Conflict Theory - A Game Theoretic Analysis of Social Conflict, Norms and Conventions

SEBASTIAN ILLE

Submitted in total fulfilment of the requirements
of the degree of Doctor of Philosophy in Economics

2010

Scuola di Dottorato in Economia Politica
Università degli Studi di Siena

Contents

O speculatore delle cose, non ti laudare di conoscere le cose, che ordinariamente, per sé medesima la natura conduce; ma rallegrati di conoscere il fine di quelle cose, che son disegniate dalla mente tua.

Leonardo da Vinci (1452-1519)

Homo sum, humani nil a me alienum puto.

Menandros (341-293 BC)

0 Preface

Acknowledgement

This thesis would have been impossible without the generous financial support from the University of Siena. Warmest thanks goes to my director Alan Kirman for valuable discussions and his remarkable ability to pinpoint weaknesses. I also thank Ugo Pagano for his administrative help and flexibility, and also the GREQAM for hosting me and for providing a creative and productive work environment.

I greatly benefited from discussions with Leonardo Boncinelli, Frédéric Deroian, Jean-Sébastien Garbi, Eric Guerci, Nobuyuki Hanaki, Nicodemo Massimo De Vito, Magali Orillard, Olivier Renault, Thomas Rongiconi, David Roubaud, Cléa Sambuc, Sylvie Thoron, Pascal Toquebeuf, Jörgen Weibull, Jean-Benoît Zimmermann. I am indebted to all the scholars cited in this thesis, whose work provided the theoretical foundation of this thesis and a resource of inspiration, as well as to Magali Orillard for granting access to her "personal library".

I am especially grateful to Samuel Bowles, who awakened my interest in game theory and social interactions. Without his inspiring lectures, I might have never chosen this field of research.

My deepest and warmest gratitude goes to my family, especially my parents and Orianne, for their continuous emotional support and counterbalance.

Foreword

The omnipresence of conflict in human history and on various social levels renders its analysis economically significant, not only due to its direct impact on the interactions and decisions of economic agents, but also by shaping the underlying social and economic environment and institutions. Social conflict constitutes thus the primary link between the five articles presented in this theses. This work is separated into three main parts, each analysing a different aspect of social conflict.

The first chapter lays the foundation for the following chapters, by developing a general analytical model of conflict between two player populations. The model incorporates an arithmetic presentation of the aggression resulting from violations of social norms and conventions. This weakens the free-rider problem, which would obstruct the occurrence of a conflict. Since an evolutionary approach is especially apt in this context, the model relies heavily on the mathematical tools provided by evolutionary game theory. Yet in this model, social norms and conventions are exogenous and universally defined.

The second part concentrates on the dynamics underlying social norms and conventions, and provides indications, which norms or conventions are most likely observable.[1] Social norms and conventions do not only serve as a coordination mechanism, but also provide the setting for social and economic interactions. This part is hence not only important with regard to the informative value of the model presented in chapter 1, but has also direct economic significance. Chapter 2 concentrates on the work of Peyton Young (1993, 1997) and the stochastic stability approach by examining the issue of state independence of error and sample size. Social norms and conventions are characterised by an environment that *works towards* a particular behaviour. Hence, approval and disapproval are vital for the dynamics defining social conventions and norms. The disapproval of a certain strategy or action is captured by the pay-off loss resulting from the lack of coordination in a 2×2 coordination game, i.e. if one player does not adhere to the conventional strategy. Consequently, chapter

[1] I do not not differentiate between norms and conventions. Max Weber distinguishes between convention, defining a mechanism that urges individuals to exhibit a certain behaviour by approval and disapproval, and custom, characterising consuetude and regular behaviour, defined by inconsiderate imitation. A norm is based on custom and convention, whereupon the conventions turn customary behaviour into norms, thus creating traditional behaviour. The transition between these concepts - norm and convention - are hence completely smooth and seamless (in fact Weber speaks of conventional norm), and a differentiation is non-essential (see Weber, 1922, Ch. VI.) In fact, in the context described here, the concept is equal to what Richard Nelson and Bhaven Sampat called *social technology* (Nelson, 2008).

2 expands Peyton Young's original stochastic stability approach in this respect. To account for the effect of approval and disapproval in the decision making process, a direct dependency between error and sample rate, on the one hand, and potential loss, on the other hand, is assumed. The results obtained from the original and modified approach overlap under certain conditions, though not necessarily for all games. Moreover, in this context, evolutionary drift plays a crucial role, requiring a state to fulfil an additional condition for being a stochastically stable state: the *one-third rule*.

Yet, it is not sufficient for the definition of a norm or convention that behaviour is approved and disapproved of. Pressure to abide to a norm or convention is exercised by a *specific group* that can be of professional, kinsmanlike, neighbourly, class-oriented, ethnic, religious, or political nature (see Weber, 1922, p. 616). Chapter 3 therefore outlines a social setting for the evolution of norms and convention that is different from Young's approach. Players interact with a reference group. In addition, adaptive play is substituted by a simplified imitation heuristic (*imitate the best action*), as norms and conventions are generally considered to be subject to emulation and reproduction. The predictions of the model differ essentially from those obtained from the former model. The comparison of chapter 2 and 3 thus shows how the prevailing norm or convention depends on the selection process.

This circumstance illustrates a fundamental issue: Is social evolution equivalent to biological evolution, such that individual decision-making behaviour is subject to a selection process that generates behaviour satisfying the axioms of expected utility theory and best-response play? If memes (Dawkins, 1976) and genes are equivalent definitions; only differentiated by the current context, i.e. biological or social, the answer might be affirmative. Yet, to say that a meme that generates maximum pay-off proliferates, provides only little information, since the logic incorporates two weaknesses: The dynamics, underlying the selection and evolution of institutions, depend on the selection mechanism and on the unit of selection.

A meme itself defines the context of interaction and forms the basis for the pay-off of memes.[2] This cast doubt on the assumption that non-best response behaviour, existent in the short-run, will be selected against in the long-term, as this short-term behaviour alters the conditions of social selection. The required stability of the underlying institutions and thus the character of selection is not mandatory.[3] Additionally

[2]See a number of studies on the time variance of institutions (Milgram, 1970; Zimbardo, 1970; Levine et al., 1994, 1999 & 2001), as well as on the context specificity of behaviour (Milgram & Shotland, 1973; Phillips, 1974; Areni & Kim, 1993; Bargh et al.; 1996)

[3]"[A] selection process requires some degree of stability in the institutions themselves and in the na-

in the social context, a quantifiable measure, such as fitness in the biological context, is absent. A distinct unit of selection, as well as its correlation with individual behaviour is thus indeterminate (Sugden, 2001).[4]

The third part, starting in chapter 4, illustrates and critically discusses an alternative perspective to standard game theoretic modelling. It is intended as a basis for future scientific discussion and a means to explain the occurrence of stable states that are not Nash equilibria. The persistent choice of strictly dominated strategies can be explained by modifying the assumption of rationality, of self-interest or about the social context. Modifications to the last two alter the game structure, as these points are captured by the pay-off matrix. Only changes in the first point leave the game structure intact. Under the condition that the game has relevance, i.e. is not misspecified, only changes to the rationality assumption are therefore viable. In face of this and the previous paragraph, the *Conflict Analysis* approach changes some of the basic assumption of rational choice. Individuals will be assumed to believe in a capacity to empathically anticipate other individuals' response to their strategy choice without the requirement to truly observe their action. Based thereon, the sequential reasoning renders strategies stable through the empathic correlation between the actions chosen by different players. Furthermore, the approach has a decisive advantage: The non-requirement of utility functions, along with the allowance for intransitive preferences, circumvents the problems resulting from the reduction of multi-dimensional preferences to a mono-dimensional utility measure by arbitrary quantification.

The last chapter demonstrates how the *Conflict Analysis* approach can be adapted to analytically represent dynamic interactions with endogenous preferences via time-inhomogeneous Markov chains. The approach is thereby turned into a flexible tool to approximate the setting of real-world interactions.

---

ture of the selection process. Also the latter must operate over a sufficient period of time. Otherwise selection cannot establish a consistent result." (Hodgson, 1996, p.49)

[4]Sugden (2001) points out the following problem: If utility of an action is a representation of the frequency by which this action is replicated amongst the group members, a Pareto utility improvement is no guarantee for the prosperity of this group. "For example, consider the behaviour patterns associated with the consumption of pleasure giving but harmful addictive substances, such as nicotine. Because of the nature of addiction, such behaviour patterns have a strong tendency to replicate themselves [...], then those behaviours score highly on the scale of utility. But we are not entitled to infer that the consumption of addictive drugs within a social group gives that group a competitive advantage in some process of group selection."

Nelson (2008) indirectly indicates the inaccuracy of the unit of selection: "Selection forces, including the ability of the human agents involved to learn from experience [...], usually are significantly weaker for institutions and social technologies than for physical technologies." Yet, the latter has fundamental impact on the former; think about the effect of recent developments in cloning, stem cell research and nanotechnology not only on law and economics, but also on social life.

References

1. Areni, Charles S.; Kim, David (1993): " The influence of background music on shopping behaviour: Classical versus top-forty music in a wine store", Advances in Consumer Research, Vol. 20, pp. 336-340
http://www.acrwebsite.org/volumes/display.asp?id=7467

2. Bargh, John A.; Chen, Mark; Burrows, Lara (1996): "Automaticity of Social Behavior: Direct Effects of Trait Construct and Stereotype Activation on Action", Journal of Personality and Social Psychology, Vol. 71, No. 2, pp. 230-244

3. Dawkins, Richard (1976): "The Selfish Gene", Oxford University Press

4. Hodgson, Geoffrey M. (1996):"Economics and Evolution - Bringing Life back into Economics", Ann Arbor, The University of Michigan Press

5. Levine, Robert V., Martinez, Tood S., Brase, Gary; Sorenson, Kerry (1994): "Helping in 36 U.S. Cities", Journal of Personality and Social Psychology, Vol. 67, pp. 69-82.

6. Levine, Robert V.; Norenzayan, Ara; Philbrick, Karen (2001): "Cultural differences in the helping of strangers", Journal of Cross Cultural Psychology, Vol. 32, pp. 543-560.

7. Levine, Robert V.; Norenzayan, Ara (1999): "The Pace of life in 31 countries", Journal of Cross Cultural Psychology, Vol. 30, pp. 178-205.

8. Milgram, Stanley: "The experience of living in cities", Science, Vol. 167, pp.1461-1468

9. Milgram, Stanley; Shotland, R.Lance (1973): "Television and antisocial behaviour: Field experiments", Academic Press, New York

10. Nelson, Richard R. (2008):"What enables rapid economic progress: What are the needed institutions?", Research Policy, Vol. 37, pp. 111

11. Phillips, David P (1974): "The influence of suggestion on suicide: Substantive and theoretical implications of the Werther effect", American Sociological Review, Vol. 39 (Jun.), pp. 340-354

12. Sugden, Robert (2001): “Ken Binmore’s Evolutionary Social Theory”, The Economic Journal, Vol. 111 (Feb.), pp. 213-243

13. Young, H. Peyton (1993): “The Evolution of Conventions”, Econometrica, Vol. 61, No. 1 (Jan.), pp. 57-84

14. Young, H. Peyton (1998): “Individual Strategy and Social Structure”, Princeton University Press

15. Weber, Max (1922): Wirtschaft und Gesellschaft, area Verlag (2007)

16. Zimbardo, Philip G. (1970): “The human choice: Individuation, reason, and order versus deindividuation, impulse, and chaos, in W. J. Arnold and D. Levine (eds.), 1969, Nebraska Symposium on Motivation (pp. 237-307), Lincoln, NE: University of Nebraska Press

All men can see the tactics whereby I conquer, but what none can see is the strategy out of which victory is evolved.

Sun Tzu (476-221 BC)

1

A Simple Model of Conflicts

This chapter bridges behavioural and classical economic literature by developing a simple analytical model that can be generalised to a large variety of conflicts. The model describes the clash between two sub-populations, one attempting to re-write the current (social) contract in its favour, the other to maintain the *status quo*. The free-rider problem obstructs the occurrence of a conflict, leading to a low probability of successful turn-over. Players belonging to the group, which is favourable to the conflict, will not join the revolutionaries, though doing so would be beneficial for the majority of players. Introducing an emotional component counter-acts the free-rider effect and enables the model to predict the existence of two stable equilibria; one with high and another with no conflict potential. In addition, adapting a threshold approach by Granovetter, the likelihood of transition from no to high conflict equilibrium will vary positively with group size.

Introduction

On May 23rd, a group of protestant noblemen enters the Bohemian chancellery by force and defenestrates three protestant senior officials. Yet, all three miraculously survive the seventeen meter fall. Although the three imperial representatives of Bohemia are fortunate, "the Defenestration of Prague" in 1618 marks a fatal day in the European history. This incident triggers a war between Catholics and Protestants that will last for 30 years. It will have severe economic repercussions by drawing six countries into war; bearing the cost of four million lives and depopulating entire areas in the Holy Roman Empire. Some regions will require more than a century to recover economically. The thirty years war is only one of many historical examples that illustrate the impact of conflicts both on the stability of institutions and economic development, and stresses the importance to understand the evolution of conflicts for the economic theory.

Game theory has already been used to explain social phenomena since the middle of the 1950s (for a historical account refer to Swedberg, 2001). At the beginning of the 1980's some of the most prominent members of analytical Marxism discussed the use of Game Theory as the sufficient mathematical means for a micro-founded explanation of social problems, structure and change, especially with respect to conflict and cooperation (Elster, 1982, Cohen 1982, Roemer 1982a). Commencing at this period analytical models on social problems started to draw from an increasingly sophisticated mathematical background, switching from "low-tech" to "high-tech" Game Theory (Swedberg, 2001).

The issue of social conflict has been especially prominent in the economic literature on class struggle. Przeworski and Wallerstein (1982) analysed under which circumstances and economic conditions class struggle should result in a compromise and when not. Eswaran and Kotwal (1985) developed a model on agricultural production that explained the existence of specific types of contractual arrangements. Mehrling (1986), on the other hand, showed how the class struggle should lead to different macroeconomic performance, defined by the degree of organization of different classes, via a model based on the work of Lancaster and Goodwin. Another issue considered in economic literature has explored the question of how revolutionary actions and institutional change may occur. Roemer (1985) provided axiomatic proofs of the elements necessary to maintain the current social system or to trigger a revolutionary change for a two person game, in which its players competed for coalitional support.

Other research has focused on the question of economic efficiency of certain social phenomena that provided potential reasons for conflicts. Roemer (1982b) investigated the issue exploitation itself and why it could be socially necessary and detached from the form of social system. He provided a more general theory of exploitation, which incorporated both the Marxian and the neoclassical concept. Cole et al. (1998) described how socially inefficient competition could be reduced by the existence of a class society. Holänder (1982) developed axiomatic proof of the Marxian exploitation concept based on a model economy.[1]

Another important strand of literature on social conflict has been the application of bargaining models to trade union behaviour and strikes. An interesting overview of strategic bargaining models and their comparison to empirical data was provided by Kennan & Wilson (1989). Examples of this literature were Clark (1996) and Kiander (1991). The former modelled the impact of future pay-offs, calculated via the firm's market share, on the general strike frequency and the conditional probability of future strikes. The later focused on the effect of stocks on bargaining outcomes and the credibility of strike threats. Another classic example was Ashenfelter & Johnson (1969), who developed a bargaining model, which involved three parties in labour negotiations (the management, the union leadership, and the union rank and file) illustrating the trade-offs between unemployment, wage changes, and industrial strike activity in conflict situations.

Yet, this "classical" literature has neglected the role of social conventions, norms and punishment.[2] Of special interest in this respect has been the literature on the evolution of conventions following the idea of stochastically stable equilibria and conventional change by Young (Foster & Young, 1990; Young, 1993, 1998 & 2005; Kandori, Mailath & Rob, 1993; Durlauf & Young, 2001; Stephen, 2000; Ellison, 1993 & 2000; Bergin & Lipman, 1996). A stream of recent literature has been concentrating on strong reciprocity and altruist punishment (Gintis et al., 2005; Brandt et al., 2006; Bowles & Choi, 2003; Bowles et al., 2003; Boyd et al., 2003), both providing theoretical evidence for the evolution of such traits and the stability of strong reciprocal strategies in the different versions of public good games. The latter literature, however, has

[1]In this context, the literature on the evolution of social classes (Bowles, 2006; Axtell et al., 2000; Eswaran & Kotwal, 1989; Roemer, 1982c; Starrett, 1976) provided insight into the conditions that maintain such a class society.

[2]This has not been the case for the extensive literature on much more general but simpler conflict games including variants of Prisoner's Dilemma, Battle of Sexes and Chicken Game (for an overview see Binmore, 1994; Gintis,2000, 2009). Variants of the latter were used by Brams to model the effects of potential threats on cooperation (Brams & Kilgour, 1987a,b, 1988; Brams & Togman, 1998.

mostly concentrated on the explanation of mutual cooperation. A very interesting and comprehensive approach, going beyond reciprocal altruism, was Binmore, 1998. Furthermore, threshold models (Granovetter, 1978; Granovetter & Soong, 1988; Banerjee, 1992; Orléan 1998, 2001 & 2002) offered an explanation of herding behaviour as an alternative approach to models based on individual rational choice.

This chapter bridges the "classical" and "behavioural" literature by illustrating the interdependency between the strategic choice of players during conflicts and the perceived violations of social norms and conventions. This simplified model of social conflict thus obtained is able to provide analytical proof of some of the intuitions that are observable in real-world conflicts. It also draws from literature stated above, especially from the work of Granovetter. The first section starts with a game between two player sub-populations, in which a conflict does not occur because of the free-rider effect, though a social conflict and the subsequent change is mutually beneficial for one player sub-population and collective action should result in such a change. The second section develops an analytical representation of the dynamics of emotional violence during a conflict situation that arises from norm violations. The third section then incorporates the "emotional component" into the original model, making social conflict possible under certain conditions. The fourth section adds a threshold approach to the model derived in the earlier section to illustrate how shifts between stable equilibria might occur. The sixth section constitutes the conclusion.

1.1 The Basic Model

Let there be a game $\Gamma = (S_1, S_2, .., S_{m+n}; \pi_1, \pi_2, ..., \pi_{n+m})$ played in a finite but large population N with players $i = 1, 2, ..., n+m$. Game Γ is thus defined by a set S_i of pure strategies for each player i. Given player i's strategy s_i and for each pure strategy profile $s = (s_1, s_2, ..., s_{n+m})$, in the set of pure strategy profiles $S = \times_i S_i$, the associated individual pay-off to player i's strategy choice is defined by $\pi_i(s) \in R$, implying that $\pi_i : S \to R$ for each player i. Further assume that two distinct sub-populations C_A and C_B, with $C_A \cap C_B = \emptyset$, $C_A \cup C_B = N$, participate in Γ. In addition, suppose that there are n players in C_A and m players in C_B and $n >> m >> 0$.[3]

[3]The last assumption is not strictly necessary, but deleting this assumption would complicate the model without giving additional insights. If this were not supposed, the winning probability should be influenced by the relative sub-population size. The expected relative pay-off is only affected with respect to supporters in each sub-population. Redefining the parameter values, by integrating the winning probability, should have a similar effect.

For simplicity denote each individual player in C_A as A and in C_B as B.[4] Thus, assume that all players in the same sub-population have an identical pure strategy set and pay-off function for each strategy. For an $A \in C_A$ assume that $S_A = \{R, C, S\}$, i.e. each player A has the choice between *(R)*evolting, *(C)*onforming to the current system, or *(S)*upporting it. Revolting implies an active action to change the current (social) contract written between C_A and C_B. Conforming denotes a strategy of "inaction"; a player waits for the other players to act. Supporting is diametric to revolting; a player approves of and actively supports the current social contract.[5] For $B \in C_B$ assume that $S_B = \{P, \neg P\}$, i.e. he can choose whether or not he provides a bonus payment to the supporters in C_A.[6]

Suppose that in the current state and with respect to the currently prevalent social contract, there exists an alternative allocation that constitutes a redistribution detrimental for C_B, yet favourable for As playing strategy R or C. This does not necessarily hold for As playing S, especially if they are paid by the Bs. Consequently, all players in C_B have an interest in maintaining the current (social) contract and in preventing the implementation of the alternative allocation. Strategy R and C players in C_A, on the contrary, benefit from a successful *revolt* that leads to the realisation of the alternative allocation.

As a first step, we derive the individual pay-off function $\pi_i(s)$ for each strategy and player type. First concentrate on the As: Label the frequency of revolutionaries in C_A as x, the frequency of supporters as y, and the frequency of conformists as z, for which it must hold $1 \equiv x + y + z$ (with each frequency lying within the unit interval). Define the additional pay-off derived from the alternative allocation after a successful turn-over as a positive constant δ^r. Further assume that if the attempt of revolution failed, revolutionaries face a negative pay-off defined by a function decreasing in the share of supporters, as those will impose the punishment. For simplicity assume that this is the linear function $\delta^p y$, with δ^p being a negative constant. This negative pay-off

[4]The conventional notation assigns lower-case letters to individuals. In later chapters, conflicts will be modelled both on a group and individual level. Hence, for reasons of consistency I will break with conventional notation and generally use upper-case letters (generally A and B) to indicate the conflicting players.

[5]In order to give a concrete meaning to the rather abstract definition of these three strategies take again the example of the Thirty Years' War. In the events following the "the Defenestration of Prague" R-players are comparable to the Hussites of Bohemia, S-players are mercenary soldiers supporting Ferdinand of Habsburg, and C-players are mostly peasants.

[6]This payment is considered as being general. It is not necessarily a direct monetary benefit, but can also be considered as workplace or social amenities, an easier career in a firm controlled by a B, a better reputation or higher social status among the Bs.

can be through death, punishment, imprisonment, social shunning, discrimination, or mobbing etc. This cost is absent if the player has chosen strategy C.[7] Hence, the expected pay-offs of revolutionaries and conformists are equal to

$$\begin{aligned} \pi_i(s_i, s_{-i}|s_i = R) &= P(win|s_i = R)\delta^r + (1 - P(win|s_i = R))\delta^p y \\ \pi_i(s_i, s_{-i}|s_i = C) &= P(win|s_i = C)\delta^r \end{aligned} \tag{1.1.1}$$

where $P(win|s_i)$ defines the probability of realising the alternative allocation, if player i chooses s_i, given the strategies s_{-i} of all players other than i.[8]

It remains to derive the functional form of probability P. As a first assumption suppose that in order to engage in a conflict, groups are formed at random from players in C_A, but that conformists never actively join a group. This reflects the general situation prior to a conflict. People congregate to discuss new labour contracts, to meet at a summit, to protest on the streets, to rally forces for battle or covert assaults etc., without exact prior knowledge of who will participate. Supporters, on the contrary, *join* the group to "sabotage a revolutionary attempt", e.g. in the form of police forces, the opposing battalion, the members supporting the counter-faction in the summit.

Since groups are formed at random, a player cannot tell the exact group's composition prior to his choice to participate. He does not know how many players actively engage in the conflict and how many of these will choose strategy R or S. His expected pay-off, however, depends on the frequency, with which each strategy is played in sub-population C_A, since groups are assumed to be defined by an unbiased sample.[9] Consequently, expected group size is determined by the expected number of individuals that join, i.e. that are not conformists. In this case, the probability of being in a group of size $s \leq n$ is simply the probability of finding $s - 1$ other individuals playing a strategy different from conforming. We obtain that the probability of being in a group of size s is:

[7] The model abstracts from collateral damage or second-order punishment, since this is not a general characteristic of conflicts, though it can be observed frequently in various conflicts (especially in the Thirty Years' War). This circumstance can be easily implemented into the model by adding an additional cost to the conformist strategy. As long as it is smaller than $\delta^p y$ the general dynamics of the original formulation should persist.

[8] Notice that s_{-i} can define a mixed strategy profile, though each component denotes a pure strategy.

[9] Notice that the replicator dynamic does not require a player to know or form expectations about the frequencies, with which each strategy is played, since strategy choice is defined by imitation. Further, the assumption of an unbiased sample excludes that a player's strategy choice directly affects the likelihood to meet another player choosing the same strategy, e.g. revolutionaries are not more likely than supporters or conformists to find other revolutionaries.

$$\binom{n-1}{s-1}(1-z)^{s-1}z^{n-s} \tag{1.1.2}$$

for both strategies S and R. Group size is thus defined by the frequency of conformists, whereas the composition of a group of size s is only defined by the frequencies of revolutionaries and supporters. Suppose that each revolutionary adds a marginal unit to the probability that the group revolts successfully, but that this marginal additional unit of probability diminishes in the number of supporters. Assume that this marginal unit is simply $\frac{1}{n}$ minus a constant weighted by the share of supporters.

If a player chooses strategy R, a group of size s can include 1 (only himself) to s (all) revolutionaries. Let τ be the number of revolutionaries in a group. The probability of drawing $\tau - 1$ other revolutionaries is $\left(\frac{x}{x+y}\right)^{\tau-1}$ and the probability to draw the remaining $s - \tau$ supporters is $\left(\frac{y}{x+y}\right)^{s-\tau}$. Furthermore, the share of supporters in a group of size s with τ revolutionaries is equal to $\frac{s-\tau}{s}$. If we assume that the marginal negative effect of a supporter in such a group is a, we obtain

$$G_R(s) = \frac{1}{n}\sum_{\tau=1}^{s}\binom{s-1}{\tau-1}\left(\frac{x}{x+y}\right)^{\tau-1}\left(\frac{y}{x+y}\right)^{s-\tau}\tau\left(1-a\left(\frac{s-\tau}{s}\right)\right) \tag{1.1.3}$$

The first part defines the expected composition of a group of size s, the second the marginal effect of the revolutionaries minus the marginal effect of supporters on the winning probability of a group of size s. Thus, $G_R(s)$ denotes the probability, with which a group of size s can impose the alternative allocation. It must hold $a \in (0,1)$ for the probability to be restricted to the unit interval. If $a = 0$ supporters have no additional negative effect on the winning probability, except for their inaction (i.e. they do not contribute to the revolutionary attempt). Given $a = 1$, we observe that the marginal effect of a revolutionary is negligible, if the group is composed by a high number of supporters. From 1.1.3 derives that

$$G_R(s) = \frac{ay(-x+y) + s^2x(x+y-ay) + sy(x+2ax+y-ay)}{ns(x+y)^2} \tag{1.1.4}$$

The probability of a successful revolt of a group of size s, defined by equation 1.1.4, can then be placed into the equation 1.1.2 for the expected group size in order to determine the probability of imposing the alternative allocation:

$$P(win|s_i = R) = \sum_{s=1}^{n} \binom{n-1}{s-1} (1-z)^{s-1} (z)^{n-s} \left(G_R(s)\right) \tag{1.1.5}$$

from which we obtain a rather complex solution.[10] We consider, however, social conflicts that concern a large population size. The previous equation 1.1.5 can thus be approximated by

$$\lim_{n \to +\infty} P(win|s_i = R) = x - \frac{axy}{x+y} \tag{1.1.6}$$

Consequently, for $a = 0$ the probability is simply $P = x$. If $a = 1$ then $P = \frac{x^2}{x+y}$, implying increasing returns to scale, i.e. the winning probability increases quadratically in the share of revolutionaries for a given share of conformists.

