipartite Hypergraph) *A hypergraph* $H = (V, E)$ *is called a bipartite hypergraph if its vertex set* V *can be partitioned into two disjoint sets* V_1 *and* V_2 *(called parts) in such a way that each hyperedge of cardinality* ≥ 2 *contains vertices from both parts.*

3.2 Network Concepts and the Concept of Polarization

There is an agreement in specialized literature about three basic elements that must be present when talking about social polarization and to consider it as a threat for social order:

First, there must be a small number of significantly *sized groups*[21]. Second, an individual belonging to any of the groups must feel a sense of *identification* or

[21] It is possible to think about non-significantly sized groups or even isolated individuals, however, the probability for social conflict to arise in such a cases is neglectable.

within-group homogeneity with the rest of individuals belonging to the same group and, third, individuals belonging to the same group must feel some degree of *distance* or *between-group heterogeneity* from individuals belonging to other groups.

In this section I associate some basic axioms defined in terms of the networks' theory with these elements underlying the concept of polarization.

Axiom 4 *Consider any two networks* $\Gamma^1 = \{N^1, G^1\}$ *and* $\Gamma^2 = \{N^2, G^2\}$ *with the same population i.e.* $N^1 = N^2$ *and let* $C^i = \left\{C_1^i, ..., C_{m_i}^i\right\}$ *with* $i = 1, 2$ *be the set of ordered components, in terms of size, from higher to lower, of network* i. *Then, other things being equal,* $\Gamma^1 \succcurlyeq^p \Gamma^2$ *whenever* $\#C_j^1 \geq \#C_j^2$*, for* $j = 1, 2$. *i.e. the cardinality for one of the two largest components in* Γ^1 *is greater or equal than that in* Γ^2.

Axiom 1 captures the idea of significantly sized groups through the concept of network components and its relative size. Each component represents a different social group. The fact that, by definition, there are no links at all between components represents the idea of maximum distance between the groups or between-group heterogeneity. This axiom focuses on the cardinality or relative size of the first two largest components taking account of the idea of small number of significantly sized groups. In fact, the axiom establish that the greater the size of at least one of the two largest components then, other things being equal, the larger the degree of polarization.

Axiom 5 *Consider any two networks* $\Gamma^1 = \{N^1, G^1\}$ *and* $\Gamma^2 = \{N^2, G^2\}$ *with the same population i.e.* $N^1 = N^2$ *and let* $C^i = \{C_1^i, ..., C_m^i\}$ *with* $i = 1, 2$ *be the set of ordered components of network* i, *ordered in terms of size from higher to lower. Then,* $\Gamma^1 \succsim^P \Gamma^2$ *if* $Cl\left(C_1^1\right) > Cl\left(C_1^2\right)$ *and* $Cl\left(C_2^1\right) > Cl\left(C_2^2\right)$.

Axiom 2 captures the idea of within-group homogeneity through the concept of overall clustering. The intuition behind overall clustering is the following: first, we consider the total amount of pairs of nodes j, k having a link with a common third node i. Then, we look at how many of those pairs j, k are actually connected among them. If we repeat the exercise for all i then we have the overall clustering of the network. Thus, the axiom focuses on within-group homogeneity by measuring how tightly clustered individuals are within each of the two largest components.

Axiom 6 *Maximum amount of polarization. Let* $\Gamma = \{N, G\}$ *be such that* $Q_1 = (N_1, G_1)$ *and* $Q_2 = (N_2, G_2)$ *are the only two cliques of* Γ *with relative size equal to a half i.e.* $S_{Q_1} = S_{Q_2} = \frac{1}{2}$, *and* $N_1 \cap N_2 = \varnothing$; *then,* $\Gamma \succsim^P \Gamma'$ *for any* $\Gamma' = (N, G')$.

Axiom 3 captures the idea of a society divided only into two groups of equal size (each of them equal to a half) with the largest distance among them and with all individuals in each group having communication among them (represented by means of completely connected components or cliques) A society like this exhibits the maximum amount of polarization

Axiom 7 *One component networks exhibit minimal polarization.*

Axiom 4 focuses on cases where society can be seen as having only one social group in the sense that every one communicates with each other directly, they share some affiliations, or indirectly they communicate through someone else.