The probability for a conformist can be derived in the same way by adapting the possible compositions and group sizes to his strategic choice. Since a conformist does not join a group, a group of size s can be composed of 0 to s revolutionaries. For τ revolutionaries in a group of size s, we need to draw τ times a revolutionary, each with probability $\left(\frac{x}{x+y}\right)$ and $s - \tau$ supporters, each with probability $\left(\frac{y}{x+y}\right)$. This gives

$$G_C(s) = \frac{1}{n} \sum_{\tau=0}^{s} \binom{s}{\tau} \left(\frac{x}{x+y}\right)^{\tau} \left(\frac{y}{x+y}\right)^{s-\tau} \tau \left(1 - a\left(\frac{s-\tau}{s}\right)\right) \tag{1.1.7}$$

and we obtain

$$G_C(s) = \frac{x(ay + s(x+y-ay)}{n(x+y)^2} \tag{1.1.8}$$

This result is again placed into equation 1.1.2, though the equation requires to be adapted, as conformists affect group size s. It can only range from 0 to $n-1$. Furthermore, a conformist needs to *draw* s revolutionaries or supporters, which occurs with probability $(1-z)^s$. Taking this into account, the probability to impose the alternative allocation if being a conformist is defined by

$$P(win|s_i = C) = \sum_{s=0}^{n-1} \binom{n-1}{s} (1-z)^{s} (z)^{n-1-s} \left(G_C(s)\right) \tag{1.1.9}$$

[10] i.e. $\frac{a(x-y)y(z^n-1)+n(1-z)\,(y(x+2ax+y-ay)+nx(x+y-ay)(1-z)+x(x+y-ay)z)}{n^2(x+y)^2(1-z)}$

giving $P(win|s_i = C) = \frac{x(ay+(n-1)(x+y)(x+y-ay))}{n(x+y)^2}$ and hence for large n

$$\lim_{n \to +\infty} P(win|s_i = C) = x - \frac{axy}{x+y} \quad (1.1.10)$$

Comparing equation 1.1.6 and 1.1.10 shows that for large populations individual strategy choice is irrelevant.

Further let $\pi_i(s_i, s_{-i}|s_i = R) = \pi_R$ and $\pi_i(s_i, s_{-i}|s_i = C) = \pi_C$ for notational simplicity. Putting in the previous results into equation 1.1.1 on page 12 yields the expected pay-off function for both strategies:[11]

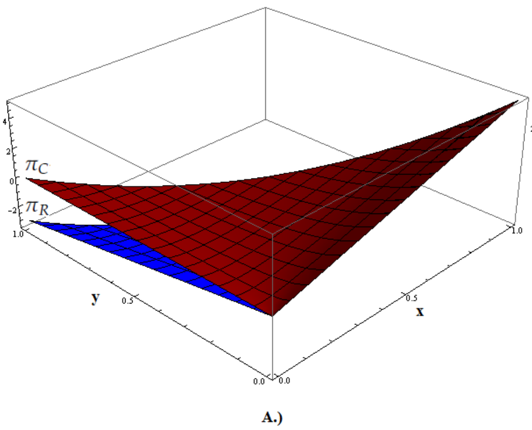
$$\pi_R = \frac{x(x+y-ay)}{x+y}\delta^r + \left(1 - \frac{x(x+y-ay)}{x+y}\right)\delta^p y$$

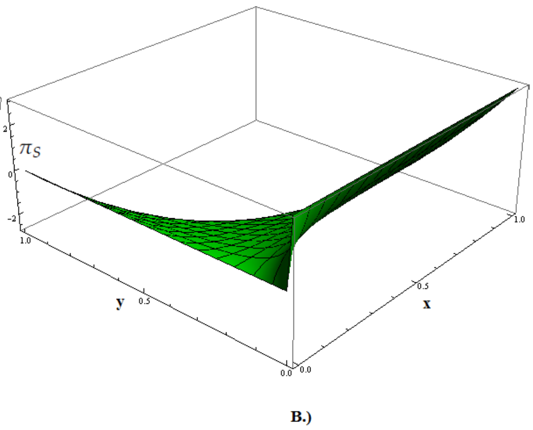
and (1.1.11)

$$\pi_C = \frac{x(x+y-ay)}{x+y}\delta^r$$

Hence, strategy C weakly dominates strategy R for sufficiently large player populations ($n >> 0$) and strictly dominates in the presence of at least one supporter (see simulation figure 1.1.AA.)).

Figure 1.1.A: Simulation of expected pay-offs for each strategy: A.) π_R-Blue, π_C-Red; B.) π_S-Green, Parameters: $a = 1$, $\delta^r = 5$, $\delta^p = -3$, $d = 3$, $c = 4$, $w = 1$





If an A chooses to support the current allocation, he is subsidised by sub-population C_B, i.e. he is paid d by the share w in C_B, which plays strategy *pay*. This payment

[11] Remember that δ^p is a negative constant

increases in the number of revolutionaries in his group. The increase in revolutionaries can be either considered as an individual danger bonus or it simply means that the fixed bonus has to be shared by the supporters of the group (since obviously $1 - \frac{s-\tau}{s} = \frac{\tau}{s}$). Strategy S also incurs a cost that increases in the number of revolutionaries, but also with the total number of supporters in C_A. The idea is that the cost of sabotage and to punish is higher the more revolutionaries are in a group. In addition, the more supporters exist, the less likely the individual player will be honoured for supporting the system.[12]

The share of revolutionaries for a group of size s is given by $\frac{\tau}{s}$. The share of supporters in the entire sub-population C_A is $\frac{s-\tau}{n}$. Putting all together gives

$$G_S(s) = \sum_{\tau=0}^{s-1} \binom{s-1}{\tau} \left(\frac{x}{x+y}\right)^{\tau} \left(\frac{y}{x+y}\right)^{s-1-\tau} \left(wd\frac{\tau}{s} - c\left(\frac{\tau}{s}\frac{s-\tau}{n}\right)\right) \tag{1.1.12}$$

from which is derived that

$$G_S(s) = \frac{d(s-1)wx}{S(x+y)} - \frac{c(s-1)x(x+(s-1)y)}{nS(x+y)^2} \tag{1.1.13}$$

Note that $G_S(s)$ is in that case only indirectly related to the expected probability of a successful revolt, but defines the expected pay-off of an S-player in a group of size s. The result of equation 1.1.13 is again placed into the equation 1.1.2 for the expected group size. Defining again $\pi_i(s_i, s_{-i}|s_i = S) = \pi_S$ gives

$$\pi_S = \sum_{s=1}^{n} \binom{n-1}{s-1} (1-z)^{s-1} (z)^{n-s} \left(G_C(s)\right) \tag{1.1.14}$$

which again yields an unwieldy solution.[13] Yet, for $n \to \infty$ the expected pay-off of an individual choosing strategy S is

$$\pi_S = \frac{x(dw - cy)}{x+y} \tag{1.1.15}$$

Provided there exists at least one supporter, it must hold that $x = 0$, since strategy C strictly dominates R. In this case $\pi_C = \pi_S = 0$ (see simulation of π_S for $w = 1$ in

[12] If one thinks of the bonus payment as moving up the hierarchy in a company or receiving social honours or just a monetary payment, an increase in the number of supporters will have a negative external effect on the individual playing this strategy.

[13] i.e. $= \frac{dwx(z^n+n(1-z)-1)}{n(x+y)^2} - \frac{cx(y+ny(n(x+y)+z-2)(1-z)-yz^n+x((1-n)z-1+z^n))}{n^2(x+y)^3}$

figure 1.1.A B.)). Hence, all points in $y + z = 1$ are Lyapunov stable equilibria that cannot be invaded by revolutionaries, but perturbations, i.e. random drift, along the axis, where $x = 0$, are not self-correcting. Any point at $y = 0$ (no supporters) provides equal pay-off to revolutionaries and conformists, yet in the presence of at least one supporter the pay-off of the former is strictly smaller than the one of the latter.

To define under which conditions a B will decide to provide the bonus payment to the supporting As, let again w be the share of players in C_B choosing strategy p and, hence $1 - w$ be the share of those not paying supporters. Assume that paying supporters provides an increasing benefit with the share of revolutionaries. We might consider for example a small number of security men that protect a factory owner against his revolting labourers protesting for higher loans. Yet, the higher the numbers of protesters the higher the need for protection. Further, if too many As join the revolutionaries, paying supporters might backfire. The conflict situation between revolutionaries and supporters creates collateral damage to the detriment of the Bs, meaning that if revolutionaries are present in great numbers, it is best to be amongst those that less strongly resisted the attempt. Assume that the benefits increase linearly in x, but fall quadratic in x. Hence, let benefits be described by $bx - bx^2$, with $b > 0$. Let there also be a spillover effect on those not paying the supporters that increases linearly in the share of payers w. Protests are not localised and a probability of spreading violence to other groups exists. Hence, paying Bs have also an incentive to "suppress" revolutionaries, when they are only indirectly concerned.

Furthermore, let there be a cost k that also increases with the share of non-payers in a group, since some economies to scale arise, e.g. two factory owners share security personnel, they also share costs. Assume also that costs increase in the number of payers. The underlying idea is that as the number of payers increases, the complexity to set up a security system that satisfy all payers at the same time becomes more complex and thus costly. If a larger number of factory owners invests in security personnel, some amateur security men, who are only recruited in the case of incidents, are substituted by well-trained professional security personnel. It is necessary to set up an infrastructure sufficient to guarantee quick information exchange and access to the various factory sites. Non-payers do not bear these costs, but suffer cost e by those who chose to pay. e signifies the negative effect of social shunning that increases in the number of payers. The more player choose to pay, the more social pressure is exercised on non-payers. For a player population of size m and ρ being the number of payers, the expected pay-off functions for each strategy can be defined as:

$$\pi_P = \sum_{\rho=1}^{m} \binom{m-1}{\rho-1} (w)^{\rho-1}(1-w)^{m-\rho} \left((1-x)xb - k\left(\frac{m-\rho}{m}\frac{\rho}{m}\right)\right)$$
$$\pi_{\neg P} = \sum_{\rho=0}^{m-1} \binom{m-1}{\rho} (w)^{\rho}(1-w)^{m-1-\rho} \left((1-x)xb\frac{\rho}{m} - e\frac{\rho}{m}\right) \tag{1.1.16}$$

where the first part of both equations defines the expected composition, and the second part (in brackets) the net pay-off of each strategy. Again for $m >> 0$, approximating by $m \to \infty$, gives

$$\pi_P = bx(1-x) - k(1-w)w$$
$$\pi_{\neg P} = w(bx(1-x) - e) \tag{1.1.17}$$

Solving 1.1.17 shows that the set of equilibria consists of four components; two interior and two pure equilibria. The interior equilibria are defined by the two roots, at which both strategies have equal pay-off, namely

$$w^{*1} = \frac{1}{2k}(k + bx - e - bx^2 - \sqrt{(e-k-b(1-x)x)^2 - 4bk(1-x)x}) \tag{1.1.18}$$

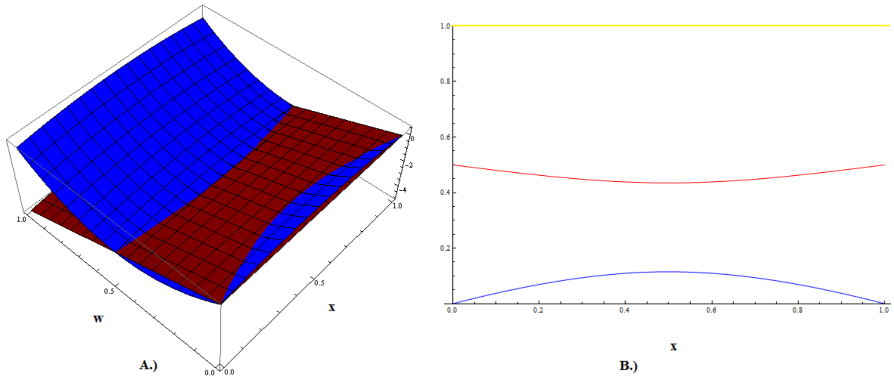
and

$$w^{*2} = \frac{1}{2k}(k + bx - e - bx^2 + \sqrt{(e-k-b(1-x)x)^2 - 4bk(1-x)x}) \tag{1.1.19}$$

under the constraint that $w^{*1}, w^{*2} \in (0,1)$. The former defines the stable interior equilibrium (henceforth called the low w-equilibrium), the latter the unstable equilibrium and frontier between the basin of attraction of the stable pure equilibrium defined by $w^{*3} = 1$, for $x \in (0,1)$ (henceforth called the high w-equilibrium), and the low w-equilibrium. Figure 1.1.B shows an example of the pay-off functions and the set of equilibria.

The lower set of equilibria illustrates that below a certain threshold of w, in the absence of any revolutionaries, no B has an incentive to pay, since the only motivation is provided by the social shunning of *payers*. Similarly if all As choose to revolt, a single B faces a situation, in which he only pays his marginal costs without any benefit, but can evade the costs from social shunning if he did not pay. Hence, no incentive to pay arises. For an intermediate share of revolutionaries an incentive to

Figure 1.1.B: Pay-off Structure Bs: Strategy π_P- blue, $\pi_{\neg P}$- red, Parameters: $b = 2$, $k = 10$, $e = 4$, low w-equilibrium: blue, high w-equilibrium: yellow



protect his property exists. If a sufficient number, however, pays supporters the spill over is adequate, i.e. it compensates for the lack of a direct net benefit from paying and the cost of social shunning, and he will have no motivation to play strategy P. On the contrary, in the high w-equilibrium at which all Bs play P, it is also always best response given any x to pay because of cost e.

If the conflict described by the current model is not considered to be defined by a single event but a sequence of repeated events, individuals are able to dynamically re-evaluate their strategy choice between events.[14] Since players meet at random in a group, they might feel unsatisfied with their current strategy and re-assess. In this case a player compares his strategy choice to a random "model" player. The player adopts the strategy of his "model" with a probability proportional to the positive difference between their model's and their own pay-off, which has been generated in the course of this event (if the difference is negative, i.e. if a player has chosen a better strategy than his model, the player will not switch). Hence, if a player observes another player faring much better with another strategy, he will very likely change strategies. If the model player only received a marginally higher pay-off, the player will less likely to switch. Notice, that if an S-player observes an R-player, who obtains higher pay-off, he will adopt this strategy with a positive probability, though strategy R is strictly dominated by C. Nevertheless, more S-players will switch to strategy C than to strategy R, since the pay-off difference between players choosing C and S is greater than between those choosing R and S.

The dynamics based on this type of assumption are best described by the replicator

[14] Think, for example, of the Monday demonstrations in the GDR in 1989 or the skirmishes during the Thirty Years' War.

dynamics (see also Bowles, 2006). The replicator dynamics are generally defined by $\dot{\sigma}_i = \sum_j \sigma_i \sigma_j (\pi_i - \pi_j)$, where σ_i denotes the frequency of strategy/trait i (i.e. in this case x, y or z) in the population.[15] Hence, $\sigma_i \sigma_j$ defines the probability, with which a player of trait σ_i (i.e. a player choosing the strategy associated to σ_i) meets another player of trait σ_j. The switching probability is a multiple of the pay-off difference $\pi_i - \pi_j$. For the limited strategy set, the replicator dynamics can be simplified to

$$\begin{aligned} \dot{x} &= x(\pi_R - \phi) \\ \dot{y} &= y(\pi_C - \phi) \text{ and} \\ \dot{w} &= w(1-w)(\pi_P - \pi_{NP}) \end{aligned} \tag{1.1.20}$$

where ϕ denotes the average pay-off of the players in C_A, defined by $\phi = x\,\pi_R + y\,\pi_C + z\,\pi_S$. The pay-offs are determined by equations 1.1.11, 1.1.15, and 1.1.17. The dynamics with respect to any distribution of the players in C_A are illustrated in figure 1.1.C, where $\Delta\sigma_i \equiv \pi_i - \phi$. Consequently, σ_i is stationary at $\Delta\sigma_i = 0$.[16] The vectors thus indicate the direction of movement relative to these loci.

The figure at the left illustrates the case, when C_B is in the low w-equilibrium region, the figure at the right shows the case, in which all B choose to pay, i.e. the high w-equilibrium. Any population in region *II* will converge to the x-axis and any population in regions *I*, *III*, *IV* and *V* converges to the y-axis.[17] Regions *IV* and *V* are absent in the low w-equilibrium as their size becomes vanishingly small for populations close to the y-axis, as $w \to 0$ and $x \to 0$.

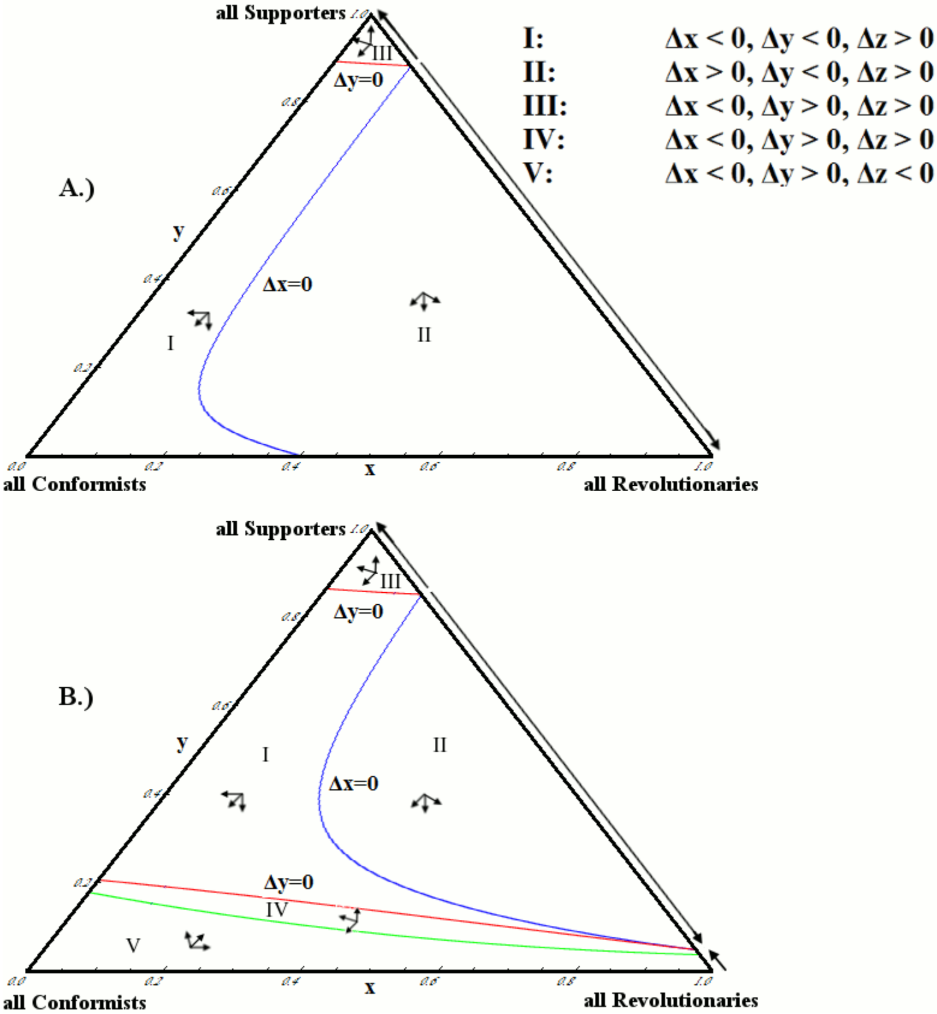
In case A.) an equilibrium on the x-axis with $y = 0$ cannot be invaded by supporters. Notice that C weakly dominates R. If non-best responses occur infrequently, evolutionary drift will push any population towards the left along the simplex axis. The number of revolutionaries will converge to zero and the population moves along the y-axis as small perturbations along this axis (i.e. for perturbations with $x = 0$) are not self-correcting. Invasion by revolutionaries is impossible at any point on y. Small perturbations of non-best response play will push a population back to the equili-

[15] For literature on the replicator dynamics, refer to Taylor & Jonker (1978), Taylor (1979), Schuster & Sigmund (1983), and especially Hofbauer & Sigmund (1988), Weibull (1997) and Nowak (2006).

[16] Solving gives 2 roots for every variable x, y and z. The frontiers in the figure only show those within the unit interval, i.e. $x + y \in (0, 1)$.

[17] Notice, however that this is not generally the case for region *I* in the low w-equilibrium. For different parameter configurations the population can converge to the x-axis, both remaining in region *I* or switching to region *II*, if the initial allocation is sufficiently close either to the x-axis or region *II*.

Figure 1.1.C: Projection of the unit simplex and the dynamics for fixed w: A.) $w = w^{*1}$, B.) $w = 1$, Parameters: $a = 1$, $\delta^r = 4$, $\delta^p = -3$, $d = 5$, $c = 25$, $b = 2$, $k = 5$, $e = 0.5$



brium $z = 1$.[18] The unstable equilibrium for $z = 0$ is defined by point $(0.12, 0.88)$; the former value denotes the share of revolutionaries, the latter the share of supporters.

[18]Notice that evolutionary drift, through small perturbations of non-best response play, pushes the equilibrium to the extremes on the y-axis, namely $y = 1$ or $z = 1$ (not in this simulation) for the low w-equilibrium, and to an interior equilibrium or $y = 1$, for the high level equilibrium, in which $w = 1$. The dynamics of the low w-equilibrium are determined by $\frac{\partial \pi_S}{\partial x}|_{x=0} = \frac{1}{2ky}(-d(|[e-k]|+e-k)(1-y)+2ky((a-1)\delta^r(1-y)y-\delta^p-(1-y)c))$ and $\frac{\partial \pi_C}{\partial x}|_{x=0} = \frac{d(|e-k|+e-k)+k(-\delta^p+y(c-(a-1)\delta^r y))}{k}$, where the latter is strictly greater than the former for small y, but the inverse may occur for large y, depending on the relative size of c, a and δ^r. For $w = 1$ the dynamics are defined by $\frac{\partial \pi_S}{\partial x}|_{x=0} = \frac{d-dy-y(c+\delta^p-cy-(a-1)\delta^r(1-y)y)}{y}$ and $\frac{\partial \pi_C}{\partial x}|_{x=0} = -d-\delta^p+y(c-(a-1)\delta^r y)$, with mixed equilibrium $y = \frac{c-\sqrt{c^2-4(a-1)d\delta^r}}{2(a-1)\delta^r}$ and $z = 1-y$.

In case B.) the situation is somewhat different. Any perturbation of y of an allocation on the x-axis will push the population out of region V into IV, in which the population converges to the y-axis but remains in this region. In addition to the higher mixed equilibrium at $(0.12, 0.88)$ a second mixed equilibrium exists at $z = 0$ and point $(0.95, 0.05)$, which is also unstable as z-perturbations will again push it into region IV.[19] In the case of infrequent non-best response play the long term equilibria will be defined by the line segment of region IV. The model is therefore insufficient to effectively model conflicts. Conflicts would occur with quasi null probability, even if their is little incentive to support the current (social) contract and a favourable alternative allocation exists. A conflict would require a large number of non-best responses that pushes the population from the y-axis into region I and sufficiently close to the x-axis or II. Further, we observe that all equilibria are only defined by two strategies. A population in a completely mixed equilibrium is unobservable, as the model neglects specific characteristics innate to conflicts. One important aspect is the emotional reaction to the violation of social norms. The next section will provide an initial attempt to analytically represent this issue.

1.2 The Effect of Social Norms

The literature on behavioural and neuro-economics (see for example Rabin, 1993, 1998, 2002; Smith, 1994; Camerer et al., 1995 & 2002; Fehr & Gächter, 2000; Frohlich et al., 1987a, 1987b; Cooper et al., 1992) has shown that fairness is a complex concept. In general a *fair share* is determined by a reference point. An individual's evaluation of whether an interaction is fair or not is determined by past-interactions, status, expectations etc.. These elements define a reference framework that can be collapsed into a set of social norms and conventions, which govern everyday interactions. Whenever a social norm is violated, individuals feel unjustly treated. Hence, a simple individual pay-off comparison as in Fehr and Schmidt (1999) is only part of the story. Fair does not necessarily mean equal. As Binmore writes: "A person's social standing, as measured by the role assigned to him in the social contract currently serving a society's status quo, is therefore highly relevant to how his worthiness is assessed by those around him". (Binmore,1998, p. 459; see also variants of the Ultimatum Game, in which *contest winners* successfully offer lower shares than in traditional version of the game, such as Frey & Bohnet, 1995; Hoffman & Spitzer, 1985.)

[19]This implies that the equilibrium can be destabilised by random mutation, but not by imitation.

The way in which social norms, and culture in general, are determined is as vague as the concept of fairness. Many attempts have been made in the past to model the evolution of social and cultural norms (refer to *chapter 2*), yet, not only each social/human science has its own definitions of culture, but these definitions are dependent on the current Zeitgeist (Geertz, 1987). Therefore no unique normative basis exists. This renders a precise description of a social norm on an analytical basis very difficult.[20] It is even unclear whether our actions are determined by global norms or rather by highly local norms (Patterson, 2004).

To circumvent this issue, it is sufficient to notice that indeed it is not strictly necessary to explicitly model social norms, since revolutionary behaviour is defined by an aggressive reaction to the *violation* of social norms. Hence, assume that a certain set of social norms, not closer specified, exists and that whenever a norm is violated, it gives rise to an aggressive feeling. This assumption keeps the model general enough, so that it is applicable to a large variety of conflicts, and norms and conventions. There are two aspects that the model should take into account:

Lorenz (1974)[21] has illustrated that, though aggression is immanent to every species and to most interactions, it is nevertheless defined by a high level of ritualisation that minimises the frequency of direct hostile conflicts, and the potential cost they would incur. Furthermore, in contrast to the prevalent conviction, mankind is amongst the least aggressive species. We have a general tendency to avoid hostility even in extreme situations.[22] Consequently, something what might be termed a "non-aggression norm" should have developed. To include this into the model, assume that a general norm exists that dampens violent responses for all kinds of arising conflicts. On the one hand, the stronger the norm, the less likely an individual will choose a violent response. On the other hand, the more violence an individual observes, the more likely he will also respond aggressively, and the lower the dampening effect of the non-aggression norm.

The example, given earlier, of *the Defenestration of Prague* illustrates the second important aspect of conflicts. A single event suddenly triggers a conflict, though the

[20]Some examples that incorporate norms into mathematical models are Bernheim, 1994; Lindbeck, 1999, 2002; Huck, 2001, 2003.

[21]Konrad Lorenz, Nobel laureate in 1973, was one of the first to analyse aggressive behaviour in different species.

[22]The space in which we normally do not tolerate others is about 1 to 2m, much smaller than for most animals. We therefore dislike a seat close to another person. Yet, during rush hour conflict situations are rarely observed. For a very recent example of the behaviour of other species (the territorial behaviour of Chimpanzees), refer to Mitani et. all (2010)

actual reasons are much more complex and long-range. The Peace of Ausgburg in 1555 warranted the peaceful coexistence of both confessions. The existential fear generated by the small ice age and catholic aggressiveness led to a seething conflict for more than 60 years, that intensified from 1600 and suddenly erupted in 1618. Such trigger events can be frequently observed in history: the closure of Louth Abbey, the storming of the Bastille, the Assassination of Archduke Franz Ferdinand of Austria, the Montgomery Bus Boycott. All these events triggered conflicts that had a wide range of economical, political and social reasons. The underlying conflict smouldered for several years, but an open clash was triggered by a single event that taken on its own, was insignificant.[23] To understand the dynamics of (social) conflicts it is necessary to incorporate these two specific features into the model.

Let there be l different existing social norms and define the violence level, which arises from a violation of norm j, as v_j, with $j = 1, ..., l$. Define the total violence level summed over all violations as $v = \sum_j v_j$. Suppose further that for every such norm violation a specific measure is taken that decreases violence.[24] Examples of such measures are wage increases, a favourable change in the social security system, work amenities, a greater right of co-determination, but it can also include propaganda that is used to misinform and to shroud the norm violation. Call this measure α_j. In addition, the non-aggression norm, defined by β reduces the violent reactions to all violations. Assume that its effect is identical for all v_j, thus β does not require an index. Consider the following system of differential equations[25]:

$$\begin{aligned} \dot{v}_j &= v_j \left(r(1 - y(1 - y)) - s\alpha_j - q\beta - v\epsilon \right) \\ \dot{\alpha}_j &= hv_j - u\alpha_j(1 + v) \\ \dot{\beta} &= (1 - x)\kappa - u\beta \end{aligned} \tag{1.2.1}$$

where all the variable not previously defined are assumed to be constant. The first equation defines the dynamics of the violence levels. Violence exponentially increases

[23]Another example, which shows how context specific these trigger events are, is the schism that was initiated by Luther's theses, though similarly revolutionary approaches of Hans Böhm or the Dominican Girolamo Savonarola and even of Luther's contemporaries Bodenstein and Müntzer possessed much less explosive power.

[24]The model simply assumes that those measures appear exogenously and are not subject to strategic choice. An additional strategy for players in C_B could be included in the model. Yet, I do not believe that this increases the clarity of the model, since the underlying argument should be similar to the decision of whether or not to pay supporters. It might be still interesting for future research to create a trade-off between paying supporters and paying for anti-aggression measures.

[25]This model has been inspired by Nowak & May, 2000.

in $r(1-y(1-y))$, where the net growth rate is equal to r in the absence of any supporters. The idea behind the non-monotonic growth of violence in the share of supporters is that low levels of supporters will suppress violent behaviour. Beyond a certain threshold, however, the increasing number of supporters does not scare off revolutionaries, but has the opposite effect, as it ignites violence.[26] Violence also decreases exponentially in $s\alpha_j v_j$ and $q\beta v_j$. Since more violent reactions have the tendency to wear off more quickly, when dampened, the effect of α_j and β increases in v_j. This form of representation models a sudden spiked upsurge of violence after a norm violation that dampens in the following periods. Furthermore, assume that violence naturally "cools down" at the rate $v\epsilon$, with ϵ relatively small. The violence level thus has a saturation point in the absence of a non-aggression norm and further counter-measures, due to its decrease in $v_j v\epsilon$. The last term in $\dot{v}_j$ bounds the total value to $v = \frac{r(1-y(1-y))}{\epsilon}$, since violence levels cannot be infinite.

Assume that a higher level of violence requires a stronger counter-measure, i.e. α_j is increasing in v_j. A high α_j is, however, costly and difficult to maintain and the positive effect of a larger measure wears off more quickly. Furthermore, it is expected that counter-measures absorb resources, and also that their efficiency depends on the acceptance by the individuals concerned. The total violent response v thus decreases α_j. The counter-measure therefore wears off more quickly if it is more efficient and if the total violence level is high.[27]

Finally, the social norm of non-aggression is only an indirect function of the total violence v, since its size is affected by the share of revolutionaries. As discussed above; if a player more frequently meets a revolutionary, who shows a high level of aggression, the player becomes accustomed to this and also shows a higher propensity for violence. If the population consist mostly of revolutionaries, the non-aggression norm can be expected to be much lower than in a population with only few revolutionaries.

First consider the equilibrium values, determined by setting the last two equations in

[26] This can be often observed on TV during strikes or demonstrations. Too much police force may lead to escalation. A recent example are the protests in Stuttgart (Germany) against the construction of a new central station.

[27] It might be more intuitive to write $\dot{\alpha}_j = hv_j - u\alpha_j(1 + v + \sum_j \alpha)$. Notice, however, that α is approximately proportional to v, thus the addition of another variable is not required.

1.2.1 to zero. These equilibrium values are given by

$$\begin{aligned} \alpha_j^* &= \frac{hv_j}{u+vu} \\ \beta^* &= \frac{\kappa(1-x)}{u} \end{aligned} \tag{1.2.2}$$

Putting in equations 1.2.2 into the first equation of 1.2.1 defines the equilibrium dynamics of an individual violence level

$$\dot{v}_j = v_j(r(1+x) - s\frac{hv_j}{u(1+v)} - q\frac{\kappa(1-x)}{u} - \epsilon v) \tag{1.2.3}$$

We are, however, interested in the aggregate value v. Hence, this can be expressed as

$$\dot{v} = \left(\sum_j \frac{v_j}{u(1+v)}\left((r(1-y(1-y)) - \epsilon v)u(1+v) - q(1-x)\kappa(1+v)\right)\right) - \frac{v}{u(1+v)} vsh \sum_j \left(\frac{v_j}{v}\right)^2 \tag{1.2.4}$$

Define $D = \sum_j \left(v_j/v\right)^2$, with $D \in (\frac{1}{n};1)$. If only one violation of a social norm currently occurs, $D = 1$, whereas in the case of n different violations and for identical levels of violent reactions, $D = \frac{1}{n}$. Substituting and rearranging equation 1.2.4 gives

$$\begin{aligned} \dot{v} = \frac{v}{u(1+v)}&(u(r(1-y(1-y)) - v\epsilon) - q(1-x)\kappa \\ &+v\left[u(r(1-y(1-y)) - v\epsilon) - q(1-x)\kappa - shD\right]) \end{aligned} \tag{1.2.5}$$

Remember that ϵ is generally expected to be very small and that $v\epsilon$ is of the same order as the other variables. It must hold by definition that $r(1-y(1-y)) \geq \epsilon v$, where equality only holds at the maximum level of v. Consequently, as total violence increases, the dynamics of the system are determined by the second term in equation 1.2.5, which is multiplied by v. Notice that $\hat{r} = r(1-y(1-y)) - \epsilon v$ equals the net growth rate of violence. Three cases may occur:

1. Immediate high level of violence: $\hat{r}u > q(1-x)\kappa + shD$.

   In this case, the combined effect of the non-aggression norm and the counter-measures, $q(1-x)\kappa + shD$ is too weak to counter-act the net growth of violence, augmented by the adverse effect of violence on the counter-measures. One might think of a society with a general tendency towards violence, little capacity to counter-act violence or with limited resources for programmes that increase social equity.

2. No violence: $\hat{r}u \leq q(1-x)\kappa$.

   In this case, the non-aggression norm alone is strong enough to stabilise the population. Violence levels exhibit non-positive growth. This relation describes a society that has a high degree of democracy and a sense for non-violent conflict solutions (high non-aggression norm) or a pacifist mentality (low net growth rate of violence). If the share of revolutionaries is very close to zero and ϵ is very small, the inequality can be approximated by $r(1-y(1-y)) < \frac{q\kappa}{u}$. Hence, a pacifist society will never experience a high level of violence, if the growth of the violent response is smaller than the direct effect of the non-aggression norm, particularly for intermediate levels of supporters.

3. Low levels of violence, followed by a sudden upsurge:
   $q(1-x)\kappa < \hat{r}u < q(1-x)\kappa + shD$.

   This is the most interesting case. If the violence can be counter-balanced only by the joint effect of counter-measures and the non-aggression norm, but not by the norm alone, the stability depends on the frequency of norm violations. An increase in the number of simultaneous or contemporary violations of social norms decreases D. This can be interpreted, for example, as a society that suffered a large number of strikes of workers as a response to a deterioration of working conditions in several sectors or a series of public spending cuts that are mostly at the expense of one social group or class. Furthermore, since the share of revolutionaries x decreases the right-hand side of the inequality (and indirectly also increases the central part of the inequality), the right part of the condition becomes less likely to be fulfilled, leading to a situation described in point 1. This implies that populations, which are already in a state of conflict, will observe new conflicts with increasing probability.

A threshold can therefore be derived that defines the minimum share of revolutionaries necessary for violence levels to increase, given by[28]

$$x^* = \frac{\kappa q + Dhs - u\hat{r}}{\kappa q} \tag{1.2.6}$$

If it is assumed that $v = lv_j$ the dynamics are defined by

$$\dot{v}_j = v_j \left(r(1 - y(1 - y)) - s\frac{hv_j}{u(1 + lv_j)} - q\frac{\kappa(1 - x)}{u} - \epsilon l v_j \right) \tag{1.2.7}$$

Solving for $\dot{v}_i = 0$ defines the aggregate equilibrium value for the deterministic approximation

$$\hat{v} = \frac{-hs - \epsilon lu - \kappa^* l + (lr^* u) + \sqrt{-4\epsilon l^2 u(\kappa^* - r^* u) + (-hs - \kappa^* l + lu(-\epsilon + r^*))^2}}{2\epsilon lu} \tag{1.2.8}$$

with $r^* \equiv \hat{r} + \epsilon v = r(1 - y(1 - y))$ and $\kappa^* \equiv \kappa q(1 - x)$. For very small ϵ, the total violence level is approximated by $\tilde{v} = l\frac{r^* u - \kappa^*}{hs - l(r^* u - \kappa^*)}$. The violence level is thus expected to explode, when approximately

$$l^* = \frac{hs}{r^* u - \kappa^*} \tag{1.2.9}$$

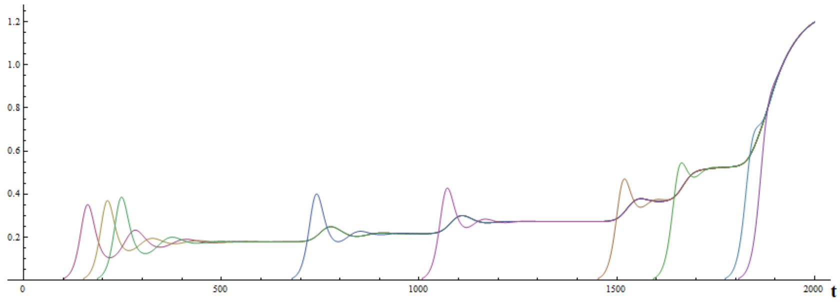
violations occur. Figure 1.2.D shows a simulation, where $v = lv_i$ holds.[29] We observe that at the 9^{th} violation, violence levels explode (according to equation 1.2.9 the maximum number of violations is $l^* = 8.52273$). After each new violation the individual violence levels jump to their new equilibrium values, approximated by $(r^* u - \kappa^*)/(hs - l(r^* u - \kappa^*))$. Notice that the total violence level is bounded by $v = r^*/\epsilon$. It attains its highest level for $x = 1$ and $v = lv_j$. Simplifying equation 1.2.8 gives

$$\begin{aligned} v^{max} &= \lim_{l \to \infty} \hat{v} = \frac{r^* u - \epsilon u - \kappa q(1 - x) + \sqrt{(u(\epsilon + r^*) - \kappa q(1 - x))^2}}{2\epsilon u} \\ &= \frac{r^* u - \kappa q(1 - x)}{\epsilon u} \leq \frac{r^*}{e} \end{aligned} \tag{1.2.10}$$

[28]This implies again that x lies in the interior of the unit interval if the net growth rate of violence is greater than the relative suppressing effect of the counter-measures weighted by the adverse effect of violence on the latter ($\hat{r} > \frac{Dhs}{u}$).

[29]The figure shows a discrete approximation of the system of ordinary differential equations. Thus slight overshooting occurs.

Figure 1.2.D: Simulation of $\hat{v}$ for 2000 periods: Probability of a new violation P=0.01, Parameters: $r = 2$, $s = 3$, $q = 1$, $h = 0.5$, $u = \kappa = x = y = 0.2$, $\epsilon = 0$



1.3 The Emotional Conflict Model

Obviously, emotions are difficult to quantify. Well-defined and established approaches of how emotions analytically affect best-response play are thus lacking.[30] If an emotional reaction is considered to "blur" the salience of certain pay-off values (or more general utility values), emotions can be directly incorporated into the pay-off functions. This might take the form of a threshold value, imposed by an emotion, below which an individual is no longer concerned with the potential loss he faces, when playing this strategy. Hence, only if the pay-off difference between best and non-best response is higher than this threshold value, will a player choose his rational best response. Alternatively, giving in to an emotion might directly provide an additional utility, thus influencing the pay-off associated to a certain strategy that channels this emotion.

Assume that in this context the violence level dampens the salience of the potential punishment that a revolutionary faces in the case, where the conflict does not end in his favour. In general this would change the pay-off of strategy R given by equation 1.1.11 to[31]

$$\pi_R = \frac{x(x+y-ay)}{x+y}\delta^r + \left(1 - \frac{x(x+y-ay)}{x+y}\right)(\delta^p y + \nu(v)) \tag{1.3.1}$$

where $\nu(v)$ transforms the value of the aggregate violence level into a pay-off or utility measure. In this configuration, the aggregate violence level directly affects the absolute value of the punishment δ^p and is unaffected by the loss probability. In the

[30] For a general discussion of the relationship between emotions and rationality, see Kirman et al. (2010).

[31] Remember that δ^p is a negative constant.

given context assume that a player under-evaluates the negative effect, if he observes uncontested revolutionaries, who do not inter-act with supporters, more frequently. If he perceives, however, too many uncontested rioting revolutionaries on the street, his affinity to them decreases or as Granovetter explained: "Individuals who would not speak out until some minimum proportion of those expressing opinions were in their camp might no longer feel the need to speak once a more substantial proportion agreed with them and the situation seemed more securely in hand. This seems even more likely when the action in question is more costly than just expressing an opinion."(Granovetter & Soong, 1988). To take account of this group effect with *decision reversals* assume that the transformation function $\nu(.)$ has the form $\nu(v) = \sigma\, v\, (xz)^2$, where σ is a constant that scales the violence level appropriately into a utility measure. Thus, the compensating effect of v increases in the absence of supporters and has its highest level at the intermediate level of x.

Remember that the system of equations governing the violence levels is stochastic. Yet, equations 1.2.8 and 1.2.10 provide approximate values for v if the individual v_j's have similar scales. The system's dynamics can thus be adequately modelled, if v is substituted by $\hat{v}$ or v^{max}. Since the former generates values similar to the latter already for relatively few violations, and situations with a high conflict potential – case 1. and especially case 3. – are of greater interest, the analysis can be reduced to the latter value without much loss of generality. Define the probability of winning as $P = \frac{x(x+y-ay)}{x+y}$, the dynamics are thus sufficiently determined by

$$\begin{aligned} \dot{x} &= x\left(P\delta^r + (1-P)\left(\delta^p y + \sigma\frac{r^* u - \kappa q(1-x)}{\epsilon u}(xz)^2\right) - \phi\right) \\ \dot{y} &= y\left(\frac{dw^* x - cxy}{x+y} - \phi\right) \\ \dot{z} &= (1-x-y)\left(P\delta^r - \phi\right) \end{aligned} \tag{1.3.2}$$

and ϕ is defined as the average pay-off as before and $w^{*3} = 1$ or $w^{*1} = \frac{1}{2k}(k + bx - e - bx^2 - \sqrt{(e-k-b(1-x)x)^2 - 4bk(1-x)x})$, depending on the equilibrium, to which sub-population C_B is associated. The equilibria are defined at those points, in which the equations in 1.3.2 are equal to zero. This obviously holds, if the same strategy is played by all As. In contrast to the approach that neglects the emotional component of conflicts, interior equilibria can evolve. These are unlikely to be observed in the low w-equilibrium, since as x approaches 1 and thus w tends to zero, high values of d, and low values of c were necessary. In the high w-equilibrium, the solution to $\Delta\sigma_i \equiv \pi_i - \phi = 0$ gives 5 roots, each for Δx, Δy and Δz. Some of the roots have values outside the unit interval of $x + y \in (0,1)$.[32]

Figure 1.3.E: Projection of the unit simplex and dynamics for $w = 1$ and $v = v^{max}$: Parameters: $a = 1$, $\delta^r = 4$, $\delta^p = -3$, $d = 5$, $c = 25$, $r = 2$, $u = 2$, $\kappa = 0.2$, $q = 1$, $\epsilon = 0.05$,$\sigma = 5$

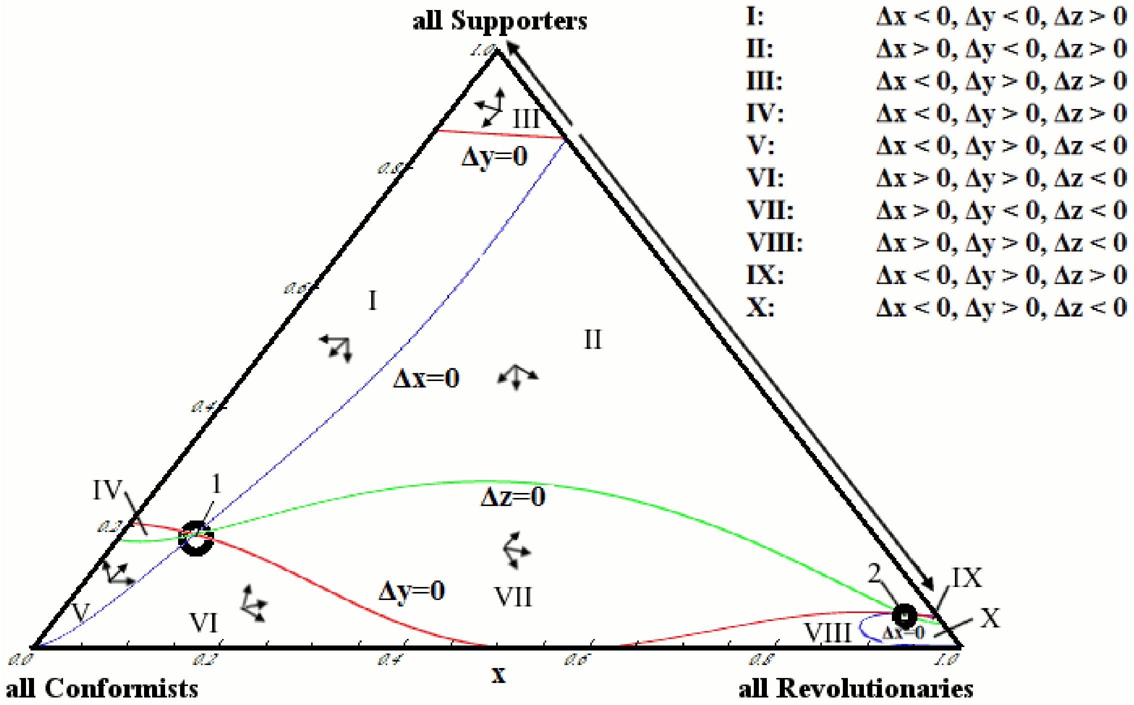


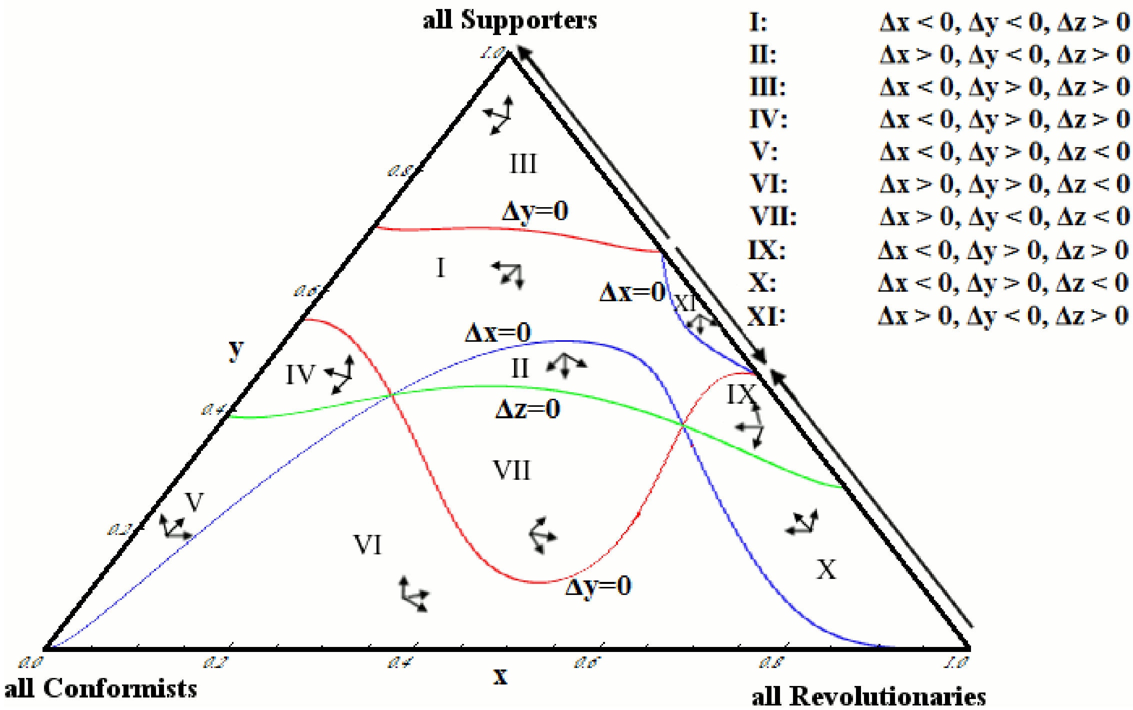
Figure 1.3.E shows a simulation, which uses identical parameter values as in figure 1.1.C on page 21 for the high w-equilibrium case. Equilibria on the y-axis are Lyapunov stable equilibria that cannot be invaded by revolutionaries. Since small perturbations through non-best response play are not self-correcting, evolutionary drift will push the population into region *IV*, representing roughly 20% supporters and 80% conformists (note that in region *III*: $\pi_S < \pi_C$). The x-axis, on the contrary, does not define any weak equilibrium points, since regions *I* and *II* do not touch the x-axis, at which $\Delta y = 0$. As in the simulation shown in figure 1.1.C two unstable equilibria exist, at which conformists are entirely absent, defined by $(0.15, 0.85)$ and $(0.95, 0.05)$.

In contrast to the previous simulation, two additional completely mixed (interior) equilibria occur, denoted by 1 and 2. The former (situated at $(0.08, 0.19)$) is unstable as pay-off of revolutionaries is increasing in their share. Thus, to the right of the

[32]Excluding these, we are left with two roots each for Δx and Δy and one root for Δz.

equilibrium point a revolutionary benefits from more players choosing strategy R and the equilibrium is not self-correcting for small perturbations to the right or left. The second completely mixed equilibrium (situated at $(0.91, 0.05)$) is stable, since at this point the pay-off is decreasing in x. A player's choice of strategy R is therefore detrimental to the pay-off of other revolutionaries at this point and the equilibrium is self-correcting. For the given case, a population converges either to the y-axis in region IV or to equilibrium 2 in the case, where at least a critical mass of revolutionaries (defined by the blue line, at which $\Delta x = 0$) exists. Intuitively the critical mass increases in the number of supporters.

Figure 1.3.F: Dynamics for $w = 1$ and $v = v^{max}$: Parameters: $a = 1$, $\delta^r = 3$, $\delta^p = -3$, $d = 9$, $c = 20$, $r = 2$, $u = 2$, $\kappa = 0.2$, $u = 1$, $\epsilon = 0.05$,$\sigma = 5$



The chosen parameters define the stable equilibrium with roughly 91% revolutionaries. Changing the parameters in favour of the supporters, i.e. by decreasing the potential gain from revolution δ^r by one unit, and setting the bonus payment d to 9 and costs to 20, gives dynamics as shown in figure 1.3.F. Region IV, defining the set of long-term equilibria in the absence of sufficient revolutionaries, moves up along the y-axis by 0.25 points. $\Delta x = 0$ and $\Delta y = 0$ intersect at the right edge at $(0.33, 0.67)$ and $(0.54, 0.46)$, and define again the two unstable equilibria, in which conformists are entirely absent. The completely mixed stable equilibrium moves towards the triangle's centre to approximately $(0.50, 0.37)$, as the unstable mixed equilibrium, which moved up the y-axis in relative position with region IV to point $(0.16, 0.43)$. The general results, however, remain. We observe the existence of a stable interior equilibrium, as well as an interval, in which the equilibrium level with $x = 0$ is defined by the dynamics of region IV, leading to a stable point on the left edge in this region.[33] Adding the emotional element, though highly simplified and abstract, generates more intuitive results than the model described in the first section.

The model can show a further characteristic of conflicts: It is often observed that beneath a threshold people remain bystanders in a conflict situation, although they feel a desire to revolt, but are afraid of being the only one to participate. Only if a sufficient number of other individuals, joining the conflict, is perceived, individuals choose sides and enter the conflict. "The power of the mighty hath no foundation but in the opinion and belief of the people." (Hobbes, 1668, Behemoth)

1.4 A Threshold Model

One issue remains to be resolved: The given model generates a set of equilibria defined by region IV, in which revolutionaries are absent, and a second interior stable equilibrium. Both are separated by the ($\Delta x = 0$)-locus that indicates the critical mass of revolutionaries necessary to induce the population to switch to the completely mixed equilibrium. How does such a transition, provoked by non-best response play, occur? Reasoning on the basis of stochastic stability is not justified in this context, since this would imply that a revolution is only triggered, if a sufficiently large number of players idiosyncratically chooses a *non-best* response strategy. Though the simulation in figures 1.3.E and 1.3.F show a relatively low critical mass of 8% and 16%, respectively, this can still imply a fairly high number of players. For a small group of 50 players the event of passing the critical mass might be observed with fairly high probability. If the model refers to larger groups, e.g. large firms with a substantial labour force or social turnovers, this event seems less likely. The issue, it seems, results from the simplifying assumption that players of identical type also have identical pay-off functions. This section adapts the threshold model approach of Granovetter (1988), which implicitly assumed varying pay-off functions across individuals, to

[33] In contrast, the system in the low w-equilibrium will generally have no interior equilibrium and, as one might expect, a lower critical mass necessary to induce players to choose strategy R. Since the lower $\Delta y = 0$-locus is absent, drift will push the population towards the ($z = 1$)-point, at which revolutionaries have a low entry barrier.

provide an explanation on how the transition over the critical mass frontier can occur.

The underlying idea is that each player needs to observe a certain number of other individuals playing a strategy in order to choose the same strategy. Using Granovetter's wording, assume that in a group a number of *instigators* arises, who do not need any player to start a riot. Upon seeing these instigators causing a disturbance, those individuals, whose threshold is equal or lower than the number of instigators will join the rioters. For an individual threshold R_t of rioters at time t, group size s and c.d.f. $F(\frac{R_t}{s})$, the system is defined by $R_{t+1} = sF(\frac{R_t}{s})$ and the equilibria are determined by $\frac{R_t^*}{s} = F(\frac{R_t^*}{s})$. An equilibrium is only directly attainable if the chain is unbroken, i.e. if it holds that $\frac{R_t}{s} \leq F(\frac{R_t}{s})$, $\forall R_t < R_t^*$. If a group has 3 instigators, but other players require at least 4 rioters, the chain is broken, no further increase in rioters is observed, and only the 3 instigators will participate in the insurrection.

In the context of the given model assume that

$$\nu(v) = \sigma\eta^2 v^{max}(xz)^2 \tag{1.4.1}$$

where η is a random variable. Suppose as before that the number of revolutionaries is given by R_t and hence, for a group of size s, it must hold that $x_t = \frac{R_t}{s}$. An individual will keep his strategy if it gives him the highest pay-off. The number of instigators is consequently determined by the number of players for whom $\hat{\pi}_R \geq \max(\pi_S, \pi_C)$, given $R_0 = 1$ and the adjusted revolutionary pay-off $\hat{\pi}_R$ that includes the random variable η and is obtained by putting equation 1.4.1 into 1.3.1. Instigators need only "themselves" to choose the revolutionary strategy. Their number is defined by $(1 - F(\eta_t^*))\, s$ (i.e. all those players defined by a $\eta > \eta^*$ in the current period), where η_t^* is identified by the solution to $\hat{\pi}_R(\eta_t^*, R_t; s) = \max(\pi_S, \pi_C)$. Define the positive root of this solution (and for $x, z > 0$) by the function $\Theta(R_t)$. Since the increase in x_t is no longer exclusively defined by the replicator dynamics, the values for y_t and z_t need to be adapted, so that population size remains normalised. Assume for convenience that both conformist and supporters have an identical disposition to choose the revolutionary strategy.[34] It must then hold, $y_{t+1} = y_t - \frac{y_t}{y_t + z_t}(x_{t+1} - x_t)$.

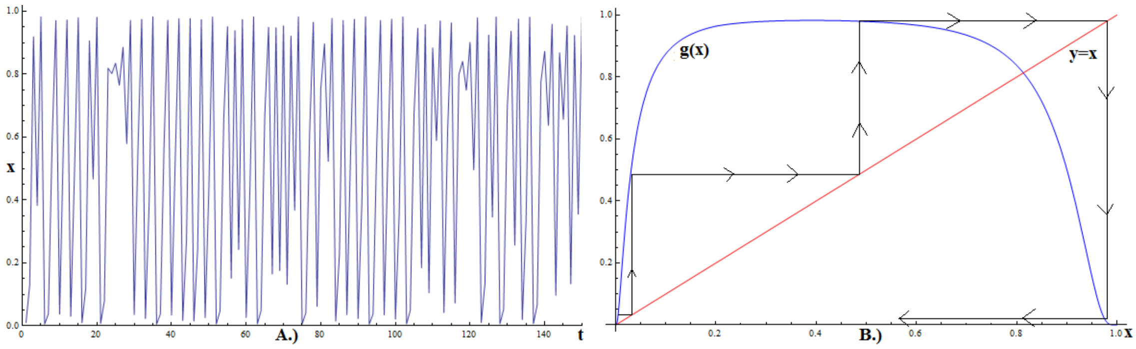
[34] It is indeed more realistic to assume that the relative size of Δy and Δz defines the proportions. This should not pose a problem as long both values are positive. If both values are of different sign, the relation is unclear.