Axiom 8 *For networks with only one component then* $\Gamma' \succsim^{P} \Gamma$ *if and only if* $\Gamma \succsim^{C} \Gamma'$ *i.e.* Γ *has greater clustering coefficient.*

Axiom 5 is a refinement for networks with only one component. The network with the greater clustering coefficient or within group homogeneity is considered as less polarized, people communicates more among themselves.

3.3 The Model

Let $H = (N, H)$ be a social affiliation hypergraph where $N = \{1, ..., n\}$ denotes the finite set of individuals in society and the set $H = \{E_1, ..., E_m\}$ denotes the set of all possible affiliations in that society.

For each $i \in N$, $H_{(i)} = \{E_k \in H : i \in E_k\}$ is the set of all non trivial affiliations, E_k, (i.e. affiliations with cardinality greater than 2, $\#E \geq 2$) such that individual i belongs to them.

Each individual is assumed to have a binary *dichotomic* system of preferences. A binary system of preferences $\succsim_i^H \subseteq P(H) \times P(H)$ is dichotomic if and only if there exists a partition $\left(G_i^H, B_i^H\right)$ of $P(H)$ (the power set of H) such that for any $A, B \in P(H)$, $A \succsim_i^H B$ if and only if $A \in G_i^H$ or $B \in B_i^H$, or both. Here, G_i

represents the set of affiliations considered by individual i as "good" affiliations and B_i represents the set of affiliations considered by individual i as "bad" affiliations.

First, it is assumed that preferences can be either "upward" or "downward" monotonic.

Preferences are "*upward*" *monotonic* if and only if dichotomic and there exist $T_i = \left\{T_i^1, \text{ ..., } T_i^k\right\}$ such that $T_i^h \subseteq P(H)$ and $A \in G_i^H$ if and only if $A \supseteq T_i^{h'}$ for some h', $h' = 1, ..., k$.

Preferences are "*downward*" *monotonic* if and only if dichotomic and there exist $T_i' = \left\{T_i'^1, \text{ ..., } T_i'^k\right\}$ such that $T_i'^h \subseteq P(H)$ and $A \in B_i^H$ if and only if $A \supseteq T_i'^{h'}$ for some h', $h' = 1, ..., k$.

This monotonic structure of the preferences implies that supersets of $T_i^k \in T_i$ belong to the set G_i, i.e. to the set of affiliations considered by i as "good". This property can miss an important real issue. Consider for instance that one affiliation in the affiliation set X is that of belonging to a mafia. Under the monotonic assumption, as far as $T_i^k \subseteq X$ then $X \in G_i$, meaning that i establishes a tie with j without any consideration about the fact of j being in mafia! Of course this is unrealistic. In order to cope with this important real issue I also model preferences in an alternative "mixed" non-monotonic way.

Preferences are "*mixed*" *non-monotonic* if and only if dichotomic and there exist $T_i = \left\{T_i^1, \text{ ..., } T_i^k\right\}$ and $T_i' = \left\{T_i'^1, \text{ ..., } T_i'^k\right\}$ such that $A \in G_i^H$ if and only there exists T_i^l and $T_i'^m$ such that $A \supseteq T_i^l$ and $A \not\supseteq T_i'^m$ for all $m = 1, ..., k$.

Finally, I assume that any social system $\left(H, \left(\succcurlyeq_i^H\right)_{i\in N}\right)$ induces a communication network (graph) $\Gamma(H)$. Specifically, given a social system $\left(H, \left(\succcurlyeq_i^H\right)_{i\in N}\right)$ then, for all $i, j \in N$; $ij \in \Gamma(H)$ if and only if $\left[H_{(i)} \in G_j^H \text{ and } H_{(j)} \in G_i^H\right]$.

If there is not link among individuals i and j then, it is assumed that individuals either do not know each other or they are not able to cope with their different affiliations set in order to establish communication, in any of both cases and to simplify matters, the result is the same they feel apart from each other.