The dynamics are defined by the following system of difference equations

$$\begin{aligned} \eta_t^* &= \Theta(R_t, y_t; s) \\ R_{t+1} &= (1 - F(\eta_t^*))\, s \\ y_{t+1} &= \frac{y_t(s - R_{t+1})}{s - R_t} \end{aligned} \tag{1.4.2}$$

Since $x_t = \frac{R_t}{s}$, the equilibria of this system are given by $x = g(x)$, where $g(x) = 1 - F(\eta(x, y(x)))$. Assume that η is $lnN(1,1))$. The following figure shows a simulation and represents the solution to equations 1.4.2 using identical parameter values as in the simulation of figure 1.3.E on page 31. The population starts at the left edge in region IV (here defined by point $y = 0.2, z = 0.8$).

Figure 1.4.G: A.) Simulation of equations 1.4.2, B.) Continuous Solution: Parameters as in Figure 1.3.E, starting at $y = 0.2, z = 0.8$



A.) illustrates the system's behaviour, which turns out to be cyclic as a result of its discreteness. Figure B.) shows the reason for this behaviour. $g(x)$ defines the share of players, with a threshold lower or equal to x. The stable equilibrium is defined at the point, at which both functions (red and blue) intersect, namely at $x^* = 0.86$.[35] It is stable since to the left of x^*, $g(x)$ exceeds x, indicating that more players are willing to choose strategy R than the current number of revolutionaries. Yet, the slope of $g(x)$ is not strictly positive and not bijective (because of the decision reversals), since η^* is non-monotonic in R_t. The black line shows how the discrete system behaves. Starting at an initially low level of x (i.e. $R_0 = 1$), the system over-shoots after two periods.

[35] As an alternative, instead of $\hat{\pi}_R(\eta^*, R_t) = \max(\pi_S, \pi_C)$, the value of η^* can be determined by $\hat{\pi}_R(\eta^*, R_t) = \phi$. If, however, R is not the only strategy with higher than average pay-off, a proportion of the players will shift to the other strategy. Therefore I have chosen the former relation. The dynamics are similar and for the given parameter values the predicted equilibrium value of x is identical to the value of the interior stable equilibrium in simulation 1.3.E, i.e $x = 0.86$, though the relative size of y and z is different.

For low and high levels of x the slope is very steep, amplifying the over-shooting and creating a cycle with a stable fixed point x^*. For the original replicator dynamics, it was assumed, however, that players update their strategy at random. This should also hold in the case of the threshold model. Over-shooting will thus be much weaker than in the discrete case, in which all players update strategies simultaneously. The system will eventually settle down at the equilibrium point.

What has been explained so far still neglects an important issue. In section 1.1 individuals have been assumed to meet at random in groups of random size. The size does not affect the position of the interior equilibrium of system 1.4.2, as x is independent of s, yet it will change the initial value x_0 and the probability that an *unbroken chain*, leading to x^*,occurs.

Given group size s, assume that the equilibrium value is defined by a minimum number of R^* revolutionaries. The equilibrium number of revolutionaries can occur in different ways. An obvious way is to simply draw R^* times an instigator or to pick one instigator, a player with threshold 1, another player with threshold 2 etc. Mixed combinations will also lead to the equilibrium number. If, for example, a group includes 4 instigators, an unbroken chain does not require another individual with threshold between 0 and 4. Drawing one of these moves the minimum threshold to 5. Thus the group might contain 4 instigators, two players with threshold 3, but none with threshold 1,2,4, or 5 and will still be able to attain the equilibrium number R^*.

The quantity and complexity of potential threshold combinations renders the calculation of the probability of an unbroken chain very cumbersome. Yet, some general results are observable.[36] When people gather in larger groups, we will observe that the expected number of players choosing to *revolt* increases in absolute terms, but decreases in relative terms. In a large group, it is less likely that the majority of players chooses this strategy than in a very small group. Some examples make the intuition clear. Assume that only one instigator is required to reach the equilibrium threshold, i.e. $R^* = 1$. The probability of this event is simply given by $P(R^* = 1) = 1 - (1 - \mu_0)^s$, where μ_0 defines the probability of drawing an instigator. If group size increases, it is more likely that at least one of the group members is an instigator. Furthermore, the number of combinations, which constitute an unbroken chain, also enlarges in group size. Assume, for example that $R^* = 2$ and that the probability to draw an individual of threshold ξ is discrete uniformly distribu-

[36]If μ_ξ defines the probability of drawing a player with threshold lower or equal to ξ, the probability is roughly approximated by $P(R^*) = \prod_{\xi=0}^{R^*-1} \binom{s}{\xi+1} \left(1 - (1 - \mu_\xi)^s\right)$ for small groups.

ted with $\mu_\xi = 1/100$. For $s = 2$ the probability equals $3/100^2$, and for $s = 4$ the probability is approximately $17.68/100^2$. Yet in the given context, the equilibrium threshold R^* is relative to group size s. It is thus reached with lower probability as s increases. For $R^* = \lceil 0.5s \rceil$, the equilibrium threshold occurs with probability 0.0199 for $s = 2$ and with probability 0.00177 for $s = 4$. It is even less likely that the entire group chooses revolt, if s is large. For $s = 4$ the probability lies at $125/100^4$, which is much lower than the probability for $s = 2$.[37] A group thus reaches x^* only with a probability determined by the group size and the underlying threshold distribution. The chain might be broken before, leading to a lower stable share of revolutionaries within a given group.

The threshold approach can thus generate the following intuitive results. It is more likely that a small group is incited than a larger. Furthermore, if group size is large, a certain number of revolutionaries will be observable with high probability. The transition probabilities in the examples are, however, very small. Yet, a large group can consist of smaller sub-groups which again are composed of smaller subsections. This illustrates that the internal structure, and the relations between groups play a crucial role. If groups are linked in a network, its structure can again give rise to a correlative effect between groups. A small group can provoke a larger one, which again stimulates an even larger group and so forth, creating a domino effect not only amongst players within a single group, but also between groups. Thus, the probability of a conflict will depend also on the network structure. This is, however, beyond the scope of this chapter and will be left to future research.

[37] We have observed that the transition to the interior equilibrium in region *IV* requires either 8% or 16% of non-best response play (hence $R_1^* = \lceil 0.08s \rceil$ and $R_2^* = \lceil 0.16s \rceil$). Applying the equation in the previous footnote 36 to the case, in which $\mu_\xi = 1/100$, shows that, for group size $s = 100$, the transition probability has very small values $4.633 * 10^{-11}$ and $6.466 * 10^{-47}$, respectively. For a group of size $s = 20$ both events occur with a probability of 3.026^{-2} and $1.605 * 10^{-4}$. Hence, we will observe a transition into the basin of attraction to the high conflict equilibrium in 1 out of 33 of these small groups.

The probabilities are even higher in case the assumption of the main model apply, i.e. if the parameters are as in the simulation presented in figure 1.4.G, and η^* is log-normal distributed. A transition to $R_1^* = \lceil 0.08s \rceil$ and $R_2^* = \lceil 0.16s \rceil$ occurs with probability 0.5 and $1.042 * 10^{-2}$ in the small group, and with probability $9.079 * 10^{-11}$ and $1.267 * 10^{-46}$ in the large group. A transition to the stable equilibrium at $R^* = \lceil 0.86s \rceil$ is, however, very unlikely (probabilities are at $8.866 * 10^{-60}$ and $9.984 * 10^{-}1526$ for the small and large group - notice, however, the the applied equation significantly underestimates the probability for larger R^*.). The log-normal distribution of η^* augments the probability of reaching a medium number of revolutionaries, but diminishes the probability of the occurrence of a very low or high share of revolutionaries, which can be seen directly from the inverted u-shaped function in figure 1.4.G.

1.5 Conclusion

In this chapter, an intuitive approach has been derived to model the general dynamics of conflicts. Though the assumptions highly abstract from the richness of real world conflicts, the model re-creates specific properties that are inherent to conflicts. It can explain why violence will break out more rapidly in some populations than others and why an open conflict may only be perceived after some triggering event, although reasons for this conflict are to be found in a longer period pre-dating the event. It also illustrates why small groups are more prone to start and participate in conflicts than larger groups.

The model also raises two starting points for future research. Fully integrating the threshold model into the replicator dynamics might create an interesting alternative to the stochastic stability approach (which is discussed in the next chapter). Further, placing the conflict model into a context, in which the links between players and groups matter, will allow us to analyse the effect of the network structure on the likelihood of conflict events.

1.A References

1. Ashenfelter, Orley; Johnson, George E. (1969): "Bargaining Theory, Trade Unions, and Industrial Strike Activity", The American Economic Review, Vol. 59, No. 1, pp. 35-49

2. Axtell, Robert; Epstein, Joshua M.; Young. H. Peyton (2000): "The Emergence of Classes in a Multi-Agent Bargaining Model", Center on Social and Economic Dynamics, Working Paper No. 9 (Feb)

3. Banerjee, Abhijit V. (1992):" A Simple Model of Herd Behavior", Quarterly Journal of Economics, Vol. 107, pp. 797-817

4. Bernheim, B. Douglas (1994): "A Theory of Conformity", The Journal of Political Economy, Vol. 102, No. 5 (Oct.), pp. 841-877

5. Binmore, Ken G. (1994):"Game Theory and Social Contract - Volume 1: Playing Fair", MIT Press, Cambridge, London

6. Binmore, Ken G. (1998):"Game Theory and Social Contract - Volume 2: Just Playing", MIT Press, Cambridge, London

7. Bergin, James & Lipman, Barton L. (1996): " Evolution with State-Dependent Mutations", Econometrica, Vol. 64, pp. 943-56

8. Bowles, Samuel (2006): "Microeconomics: Behavior, Institutions, and Evolution" (The Round Series in Behavioral Economics), Princeton University Press, Princeton, Oxford

9. Bowles, Samuel; Choi, Jung-Kyoo 2007: "The Coevolution of Parochial Altruism and War", Science, Vol. 318, No. 26 (Oct.), pp.636-640

10. Bowles, Samuel; Choi, Jung-Kyoo; Hopfensitz, Astrid (2003): "The Coevolution of Individual Behaviors and Group Level Institutions", Journal of Theoretical Biology, Vol. 223, No. 2, pp. 135-147

11. Boyd, Robert; Gintis, Herbert; Bowles, Samuel, Richerson, Peter (2003):"The Evolution of Altruistic Punishment", Proceedings of the National Academy of Science, Vol. 100, No. 6, pp. 3531-35

12. Brams, Steven J.; Kilgour, D. Marc (1987a): "Optimal Threats", Operations Research, Vol. 35, No. 4 (Jul. - Aug.), pp. 524-536

13. Brams, Steven J.; Kilgour, D. Marc (1987b): "Threat Escalation and Crisis Stability: A Game-theoretic Analysis", The American Political Science Review, Vol. 81, No. 3 (Sep.), pp. 833-850

14. Brams, Steven J.; Kilgour, D. Marc (1988): "Deterrence versus Defense: A Game-Theoretic Model of Star Wars", International Studies Quarterly, Vol. 32, No. 1 (Mar.), pp. 3-28

15. Brams, Steven J.; Togman, Jeffrey M. (1998): "Cooperation through Threats: The Northern Ireland Case", PS: Political Science and Politics, Vol. 31, No. 1 (Mar.), pp. 32-39

16. Brandt, Hannelore; Hauert, Christoph; Sigmund, Karl (2006): "Punishing and Abstaining for Public Goods", Proceedings of The National Academy of Science, Vol. 103, No. 2 (Jan.), pp. 495-497

17. Camerer, Colin F.; Loewenstein, George (2002):"Behavioral Economics: Past, Present, Future", draft: 10/25/02, http://www2.econ.iastate.edu/tesfatsi/camererintrochapter.advancesbe.pdf

18. Camerer, Colin F.; Loewenstein, George; Prelec, Drazen (2005):"Neuroeconomics: How Neuroscience Can Inform Economics", Journal of Economic Literature, Vol. 43 (Mar.), pp. 9-64

19. Clark, Simon (1996):"Strike Behaviour When Market Share Matters", Oxford Economic Papers, New Series, Vol. 48, No. 4 (Oct.), pp. 618-639

20. Cohen, G. A. (1982): "Reply to Elster on "Marxism, Functionalism, and Game Theory", Theory and Society, Vol. 11, No. 4 (Jul.), pp. 483-495

21. Cole, Harold L.; Mailath, George J.; Postlewaite, Andrew (998): "Class systems and the enforcement of social norms", Journal of Public Economics, Vol. 70, pp. 5–35

22. Cooper, Christine L.; Dyck, Bruno; Frohlich, Norman (1992):"Improving the Effectiveness of Gainsharing: The Role of Fairness and Participation", Administrative Science Quarterly, Vol. 37, No. 3 (Sep.), pp. 471-490

23. Durlauf, Steven N.; Young, Peyton (2001): “Social Dynamics”, MIT Press, Cambridge, London

24. Ellison, Glenn (1993): “Learning, Local Interaction, and Coordination”, Econometrica, Vol. 61, No. 5 (Sep.), pp.1047-1071

25. Ellison, Glenn (2000): ”Basins of Attraction, Long-Run Stochastic Stability, and the Speed of Step-by-Step Evolution“, Review of Economic Studies, Vol. 67, pp. 17-45

26. Elster, Jon (1982): “Marxism, Functionalism, and Game Theory: The Case for Methodological Individualism”
http://www.geocities.com/hmelberg/elster/AR82MFGT.HTM

27. Eswaran, Mukesh; Kotwal, Ashok (1985): “A Theory of Contractual Structure in Agriculture”, The American Economic Review, Vol. 75, No. 3 (Jun.), pp. 352-367

28. Eswaran, Mukesh; Kotwal, Ashok (1989): “Why are Capitalists the Bosses?”, The Economic Journal, Vol. 99, No. 394 (Mar.), pp. 162-176

29. Fehr, Ernst; Gächter,Simon (2000): “Fairness and Retaliation: The Economics of Reciprocity”, The Journal of Economic Perspectives, Vol. 14, No. 3. (Summer), pp. 159-181.

30. Fehr, Ernst; Schmidt, Klaus M. (1999): “A Theory Of Fairness, Competition, And Cooperation”, The Quarterly Journal of Economics, MIT Press, Vol. 114, No. 3 (Aug.), pp. 817-868

31. Foster, Dean; Young, H. Peyton: “Stochastic Evolutionary Game Dynamics”, Theoretical Population Biology, Vol. 38 (1990), pp. 219-232

32. Frey, Bruno S.; Bohnet, Iris (1995): “ Institutions Affect Fairness: Experimental Investigations”, Journal of Institutional and Theoretical Economics, Vol. 3, No. 2, pp. 347-369

33. Frohlich, Norman; Oppenheimer, Joe A.; Eavey,Cheryl L. (1987a): “Laboratory Results on Rawls’s Distributive Justice”, British Journal of Political Science, Vol. 17, No. 1, (Jan.), pp. 1-21

34. Frohlich, Norman; Oppenheimer, Joe A.; Eavey,Cheryl L. (1987b): "Choices of Principles of Distributive Justice in Experimental Groups",American Journal of Political Science, Vol. 31, No. 3, (Aug.), pp. 606-636

35. Geertz, Clifford (1987): "Dichte Beschreibung. Beiträge zum Verstehen kultureller Systeme", Frankfurt a. Main

36. Gintis, Herbert (2009): "Game Theory Evolving", Princeton University Press, Princeton, Oxford

37. Gintis, Herbert (2000): "The Bound of Reason - Game Theory and the Unification of the Behavioral Sciences", Princeton University Press, Princeton, Oxford

38. Gintis, Herbert; Bowles, Samuel; Boyd, Robert; Fehr, Ernst (2005): "Moral Sentiments and Material Interests - The Foundation of Cooperation in Economic Life", The MIT Press Cambridge, Cambridge, London

39. Granovetter, Mark (1978):"Threshold Models of Collective Behavior", The American Journal of Sociology, Vol. 83, No. 6 (May), pp. 1420-1443

40. Granovetter, Mark; Soong, Roland (1988): "Threshold Models of Diversity: Chinese Restaurants, Residential Segregation, and the Spiral of Silence", Sociological Methodology, Vol. 18, pp. 69-104

41. Hobbes, Thomas (1668): "Behemoth, or The Long Parliament", Marshall Simpkin, 1889
http://www.archive.org/download/
behemothorlongp00hobbgoog/behemothorlongp00hobbgoog.epub

42. Hofbauer, Josef; Sigmund, Karl (1988):" The Theory of Evolution and Dynamical Systems", Cambridge University Press, Cambridge

43. Hoffman, Elizabeth; Spitzer, Matthew L. (1985): "Entitlements, Rights, and Fairness: An Experimental Examination of Subjects' Concept of Distributive Justice", Journal of Legal Studies, Vol. 2, pp. 259-297

44. Holländer, Heinz (1982): "Class Antagonism, Exploitation and the Labour Theory of Value ", The Economic Journal, Vol. 92, No. 368 (Dec.), pp. 868-885

45. Huck, Steffen; Kübler, Dorothea; Weibull, Jörgen (2001): "Social Norms and Optimal Incentives in Firms", The Research Institute of Industrial Economics, Working Paper No. 565

46. Huck, Steffen; Kübler, Dorothea; Weibull, Jörgen (2003): "Social Norms and Economic Incentives in Firms", Revised version of IUI WP 565, 2001, 10 March

47. Kandori, Michihiro; Mailath, George J.; Rob, Rafael (1993): "Learning, Mutation, and Long Run Equilibria in Games", Econometrica, Vol. 61, No. 1 (Jan.), pp. 29-56

48. Kennan, John; Wilson, Robert (1989):"Strategic Bargaining Models and Interpretation of Strike Data", Journal of Applied Econometrics, Vol. 4, Supplement: Special Issue on Topics in Applied Econometrics (Dec.), pp. 587-5130

49. Kiander, Jaakko (1991):"Strike Threats and the Bargaining Power of Insiders", The Scandinavian Journal of Economics, Vol. 93, No. 3 (Sep.), pp. 349-362

50. Kirman, Alan; Livet, Pierre; Teschl, Miriam (2010): "Rationality and Emotions", Philosophical Transactions of the Royal Society series B, Vol. 365, pp.215-219 http://www.filesonic.com/file/28715741/Rationality and Emotions.pdf

51. Lindbeck, Assar; Nyberg, Sten; Weibull, Jörgen W. (2002): "Social Norms and Welfare State Dynamics", The Research Institute of Industrial Economics, Working Paper No. 585

52. Lorenz, Konrad (1974):"Das sogenannte Böse - Zur Naturgeschichte der Agression", Deutscher Taschenbuch Verlag GmbH & Co. KG, Mnchen, 2007

53. Martin, John (1909): "Socialism and the Class War", The Quarterly Journal of Economics, Vol. 23, No. 3 (May), pp. 512-527

54. Mehrling, Perry G. (1986): "A Classical Model of the Class Struggle: A Game-Theoretic Approach", The Journal of Political Economy, Vol. 94, No. 6 (Dec.), pp. 1280-1303 Published by: The University of Chicago Press

55. Mitani, John C.; Watts, David P., Amsler, Sylvia J. (2010): "Lethal intergroup aggression leads to territorial expansion in wild chimpanzees", Current Biology, Vol. 20, No. 12 (Jun.)

56. Nowak, Martin A. (2006): ”Evolutionary Dynamics - Exploring the Equations of Life“, Harvard University Press, Cambridge, London

57. Nowak, Martin A.; May, Robert M. (2000): ”Virus dynamics: Mathematical principles of immunology and virology”, Oxford University Press, Oxford

58. Orléan, André (1998): “The evolution of imitation”, in ”The Economics of Networks”, Patrick Cohendet, Patrick Llerena, Hubert Stahn, Gisèle Umbauer (eds.), Springer Verlag, Berin, Heidelberg, New York, Ch. 15, pp. 325-339

59. Orléan, André (2001): “Psychologie des marché - Comprendre les foules spéculative”, in “Crises financières”, Jacques Graverau, Jacques Trauman (eds.), Economica, pp. 105-128
http://www.pse.ens.fr/orlean/depot/publi/CHA2001tCOMP.pdf

60. Orléan, André (2002): “Les Interactions Mimétiques”, in “Leçons de microéconomie évolutionniste“, Jacques Lesourne, André Orléan, Bernard Walliser (eds.), Édition Odile Jacob, Paris, pp.119-157

61. Patterson, Orlando (2004):“Kultur ernst nehmen: Rahmenstrukturen und ein afroamerikanisches Beispiel“, in Huntington, Samuel P.; Harrison, Lawrence E.:“Streit um Werte”, Goldman Verlag, München,

62. Przeworski, Adam; Wallerstein, Michael (1982): “The Structure of Class Conflict in Democratic Capitalist Societies”, The American Political Science Review, Vol. 76, No. 2 (Jun.), pp. 215-238

63. Rabin, Matthew (1993): “Incorporating Fairness into Game Theory and Economics”, The American Economic Review, Vol. 83, No. 5 (Dec.), pp. 1282-1302

64. Rabin, Matthew (1998): “Psychology and Economics”, Journal of Economic Literature, Vol. 36, No. 1. (Mar.), pp. 11-46

65. Rabin, Matthew (2002): “A Persepctive on Psychology and Economics”,Department of Economics, University of California, Berkeley
http://repositories.cdlib.org/iber/econ/E02-313

66. Roemer, John E. (1982a): “Methodological Individualism and Deductive Marxism”, Theory and Society, Vol. 11, No. 4 (Jul.), pp. 513-520

67. Roemer, John E. (1982b): “Exploitation, Alternatives and Socialism”, The Economic Journal, Vol. 92, No. 365 (Mar.), pp. 87-107

68. Roemer, John E. (1982c): “Origins of Exploitation and Class: Value Theory of Pre-Capitalist Economy”, Econometrica, Vol. 50, No. 1 (Jan.), pp. 163-192

69. Roemer, John E. (1985): “Rationalizing Revolutionary Ideology”, Econometrica, Vol. 53, No. 1 (Jan.), pp. 85-108

70. Schuster, Peter; Sigmund, Karl (1983): “Replicator Dynamics”, Journal of Theoretical Biology, Vol. 100, pp. 533-538

71. Smith, Vernon L. (1994): “Economics in the Laboratory”, Journal of Economic Perspectives , Vol. 8, pp. 113-131

72. Starrett, David (1976): “Social Institutions, Imperfect Information, and the Distribution of Income”, The Quarterly Journal of Economics, Vol. 90, No. 2 (May), pp. 261-284

73. Swedberg, Richard (2001): “Sociology and Game Theory: Contemporary and Historical Perspectives”, Theory and Society, Vol. 30, No. 3 (Jun.), pp. 301-335

74. Taylor, Peter D. (1979): “Evolutionary Stable Strategies with two types of player”, Journal of Applied Probability, Vol. 16, pp. 76-83

75. Taylor, Peter D.; Jonker,Leo B. (1978): “Evolutionary Stable Strategies and Game Dynamics”, Mathematical Biosciences, Vol. 40, pp. 145-56

76. Weibull, Jörgen (1995):“Evolutionary Game Theory”, Cambridge, MIT Press, Cambridge, London

77. Young, H. Peyton (1993): “The Evolution of Conventions”, Econometrica, Vol. 61, No. 1 (Jan.), pp. 57-84

78. Young, H. Peyton (1998): “Individual Strategy and Social Structure”, Princeton University Press, Princeton, Oxford

79. Young, H. Peyton (2005): “Diffusion of innovations in social networks”, in “Economy as a Complex Evolving” System, Vol. 3 (ed. L. E. Blume & S. N. Durlauf), Oxford University Press, Oxford

Il y a l'histoire des opinions, qui n'est guère que le recueil des erreurs humaines.

Voltaire (1694-1778)

2 Dynamics of Conventions & Norms in Integral Space: A Critical View on Stochastic Stability

This chapter critically analyses the well-known Stochastic Stability approach developed by Peyton Young. The focus is on 2×2 coordination games. The original approach has been criticized for the assumption of state independent error and sample rate, and is thus extended to take this into account. Both, error and sample size are supposed to be correlated with the loss that occurred, if a player chooses a non-best response strategy. The original predictions are robust to this change, if the game's pay-off matrix exhibits some form of symmetry or if the correlation between potential loss from idiosyncratic play and the state dependent variable has a specific functional form. Yet, the state dependent version will not necessarily determine the same Stochastically Stable State (SSS) as the original approach if neither of these conditions is met. Further, adopting an approach by Nowak et al. it is illustrated that, in this context, the minimum stochastic potential is a necessary but insufficient condition for an SSS. The state dependency of error and sample size requires that each equilibrium on the path towards an SSS is accessible with a number of errors lower than one-third of the sample.

Introduction

In the spirit of Max Weber's "Wirtschaft und Gesellschaft" (1922), the role of culture, as an important economic determinant, has experienced a strong revival in the scientific literature since the late 80s (Huntington & Harrison, 2000 & 2004; Welzel & Inglehart, 1999; Huntington, 1997; Ades & Di Tella, 1996; Bollinger & Hofstede, 1987), illustrating the fundamental impact of social norms and conventions on economic development. Culture does not only determine the institutional framework, but also the behaviour of economic agents. Consequently, social processes and culture, though often neglected, are essential variables of economic theory. Daniel Etounga-Manguelle stated "Culture is the mother, institutions are the children".[1] The dynamics determining social conventions and norms must be therefore of special economic interest.

Along this line, "The Evolution of Convention" by Peyton Young (1993) is a well-known approach that allows one to discriminate between potential conventions, being synonymically defined as pure Nash equilibria.[2] In his approach, Young does not try to explain the subtle workings of how a transition between conventions occurs, but why we observe certain conventions to be more stable and to persist longer than others. This chapter takes Young's approach as a basis and guideline. Consequently, the sociological intricacies of social norms and conventions are not the issue of this chapter, but conventions in the abstract form of the stochastic stability approach. Since the assumption of state independent error size has been criticised, the focus will be on the state dependency both of the sample process and the error probability by relaxing and altering some of the assumptions of the original approach. I will show under which conditions the original results are maintained. This approach offers an advanced equilibrium refinement criterion that provides a reasonable explanation for the occurrence of certain long-term conventions.

This chapter constitutes the first part of a critical analysis of the stochastic stability approach by Peyton Young. With the exception of its focus on the issue of state independence, the method described herein will keep most of the assumptions of the original approach. Each player samples any previous interaction and can be paired with any other player in his population[3], both with positive probability. Players are

[1]Manguelle in Huntington & Harrison, 2004, page 135

[2]This results from the underlying concept that best response play leads to adopting conventional behaviour as long as an individual believes that a sufficiently large number of other individuals follows the same behaviour, thus forming a pure Nash equilibrium of an n-person game.

[3]Player pairing is only restricted by their affiliation to types. If types exist, a player can only be paired

hence completely connected. Following the definition of Jason Potts (2000), this is defined as an interaction in "integral space", in order to separate the criticism here from that of *chapter 3*. The first section will illustrate that reasonable assumptions on the error size will strengthen the original results for symmetric games, though generally not for asymmetric games.

The second section points out a further issue in this context. The original framework neglects the effect of random drift that occurs in the presence of a state dependent error size, since error rates are assumed to converge to zero.[4] If, on the contrary, error rates are high, the subsequent drift will necessitate a larger basin of attraction as a counter-force, in order for a state to be a Stochastically Stable State (*SSS*). Hence, for interactions with a relatively low potential loss owing to erroneous play, stochastic stability approach cannot be applied directly and requires a second condition to be fulfilled: the two third rule.

2.0.1 A short Introduction to Stochastic Stability

In the context of social conventions several questions arise: Which strategies constitute a possible convention? Why do certain conventions persist, whilst others are rather short-lived? Why does a specific convention emerge and not another, i.e. why do we see both similar[5] but also entirely different behavioural patterns in locally separated parts of the world? Classical game theory provides an answer to the first question. A convention or norm is described by a stable Nash equilibrium in pure strategies. The strategy profile defined by the convention consists of the best response strategies of each player (type), implying that if a sufficient number of individuals follows the convention, it is pay-off maximising to do the same. The second question, however, can only be insufficiently answered. Obviously, since the conventional strategy is best response to the strategies played by all other individuals, it is best not to deviate from the strategy prescribed by the convention. Yet, when it comes to interactions, in which more than one Nash equilibrium in pure strategies exist, the determination of a long-term convention is ambiguous. Why is money accepted in exchange for goods and services? Why are economic interactions determined by certain informal rules and not others? Why do people first let others exit the coach and only after that enter

with players of a different type. In "the Battle of Sexes" a man can be paired with any woman in his population, but not with another man.

[4]Though, Theorem 4.2. in Young,1998, provides a condition for the case of non-zero error rates, the results obtained here are different as some of the original assumptions are changed.

[5]So called *evolutionary universals*, see Parsons, 1964

the train? The inverse behaviour could also define a convention. The answer often given to the third question is that the choice between conventions follows a non-ergodic process. This answer leaves to much space to chance events and unknown exogenous variables to be able to explain the similarity of conventions in separate regions of the world.

A more adequate explanation to this question is, however, strongly connected to the second question. The history leading to a new convention is fundamentally shaped by the underlying conventions and norms that currently prevail. Hence, social conventions and norms at one point in time will define the historical circumstances that determine future norms or conventions.[6] The third question thus collapses to the second. This circumstance requires an approach to discriminate between various conventions, answering the question of why certain types of conventions prevail over others. Kandori, Mailath and Rob (1993) and Young (1993) have developed similar approaches to this question. Since it constitutes the basis for subsequent derivations, this chapter will focus on Young's approach on stochastic stability and will elaborate its basic reasoning in the following. Readers familiar with the concept can skip to the following section 2.1.

Assume that for a finite player population, n different sub-populations exist, each indicating a player type. Strategies and preferences are identical for all individuals in the same sub-population. The game is defined by $\Gamma = (X_1, X_2, ..., X_n; u_1, u_2, ..., u_n)$, where X_i indicates the strategy set and u_i the utility function of individuals of type i. Hence for simplicity, define each individual in such a sub-population C_i as player i. Assume that one individual from each sub-population C_i is drawn at random in each period to play the game. Each individual draws a sample of size $s < \frac{m}{2}$ from the pure-strategy profiles of the last m rounds the game has been played.[7] The idea is that the player simply asks around what has be played in past periods. Hence, the last m rounds of play can be considered as the collective memory of the player population. In addition to Young's assumptions, I assume explicitly that m and s are large.[8]

[6]see Bicchieri (2006) for an overview of the current literature on how existing norms affect players' choices

[7]In fact the general condition is $s \leq \frac{m}{L_\Gamma+2}$, with L_Γ being the maximum length of all shortest directed paths in the best reply graph from a strategy-tuple x to a strict Nash equilibrium (see Young, 1993). Since here I restrict the analysis to 2×2 matrix games with two strict Nash equilibria in pure strategy, the simplified assumption suffices.

[8]This assumption is made to guarantee that the values, which the basin of attraction can take, are approximately continuous. The example in appendix 2.A on page 63 illustrates an instance, in which this is not the case. Young generally defines the resistance of a switch from one absorbing state to another as the least integer greater or equal than the product of the resistance value and the

Each state is thus defined by a history $h = (x^{t-m}, x^{t-m+1}, \ldots, x^t)$ of the last m plays and a successor state by $h' = (x^{t-m+1}, x^{t-m+2}, \ldots, x^t, x^{t+1})$ for some $x^{t+1} \in X$, with $X = \prod X_i$, which adds the current play to the collective memory of fixed size m, deleting the oldest. Each individual is unaware of what the other players will choose as a best response. He thus chooses his best reply strategy with respect to the strategy frequency distribution in his sample (fictitious play with bounded memory, which Young called *adaptive play*). He chooses, nonetheless, any strategy in his strategy profile with a positive probability. Consequently, suppose that there is a small probability that an agent inadequately maximizes his choice, and commits an error or simply experiments. The probability of this error equals the rate of mutation $\varepsilon > 0$, i.e. with probability ε an individual j in C_j does not choose his best response $x_j^* \in X_j$ to his sample of size s from a past history of interactions.[9] Instead he chooses a strategy at random from X_j. Since each state is reachable with positive probability from any initial state if $\varepsilon > 0$, the process is described by an irreducible Markov chain on the finite state space $\Omega \subset (X_1 \times X_2 \times ... \times X_n)^m$. Not all states are, however, equally probable. In order to shift a population from some stable equilibrium (i.e convention), at which players only remember to have always played the same strategy, defined by strategy profile $x^{*t} = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))$ and history $h_k = (x^{*t-m}, x^{*t-m+1}, \ldots, x^{*t})$ to some other stable equilibrium defined by $x^{'t}$ and $h_l = (x^{'z-m}, x^{'z-m+1}, \ldots, x^{'z})$ in time z, requires that a sufficiently large number of individuals idiosyncratically chooses a non-best response strategy to move the population out of the basin of attraction of the equilibrium defined by h_k into the basin of attraction of another equilibrium, so that $x_i^{'}$ is eventually a best response to any sample drawn from m.

For each pair of recurrent classes E_i, E_j from the set of recurrent classes $E_1, E_2, \ldots, E_k$ in the non-perturbed Markov process, a directed ij-path is defined by a sequence of states $(h_1, h_2, ..., h_z) \in \Omega$ that goes from E_i to E_j.[10] Define the resistance $r(h, h^{'})$ as the number of mistakes (perturbations) necessary to cause a transition in each period

sample size; an unnecessary requirement if s is considered to be large.

[9] Strictly speaking the error rate is given by $\lambda_j\varepsilon$ for player j and has full support, i.e. all strategies in X_j are played with positive probability whenever an error occurs or the player experiments. Note, however, in the standard case the *SSS* is independent of λ_j and the probability, with which a strategy is randomly chosen.

[10] A recurrent class E_i is simply defined by the set of states that cannot access any state outside the class, but only any other inside. Hence, if a state is absorbing, like the equilibria of the coordination game in the non-perturbed Markov process, the recurrent class is defined by a single state - the pure equilibrium. In the perturbed Markov process, a single recurrent class spans the entire state space, since each state is accessible from any other in a finite number of periods.

from any current state h to a successor state h' connected by a directed edge.[11] The resistance of this path is given by the sum of the resistances of its edges, $r_\sigma = (r(h_1,h_2) + r(h_2,h_3) + \ldots + r(h_{z-1},h_z))$. Let r_{ij} be the least resistance over all those ij-paths. Hence, there exists a tree rooted at vertex j for each recurrent class E_j that connects to *every* vertex different from j. Notice that connections can be defined by a direct or indirect path leading from any other vertex k for E_k to j for E_j, with $k \neq j$, in the perturbed process. A path's resistance is thus given by the sum of the least resistances r_{ij} over all the edges in the tree. The stochastic potential for any E_j is defined as the least resistance among all these trees leading to vertex j.[12] The recurrent class with least stochastic potential determines the Stochastically Stable State. Remember the least resistance path can be direct or indirect, and takes further account of all strategies in the strategy set. In other words, an *SSS* is the equilibrium that is the easiest accessible from *all other states combined*.[13]

The assumptions of the stochastic stability approach have been discussed in the literature and the assumption that errors are state and pay-off independent has been criticized (Bergin and Lipman, 1996). Yet, it does not need major changes in Young's approach to take account of this. The following section will thus include pay-off dependent sample and error size into the calculation of the resistances. The method is based on an approach of Young (see Theorem 4.1 in " Individual Strategy and Social Structure", 1998) and the work of van Damme and Weibull (1998). This chapter will show that stochastic stability still holds under the assumption of state dependency under most conditions. It also illustrates that there is a significant difference between assuming state dependent sample size and error size, if different player types interact. Sample size affects the rate at which a player type directly *observes* mutations and is dependent on the pay-off the player type has at the current equilibrium state. In contrast, error size affects the rate at which an error is *committed* by the other player types and is therefore dependent on the pay-off of these other player types. Section

[11] That means that the transition from h to the successor state h' in an n-person game is of order $\varepsilon^{r(h,h')}(1-\varepsilon)^{n-r(h,h')}$. If h' is a successor of h in the non-perturbed process, e.g. they are in the same basin of attraction, resistance is 0. If h' is not a successor state both in the perturbed and unperturbed process, the resistance is equal to ∞.

[12] Given the general 2×2 coordination game and normalising the sample rate to 1, the stochastic potential of j to i is then given by the minimum sample share along each path necessary for both player types to choose strategy i as best response.

[13] Hence, the stochastic potential for an equilibrium in a coordination game with three strategies and three pure Nash equilibria is the sum of two least resistances. The path is either defined by two direct edges or one indirect and one direct edge, depending on whether the indirect path involves a lower resistance than the direct path.

2.2 will show, however, that these assumptions raise a different issue, since the error rate can be expected to be high in certain states h, causing a potential disruption in the transition from one convention to another.

2.1 State Dependent Sample and Error Size

This section will follow the approach of Eric van Damme and Jörgen Weibull (1998) to some extent. It only considers sample and error size as loss-dependent, but generally assumes type independence (if not mentioned otherwise), i.e. error and sample size are defined by a function that has only the pay-offs as its arguments and is not type specific. Findings are rather similar, but will differ in various details. Van Damme and Weibull assume that an individual can choose his error level, but has to pay a control cost. The *control cost function* $v(\epsilon_i(\omega))$ is defined as a function of individual's i error size at the current state ω. Furthermore, the *control cost function* is supposed to be decreasing, strictly convex, and twice differentiable. As in van Damme and Weibull, this chapter will only consider 2×2 coordination games with two strict Nash equilibria in pure strategies, generally of the form presented in matrix 2.1.1, with $a_i > c_i$ and $d_i > b_i$.[14]

$$\begin{array}{c} \\ A_{row} \\ B_{row} \end{array} \begin{array}{c} \begin{array}{cc} A_{column} & B_{column} \end{array} \\ \begin{pmatrix} a_1, a_2 & b_1, c_2 \\ c_1, b_2 & d_1, d_2 \end{pmatrix} \end{array} \tag{2.1.1}$$

Define $g_i(\omega) = max[\pi_i(A, \omega), \pi_i(B, \omega)]$, and $w_i(\omega) = min[\pi_i(A, \omega), \pi_i(B, \omega)]$ and thus $l_i(\omega) = g_i(\omega) - w_i(\omega)$, given the current conventional state ω. The expected pay-off is then determined by $\pi_i = g_i(\omega) - \epsilon_i(\omega) l_i(\omega) - \delta v(\epsilon_i(\omega))$, where l defines the loss in the case, where an error is committed and the non-conventional (non-best

[14] In the case of more pure Nash equilibria the reduced resistances are given by the the minimum sum of the resistances of the edges of each directed graph towards an equilibrium in the set of all i-trees. Yet, as is done in Young's approach, a mere summation of the least resistances without weighting them seems problematic. If a society moves along an indirect path, it will spend time in the basin of attraction of an equilibrium that lies on that path, making it a temporal convention. During this time, the society will play a strategy profile close to that dictated by the equilibrium. This might be one that strongly inhibits idiosyncratic play and will be robust against individual errors. The state dependent error and sample size thus also applies to the resistances along the indirect paths. Hence, the approach that will be described can be easily applied to games with larger strategy sets and more equilibria by weighting the individual resistances in each sum accordingly.

response) strategy is played. In addition to the approach of van Damme and Weibull, ϵ_i is assumed either to be a function of the sample size $s_i(\omega)$, implying that the larger $s_i(\omega)$ the lower the probability of drawing a skewed sample from the collective memory m. Alternatively the error probability is assumed to be directly controllable by each individual and is determined by an exponent $\gamma_i(\omega)$ and the exogenous "baseline error ε", such that $\epsilon_i(\omega) = \varepsilon^{\gamma_i(\omega)}$. The idea is that individuals try to "stabilise their trembling hand" if stakes are high, whereas they are more inclined to explore alternative strategies if potential loss is small. Further let us assume δ equals 1.[15] Therefore expected profit is given by

$$\pi_i = g_i(\omega) - \epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon) l_i(\omega) - v(\epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon)).$$

Given the previous assumptions, $v(\epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon))$ is strictly convex and twice differentiable in ϵ_i, and ϵ_i is strictly decreasing both in $s_i(\omega)$ and $\gamma_i(\omega)$. Hence the marginal cost function $-v^{'}(.)$ will be decreasing in ϵ_i and increasing in s_i and γ_i. Maximizing the expected pay-off yields $l = -\frac{\partial v[\epsilon_i(s,\gamma;\varepsilon)]}{\partial \epsilon_i}$.

For the general 2×2 coordination game assume that ω and ω' denote the two possible conventional states of the world. As before $\epsilon_i(\omega)$ is the mutation probability of type $i = 1, 2$ in states ω, $s_i(\omega)$ is the corresponding sample size, and $\gamma_i(\omega)$ is defined as such that $\epsilon_i(\omega) = \varepsilon^{\gamma_i(\omega)}$. Given the definition above $l_i(\omega)$ defines the loss function of player i in state ω as the loss that occurs if player i erroneously plays his non-best response with respect to state ω. Thus the loss function is defined as $l_1(\omega) = \pi^{row}_{\omega\omega} - \pi^{row}_{\omega'\omega}$ for the row players, and $l_2(\omega) = \pi^{column}_{\omega\omega} - \pi^{column}_{\omega'\omega}$ for column players, if π^{row} and π^{column} indicate the corresponding pay-offs in the pay-off matrix for each player type and the first part of the index the state to which the player chooses the best response strategy and the second part the actual state. Error and sample rate are state dependent, but type independent; and the expected relation holds:[16]

$$l_i(\omega) < l_j(\omega') \Leftrightarrow \epsilon_i(\omega) > \epsilon_j(\omega') \Leftrightarrow s_i(\omega) < s_j(\omega') \Leftrightarrow \gamma_i(\omega) < \gamma_j(\omega'), \quad \text{for } i, j = 1, 2. \tag{2.1.2}$$

[15] δ can be dropped, since it is unnecessary as ε is exogenous.

[16] This is a direct consequence of $l = -\frac{\partial v[\epsilon_i(s,\gamma;\varepsilon)]}{\partial \epsilon_i}$ and the previous definition of the function $l_i(\omega)$. Sample and error size are state dependent, since ϵ varies with the pay-off, which an individual attributes to the best and non-best response strategy in each equilibrium state. Sample and error size are type independent, since the cost control function is only a function of the error size and not of the type. This implies that ($s_1(\omega) = s_2(\omega)$ and $\gamma_1(\omega) = \gamma_2(\omega)$ iff $l_1(\omega) = l_2(\omega)$), and $s_1(\omega) < s_2(\omega)$ and $\gamma_1(\omega) < \gamma_2(\omega)$ iff $l_1(\omega) < l_2(\omega)$ and the inverse.

Since only two equilibria in pure strategies exist, both equilibria are connected by only direct paths. If sample and error size are state and type independent, the "reduced resistances" will be equal to the stochastic potential. So it suffices to compare only the two reduced resistances along the direct paths (one for each player type). The reduced resistances, in the case of state and type independent error and sample sizes, are defined by the size of the basin of attraction, as these constitute the minimum share in the sample, necessary to change best response play. Thus, in this case and for 2×2 coordination games, the reduced resistances for the first and second equilibrium, i.e. the minimum frequency of non-best response players in the standardised sample to induce best response players to choose the same strategy, are defined as follows:

$$r_{AB} = min\left(\frac{a_1 - c_1}{a_1 - b_1 - c_1 + d_1}, \frac{a_2 - c_2}{a_2 - b_2 - c_2 + d_2}\right) \text{ and}$$
$$r_{BA} = min\left(\frac{d_1 - b_1}{a_1 - b_1 - c_1 + d_1}, \frac{d_2 - b_2}{a_2 - b_2 - c_2 + d_2}\right) \qquad (2.1.3)$$

or more succinctly:

$$r_{AB} = \alpha \wedge \beta \text{ and } r_{BA} = (1 - \alpha) \wedge (1 - \beta)$$

where A and B describe the pure Nash equilibria defined by their corresponding strategies, and α and β define the minimum population frequencies in the sample, necessary to induce best-response players to switch to strategy B. Obviously in this case the *SSS* is equivalent to the risk dominant Nash equilibrium. (for detailed proofs, refer to Young; 1993, 1999).

The symmetric case describes a game, in which a player's position is irrelevant, i.e. pay-offs are independent of the indices in matrix 2.1.1. Given these assumptions the following two propositions for state dependent sample size can be shown (all proofs can be found in appendix 2.A):

Proposition 2.1.I. *For the symmetric case with state dependent sample size the resistances are determined by* $r^s_{AB} = \alpha s(A)$ *and* $r^s_{BA} = (1 - \alpha)s(B)$, *where* $s(\omega)$ *defines the sample rate in convention* ω.

Proposition 2.1.II. *In the case of two different player types* $i = 1, 2$ *and state dependent sample size* $s_i(\omega)$, *the resistances are defined by* $r^s_{AB} = \alpha s_1(A) \wedge \beta s_2(A)$ *and* $r^s_{BA} = (1 - \alpha)s_1(B) \wedge (1 - \beta)s_2(B)$.

Suppose that in the case of symmetric pay-offs it holds that $s(A) \neq s(B) = 1$.[17] Then the equilibrium sample size s^*, at which both equilibria are stochastically stable, is given by $s^* = \frac{1-\alpha}{\alpha}$. For all $s(A) > s^*$, h_A is the sole Stochastically Stable State. In the case of $s(A) < s^*$ the SSS is defined by h_B.

For state dependent error size, defined by $\varepsilon^{\gamma_i(\omega)} = \epsilon_i(\omega)$, and normalised state independent sample size ($s_i = 1$), the following two propositions hold:

Proposition 2.1.III. *In the symmetric case with state dependent error size, resistances are given by* $r^{\gamma}_{AB} = \alpha\gamma(A)$ *and* $r^{\gamma}_{BA} = (1-\alpha)\gamma(B)$.

Hence, a decrease (increase) in error size from $\varepsilon(\omega)$ to $\varepsilon(\omega^{'})$, with $\varepsilon(\omega') = \varepsilon(\omega)^{\zeta}$ and $\zeta > 1$ ($\zeta < 1$), is equivalent to an increase (decrease) in the relative sample size $s_i(\omega)$ by ζ.

Proposition 2.1.IV. *In the general case with state dependent error size, the resistances are given by* $r^{\gamma}_{AB} = \alpha\gamma_2(A) \wedge \beta\gamma_1(A)$ *and* $r^{\gamma}_{BA} = (1-\alpha)\gamma_2(B) \wedge (1-\beta)\gamma_1(B)$.

In the symmetric case, the "*speed*", at which the boundary of the basin of attraction $\mathfrak{B}_A$ is approached, directly depends on $\gamma(A)$. A $\gamma(A) > 1$ reduces the error rate and decreases the "*step size*" and thus steepens the basin of attraction. The relation between sample size and error size in the symmetric case is reasonable. A higher sample rate should decrease the probability of an error occurring in a symmetric game.

It follows that, for the symmetric case, the unique invariant distribution satisfies[18]

$$\frac{h^{'}_A}{h^{'}_B} = \varepsilon^{m-i^*+1-\gamma i^*} \frac{k_A[1+f_A(\varepsilon)]}{k_B[1+f_B(\varepsilon)]}$$

where i^* indicates the interior mixed equilibrium state at which the error rate changes from some error rate ε^{γ} to another defined by ε.[19] For m very large and $\varepsilon \to 0$ this can be normalized and rewritten as: $\frac{h^{'}_A}{h^{'}_B} = \varepsilon^{1-\alpha-\gamma\alpha}\frac{k_A}{k_B}$, for α defined as before. In the case $\gamma > \frac{1-\alpha}{\alpha} = \gamma^*$ the exponent is negative and the ratio goes to ∞. Hence $h^{'}_A \to 1$. In the case of $\gamma < \gamma^*$ the ratio goes to zero and $h^{'}_B \to 1$. For $\gamma = \gamma^*$, then $\frac{h^{'}_A}{h^{'}_B} \to \frac{k_A}{k_B}$.

[17] This normalisation is applicable to any game with symmetric pay-offs, since the matrix's pay-offs can be changed by a positive affine transformation as such that the function of the sample rate equates to 1, and only the relative values are of interest.

[18] The invariant (or stationary) distribution of a Markov process is described by $h^* = h^*P$, for transition matrix P and history h^*. -For details, see Bergin and Lipman, 1996.

[19] More formally: If p_{ij} represents the probability of moving from state i to state j in the unperturbed Markov process, then $\exists i^*$ such that $p_{i0} = 1$ if $i < i^*$, and $p_{im} = 1$ if $i > i^*$).

The general pay-off matrix in 2.1.1 can have 4 different pay-off structures. The symmetric case is generally defined in the literature as above, i.e. it does not matter whether an individual is a column or row player. This situation occurs in a population with only one player type. If two player types exist, their interests can be diametrically opposed, i.e. pay-offs are defined by a matrix, in which $a_i = d_j$ and $c_i = b_j$ for $i \neq j$. In such games pay-offs for both players are identical, but mirrored on both diagonals of the pay-off matrix. Hence, I define such pay-off matrices as "double mirror-symmetric". Finally, a third type of pay-off symmetry may occur. If $a_i = d_j$, $b_i = b_j$ and $c_i = c_j$, pay-offs are only mirrored on the main diagonal. I define such a pay-off matrix as "mirror-symmetric". In the case, where the original pay-off matrix cannot be transformed into one of the previous structures by the *same* positive affine transformation of all pay-off values (i.e. one that maintains the relative loss values), the pay-off matrix is defined as "asymmetric".[20]

In the following the main proposition of this section is stated. The proof (as all the others) are to be found in the appendix. Using the previous definitions of the pay-off matrices:

Proposition 2.1.V. *Given the case, where condition 2.1.2 on page 53 holds and resistances are defined by:*

$r^s_{AB} = \alpha s_1(A) \wedge \beta s_2(A)$ *and* $r^s_{BA} = (1-\alpha)s_1(B) \wedge (1-\beta)s_2(B)$ *in the case of state dependent sample size or*

$r^\gamma_{AB} = \alpha\gamma_2(A) \wedge \beta\gamma_1(A)$ *and* $r^\gamma_{BA} = (1-\alpha)\gamma_2(B) \wedge (1-\beta)\gamma_1(B)$ *in the case of state dependent error size.*

In the case of state dependent sample size, the original results of stochastic stability are confirmed if the pay-off structure exhibits some form of symmetry, i.e. if it is either symmetric, mirror-symmetric or double mirror-symmetric. Results do not necessarily coincide if pay-offs are asymmetric. Yet, the asymmetric case confirms the results if the sample size is a function of the relative instead the absolute potential loss, i.e. if it is independent of any positive affine transformation of the pay-off matrix. This also holds for the state dependent error size.

In the case, where both error and sample size are state dependent, the reduced resistances for pay-off matrix 2.1.1 can be generalised to

[20] A similar definitions can be found in Weibull, 1995. For examples of these matrices, see pages 67ff.

$r^{s\gamma}_{AB} = \alpha s_1(A)\gamma_2(A) \wedge \beta s_2(A)\gamma_1(A)$ and $r^{s}_{BA} = (1-\alpha)s_1(B)\gamma_2(B) \wedge (1-\beta)s_2(B)\gamma_1(B)$. Furthermore, note that this approach also yields a positive relation between risk-aversion and surplus share, if the assumption of type independence of error and sample size is relaxed:

Proposition 2.1.VI. *In a double mirror-symmetric coordination game with two pure Nash equilibria, the player type that is less risk-averse can appropriate the greater share of the surplus in the case, where sample size is state dependent. In the case, where error size is state dependent, this result holds if $\gamma_i(\omega)$ is strictly convex in $l_i(\omega)$. If the function is strictly concave, the more risk-averse player appropriates the greater surplus share.*

Being more open to taking risks can, ceteris paribus, benefit the player type. This result is coherent with findings in the economic literature (King, 1974, Rosenzweig & Binswanger, 1993, Binmore, 1998 and for a critical discussion of empirical studies, see Bellemare & Brown, forthcoming) on the positive correlation between wealth and risk. If we take risk as a measure of need[21] , the analytical result of this approach is the obvious relation that the needier one social group is and the less it has to lose (from punishment, social shunning, non-conformity etc.) the more likely the social convention will be defined in its favour.[22]

Intuitive assumption about the sample size, thus lead to the confirmation of the approach of Peyton Young for most interactions. If equation 2.1.2 holds, the approach of Young is unaffected in the case of state dependency for symmetric pay-off configurations or if loss is regarded in relative and not absolute terms. Yet, the results of the state dependent approach, only given equation 2.1.2, will not coincide with the standard approach in all pay-off configurations.[23] The state dependency of the error

[21] "Recall that need is to be measured in terms of the risks that people are willing to take to satisfy their lack of something important to them." See Binmore,1998, p. 463.

[22] This also conforms with Theorem 9.1. in Young, 1996, which shows that conventions are close to a social contract that maximises the relative pay-off of the group with the least relative pay-off.

[23] Also note the impact that state dependency has on the time that a player population requires on average until the equilibrium is upset, i.e. the time a convention endures.

Since the waiting time w is given by $\sum_{t\geq 1} tp(1-p)^{t-1} = \frac{1}{p}$ it holds:

$w = \left[\sum_{i=\alpha N}^{N}\left(\binom{N}{i}\varepsilon^{s\gamma i}(1-\varepsilon)^{s\gamma(N-i)}\right)\right]^{-1}$. First order derivatives for s and γ of this expression are positive. As a consequence, transition time will depend both on the frequency of interactions, and on the loss levels, and hence on absolute pay-off values. Thus, it is even more apparent that we cannot make absolute qualitative statements about the time a population will stay in a certain equilibrium. The critique often raised against this framework that transition will take more time than institutional change might thus miss the point, since all variables that affect transition time are ordinal and can be subjected to positive affine transformations. Further, the interpretation of what defines an interaction or interaction period affects the transition time.

and sample size entails, however, a fundamental issue that will be discussed in the following section.

2.2 The one-third Rule and State Dependency

In the previous analysis, I have only considered pay-off losses in the case, where a society is located close to one of the equilibria, i.e. if a distinct convention prevails. Hence, the loss $l_i(\omega) = \pi^i_{\omega\omega} - \pi^i_{\omega'\omega}$ has been defined as the pay-off difference that occurs if a player chooses his best strategy with respect to the absorbing state ω', though the actual current state is defined by the pure Nash equilibrium state ω. It has been thus assumed that a player only considers maximal potential loss and assigns unit probability to the strategy profile defining the equilibrium. On the one hand, this is a reasonable assumption if a player considers only pure conventional strategies (or in the case of very high discount rates). On the other hand, it might be more realistic to assume that a player evaluates his potential loss according to his sample.

Suppose that a player has to decide whether to experiment or not. Whenever the player population is in a state of transition and moves against the force of the basin of attraction of the current convention towards the other equilibrium, the player observes that other players have been experimenting before. Assume that he observes previous "experimenters" at a rate of p in his sample. Based on this, he might expect that in the current play his counterpart will also experiment with probability p. He thus evaluates his potential loss based on the mixed state, defined by his sample, and on the expected loss value, by considering the loss function $l_1(\omega^p) = \pi_{\omega\omega^p} - \pi_{\omega'\omega^p}$, given that ω^p indicates a state in which players experiment at rate p. Hence, for pay-offs as in matrix 2.1.1 a player i will compare $l_i(A^p) = (1-p)a_i + pb_i - (1-p)c_i - pd_i$ and $l_i(B^p) = pc_i + (1-p)d_i - pa_i - (1-p)b_i$. Note that the previous case is obtained by setting $p = 0$, i.e. the player does not expect his counterpart to experiment. We only apply, however, a linear function to the loss. Not playing the conventional strategy in h_A will always incur an expected loss greater than playing the non-conventional strategy in h_B as long as $a - c > d - b$.