3.4 Comparative Statics

In this section I analyze how changes in the affiliation sets for each individual and changes in social preferences may alter the degree of social polarization. Specifically, I focus on how enlargements of the affiliations sets of each individual affect polarization under the three different types of preferences proposed into the model i.e. upward and downward monotonic and mixed non-monotonic preferences. Additionally, I also analyze how such an enlargement alter polarization for centralized societies, characterized here by hypergraphs called "stars".

Proposition 1 *Let* $\mathbf{H} = (N, H)$ *be a social affiliation hypergraph with "upward monotonic" dichotomic preference profile* $(\succcurlyeq_i)_{i\in N} = \left(\succcurlyeq_{i(H)}\right)_{i\in N}$ *and with systems of thresholds given by* $T_i = \left\{T_i^1, ..., T_i^k\right\}$ *for each* $i \in N$*. Then, for any* $i, j \in N$, $(i, j) \in \Gamma(H)$ *–also written* $ij \in \Gamma(H)$*– if and only if* $H_{(i)} \cap H_{(j)} \neq \varnothing$.

Proof. *Suppose* $ij \in \Gamma(H)$ *then by definition* $H_{(i)} \in G_j^H$ *and* $H_{(j)} \in G_i^H$ *i.e. there exist* T_i^l *and* T_j^m *such that* $H_{(j)} \supseteq T_i^l = \{x_i^l\} \subseteq H_{(i)}$ *and* $H_{(i)} \supseteq T_j^m = \{x_j^m\} \subseteq H_{(j)}$. *Hence,* $H_{(i)} \cap H_{(j)} \neq \oslash$. ■

Proposition 1 shows that with upward monotonic dichotomic preferences two individuals communicate if and only if they share at least one affiliation of those who individually they consider as a good or desirable affiliations in order for them to establish communication.

Proposition 2 *Under "upward monotonic" dichotomic preferences* $(\succcurlyeq_i)_{i\in N} = \left(\succcurlyeq_{i(H)}\right)_{i\in N}$, *with systems of thresholds given by* $T_i = \{T_i^1, ..., T_i^k\}$ *for each* $i \in N$. *If* $\mathbf{H} = (N, H)$, $\mathbf{H}' = (N, H')$ *such that* $H_{(i)} \subseteq H'_{(i)}$ *for each* $i \in N$ *then* $\Gamma(H) \subseteq \Gamma(H')$

Proof. *If* $ij \in \Gamma(H)$ *then, there exist* T_i^k, T_j^h *such that* $H_{(i)} \supseteq T_j^h$, $H_{(j)} \supseteq T_i^k$. *Hence* $H'_{(i)} \supseteq T_j^h$ *and* $H'_{(j)} \supseteq H_{(j)} \supseteq T_i^k$ *and thus* $ij \in \Gamma(H')$ ■

Proposition 2 shows that if there is an enlargement of each individual affiliations' set then, under upward monotonic dichotomic preferences the new associated communication graph, the one after enlargement, must include the initial one. The intuition is that if all individuals expand its affiliation sets then there would be more or at least the same communication links among individuals in society.

Proposition 3 *Under "upward monotonic" dichotomic preference profile* $(\succcurlyeq_i)_{i\in N} = \left(\succcurlyeq_{i(H)}\right)_{i\in N}$, *with systems of thresholds given by* $T_i = \{T_i^1, ..., T_i^k\}$ *for each* $i \in N$.

If $\mathbf{H} = (N, H)\,, \mathbf{H}' = (N, H')$ *such that* $H_{(i)} \subseteq H'_{(i)}$ *for each* $i \in N$ *then* $\Gamma(H') \preccurlyeq^P \Gamma(H)$

Proof. *If* $H_{(i)} \subseteq H'_{(i)}$ *for each* $i \in N$ *then,* $\Gamma(H) \subseteq \Gamma(H')$ *(Proposition 2). Hence there exist at least one* $ij \in \Gamma(H')$ *such that* $ij \notin \Gamma(H)$. *Therefore* $\Gamma(H') \succcurlyeq^C \Gamma(H)$. *Hence,* $\Gamma(H') \preccurlyeq^P \Gamma(H)$. ■

Proposition 3 shows that under upward monotonic dichotomic preferences one graph is less polarized than another if the affiliation sets of the former (one for each individual) includes the affiliation sets of the latter. The intuition grounds on the previous result, if all individuals expand its affiliation sets then there would be more or at least the same communication links among individuals in the new society. Therefore, other things being equal, there would be more communication among individuals who did not communicate before and as a consequence the graph will be less polarized.