The propositions could be extended from the case of $l_1(\omega)$ to the extended case $l_1(\omega^p)$, and the general results with respect to stochastic stability should persist. This is not done here, since I believe a more fundamental issue is raised by the transition: Relaxing Young's condition of a state independent error, entails that its probability cannot generally assumed to be small, if we consider $l_i(\omega^p)$, since the absolute size of the potential expected loss varies with the strategy distribution in the sample, i.e. with the number of experimenters. That implies that $l_i(\omega^p) \to 0$, as the distribution approaches the interior equilibrium. As the loss grows smaller, the error size increases. At the interior equilibrium expected pay-off from both strategies is identical and hence, no loss results from choosing any of the two strategies. As a consequence, error rates will be high close to the mixed equilibrium distribution.[24] Hence, the zero limit of the error size is inapplicable. Also in other situations error rate can generally be expected to be high and expected loss is generally low.[25] In these cases a stable convention requires additional properties.

Random drift plays a substantial role in the determination of a convention. Furthermore, it is more realistic to assume that it is not necessarily the last element in the collective memory, which is forgotten, since this also requires the individual capability to exactly define the sequence of interactions.[26] The process by which the collective memory of size m is updated can then be described by a stochastic death and birth process - a Moran process (for details, refer to section 2.B in the appendix).

By adapting the approach of Nowak et al. (2006) it can be shown that relaxing the assumption of overall small error rates will require that an equilibrium generates a larger basin of attraction to define the long-term conventional strategy. A predominance of random error creates an additional invasion barrier and further, once sufficiently mixed, drift will push the society towards a completely mixed strategy profile. This counter-acts the selection process, which gravitates the society towards

[24]Notice that equation 2.1.2 on page 53 does not necessary imply that $\gamma_i(\omega) \to 0$ and thus $\epsilon_i(\omega) \to 1$ if $l_i(\omega^p) \to 0$. Yet without any additional assumptions, which limit the error to an upper bound, we can assume that a player chooses his strategy completely at random whenever he observes a distribution, in which he is ambiguous about the prevalent convention.

[25]A high error or mutation rate can be expected if potential loss is generally relatively low in comparison to the pay-offs received in both equilibria. Another examples are cases in which sampling of information is very costly and individuals are only weakly affected by expected pay-offs.

Consider the example of driving on the left or right. Since most people are right handed, keeping left was indeed risk dominant. Nowadays, both conventions can be observed and are stable. They are imposed by law and risks are high to be punished in the case of infraction. For pedestrians this is not the case. Although to keep on the same side as driving a car is the marginal risk dominant strategy, the costs of walking on the same side as the vis-à-vis are low and people pass both on the left and right. Thus, we can observe a mixed equilibrium. Similar reasoning holds for the convention to stop at a red light.

[26]In the original approach, it is supposed that an individual can sample at most half of the history of past interactions, but is still capable to assign a time frame to each interaction. The Moran process, underlying the approach described herein, assigns equal "death" probability to all elements. It takes into account the tendency that individuals remember and over-evaluate rare events more strongly than common events. A rare event is thus less likely to be chosen for death, since it's rate of occurrence is lower in m.

the Nash equilibrium inside the basin of attraction. Hence, a certain minimum basin of attraction is necessary for an equilibrium to exercise sufficient gravitational pull on a society at the 50 : 50-state to overcome the adverse effect of drift. This implies that the resistance on the path towards this equilibrium must be sufficiently small. It holds:[27]

Proposition 2.2.I. *Consider a large population, playing a coordination game with two pure-strategy Nash equilibria, a symmetric pay-off matrix and normalised sample rate* $s_i(\omega) = 1$*. The unstable interior equilibrium is given by a frequency of* $\alpha = \frac{a-c}{a-b-c+d}$ *players choosing strategy B. If error rate is large with respect to selection by adaptive play, the reduced resistance* r_{AB} *(or* r_{BA}*) must be smaller than* $\frac{1}{3}$ *for selection to favour the convention* h_B *(or* h_A*). If* $\alpha \in (\frac{1}{3}; \frac{2}{3})$*, the fixation probability is less than* $\frac{1}{m}$ *for both strategies, and random drift superimposes selection. The fixation probability thereby defines the probability of a mutant strategy to cause a switch from some convention to that defined by the mutant strategy.*

In other words, stochastic stability is a necessary condition for a stable convention, but will not suffice in the case of a high stochastic error rate. If the basin of attraction of all equilibria is insufficiently large, i.e. smaller than 2/3 of the distance between between both equilibria, and hence the transition has a reduced resistance larger than 1/3, the stochastic strategy choice does not favour any strategy. If this stochastic replacement is strong with respect to the best response play, replacement will superimpose selection by invasion. The question to be answered is indeed, whether or not it is a reasonable assumption to expect a low mutation rate, when explaining the evolution of conventions. The higher the risk and hence the potential loss, the longer a convention will persist, since its sample size will be proportionally greater and its error size proportionally smaller. The idea that a society shifts between equilibria, however, requires a population to move through the interior equilibrium during the turnover process. Only if selection based on best response play is strong enough, will the population be able to attain the Stochastically Stable State.[28] If this is not the case, a society ends up in a mixed equilibrium. In such a case a society can only escape this equilibrium by exogenous intervention.[29]

[27]The proof, based on Nowak et al., as well as some extensions can be found in the appendix 2.B.

[28]Remember that in the underlying approach, transition only occurs spontaneous and involuntarily, and is not subject to conscious and deliberate (revolutionary) choice; or following Carl Menger, the institutions in this approach are purely organic.

[29]Coming back to the example of driving on one side. In the absence of public law both strategies can be considered as fairly risk equal. That does not necessarily mean that a transition from one convention to the other can be observed, since potential loss is large, error rate is low and transition

The one-third rule applies to the extreme case, in which error rate is generally considered to be large. This will not be the case close to the pure equilibrium states, at which only very few players experiment, and whenever the potential loss is high. In these cases, the minimum basin of attraction lies between two thirds and one half. A solution to this issue is postponed to later research.[30] Yet, it may be prudent to constrain consideration to direct or indirect paths with edges of reduced resistances smaller than one third as valid potential paths towards a convention defining the stochastic potential.

2.3 Conclusion

Overall, the discussion in this chapter does not fundamentally challenge the stochastic stability approach in general, since it has found identical results under most conditions, yet it questions the *general* viability of the original results. In the context of state dependence, findings in this chapter contrast with the original stochastic stability approach of Peyton Young in two important points: First, we observe that Stochastically Stable States, defined by the state dependent and independent approach, do not necessarily coincide. Both approaches predict the same *SSS* in the case of some symmetry between player types in the pay-off matrix or if the relation between potential loss on the one hand and error and sample rate on the other hand has certain functional form. Yet, in the general case the *state dependent SSS* might not be the one predicted by the original approach.

Second, the correlation between error and sample rate, and individual pay-off casts doubt on whether all possible paths towards an equilibrium can be taken into account, when calculating the *SSS*. If a path is defined by two equilibria that generate approximately the same average pay-offs and thus risk rates, one Nash /conventional strategy cannot successfully invade another. It turns out that in this context, minimum stochastic potential is a necessary, but not a sufficient condition for an *SSS*.

There are two possible interpretations of the one-third rule and thus extensions of

time is consequently high (independent of state intervention). The question is, whether a strategy profile, once completely mixed, would end up in a pure strategy profile, where all individuals drive on identical sides. Habits of Italian and southern French road users may lead to some doubt.

[30] A straight-forward solution is to define the error as a function of the pay-off slope. The higher the slope, the more likely the error approaches zero in the long run. Pay-offs are, however, not absolute and games that show the same structure after a positive affine transformation are considered equivalent. Yet, this transformation should change the pay-off slope and any arbitrary positive affine transformation will thus have an impact on the error convergence.

the stochastic stability criterion that instantly come to mind. Either the rule implies that a Nash equilibrium with resistance larger than one third cannot be invaded by another Nash strategy. It follows that any path (i.e edge) with too high resistance is considered invalid and an i-tree can only be constituted by valid edges. This might eventually lead to a situation, in which no equilibrium is *SSS* as no valid i-tree exists. Alternatively, the rule also entails that if the unstable interior equilibrium is sufficiently close to the centre (i.e. at $1/2$; the $50:50$-state), random replacement superimposes selection. The player population will be continuously drawn back to the interior. Hence, a completely mixed interior equilibrium at $1/2$ can be considered as a potential *SSS*, if it lies on such an invalid path. In order to calculate the stochastic potential of the mixed equilibrium define the two equilibria that are on both vertices of the mixed equilibrium's edge. Then determine the resistances of the valid paths from any other equilibrium towards either of these two equilibria. The smallest sum of these resistances constitutes the stochastic potential of the mixed equilibrium. The feasibility and plausibility of the results obtained, if one of these extensions is applied, as well as the search for alternatives leave room for further research.

This chapter further illustrates the difference between sample and error size. In the case of more than one player type, both variables affect the stochastic potential of an equilibrium in a different way. This difference is of special interest for the determination of an *SSS* for general games without any symmetry in the pay-off matrix. The sole focus on a state dependent error rate is insufficient in these cases.

Yet, the one-third rule only defines the boundary (limit) condition, in which conventions are completely subject to random drift. Usually, this is the case for a small region around the interior equilibrium. One half and one third are consequently the upper and lower bound of the condition that defines the maximum viable reduced resistance. The actual threshold value should be adapted to the type of game and its context. It is therefore necessary to expand the rule in a way that defines a contextual threshold value for the resistances that lies between both values.

2.A Proofs for Stochastic Stability

Before coming to the proofs an example will help to understand the intuition. In order to simplify as much as possible, for the length of this example, I will abstract from the loss - error rate relation 2.1.2 on page 53, and from the assumption of a relatively large sample size as well as the condition that $s \leq \frac{m}{2}$. (The example will also make it evident why this has been initially assumed.)

Example: Consider two players, who meet each other on a road once a day, and have to decide whether to cross on the left or right. Hence, they play a 2×2 coordination game. Assume that players have a very short memory and remember only the last 2 moves ($m_{i,t} = (x_{j,t-1}, x_{j,t})$). Memory size is identical to sample size. Each state of the game can thus be represented by a vector of four components ($h_t = (m_i, m_j)$). Further assume that players are symmetric, therefore $h_t = (m_i, m_j) = (m_j, m_i)$. The 10 possible states are then defined as *(ll,ll), (ll,lr), (ll,rl), (ll,rr), (lr,lr), (lr,rl), (lr,rr), (rl,rl), (rl,rr), and (rr,rr)*. Each player chooses his best response to his memory of the opponent's last two actions. Obviously *(ll,ll)* and *(rr,rr)* are absorbing states, as the best response to rr is always r and to ll always l. Assume that both equilibria provide the same strictly positive pay-off, and that mis-coordination gives zero pay-off. In the case a player has a "mixed memory" of the opponent's play, i.e. rl or lr, he chooses l or r both with probability $\frac{1}{2}$. In the unperturbed Markov process, states *(ll,ll)* or *(rr,rr)* will persist forever, once they are reached. State *(ll,lr)* will move to state *(ll,rl)* or *(lr,rl)*, each with probability $\frac{1}{2}$.

Now assume that a player commits an error with a low probability and does not choose his best response strategy. Let the case, in which he has memory ll and chooses r, occur with probability λ and the second case, in which he has memory rr and chooses l, occur with probability ε. Let the states' position be as in the previous enumeration, starting with *(ll,ll)* and ending with *(rr,rr)*. The transition matrix of the perturbed Markov process is then defined as:

$$P^{\varepsilon} = \begin{pmatrix} (1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\lambda)/2 & \lambda/2 & 0 & (1-\lambda)/2 & \lambda/2 & 0 & 0 & 0 \\ (1-\lambda)/2 & \frac{1}{2} & 0 & 0 & \lambda/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon/2 & \frac{1}{2} & (1-\varepsilon)/2 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon/2 & \varepsilon/2 & 0 & (1-\varepsilon)/2 & (1-\varepsilon)/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon^2 & 2(1-\varepsilon)\varepsilon & (1-\varepsilon)^2 \end{pmatrix} \quad (2.A.1)$$

$P^{\sigma} = \lim_{n \to +\infty; \varepsilon, \lambda \to 0} P^{\varepsilon}$ defines the limit distribution with ε and λ approaching zero at the same rate. If $\lambda = \varepsilon$ each row vector of P^{σ} has components $(0.5\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.5)$. Thus, both equilibrium states occur with equal probability. If $\lambda < \varepsilon$ state (ll,ll) is *SSS*, if $\lambda > \varepsilon$ state (rr,rr) is *SSS*.[31]

If we assume that equilibrium (l,l) generates a larger pay-off than equilibrium (r,r), all states, except (rr,rr), will converge to state (ll,ll) in the unperturbed Markov process. Ceteris paribus, the transition matrix looks as:

$$\begin{pmatrix}
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1-\lambda)^2 & (1-\lambda)\lambda & 0 & (1-\lambda)\lambda & \lambda^2 & 0 & 0 & 0 \\
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\lambda)^2 & 2(1-\lambda)\lambda & \lambda^2 \\
0 & 0 & (1-\lambda)^2 & (1-\lambda)\lambda & 0 & (1-\lambda)\lambda & \lambda^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon(1-\lambda) & (1-\varepsilon)(1-\lambda)+\varepsilon\lambda & (1-\varepsilon)\lambda \\
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon^2 & 2(1-\varepsilon)\varepsilon & (1-\varepsilon)^2
\end{pmatrix} \tag{2.A.2}$$

Since state *(rr,rr)* has no basin of attraction for $m = 2$, we cannot calculate the resistances for various pay-offs. Yet, a change in the relative error size can still shift the *SSS*. For $\lim_{n \to +\infty} P'^{\varepsilon,\lambda}$, $\varepsilon = 0.00001$ and $\varepsilon^{\frac{1}{5}} = \lambda$ each row vector is defined by $(0.02611\ \ 0.0058\ \ 0.00586\ \ 0.00065\ \ 0.00032\ \ 0.00144\ \ 0.00016\ \ 0.00026\ \ 0.00022\ \ 0.95918)$. We observe that though state *(ll,ll)* is risk dominant, the players will spend approximately 96% of the time in state *(rr,rr)*.[32]

Proof of proposition 2.1.I:. (This proof is with the exception of minor changes identical to Theorem 4.1 in Young 1998) Let G be a 2×2 coordination game with the corresponding conventions (pure Nash equilibria) $h_A = (A, A)$ and $h_B = (B, B)$. Let $\mathfrak{B}_i$, with $i = A, B$ represent the equilibria's basins of attraction. In addition, let the pay-offs of the game be symmetric (no sub-population exists, i.e. each player has a positive probability to interact with any other player in his population). Assume that sample size is dependent on the pay-offs at the current convention. Hence, as long as the population is inside the basin of attraction of convention h_A, players sample at a size $s(A)$, in the case they are in $\mathfrak{B}_B$, sample size is $s(B)$. Further, let the memory m be sufficiently large ($s(\omega) \leq m/2$). Let r_{AB} denote the reduced resistance for every path on the z-tree from h_A to h_B as a function of the sample size $s(A)$. Since after entering $\mathfrak{B}_B$ the system converges to h_B without further errors, r_{AB} is the same as the reduced resistance for

[31] e.g. if $\varepsilon = 0.0001$ and $\lambda = \varepsilon^{1.5}$, the population remains in state (ll,ll) almost all time (99%) and basically never in state (rr,rr) ($< 1\%$).

[32] Notice that, however, in this example the *SSS* will ultimately switch to *(ll,ll)* as $\varepsilon \to 0$, since *(rr,rr)* has no basin of attraction. A larger memory of 3 would require a transition matrix of size 36×36

all paths from h_A to $\mathfrak{B}_B$. Let α be defined as above and suppose that the population is in h_A for a sufficiently large time, so that all players have chosen strategy A for m periods in succession. For a player to choose strategy B and for the system to enter $\mathfrak{B}_B$ there must be at least $\alpha s(A)$ times strategy B in the player's sample. This can only happen with positive probability if $\alpha s(A)$ players successively commit the error of choosing action B. The probability of this to occur is at least $\varepsilon^{\alpha s(A)}$. The same logic holds for convention h_B, only that $(1-\alpha)s(B)$ players successively have to make the mistake. This event then happens with order $\varepsilon^{(1-\alpha)s(B)}$. It follows that the resistance from h_A to h_B is thus $r^s_{AB} = \alpha s(A)$ and from h_B to h_A is $r^s_{BA} = (1-\alpha)s(B)$. h_A is stochastically stable iff $r^s_{AB} \geq r^s_{BA}$. □

Proof of proposition 2.1.II:. Assume the same conditions as before except that row players have sample size $s_1(A)$ near h_A and $s_1(B)$ near h_B, and the column players have sample size $s_2(A)$ and $s_2(B)$ respectively and pay-offs are not necessarily symmetric (i.e interaction pairs are given by one row and one column player). Keep in mind that α refers to the share of column players and β the share of row players. Hence, a row player 1 currently playing strategy $x_1 = A$ will only change strategy if there is a sufficient number of column players playing $x_2 = B$ in his sample. For a positive probability of this to happen there must be at least $\alpha s_1(A)$ players committing an error in subsequent periods, occurring with probability $\varepsilon^{\alpha s_1(A)}$. For a column player 2 with $x_2 = A$ to switch there must be a sufficient number of row players playing $x_1 = B$ in his sample. Hence, there must be again at least $\beta s_2(A)$ of these players in m, happening with probability $\varepsilon^{\beta s_2(A)}$. The same reasoning holds for the transition from h_B to h_A. Hence, $r^s_{AB} = \alpha s_1(A) \wedge \beta s_2(A)$ and $r^s_{BA} = (1-\alpha)s_1(B) \wedge (1-\beta)s_2(B)$. □

Proof of proposition 2.1.III:. Now suppose that the rate of mutation is $\epsilon(A) = \varepsilon^{\gamma(A)}$ in $\mathfrak{B}_A$ and $\epsilon(B) = \varepsilon^{\gamma(B)}$ in $\mathfrak{B}_B$ and that pay-offs are symmetric. Assume that sample size is constant and normalised at $s(A), s(B) = 1$, thus is state and pay-off independent. Other conditions are equal to the first proof. Starting in h_A for a system to enter $\mathfrak{B}_B$ with positive probability, again a share of α players successively has to commit the error of choosing action B. For a player to change strategy from A to B there must be thus at least αs players playing strategy B in m, in order to sample a share of α B players with positive probability. By the same logic as above this event occurs with probability $\varepsilon^{\gamma(A)\alpha}$. Congruently, a switch from h_B to h_A happens with probability $\varepsilon^{\gamma(B)(1-\alpha)}$. The resistance from h_A to h_B is thus $r^\gamma_{AB} = \gamma(A)\alpha$ and from h_B to h_A is $r^\gamma_{BA} = \gamma(B)(1-\alpha)$. □

Proof of proposition 2.1.IV:. As in the second proof suppose that pay-offs are not necessarily symmetric and that there exist two inter-acting types of players with state dependent error size $\epsilon_i(\omega)$. Row players have error size $\varepsilon^{\gamma_1(A)} = \epsilon_1(A)$ near h_A and $\varepsilon^{\gamma_1(B)} = \epsilon_1(B)$ near h_2, and column players have error size $\varepsilon^{\gamma_2(A)} = \epsilon_2(A)$ and $\varepsilon^{\gamma_2(B)} = \epsilon_2(B)$ respectively. For convenience assume that sample rate is normalised to $s(A), s(B) = 1$. A row player 1 currently playing strategy A will only change his strategy if there is a sufficient number of column players playing B, i.e. if he encounters a proportion of at least α column players choosing strategy B in his sampled set. For this event to happen with positive probability, there must be $s_1(A)\alpha$ of this column players in m. For a normalised sample size $s_{1,2} = 1$ this happens with a probability of $\epsilon_2(A)^\alpha = \varepsilon^{\alpha\gamma_2(A)}$. A column player has to meet a portion of β row players erroneously playing strategy B. Hence, there must be at least βs_2 such players in m, which occurs with probability $\epsilon_1(A)^\beta = \varepsilon^{\beta\gamma_1(A)}$. For h_B the argument is analogous. Thus $r^\gamma_{AB} = \gamma_2(A)\alpha \wedge \gamma_1(A)\beta$ and $r^\gamma_{BA} = \gamma_2(B)(1-\alpha) \wedge \gamma_1(B)(1-\beta)$. □

If we define the state dependent sample size as in the proof of proposition 2.1.III and error size as in proposition 2.1.IV, it follows that in the case of both state dependent error and sample size the least resistances are given by

$$r^{\gamma s}_{AB} = \gamma_2(A)s_1(A)\alpha \wedge \gamma_1(A)s_2(A)\beta \text{ and } r^{\gamma s}_{BA} = \gamma_2(B)s_1(B)(1-\alpha) \wedge \gamma_1(B)s_2(B)(1-\beta).$$

The following proof is divided into 4 parts, one for each possible pay-off structure. Similarly to the previous proof structure, I will illustrate the easiest case at first, so that the intuitions of the later proofs are more accessible to the reader. The first three show the cases with some sort of symmetry, whereas the last illustrates the asymmetric case.

Proof of proposition 2.1.V:. Assume condition 2.1.2 on page 53 holds, defined as:

$$l_i(\omega) < l_j(\omega') \Leftrightarrow \epsilon_i(\omega) > \epsilon_j(\omega') \Leftrightarrow s_i(\omega) < s_j(\omega') \Leftrightarrow \gamma_i(\omega) < \gamma_j(\omega'), \quad \text{for } i,j = 1,2 \tag{2.A.3}$$

Since error and sample size are only assumed pay-off dependent, but type independent, it must also hold that $s_i(\omega) = s_j(\omega)$ and $\gamma_i(\omega) = \gamma_j(\omega)$ iff $l_i(\omega) = l_j(\omega)$, and $s_i(\omega) < s_j(\omega)$ and $\gamma_i(\omega) < \gamma_j(\omega)$ iff $l_i(\omega) < l_j(\omega)$ for $i \neq j$. For the easiest case with symmetric pay-offs and only a single player type assume the following symmetric pay-off structure as given in table 2.A.a.

In this case equilibrium (A, A) is risk dominant equilibrium iff $\alpha > (1-\alpha)$, thus

Table 2.A.a: symmetric pay-off structure

	A	B
A	a, a	b, c
B	c, b	d, d

iff $a + b > d + c$. Assume this to hold. From 2.A.3 we obtain $l_i(A) > l_i(B) \Leftrightarrow s_i(A) > s_i(B)$ as $a - c > d - b$. Thus $s_i(A)\alpha > s_i(B)(1 - \alpha)$. We also obtain that $\epsilon_i(A) < \epsilon_i(B) \Leftrightarrow \gamma_i(A) > \gamma_i(B)$ and hence $\gamma_i(A)\alpha > \gamma_i(B)(1 - \alpha)$.

Consider the case of a double-mirror symmetric coordination games in table 2.A.b [33]:

Table 2.A.b: double mirror-symmetric pay-off structure

	A	B
A	a, d	b, b
B	c, c	d, a

For this pay-off structure $\alpha = (1 - \beta)$ and $\beta = (1 - \alpha)$. If $\alpha < \beta$, then $(1 - \alpha) > (1 - \beta)$ and the reduced resistance is $r_{AB} = \alpha = r_{BA} = (1 - \beta)$ and both equilibria are SSS. The same holds for $\alpha > \beta$. Furthermore $l_1(A) = a - c, l_1(B) = d - b, l_2(A) = d - b, l_2(B) = a - c$. If $\alpha < \beta$, then $l_1(A) = l_2(B) < l_1(B) = l_2(A) \Leftrightarrow s_1(A) = s_2(B) < s_1(B) = s_2(A)$, or the inverse for $\alpha > \beta$. For simplicity redefine $s_1(A) = s_2(B) = s$ and $s_1(B) = s_2(A) = s^{'}$. Then $r_{AB} = s\alpha \wedge s^{'}\beta$ and $r_{BA} = s^{'}(1 - \alpha) \wedge s(1 - \beta)$. Hence, no equilibrium stochastically dominates in the case of state dependent sample size, since $r^s_{AB} = s\alpha \wedge s^{'}(\beta) = r^s_{BA}$.

Similarly, for the case of state dependent error size redefine $\gamma_1(A) = \gamma_2(B) = \gamma$ and $\gamma_1(B) = \gamma_2(A) = \gamma'$. Then $r^{\gamma}_{AB} = \gamma'\alpha \wedge \gamma\beta$ and $r^{\gamma}_{BA} = \gamma(1 - \alpha) \wedge \gamma'(1 - \beta)$ and both resistances are identical.

In the case of 2×2 conflict games, with the mirror-symmetric pay-off structure as in table 2.A.c:

[33]The pay-off structure of this game type is double mirror symmetric, since it can be transformed into the following game:

	A	B
A	a, b	$0, 0$
B	$0, 0$	b, a

$a = a_{11} - a_{21} = b_{22} - b_{21}$ and $b = a_{22} - a_{12} = b_{11} - b_{12}$.

Table 2.A.c: mirror-symmetric pay-off structure

	A	B
A	a, d	b, c
B	c, b	d, a

Assume without loss of generality that $a > d > b, c$, then $r_{AB} = \beta$ and $r_{BA} = (1 - \alpha)$. Equilibrium (A,A) will be the SSS, iff $d - c > d - b$, hence iff $c < b$. Assume further that this is the case, then $l_1(A) = a - c, l_2(A) = d - c, l_1(B) = d - b, l_2(B) = a - b$ and $s_1(A) > s_2(A) > s_1(B)$ and $s_1(A) > s_2(B)$, since 2.A.3 holds. Define a positive, continuous and strictly increasing function μ, such that $s_i(\omega) = \mu(l_i(\omega))$ and hence $r_{AB} = \mu(d - c)(\frac{d-c}{(.)})$ and $r_{BA} = \mu(d - b)(\frac{d-b}{(.)})$, where $(.) = (a - b - c + d)$. Since $d - c > d - b$, equilibrium (A, A) will be SSS. The same argument holds for $c > b$, in which case h_B is SSS.

In the case of state dependent error size and for assumption $c < b$, we obtain $\gamma_1(A) > \gamma_2(A) > \gamma_1(B)$ and $\gamma_1(A) > \gamma_2(B)$. Again define a strictly increasing, positive, continuous function η, with $\gamma_i(\omega) = \eta(l_i(\omega))$, such that $r^{\gamma}_{AB} = \eta(d - c)(\frac{a-c}{(.)}) \wedge \eta(a - c)(\frac{d-c}{(.)})$ and $r^{\gamma}_{BA} = \eta(a - b)(\frac{d-b}{(.)}) \wedge \eta(d - b)(\frac{a-b}{(.)})$, where $\eta(l_i(\omega)) = \gamma_i(\omega)$. Hence, if $c < b$ it must hold that $min \left\{\eta(d - c)(\frac{a-c}{(.)}); \eta(a - c)(\frac{d-c}{(.)})\right\} >$ $min \left\{\eta(a - b)(\frac{d-b}{(.)}) \wedge \eta(d - b)(\frac{a-b}{(.)})\right\}$ and h_A is the SSS. By the same reasoning, for $c > b$ it holds that $r^{\gamma}_{AB} < r^{\gamma}_{BA}$ and thus h_B is the SSS.

Assume the general case of 2×2 conflict-coordination games, with the asymmetric pay-off structure of the following table:

Table 2.A.d: asymmetric pay-off structure

	A	B			A	B
A	a_{11}, b_{11}	a_{12}, b_{12}	$\Rightarrow$	A	a, b	$0, 0$
B	a_{21}, b_{21}	a_{22}, b_{22}		B	$0, 0$	c, d

The first pay-off matrix is equivalent to the second, given $a = a_{11} - a_{21}$, $b = b_{11} - b_{12}$, $c = a_{22} - a_{12}$ and $d = b_{22} - b_{21}$. The definition in the right matrix will be used in the following, as the transformation will not affect the loss size and thus results, but will simplify notation. For this pay-off matrix the frequencies are given by $\alpha = \frac{a}{a+c}$, and $\beta = \frac{b}{b+d}$. Define as before $s_i(\omega) = \mu(l_i(\omega))$, and $\gamma_i(\omega) = \eta(l_i(\omega))$. If

for both player types the same equilibrium risk dominates, the solution is trivial. For $a > c$ and $b > d$, it always holds that
$min\{\alpha s_1(A); \beta s_2(A)\} > min\{(1-\alpha)s_1(B); (1-\beta)s_2(B)\}$ and also
$min\{\alpha\gamma_2(A); \beta\gamma_1(A)\} > min\{(1-\alpha)\gamma_2(B); (1-\beta)\gamma_1(B)\}$. Hence, h_A is SSS.

For $a > c$ and $d > b$ we obtain $\alpha > 1-\alpha$ and $1-\beta > \beta$. Hence, $\alpha > \beta$ and $1-\beta > 1-\alpha$. Consequently, there are two possibilities. Either $\beta > 1-\alpha$ (h_A is SSS) or $\beta < 1-\alpha$ (h_B is SSS).

State dependent sample rate: Define as before that $s_i(\omega) = \mu(l_i(\omega)$. Then by assumption $\mu(a)\alpha > \mu(c)(1-\alpha)$ and $\mu(b)\beta < \mu(d)(1-\beta)$. Under these conditions theoretically four cases can occur:
1. case: If $\mu(a)\alpha < \mu(b)\beta$, then $c < a < b < d$ and thus, $\mu(c)(1-\alpha) < \mu(d)(1-\beta)$. In this case h_A is SSS.
2. case: If $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$,then $b < d < c < a$ and thus, $\mu(a)\alpha > \mu(b)\beta$. In this case h_B is SSS.

Hence, the results for the state dependent sample size do not necessarily coincide with the state independent case.
3. case: The indeterminate case occurs, if $\mu(a)\alpha > \mu(b)\beta$ and $\mu(c)(1-\alpha) < \mu(d)(1-\beta)$. Depending on the relative size of b and c and the order of $\mu(l_i(\omega)$ the state dependent solution will differ from the original approach.
4. case: A contradiction occurs, if $\mu(a)\alpha < \mu(b)\beta$ and $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$. The case contradicts with the assumption that $a > c$ and $d > b$.

As a result only for $\mu(a)\alpha < \mu(b)\beta$ and originally $h_A = SSS$, and for $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$ and originally $h_B = SSS$, the state independent results are definitely confirmed.

State dependent error rate: As before define $\eta(l_i(\omega)) = \gamma_i(\omega)$. The reduced resistances are then given by $r^{\gamma}_{AB} = \eta(b)\frac{a}{a+c} \wedge \eta(a)\frac{b}{b+d}$ and $r^{\gamma}_{BA} = \eta(d)\frac{c}{a+c} \wedge \eta(c)\frac{d}{b+d}$. Without further assumptions on $\eta(l_i(\omega))$ no definite results can be obtained.

Assume that both $\hat{\mu}(l_i(\omega))$ and $\hat{\eta}(l_i(\omega))$ are defined as such that they are not subject to any positive affine transformation, thus $s_1(A) = \hat{\mu}(\alpha)$ and $\gamma_1(A) = \hat{\eta}(\alpha)$, and define the remaining sample and error rates equivalently. This implies that a player only regards his potential loss in relative terms and not in absolute pay-offs.

For the state dependent sample size the resistances are $r^s_{AB} = \hat{\mu}(\alpha)\ \alpha \wedge \hat{\mu}(\beta)\ \beta$ and $r^s_{BA} = \hat{\mu}(1-\alpha)\ (1-\alpha) \wedge \hat{\mu}(1-\beta)\ (1-\beta)$. For $1-\alpha < \beta$, it follows that h_A is SSS, and for $1-\alpha > \beta$, it is obtained that h_B is SSS.

For the state dependent error size the resistances are thus $r^\gamma_{AB} = \hat{\eta}(\beta)\ \alpha \wedge \hat{\eta}(\alpha)\ \beta$ and $r^\gamma_{BA} = \hat{\eta}(1-\beta)\ (1-\alpha) \wedge \hat{\eta}(1-\alpha)\ (1-\beta)$. Hence, for $1-\alpha < \beta$ and given the former assumptions, it must be that $\alpha > 1-\beta > \beta > 1-\alpha$ and thus $min\,\{\hat{\eta}(\beta)\ \alpha; \hat{\eta}(\alpha)\ \beta\} > min\,\{\hat{\eta}(1-\beta)\ (1-\alpha); \hat{\eta}(1-\alpha)\ (1-\beta)\}$. As a consequence, it follows that h_A is SSS. In the same way, if $1-\alpha > \beta$ it must hold that $r^\gamma_{AB} < r^\gamma_{BA}$ and h_B is SSS.

Consequently, in the case losses are considered relative and are independent of a positive pay-off transformation that does not change the game structure, state dependency confirms the results obtain in the standard approach. This is not necessarily the case for any function of the sample and error size, if pay-offs show no form of symmetry.[34] □

A short example will illustrate these results. Suppose the following pay-off matrix:

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 16,6 & 0,0 \\ 0,0 & 10,8 \end{pmatrix} \end{array} \tag{2.A.4}$$

Hence, $\alpha = \frac{8}{13}$, $(1-\alpha) = \frac{5}{13}$, $\beta = \frac{3}{7}$, and $1-\beta = \frac{4}{7}$. As a result it holds, that $h_A = SSS$. For the general case we obtain $r^s_{AB} = \mu(6)\ \frac{3}{7}$ and for h_A to be SSS under the assumption of state dependent sample size it must hold that $\mu(6)\ \frac{3}{7} > \mu(10)\ \frac{5}{13}$ (and $\mu(10)\ \frac{5}{13} < \mu(8)\frac{4}{7}$), which is not the case for all functional forms of $\mu(.)$.[35]

[34] The pay-off matrix 2.A.d can be further refined to $\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} a,b & 0,0 \\ 0,0 & c,c \end{pmatrix} \end{array}$. The positive affine transformation will, however, change the relative loss level. Assuming $a > c > b$, from which we obtain $\alpha = \frac{a}{a+c} > 1-\beta = \frac{c}{b+c} > \beta = \frac{b}{b+c} > 1-\alpha = \frac{c}{a+c}$. Thus, $r^\gamma_{BA} = \gamma(c)\frac{c}{a+c}$ for sure, which will always be smaller than $\gamma(a)\frac{b}{b+c}$. It is therefore sufficient to proof that $\gamma(c)\frac{c}{a+c} < \gamma(b)\frac{a}{a+c}$, if $\gamma(a)\frac{b}{b+c} > \gamma(b)\frac{a}{a+c}$. We obtain $1 < \frac{c}{b} < \frac{a+c}{b+c} < \frac{a}{c} < \frac{a}{b}$ and $\frac{\gamma(b)a}{\gamma(a)b} < \frac{a+c}{b+c}$. Unfortunately the assumptions are still insufficient to prove that $\frac{a}{c} > \frac{\gamma(c)}{\gamma(b)}$.

[35] Notice that $b < d < c < a$ is not a sufficient condition for $SSS = h_B$, see condition 2. above.

For the state dependent error size it holds $r^{\gamma}_{BA} = \eta(8)\ \frac{5}{13}$. Hence, this must be strictly smaller than min $\{\eta(6)\ \frac{8}{13}\ ,\ \eta(16)\ \frac{3}{7}\}$, which again is not fulfilled for all functional forms of $\eta(.)$.

If we restrict the form of $\mu(.)$ and $\eta(.)$ to the assumptions above, we obtain: $r^{s'}_{AB} = \min\{\hat{\mu}(\frac{8}{13})\ \frac{8}{13}\ ,\ \hat{\mu}(\frac{3}{7})\ \frac{3}{7}\}$ and $r^{s'}_{BA} = \min\left\{\hat{\mu}(\frac{5}{13})\ \frac{5}{13}\ ,\ \hat{\mu}(\frac{4}{7})\ \frac{4}{7}\right\}$. Thus, $r^{s'}_{AB} > r^{s'}_{BA}$. Further $r^{\gamma'}_{AB} = \min\{\hat{\eta}(\frac{3}{7})\ \frac{8}{13}\ ,\ \hat{\eta}(\frac{8}{13})\ \frac{3}{7}\}$ and $r^{\gamma'}_{BA} = \min\left\{\hat{\eta}(\frac{4}{7})\ \frac{5}{13}\ ,\ \hat{\eta}(\frac{5}{13})\ \frac{4}{7}\right\}$. Hence, either $\hat{\eta}(\frac{3}{7})\ \frac{8}{13} > \hat{\eta}(\frac{5}{13})\ \frac{4}{7}$ or $\hat{\eta}(\frac{8}{13})\ \frac{3}{7} > \hat{\eta}(\frac{4}{7})\ \frac{5}{13}$, resulting in $r^{\gamma'}_{AB} > r^{\gamma'}_{BA}$. Consequently, in the constrained case, $h_A = SSS$ both for state dependent sample and error size.

Proof of proposition 2.1.VI:. Assume a double-mirror symmetric game with two Nash equilibria of the form

$$\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} \text{a,b} & 0{,}0 \\ 0{,}0 & \text{b,a} \end{pmatrix} \end{array} \tag{2.A.5}$$

In this case the frequencies are as such that $\alpha = 1 - \beta$ and $1 - \alpha = \beta$. Hence in $r_{AB} = r_{BA}$ and each equilibrium is SSS. Assume without loss of generality that player type 1 (row player) is less risk averse than player type 2 (column player) and that he has a higher surplus in h_A than in h_B and the inverse for type 2, i.e. $a > b$. Since player 1 is less risk averse, it can either be expected that $s_1(\omega) < s_2(\omega')$, or $\gamma_1(\omega) < \gamma_2(\omega')$, where ω and ω' indicate state h_A or h_B. Further, we know that $\alpha > (1 - \alpha)$.

In the case of state dependent sample size the resistances are rewritten as: $r^s_{AB} = \alpha\ s_1(A) \wedge (1-\alpha)\ s_2(A)$ and $r^s_{BA} = (1-\alpha)\ s_1(B) \wedge \alpha\ s_2(B)$. It must hold that $s_1(A) > s_1(B)$ and $s_2(B) > s_2(A)$, but also that $s_1(A) < s_2(B)$ and $s_1(B) < s_2(A)$. Hence, $r^s_{BA} = (1-\alpha)\ s_1(B)$, which is smaller than both values for r^s_{AB}. Consequently, $r^s_{AB} > r^s_{BA}$ and h_A will be SSS. Hence, the less risk averse player type 1 can gain a higher surplus.

For the case of state dependent error size, define two positive and strictly increasing functions u and v as such that $u(.) > v(.)$, $u(0), v(0) = 0$ (determined by pay-off function in section 2.1) and $u'(.), v'(.) > 0$. Let $\gamma_1(\omega) = v(l_1(\omega))$ and $\gamma_2(\omega) = u(l_2(\omega))$, and hence, $\gamma_1(A) = v(a)$, $\gamma_1(B) = v(b)$, and $\gamma_2(A) = u(b)$, $\gamma_2(B) = u(a)$.

The resistances are $r^{\gamma}_{AB} = \alpha u(b) \wedge (1-\alpha)v(a)$ and $r^{\gamma}_{BA} = (1-\alpha)u(a) \wedge \alpha v(b)$.

Four possible outcomes can occur -

1. case: $r^{\gamma}_{AB} = \alpha u(b)$ and $r^{\gamma}_{BA} = \alpha v(b)$. From the minimum conditions of the resistances it must be that $\frac{a}{b} < \frac{v(a)}{u(b)} < \frac{u(a)}{v(b)}$. It must further hold that $u(.)$ and $v(.)$ are convex in pay-offs a and b. As $u(.) > v(.)$, $h_A = SSS$.

2. case: $r^{\gamma}_{AB} = \alpha u(b)$ and $r^{\gamma}_{BA} = (1-\alpha)u(a)$. In this case it must hold $\frac{u(a)}{v(b)} < \frac{a}{b} < \frac{v(a)}{u(b)}$. This contradicts the assumptions that $u(.) > v(.)$.

3. case: $r^{\gamma}_{AB} = (1-\alpha)v(a)$ and $r^{\gamma}_{BA} = \alpha v(b)$. Thus, $\frac{u(a)}{v(b)} > \frac{a}{b} > \frac{v(a)}{u(b)}$. Since $u(.) > v(.)$, no further restrictions on the functions' slope can be derived. For $h_A = SSS$ only if $\frac{a}{b} < \frac{v(a)}{v(b)}$, which holds in the case $v(.)$ is a strictly convex function. If the function is strictly concave $h_B = SSS$.

4. case: $r^{\gamma}_{AB} = (1-\alpha)v(a)$ and $r^{\gamma}_{BA} = (1-\alpha)u(a)$. For this inequality to occur, it must be that $\frac{a}{b} > \frac{u(a)}{v(b)} > \frac{v(a)}{u(b)}$. It must further hold that $u(.)$ and $v(.)$ are concave in pay-offs a and b. As $u(.) > v(.)$, $h_B = SSS$.

Hence, if $u(.)$ and $v(.)$ are strictly convex, then h_A is SSS. If both functions are strictly concave, then h_B is SSS. □

2.B The one-third rule

The following proof is based and adapted from Nowak et al. (2006):

Assume as before a general 2×2 coordination game with two strict Nash equilibria in pure strategies. For simplicity assume that pay-offs are symmetric as in the following symmetric matrix:

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} a,a & b,c \\ c,b & d,d \end{pmatrix} \end{array} \tag{2.B.1}$$

Further assume that two players are randomly paired. Since none can observe a priori the other player's strategic choice, each asks s other players among the m players in his population what strategy they have chosen in previous periods. (An alternative interpretation of adaptive play is that the same player faces an identical choice m times during his life, but recalls only for s incidences which strategies has been chosen by his counterparts.) Based on his sample, each chooses the strategy for the current game.

The collective memory of the other players' strategy choice (and his own) during $\frac{m}{2}$ past plays is sufficiently described by a matrix of size $m \times m$, since it resembles the following random Moran process: After each interaction both players retain the memory of the other's current play (alternatively they retain their own strategy), and two "old" memories from previous play are forgotten. Yet, for simplicity this process can be approximated by a sequence in which one new memory is born and added to the memory, an old is lost and deleted from the memory. Thereafter the new memory of the second player is added and an old is forgotten. Hence, each play defines two periods in the birth-death process.[36]

Furthermore the old memory, which "dies", is not necessarily the oldest memory. It is in fact supposed that any memory of previous play can be forgotten with equal probability $\frac{1}{m}$. This relaxation both simplifies the following analysis but also augments the degree of realism. The original approach by Young is overly restrictive. A general assumption that the last element in the collective memory dies, requires players to keep track of the precise order of events. Hence, in the following it is assumed that the death of an element (or rather its omission) is not deterministically defined, but is part of a stochastic process.[37]

In such a process two different events can occur in each period. Either a strategy in m substitutes an element indicating a different strategy or the same strategy. Hence, the rate, at which a certain strategy has been played in the memory of m players[38] (or $\frac{m}{2}$ previous plays) can either decrease, increase (each by one unit) or remain unchanged. Define i as the number of memories that strategy A has been played, i.e. i is equal to the frequency with which strategy A occurs in the history of past plays. Consequently, the probability that an element defining strategy A is forgotten is equal to $\frac{i}{m}$, and similarly for the strategy B, this probability is equal to $\frac{m-i}{m}$. One can also

[36]The reason for these simplifications is the following: If the process is determined by the sequence two births and subsequently two deaths the transition of a state is not bounded to its immediate successor or predecessor (one element more or one element less) but can directly transit to the second-order successor or predecessor (two elements more or two elements less), resulting in 4 state dependent transition probabilities instead of two, which tremendously complicates the following derivation. The reason for the strict pay-off symmetry is that it allows to neglect types. In the general case it is required to define two Moran processes. We then obtain two interdependent systems of equations not easily solvable in closed form. Yet, I believe the illustrative purpose of this section is not upset by these additional assumptions as the intention is to illuminate the general dynamics of the process in the case of a high level of idiosyncratic play.

[37]If the memory is updated over a large round of interactions, the process will eventually coincide with the deterministic approach.

[38]Assuming that players are only drawn once. Hence, more correctly it is m memories of some number of players. If a player had been drawn twice he retains two memories.

interpret this assumption as more unique events being less likely to be forgotten.

Given the underlying transition probabilities $p_{i,i+1}$ and $p_{i,i-1}$ to move from state i to i+1 and i-1 respectively, the Markov process is defined by a three-diagonal transition matrix of the following form

$$T = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ p_{1,0} & 1-p_{1,0}-p_{1,2} & p_{1,2} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & p_{m-1,m-2} & 1-p_{m-1,m-2}-p_{m-1,m} & p_{m-1,m} \\ 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \tag{2.B.2}$$

Hence, i=0 (corresponding to state h_B) and i=m (corresponding to state h_A) are absorbing states. Call q_i the probability to reach the absorbing state i=m, hence

$$\begin{aligned} q_0 &= 0 \\ q_i &= p_{i,i-1}q_{i-1} + (1 - p_{i,i-1} - p_{i,i+1})q_i + p_{i,i+1}q_{i+1} \\ q_m &= 1 \end{aligned} \tag{2.B.3}$$

Define the difference in subsequent transition probabilities as $y_i = q_i - q_{i-1}$. Consequently, $\sum_{i=1}^{m} y_i = q_m - q_0 = 1$. Furthermore $\sum_{i=1}^{m} y_i = \sum_{j=1}^{m-1} \prod_{k=1}^{j} \frac{p_{k,k-1}}{p_{k,k+1}} q_1 = 1$ and $q_i = q_1 \left(1 + \sum_{j=1}^{i-1} \prod_{k=1}^{j} (\frac{p_{k,k-1}}{p_{k,k+1}})\right)$.[39] We obtain

$$\begin{aligned} q_1 &= \frac{1}{1 + \sum_{j=1}^{m-1} \prod_{k=1}^{j} (\frac{p_{k,k-1}}{p_{k,k+1}})} \\ q_i &= \frac{1 + \sum_{j=1}^{i-1} \prod_{k=1}^{j} (\frac{p_{k,k-1}}{p_{k,k+1}})}{1 + \sum_{j=1}^{m-1} \prod_{k=1}^{j} (\frac{p_{k,k-1}}{p_{k,k+1}})} \end{aligned} \tag{2.B.4}$$

The fixation probability ρ_x defines the probability that an individual strategy x, mutant to the current convention of $m-1$ states that are only defined by strategy x', can sufficiently proliferate to finally reach the absorbing state, in which all individuals played strategy x in memory m, i.e the probability of switching conventions. Clearly $\rho_A = q_1$ and $\rho_B = 1 - q_{m-1}$. Thus, for the general case we obtain:

[39] As $q_i = \frac{p_{i,i-1}q_{i-1}+p_{i,i+1}q_{i+1}}{p_{i,i-1}+p_{i,i+1}}$, it follows that $y_i = \frac{p_{i,i+1}(q_{i+1}-q_{i-1})}{p_{i,i-1}+p_{i,i+1}}$ and $y_{i+1} = \frac{p_{i,i-1}(q_{i+1}-q_{i-1})}{p_{i,i-1}+p_{i,i+1}}$, hence $y_{i+1} = \frac{p_{i,i-1}}{p_{i,i+1}} y_i$. Since $y_1 = q_1$, it follows $y_i = \prod_{k=1}^{i-1} (\frac{p_{k,k-1}}{p_{k,k+1}}) q_1$. The second equation results from $q_i = y_i + q_{i-1} = y_i + y_{i-1} + q_{i-2} = \dots$.

$$\rho_A = \frac{1}{1 + \sum_{j=1}^{m-1} \prod_{k=1}^{j} (\frac{p_{k,k-1}}{p_{k,k+1}})}$$
$$\rho_B = \frac{\prod_{k=1}^{m-1} (\frac{p_{k,k-1}}{p_{k,k+1}})}{1 + \sum_{j=1}^{m-1} \prod_{k=1}^{j} (\frac{p_{k,k-1}}{p_{k,k+1}})} \tag{2.B.5}$$

Now assume that an individual calculates his pay-off according to matrix 2.B.1 on page 72 and memory size m. If he were capable to draw an unbiased sample from the entire memory, i.e. $distribution(s) = distribution(m)$, and if he expects this to be a representative account of the strategy distribution in the entire population for the coming period, the expected pay-off is simply defined by the relative frequencies of both strategies. Consequently, he expects that the population consists of i individuals playing A and $m - i$ individuals playing B. Define $\Pi_x(i)$ as the expected pay-off that an individual receives, when playing strategy x in such a population, where the frequency of strategy A is defined by i.[40]. The expected pay-offs for each strategy are then given by

$$\Pi_A(i) = \frac{a(i-1) + b(m-i)}{m-1}$$
$$\Pi_B(i) = \frac{ci + d(m-1-i)}{m-1} \tag{2.B.6}$$

Adaptive play assume that sample size is smaller than memory size ($m \geq 2s$) and, thus, distributions do not necessarily coincide. This can also be represented by a new element in m, which is not deterministically defined solely by equations 2.B.6, but by a stochastic process. An error occurs, when a player draws a skewed sample or idiosyncratically chooses an action at random. The higher the relative expected pay-off of the best response strategy, the more likely this strategy will define the new element in m. This is simply equivalent to inducing an individual to play a non-best response strategy requires a larger share of adverse states (i.e. of non-best strategies) in sample s than the share defined by the unstable interior equilibrium. In other words, the sample needs to be sufficiently skewed, which happens with decreasing probability as the collective memory includes a larger share of states to which the strategy is a

[40]Keep in mind that there are $m - 1$ other individuals and $i - 1$ strategy A players and $m - i$ strategy B players for an individual playing A and similarly for an individual playing B.

non-best response. Since all elements in m are sampled with equal probability, the likelihood of a sufficiently skewed sample will decrease as the relative pay-off of the best-response strategy increases in the current state.

Adaptive play thus supposes that with a certain probability $\varepsilon > 0$ individuals choose a response strategy at random, i.e. they experiment. With probability $\varepsilon(l_i(\omega^p))$ the process does not behave according to the relative expected pay-offs defined in 2.B.6, but completely random. In order to weight the intensity of strategic selection, based on expected pay-offs, and the random choice, write the pay-off functions as

$$\begin{aligned} \pi_A &= \varepsilon + (1-\varepsilon)\Pi_A(i) \\ \pi_B &= \varepsilon + (1-\varepsilon)\Pi_B(i) \end{aligned} \tag{2.B.7}$$

with $\varepsilon \in (0,1)$. Strong selection is thus defined by $\varepsilon \to 0$, which is the case underlying the intuition of Kandori, Mailath and Rob, and Young. Weak selection occurs in the case of a high error rate $\varepsilon \to 1$, if $l_i(\omega^p) \to 0$. These assumptions allow to define the transition probabilities of the Moran process, described in 2.B.2 and 2.B.3. The probability that a new element in the collective memory defines either strategy A or strategy B is $\frac{i\pi_A}{i\pi_A+(m-i)\pi_B}$ or $\frac{(m-i)\pi_B}{i\pi_A+(m-i)\pi_B}$, respectively. Consequently,

$$\begin{aligned} p_{i,i+1} &= \frac{i\pi_A}{i\pi_A+(m-i)\pi_B}\frac{m-i}{m} \\ p_{i,i-1} &= \frac{(m-i)\pi_B}{i\pi_A+(m-i)\pi_B}\frac{i}{m} \end{aligned} \tag{2.B.8}$$

where $\frac{m-i}{m}$ defines the probability that an element in the collective memory that defines strategy B is chosen forth death, and $\frac{i}{m}$ the probability that this element defines strategy A. For the neutral case of $\varepsilon = 1$, in which selection favours neither A nor B, we obtain a completely random process, such that fixation probability is defined as $\rho_x = \frac{1}{m}$.[41] This is intuitive, since each value in the matrix describing the Moran process, has the same probability to spread over the entire memory. The probability for one initial mutant strategy to cause the population to switch from one convention to the other, is equal to $\frac{1}{m}$, which is the probability to finally occur in all m states of

[41]Since, $p_{i,i-1} = p_{i,i+1}$ we can easily solve the system in 2.B.3.

the collective memory.

$$\rho_A = \frac{1}{1+\sum_{j=1}^{m-1}\prod_{k=1}^{j}(\frac{\pi_B}{\pi_A})} \quad (2.B.9)$$

Based on Nowak et al. (2006), the Taylor expansion for $\varepsilon \to 1$ (or $(1-\varepsilon \to 0)$ gives

$$\rho_A = \frac{1}{m}\frac{1}{1-(\gamma m-\delta)(1-\varepsilon)/6} \quad (2.B.10)$$
$$\text{with } \gamma = a+2b-c-2d \text{ and } \delta = 2a+b+c-4d$$

In order for selection to favour strategy A, its fixation probability must be greater than in the neutral case, i.e. $\rho_A > \frac{1}{m}$ and it must be that $\gamma m > \delta$. Hence,

$$a(m-2)+b(2m-1) > c(m+1)+d(2m-4) \quad (2.B.11)$$

If the memory size m is sufficiently large, we obtain

$$a-c > 2(d-b) \quad (2.B.12)$$

Setting the pay-offs from 2.B.6 or 2.B.7 equal, we obtain for a large memory size that the unstable interior equilibrium is defined by a frequency of $\alpha = \frac{a-c}{a-c-b+d}$ players choosing strategy B, defining the basins of attraction for both strict Nash equilibria. By putting in the results of the previous equation 2.B.12, we obtain

$$\alpha > 2/3 \quad (2.B.13)$$

Thus, in order for equilibrium (A, A) to be a SSS, the minimum frequency of idiosyncratic players choosing strategy A to cause a switch of best-response players to this strategy, and thus the reduced resistance to (A,A), is defined by $r_{BA} = 1-\alpha$, and has to be less than one third in the context of weak selection and high error rate. Redefining yields the same result with respect to the resistance of (B, B), i.e. $r_{AB} < 1/3$, whence proposition 2.2.I.

□

2.C References

1. Ades, Alberto; Di Tella, Rafael (1996): "The Causes and Consequences of Corruption", IDS Bulletin Vol. 27, No. 22

2. Bellemare, Marc F.; Brown, Zachary S. (2009): "On the (Mis)Use of Wealth as a Proxy for Risk Aversion, American Journal of Agricultural Economics, forthcoming (submission September 29)

3. Bergin, James & Lipman, Barton L. (1996): "Evolution with State-Dependent Mutations", Econometrica, Vol. 64), pp. 943-56

4. Bicchieri, Christina (2006): "The Grammar of Society - The Nature and Dynamics of Social Norms", Cambridge University Press, New York

5. Binmore, Ken G. (1998): "Game Theory and Social Contract - Volume 2: Just Playing, MIT Press, Cambridge, London

6. Bollinger, Daniel; Hofstede, Geert (1987): "Les Différences culturelles dans le management", Les Editions d'Organisation, Paris

7. van Damme, Eric; Weibull, Joergen W.(1998): "Evolution with Mutations driven by Control Costs", Tilburg University, Discussion Paper, 1998-94, Working Paper Version: September 29
http://arno.uvt.nl/show.cgi?fid=3769

8. Huntington, Samuel P. (1997): "The Clash of Civilizations and the Remaking of World Order", Touchstone Press, New York

9. Huntington, Samuel P.; Harrison, Lawrence E. (2000): "Culture Matters: How Values Shape Human Progress", eds., Basic Books, New York

10. Huntington, Samuel P.; Harrison, Lawrence E. (2004): "Streit um Werte", Goldman Verlag, München

11. Kandori, Michihiro; Mailath, George J.; Rob, Rafael (1993): "Learning, Mutation, and Long Run Equilibria in Games", Econometrica, Vol. 61, No. 1 (Jan.), pp. 29-56

12. King, Allan G. (1974): "Occupational Choice, Risk Aversion, and Wealth", Industrial and Labor Relations Review, Vol. 27, No. 4 (Jul.), pp. 586-596

13. Nowak, Martin A. (2006): "Evolutionary Dynamics - Exploring the Equations of Life", Harvard University Press, Cambridge, London

14. Parsons, Talcott (1964): "Evolutionary Universals in Society", American Sociological Review, Vol. 29, No. 3, pp. 339-357

15. Potts, Jason (2000): "The New Evolutionary Microeconomics", Edward Elgar

16. Rosenzweig, Mark R.; Binswanger, Hans P. (1993): "Wealth, Weather Risk and the Composition and Profitability of Agricultural Investments", The Economic Journal, Vol. 103, No. 416 (Jan.), pp.56-78

17. Weber, Max (1922): Wirtschaft und Gesellschaft, area Verlag, 2007

18. Weibull, Jörgen (1995): "Evolutionary Game Theory", MIT Press, Cambridge, London

19. Welzel, Christian; Inglehart, Ronald (1999): "Analyzing Democratic Change and Stability: A Human Development Theory of Democracy", Wissenschaftszentrum Berlin für Sozialforschung GmbH (WZB), Berlin

20. Young, H. Peyton (1993): "The Evolution of Conventions", Econometrica, Vol. 61, No. 1 (Jan.), pp. 57-84

21. Young, H. Peyton (1998): "Individual Strategy and Social Structure", Princeton University Press, Princeton, Oxford

People seldom improve when they have no other model, but themselves to copy after.