Proposition 4 *Under "upward monotonic" dichotomic preferences* $(\succcurlyeq_i)_{i \in N} = \left(\succcurlyeq_{i(H)}\right)_{i \in N}$, *with systems of thresholds given by* $T_i = \left\{T_i^1, ..., T_i^k\right\}$ *for each* $i \in N$. *If each affiliation* $a_k \in H$ *splits into two new affiliations,* $a_k, a_l \in H'$*,then,* $\Gamma(H) \preccurlyeq^P \Gamma(H')$.

Proof. *If each* $a_k \in H$ *splits into* 2 *new affiliations* $a_k, a_l \in H'$ *there is at least one couple* i, j *such that* $ij \in a_k \subseteq H$ *but* $ij \notin a_k \subseteq H'$. *Hence,* $ij \in \Gamma(H)$ *and* $ij \notin \Gamma(H')$.*Therefore* $\Gamma(H) \succcurlyeq^C \Gamma(H')$ *and,* $\Gamma(H) \preccurlyeq^P \Gamma(H')$. ■

Proposition 4 shows that, under upward monotonic dichotomic preferences, the fact of affiliations broken down into two new affiliations implies that there would be at least one couple of individuals who will loose one common affiliation and, thus there will be the same number of communication links among individuals or less. In turn, there will be less overall communication among individuals in society and therefore, other things being equal, society will be more polarized.

Proposition 5 *Under "downward monotonic" dichotomic preferences* $(\succcurlyeq_i)_{i \in N} = \left(\succcurlyeq_{i(H)}\right)_{i \in N}$, *with systems of thresholds given by* $T_i = \left\{T_i'^1, ..., T_i'^k\right\}$ *for each* $i \in N$. *If* $\mathbf{H} = (N, H)$, $\mathbf{H}' = (N, H')$ *such that* $H_{(i)} \subseteq H'_{(i)}$ *for each* $i \in N$ *then* $\Gamma(H) \supseteq \Gamma(H')$ *i.e. if each player's set of affiliations is enlarged then the resulting communication graph shrinks.*

Proof. *If* $ij \notin \Gamma(H)$ *then, there exist* $T_i'^k, T_j'^h$ *such that* $H_{(i)} \supseteq T_j'^h$ *and* $H_{(j)} \supseteq T_i'^k$. *Hence,* $H'_{(i)} \supseteq T_j'^h$ *and* $H'_{(j)} \supseteq T_i'^k$ *and thus* $ij \notin \Gamma(H')$. ■

Proposition 5 shows that, under downward monotonic dichotomic preferences, if there is an enlargement of each individual affiliations' set then, the new associated communication graph, the one after enlargement, must be included in the initial one. The intuition behind the proof is that if all individuals expand its affiliation sets then there would be less or at least the same communication links among individuals in society.

Proposition 6 *Under "downward monotonic" dichotomic preferences* $(\succcurlyeq_i)_{i\in N} = \left(\succcurlyeq_{i(H)}\right)_{i\in N}$, *with systems of thresholds given by* $T_i = \left\{T_i^{\prime 1}, ..., T_i^{\prime k}\right\}$ *for each* $i \in N$. *If* $\mathbf{H} = (N, H)\,, \mathbf{H}' = (N, H')$ *such that* $H_{(i)} \subseteq H'_{(i)}$ *for each* $i \in N$ *then* $\Gamma(H') \preccurlyeq^P \Gamma(H)$

Proof. *If* $H_{(i)} \subseteq H'_{(i)}$ *for each* $i \in N$ *then,* $\Gamma(H) \supseteq \Gamma(H')$ *(Proposition 5). Hence there may exist at least one* $ij \in \Gamma(H)$ *such that* $ij \notin \Gamma(H')$. *Therefore* $\Gamma(H') \preccurlyeq^C \Gamma(H)$. *Hence,* $\Gamma(H') \succcurlyeq^P \Gamma(H)$. ■

Proposition 6 shows that under downward monotonic dichotomic preferences one graph is more polarized than another if the affiliation sets of the former (one for each individual) include the affiliation sets of the latter. The intuition grounds on the previous proposition, if all individuals expand its affiliation sets then there would be less or at least the same communication links among individuals in the new society. Therefore, other things being equal, there would be less communication among individuals than before and as a consequence the graph will be more polarized.

Proposition 7 *Under "mixed non-monotonic" preferences* $(\succcurlyeq_i)_{i\in N} = \left(\succcurlyeq_{i(H)}\right)_{i\in N}$ *with system of thresholds and ceilings* $T_i = \left\{T_i^1, ..., T_i^k\right\}$ *and* $T_i' = \left\{T_i^{\prime 1}, ..., T_i^{\prime k}\right\}$, *if* $\mathbf{H} = (N, H)\,, \mathbf{H}' = (N, H')$ *such that* $H'_{(i)} \supseteq H_{(i)}$ *for each* $i \in N$ *then, the resulting communication graph* $\Gamma(H')$ *may grow larger, shrink, or stay the same with respect to* $\Gamma(H)$.

Proof. *Let* $\Upsilon_i = H_i' \backslash H_i' \cap H_i$. *If* $H'_{(i)} \supseteq H_{(i)}$ *such that* $\Upsilon_i \cap T_i' = \oslash$ *then* $H'_{(i)} \supseteq T_i^l$ *and* $H'_{(i)} \not\supseteq T_i^{\prime m}$ *for all* $m = 1, ..., k$.*Hence* $\Gamma(H')$ *will grow larger than* $\Gamma(H)$. *If*

$H'_{(i)} \supseteq H_{(i)}$ such that $\Upsilon_i \cap T'_i \neq \oslash$ then $H'_{(i)} \supseteq T^l_i$ and $H'_{(i)} \nsupseteq T'^m_i$ for at least one $m = 1, ..., k$.Hence $\Gamma(H')$ will shrink with respect to $\Gamma(H)$. ■

Proposition 7 shows that under non-monotonic system of preferences an enlargement of the affiliations set will alter the resulting communication network in different ways depending on the preferences itself. Thus the possibility for new social relationships depends crucially on the set of affiliations considered as good or bad for each individual

The next two propositions focus on "Stars" one relevant type of hypergraphs (societies). More precisely centralized societies i.e. societies characterized by the existence of one individual who shares at least one affiliation with every one else so she is at the center of society.

Proposition 8 *If $\mathbf{H} = (N, H)$ is a star, and $(\succcurlyeq_i)_{i\in N} = \left(\succcurlyeq_{i(H)}\right)_{i\in N}$ an "upward monotonic" dichotomic preference profile with systems of thresholds given by $T_i = \left\{T^1_i, ..., T^k_i\right\}$ for each $i \in N$. Then, $\Gamma(H)$ is connected.*

Proof. *Let $i^* \in N$ be the star-center. Since, by definition, for all $i \in N$, $H_{(i)} \neq \oslash$ then, there exist $x, x' \in H$ such that $x \in H_{(i)} \cap H_{(i^*)}$ and $x' \in H_{(j)} \cap H_{(i^*)}$. Hence, $H_{(i)} \in G^H_{i^*}$ and $H_{(i^*)} \in G^H_i$ and $H_{(j)} \in G^H_{i^*}$ and $H_{(i^*)} \in G^H_j$. Thus, $ii^* \in \Gamma(H)$ and $i^*j \in \Gamma(H)$ i.e. there is a path from i to j and $\Gamma(H)$ is connected.* ■

Proposition 8 shows that under upward monotonic dichotomic preferences the communication graph associated to a centralized society will be always connected. In other words there is either direct or indirect communication about ant two individuals in society. Intuitively, if every one communicates with the same one person (the star center) then it is always possible for any two people to communicate through her.

Proposition 9 *Under "upward monotonic" preferences* $(\succcurlyeq_i)_{i\in N} = \left(\succcurlyeq_{i(H)}\right)_{i\in N}$, *with systems of thresholds given by* $T_i = \left\{T_i^1, ..., T_i^k\right\}$ *for each* $i \in N$ *if* $\mathbf{H} = (N, H)$, $\mathbf{H}' = (N, H')$ *are* 2 *stars such that* $\#E_h \in H > \#E_k \in H'$ *for all* h, k *then,* $\Gamma(H) \succcurlyeq^C \Gamma(H')$ *hence* $\Gamma(H) \preccurlyeq^P \Gamma(H')$.

Proof. *Let* $i^* \in N$ *be the star-center. Since, by definition, for all* $i \in N$, $H_{(i)} \neq \oslash$ *then, there exist* $x, x' \in H$ *such that* $x \in H_{(i)} \cap H_{(i^*)}$ *and* $x' \in H_{(j)} \cap H_{(i^*)}$*.Hence the degree of* i^* *equals* $n-1$ *for any star. Therefore, if* $\#E_h \in H > \#E_k \in H'$ *for all* h, k *then, there exist at least* 1 *pair* ij *such that* $ij \in E_h \in H$ *and* $ij \notin E_k \in H'$*. Thus, there is at least one* x *such that* $x \in H_i \cap H_j$ *and* $x \notin H_i' \cap H_j'$*. So,* $\Gamma(H) \succcurlyeq^C \Gamma(H')$ *hence* $\Gamma(H) \preccurlyeq^P \Gamma(H')$. ■

Proposition 8 shows that under upward monotonic dichotomic preferences if the cardinality of all hyperlinks is grater in one hypergraph than in other then, the former will be less polarized. The intuition behind this result is that the more people share common affiliations then there will be more communication among them and therefore, society will be less polarized.

3.5 Concluding remarks

As explained above, polarization is highly correlated with social conflict. The model presented here may contribute to the analysis of social conflict in at least two respects. First, the explicit recognition of the fact that individuals have multiple affiliations overcomes the usual problem in economics of analyzing polarization in terms of just one dimension. In fact, proposition 1 deals with the issue that most of the time people establish social relationships after finding certain amount of commonalities with each other. Propositions 2 and 3 recognize the fact that people with longer sets of affiliations would have more opportunities to find things in common with others thus, abating the sense of antagonism and, diminishing social polarization. As an example consider a Hutu in Kigali whose sense of identity is based on the only fact of being a Hutu. Thus, for him, it may be easier to consider a Tutsi as his enemy. But, recognizing the fact that he is not only a Hutu but also an habitant of Kigali, a citizen of Rwanda an African and a human being enables him to find some commonalities with a Tutsi.

Proposition 4 tackles the opposite case i.e. a reduction of the affiliation sets. In particular it shows that if each affiliation set breaks down into two affiliation sets, then it will reduce the number of possible commonalities among people thus, increasing the chances for polarization to grow.

Second, the model also accounts for the fact that certain affiliations considered as undesirable affiliations may block any social relationship. There is an old Italian tale about the Fascism's rise in the twenties. A Fascist recruiter was trying to convince a socialist peasant to join the party. The peasant said: "I can not join your party. My father was socialist and so was my grandfather so it is impossible for me to become a Fascist. The recruiter responded: You are reasoning the wrong way. What about you if your father had been an assassin and also your grand father? The peasant answered: "Ah, in that case I would surely be in the Fascist party." Propositions 5 and 6 show that if a person considers certain affiliations as bad or undesirable and finds at least one of them in someone's else affiliation set then, there is no room for establishing communication among them independently of the number of commonalities they may have.

Proposition 7 shows that under a non-monotonic system of preferences an enlargement of the affiliations set will alter the resulting communication network in different ways depending on the preferences themselves. Thus the possibility of new social relationships depends crucially on the set of affiliations considered as good or bad by each individual.

Propositions 8 and 9 show that for centralized societies the larger the number of people sharing each affiliation the less the amount of polarization there will be. It remains for further research to explore, among other questions, how changes in the affiliation sets modify the resulting communication graphs for different well known types of hypergraphs such as bipartite hypergraphs.

3.6 References

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