Oliver Goldsmith (1728-1774)

3

The Dynamics of Conventions & Norms: - in Spatial Games -

This chapter investigates the generalisability of the predictions made by the stochastic stability approach, developed by Peyton Young, by adapting the initial assumptions to a more realistic context. Focus is placed on 2×2 Nash coordination games. Players are not paired at random nor make strategic choice based on the adaptive play assumption; instead, players consider only strategic choice of members of a reference group in the former period. Individual pay-off is further defined by the current strategic choice of a reference group, which does not necessarily coincide with the former. Specific conditions for a long-run equilibrium are derived if both reference groups are identically defined by the surrounding players on a spatial grid (Moore neighbourhood). In this context and in contrast to Young's approach, a player population converges to the Pareto dominant though risk inferior convention for a broad range of pay-off configurations. Additional general results are obtained, if both reference groups are of different size. As the reference group increases, a player population converges more likely to the risk dominant though Pareto inferior equilibrium.

Introduction

The inter-correlation between cultural and economic variables has already been emphasized by Max Weber in his œuvre "Wirtschaft und Gesellschaft" (1922), but also more recently, prominent scholars have stressed the necessity to give culture a proper recognition as an economic determinant (Huntington & Harrison, 2000 & 2004; Welzel & Inglehart, 1999; Huntington, 1997; Ades & Di Tella, 1996; Bollinger & Hofstede, 1987).[1] "Economic reality is necessarily embedded within broader social relations, culture and institutions, and the real boundaries between the 'economy', and 'society' and 'polity' are fuzzy and unclear." (Hodgson, 1996, p.8). Agents take decisions subject to cultural constraints defined by the current social norms and conventions. Besides, these decisions define the basis for new norms and conventions, thus altering the rules for future interactions (Biccheri, 2006). The dynamics of social conventions and norms are therefore of particular economic interest and the influential paper "The Evolution of Convention" by Peyton Young (1993) is of special importance under this perspective.[2] This approach allows to discriminate between the various Nash equilibria that can occur in a game, and defines the *stochastically stable state* (SSS). This equilibrium state determines the convention and hence the strategies followed by agents in the long-term.

Though the adaptive play assumption provides a more realistic setting of play than classic approaches by adding bounded memory, idiosyncratic play and random pairing, other assumptions are still overly restrictive for certain settings that constitute a basis for social interactions. This chapter analyses the generalisability and robustness of the predictions made by Young's approach, i.e. it asks the question if the general results of the stochastic stability are maintained if some of the fundamental assumptions are changed to fit a more realistic context. Two major factors are considered.

First, Young's initial framework[3] assumes that interactions are not local, but each individual samples any play from a past history of plays and is paired with any other individual in his population, both with positive probability.[4] This is, however, a fairly unrealistic assumption for large and dispersed player populations or if

[1] As Daniel Etounga-Manguelle formulated poignantly "Culture is the mother, institutions are the children" (Manguelle in Huntington & Harrison, 2004, page 135).

[2] along with the article by Kandori, Mailath & Rob (1993)

[3] This initial assumption is relaxed in Young, 1998, and Durlauf & Young, 2001

[4] In the context of various player types, the interaction of an individual is restricted only to players of the other types. Yet, also here any player in such a sub-population can be paired with the individual.

individual perception of conventions is exclusively shaped by the interaction with a reference group (parents, family, friends, colleagues etc.). In order to localize interactions, agents are placed on a torus shaped two dimensional grid and only interact with their surrounding neighbours, turning the original setting into a "spatial game", where connections between players, represented by their relative spatial position, are of relevance.

Second, under more realistic conditions, the determination of a best-response strategy requires superior mental capabilities; even under the assumption of adaptive play. Therefore adaptive (best response) play is substituted by a heuristic that imitates the most successful player.

In these simplified networks, a prevailing convention will not necessarily be defined by the SSS. The positive pay-off difference that players earn in a certain equilibrium with respect to the other equilibria, will affect its likelihood to determine the long-term social convention. In the case of local interaction and imitation, a trade-off between risk and Pareto dominance can thus be observed. This relation is a result of an interesting property of these networks: In contrast to evolutionary games that suppose global interactions and best-response play, it is redundant to assume *a priori* the assortment among players with the same strategy. On the contrary, assortment is an immanent evolving property of this network structure. Consequently, dynamics will differ, since assortment will place more weight on the diagonal elements in the pay-off matrix. In this context, Pareto dominance can prevail over risk-dominance.

The first section considers a symmetric 2×2 coordination game. The second section will generalise the approach to two types of players. The third section will look at the two dimensions of space. In this model individuals have a space which they observe, i.e. an area that defines the set of players that can be imitated, and a space which affects their benefits, i.e. the number of other surrounding players that define the individual's pay-off for each strategy in accordance to his strategic choice. In the former sections both were restricted to the adjacent players. Under various conditions, however, both spatial dimensions do not necessarily coincide and can be of various sizes. The equilibrium, to which a population will converge, is affected by the relative sizes of both dimensions of space. The higher the size of the "pay-off space" with respect to the "imitation space", the higher the probability that a population converges to the risk dominant equilibrium.

3.1 The Evolution in a Spatial Game

In Young's approach one player from each type class[5] is drawn at random to participate in the game, implying that the game can be played by any possible combination of players only restricted by the affiliation to a certain player type.[6] The approach further assumes that individuals choose their best response by maximising the expected pay-off given each available strategy in their strategy set and the actions sampled from a history of past plays. It is expected that there is a positive probability to sample any of these past plays. The player population is fully connected and interactions are global. Yet, it is more reasonable to assume that most interactions are strictly local, both regarding the pairing and the sampling process.[7]

Furthermore, the determination of a best-response strategy demands each individual to possess the mental capacity to evaluate the exact expected pay-off for each strategy in his strategy set, given an anticipated strategy profile. A player requires thus both, full knowledge of his individual strategy set, and the precise associated pay-offs. Individuals tend, however, to choose a strategy based on simplifying heuristics (see Page, 2007). "Imitate the best action" could be a reasonable heuristic to explain strategical choice (a similar rule has been applied in Robson & Vega-Redondo, 1996).[8] According to this rule each player adopts the strategy of the neighbouring player with maximum pay-off for future play.

On the one hand, several articles have addressed the first issue to some degree (Young, 1998; Young, 2005; Morris, 2000; Lee & Valentinyi, 2000; Ellison, 1993 & 2000), but neglected the second by assuming local interactions with some form of (fictitious) best response play. One the other hand, the literature on evolutionary game theory generally has assumed the inverse, i.e. global interactions and strategic choice via imitation (though probabilistic). The approach, elaborated in this chapter, supposes

[5] Young's assumption can be more generally interpreted as this is only being the case, if a strategy profile is defined by as many strategies as there are types. For example, if only a single type exists in a 2×2 Nash coordination game, obviously 2 players are drawn. However, the arguments against this assumption still remain valid.

[6] A simple example: In the "Battle of Sexes" any man can be paired with any woman in his population, but not with another man.

[7] For a critique of integral / fully connected frameworks refer to Potts, 2000.

[8] This assumption seems especially suitable for this context. As Samuel Bowles stated:"We know that individual behavioural traits may proliferate in a population when individuals copy successful neighbours. So too may distributive norms, linguistic conventions, or individual behaviours underpinning forms of governance or systems of property rights diffuse or disappear through the emulation of the characteristics of successful groups by members of less successful groups." see Bowles, 2006, p.444.

both local interactions and imitation, and generates different results than those obtained in the literature previously stated. Only if *both* assumptions (imitation and local interactions) apply, the Pareto dominant equilibrium, instead of the risk dominant equilibrium, will be selected by a population in the long-term for a broad parameter range. The basic dynamics will be illustrated in this section.

Assume the following:[9]
(I.) All individuals interact on a toroid, two dimensional grid, on which they are initially placed at random,
(II.) individuals only interact with their direct neighbours (Moore neighbourhood),
(III.) individual pay-offs only depend on the individual's strategy and on the strategies played by his neighbours,
(IV.) each individual adopts the strategy of the neighbour with maximum pay-off; if the individual already received not less than the maximum pay-off, he will keep his strategy,
(V.) all players update synchronously and once in each period,
(VI.) updating is deterministic (no mutations) and the outcome of the game is only defined by the initial conditions and distribution, and the pay-off matrix.
These assumptions will keep the analysis as simple as possible and will enable us to predict the population's evolution without the need to run explicit simulations. Yet, simulations are performed both to support results and to visualise dynamics.

3.1.1 Symmetric pay-offs

Let N be a finite population of individuals, in which each player is assigned to a unique individual position on a two-dimensional torus-shaped grid, defined by a coordination tuple (x, y) with $x, y \in \mathbf{N}$. Each individual only interacts with his Moore neighbourhood. Let $\sim$ be a binary relation on N, such that $i \sim j$ means "i is neighbour of j". For an individual i in a patch with coordinates (x_i, y_i), an individual j, with $j \sim i$, is defined as $\{j : (x_j = x_i + v, y_j = y_i + w)\}$, with $v, w = -1, 0, 1$ and $|v| + |w| \neq 0$. Consequently, it is assumed that the binary relation $\sim$ is irreflexive, symmetric and each player has 8 neighbours surrounding him. Define $\mathfrak{N}(i)$ as the set of neighbours of i, such that $\mathfrak{N}(i) \equiv \{j : j \sim i\}$. Initially assume that the pay-offs for two strategies

[9] Similar assumptions have been made in other spatial models, such as Nowak & May (1996). With respect to the approach described in the previous chapter, in this approach, the history of past play is reduced to the last interaction, and sampling and interactions are deterministic and limited to the entire set of a player's neighbours.

$s(i) = A, B$ of player i are given by a symmetric pay-off matrix with a single player type. Hence, it is irrelevant, whether an individual plays as row or column:

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a, a & b, c \\ c, b & d, d \end{pmatrix} \end{array} \tag{3.1.1}$$

with $a > c$ and $d > b$. Define equilibrium (A, A) as h_A and equilibrium (B, B) as h_B. Let A_t be the set of individuals playing strategy A in period t, and B_t the set of individuals playing B in the same period. Further, let $\Theta_t^A(i) \equiv |A_t \cap \mathfrak{N}(i)|$ and $\Theta_t^B(i) \equiv |B_t \cap \mathfrak{N}(i)| = 8 - \Theta_t^A(i)$ be the number of strategy A and B playing neighbours of i. The pay-off of player i at time t is thus defined as

$$\pi_t(i) = \begin{cases} \Theta_t^A(i)a + \Theta_t^B(i)b, & \text{if } s_t(i) = A \\ \Theta_t^A(i)c + \Theta_t^B(i)d, & \text{if } s_t(i) = B \end{cases} \tag{3.1.2}$$

Let $\Pi_t(i) \equiv \left\{ \bigcup_{j \in \mathfrak{N}(i)} \pi_t(j) \cup \pi_t(i) \right\}$ be the joint set of player i's and his neighbours' pay-offs. For the following period, this player chooses a strategy s_{t+1} based on the imitation rule

$$s_{t+1}(i) = \{s_t(k) : \pi_t(k) = sup\left(\Pi_t(i)\right)\} \tag{3.1.3}$$

Though the following analysis is local, it enables us to predict the global evolution based on the given pay-off configuration. The Pareto superior strategy is defined by the Pareto superior equilibrium. The following results are a direct consequence:

I. In the case, where a player chooses the Pareto dominant strategy, i.e. the strategy defined by the largest value on the pay-off matrix's main diagonal, his pay-off increases with the number of neighbouring players choosing the same strategy. The maximum pay-off for this strategy is obtained by individuals only surrounded by players of the same strategy. This also holds for the Pareto inferior strategy, if the matrix's main diagonal pay-off values are strictly greater than the off-diagonal values.

II. Any interior individual, only surrounded by players of the same strategy, has never an incentive to switch, since all players in his neighbourhood play the same strategy. Transitions can only occur at borders of clusters.

III. If an individual, which is completely surrounded by players of his own strategy, plays the Pareto dominant strategy, pay-off is maximal and none of his neighbours will switch to the Pareto inferior strategy.

In order for two equilibria to be risk equivalent it must hold that $a - c = d - b$. The pay-off can thus be written as $d = a + \rho$ and $b = c + \rho$. Define the pay-off difference ρ as the "pay-off premium". For $\rho > 0$ equilibrium h_B Pareto dominates h_A.

Definition: A **cluster of size** r is defined by the highest number of neighbours playing the same strategy in the set of directly connected players with identical strategy, i.e. a cluster of size r is defined as a set of neighbouring players, in which at least one player has r-1 other players with the same strategy in his neighbourhood.

For example, suppose a straight line of players where each player has a neighbour to his left and right with identical strategy, except for the corner elements. Such a straight line will always have a size of 3, since by definition each player has at least one neighbour with 2 identical players, i.e. those, who choose the same strategy, in his neighbourhood. Hence, all players in this cluster will compare any player with a different strategy either to $2a + 6b$ or to $2d + 6c$.[10] The reason for this definition is that the dynamics do not depend on the number of connected players with the same strategy, but on the element with the highest pay-off in the neighbourhood. Consequently, the length of such a line of identical players is unimportant. The same reasoning holds for larger clusters. In addition, this definition restricts cluster size to a maximum value of 9, since a player can only have a neighbouring player with a maximum of 8 identical neighbours. See figure 3.1.B on page 90 for an example: a.) and b.) show two clusters of size 4, whereas c.) shows a cluster of size 5.

Proposition 3.1.I. *Given a pay-off matrix as in 3.1.1 with two risk equivalent pure Nash equilibria, for any a, b, c, d as long as they satisfy $a > c$ and $d > b$, a population, whose convention is defined by the Pareto inferior strategy A, is successfully invaded by a minimal cluster of size r, choosing the Pareto dominant strategy B, if the pay-off premium satisfies:*

$$\left.\begin{array}{l}\rho > 3(a-c) \text{ and } r \geq 4 \text{ and square} \\ \rho > \quad a-c \quad \text{and } r \geq 5\end{array}\right\} \quad \text{for } a < b$$
$$\rho > \frac{3}{5}(a-c) \text{ and } r \geq 6 \qquad\qquad \text{for } a \geq b \tag{3.1.4}$$

[10] This holds also for the outer elements of the line cluster, if $a, d > b, c$. If this is not the case, these outer element have highest pay-off with $a + 7b$ or $7c + d$, if strategy A or B is Pareto inferior.

If $\rho < \frac{1}{2}(a-c)$ or $\rho < \frac{1}{5}(a-c)$, the population will return to the incumbent convention if the invading cluster is of size 6 or 7, respectively. If none of the conditions is fulfilled, clusters of size 6 and 7 are stable but cannot invade the population.

Proof. Assume the aforementioned conditions $a > c$ and $d > b$, and that h_B Pareto dominates h_A. Hence, $d = a + \rho$, with $\rho > 0$. Further assume that the player population is currently in h_A and that there exists a mutant cluster C playing strategy B of size r invading the player population.

For $\zeta = A, B$ define a player $i \in \zeta_t$ iff $\Theta_t^\zeta(i) = |\mathfrak{N}(i)|$ as an *internal*, playing strategy ζ and a player $j \in \zeta_t$ iff $\Theta_t^\zeta(j) < |\mathfrak{N}(j)|$ as an *external*, playing strategy ζ. In other words, an *internal* of strategy ζ is fully surrounded by player of his own kind (i.e. those who play the same strategy), whereas an *external* is situated at the border of a cluster. By the definition of cluster size r: $\exists$ *internal* iff $r = 9$. For notational simplicity define $\Theta_t = \Theta_t^A(i)$, and thus the pay-off as $\pi_t^A(i) = \Theta_t(a-b) + 8b$, if $i \in A_t$, and $\pi_t^B(i) = \Theta_t(c-d) + 8d$, if $i \in B_t$. Two cases can occur; either $a \geq b$ or $b < a$.

1. case $a \geq b$: In this case it holds that $\frac{\partial \pi_t^A(i)}{\partial \Theta_t} > 0$ and $\frac{\partial \pi_t^B(i)}{\partial \Theta_t} < 0$. Since the derivative of a strategy A player is positive in the number of players of his own kind in his neighbourhood and cluster size of the incumbent strategy is 9, then $\max \pi_t^A(i) = 8a$. For the proliferation of an invading cluster of strategy B players it must thus hold that $\pi_t^B(external) > 8a$. As the marginal pay-off of a B player is negative in the number of surrounding A players, the pay-off of such a player increases in the cluster size. For any player $l \in C$ define player k, s.t. $\pi_t(k) = \sup\left(\Pi_t^B(l) : \Pi_t^B(l) = \left\{\bigcup_{h \in \mathfrak{N}(l) \cap C} \pi_t(h) \cup \pi_t(l)\right\}\right)$.

The size of the invading cluster diminishes if l switches, thus and only if $\pi_t^A(j) > \pi_t^B(k)$, with $\pi_t^A(j) = \sup\left(\Pi_t^A(l) : \Pi_t^A(l) = \left\{\bigcup_{h \in \mathfrak{N}(l)/C} \pi_t(h)\right\}\right)$. As the relation between cluster size r and the pay-off of player k is positive, this reduces to $\pi_t(k) = (9-r)c + (r-1)d$ for $r < 9$, since for this cluster size no *internal* exists. For this size it must further hold that $\pi_t^B(k) = \max \pi_t^B(external)$. We thus obtain that the condition for proliferation is defined by $8a < (9-r)c + (r-1)d$ for $r < 9$.

This implies that a cluster of size 5 and smaller can never proliferate, i.e. expand. To see this note that $\pi_t^B(k) = 4c + 4d \not> 8a$, $\forall \rho > 0$ and $a \geq b$. Similarly a cluster of smaller size can never be sustained, i.e. never resists an invasion by the incumbent strategy, as $\pi_t^B(k) \leq 5c + 3d$, being strictly less than $\pi_t^A(j) = \{7a + b \text{ or } 5a + 3b\}$.[11] Hence, minimum cluster size for a successful invasion is 6 and it is sufficient to look at clusters of this size and above.

[11] The second value occurs if the cluster is cross-shaped.

For $r = 6$ it must hold that $3c + 5d > 8a$ in order to proliferate. Since $\pi_t^A(j) = 7a + b$, the condition for the incumbent strategy to invade the mutant cluster is $7a + b > 3c + 5d$. For $r = 7$ the condition for invasion is $7a + b > 2c + 6d$. Solving these equation provides the condition in the proposition. Notice that by definition a cluster of size 8 will persist and, furthermore that clusters will increase to size 9 if the conditions are met. Considering the possible structures of a cluster of size 9, it holds that $\pi_t^B(external) \geq 3c + 5d$. The condition for cluster size 6 is therefore sufficient and necessary for larger cluster sizes.

2. case $a < b$: In this case it holds that $\frac{\partial \pi_t^\zeta(i)}{\partial \Theta_t} < 0$, for $\zeta = A, B$. In other words, the Pareto inferior strategy benefits from the abundance of individuals playing the Pareto dominant strategy in the neighbourhood. Therefore only elements neighbouring the invading cluster have highest pay-off. The condition for proliferation and invasion by the incumbent strategy are thus identical, both are determined by $\pi_t^B(k) > \pi_t^A(j)$ for k and j defined as above.

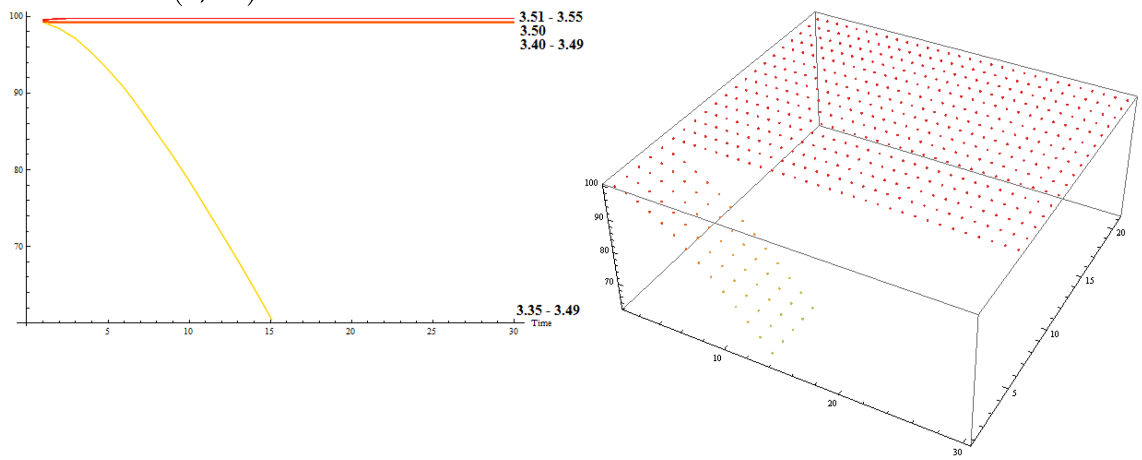
Invading clusters of size smaller than 4 cannot persist, as ρ is defined as strictly positive. For a cluster of size 3 to proliferate it must hold $6c + 2d > 5a + 3b$, which is a contradiction of $\rho > 0$. A cluster of size 4 can only prevail if its structure is such that all its elements have the same pay-off (a square). In this case it must hold that $5c + 3d > 6a + 2b$ and hence $\rho > 3(a - c)$. If it is not square shaped we require $5c + 3d > 5a + 3b$, a contradiction of assumption $a > c$. For cluster size $r = 5$ the condition is $4c + 4d > 5a + 3b$ and hence $\rho > a - c$. Any larger mutant cluster always resists invasion, since $3c + 5d > 5a + 3b$. □

Hence, a minimum pay-off premium $\rho > \frac{3}{5}(a - c)$ is sufficient for a population to converge to the Pareto strategy of the symmetric coordination game, if a cluster of size 6 or greater can evolve with positive probability. Figure 3.1.A[12] shows the result of a set of simulations for $d = 4$, $b = 3$, $c = a - 1$ and a going from 3.35 to 3.55 in steps of 0.01. The values represent the proportion of individuals playing strategy A in $t \in (1, 50)$, where their initial share is set to 85% in $t = 0$.[13] Thresholds are at their expected values. The population converges to equilibrium h_B for values of a smaller than 3.4 and to equilibrium h_A for values larger than 3.5. Stable mixed equilibria occur for intermediate values (here: one at less than 0.8%, a second at 0.4% strategy B players).

[12]All simulations can be reproduced on my homepage http://www.econ-pol.unisi.it/ille $->$ models

[13]Remember that initial seeding is random. An initial share of 15% B players generates a cluster of size 6 with positive probability for the given population size.

Figure 3.1.A: share of strategy A players; d=4, b=3, c=a-1, $a \in (3.3, 3.55; 0.01)$, $t \in (1, 30)$



Proposition 3.1.II. *Clustering is an evolving property and most clusters of at least one strategy will have a size equal to 9 after an initial period of interaction. In addition, for $b > a$ and $\rho > 7(a-c)$, stable single clusters can occur, playing the Pareto inferior strategy A. In the case of $a > b$, clusters, playing the Pareto dominant strategy B, of size 6 can be stable, if $\frac{1}{2}(a-c) < \rho < \frac{3}{5}a - c$, of size 7, if $\frac{1}{5}(a-c) < \rho < \frac{1}{3}(a-c)$, and of size 8, if $0 < \rho < \frac{1}{7}(a-c)$. Cluster of size 5 are stable iff $a = b$.*

Proof. For two clusters C_A and C_B to be neighbours there are at least two players $n \in C_A$ and $m \in C_B$ with $n \sim m$. Define a player j, s.t.
$\pi_t(j) = \sup\left(\Pi(C_A) : \Pi(C_A) = \left\{\bigcup_{i \in C_A} \pi_t(i)\right\}\right)$ and player k, s.t.
$\pi_t(k) = \sup\left(\Pi(C_B) : \Pi(C_B) = \left\{\bigcup_{h \in C_B} \pi_t(h)\right\}\right)$. This implies that player j has highest pay-off in cluster A and k in cluster B. By the definition of cluster size r, $\exists$ *internal* iff $r = 9$.

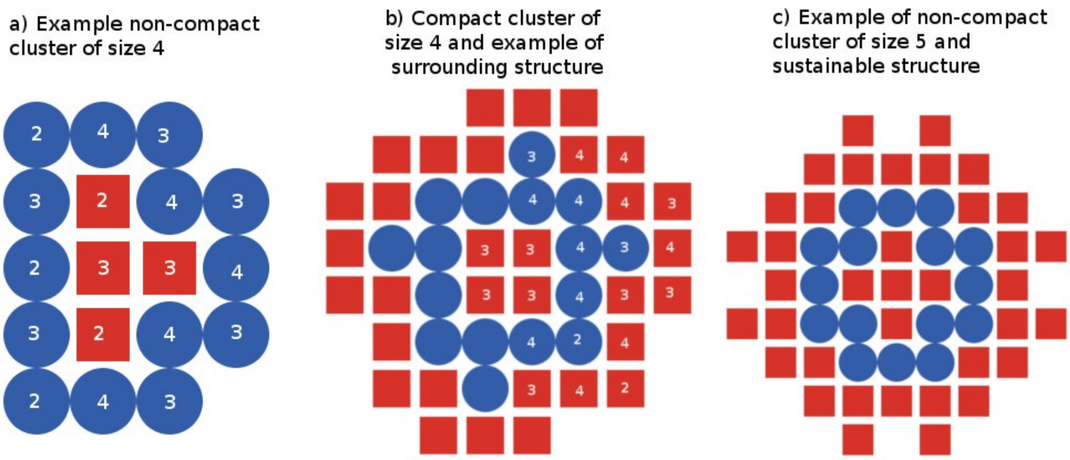
First concentrate on clusters of $r < 9$. It must be that $j, k = \mathit{external}$. With positive probability either $j \sim k$, or $l \sim j, k$ for some player l. For none of the players to switch strategy it must be $\pi_t(j) = \pi_t(k)$. The pay-offs of both elements can be rewritten as $a(\mathfrak{r}_A - 1) + b(9 - \mathfrak{r}_A) = c(9 - r_B) + d(r_B - 1)$. Notice that r_B defines the size of C_B, and $\mathfrak{r}_A = r_A$, i.e. the size of C_A, if $a \geq b$ or $r_A < 3$. If $a < b$ and $r_A \geq 3$, then $\mathfrak{r}_A$ does not necessarily coincide with the size of C_A, since the pay-off function refers to the player in C_A, which is least connected to players of the same strategy. Solving the equation shows that for some values of ρ, a set of value pairs $(\mathfrak{r}_A, r_B)$ exists for which the equation is fulfilled.[14] Define such a set of pairs for a given

[14]Yet, some of these pairs are geometrically impossible, e.g. in the case, where $\mathfrak{r}_A = 1$ and $r_B = 2$ and

ρ as $R(\rho) = \{\bigcup(\mathfrak{r}_A, r_B) : \pi_t(j, \mathfrak{r}_A) = \pi_t(k, r_b)\}$. For any two neighbouring clusters C_A and C_B and a given ρ, it must hold that their value pair $(\mathfrak{r}_A, r_B) \in R(\rho)$. This occurs with zero probability for all such neighbouring clusters under the condition of initial random distribution and at least one cluster collapses triggering the instability of others. (For an example of how a stable population with size pair (5,5) must be structured, refer to figure 3.1.B C.).)

At least one strategy will thus develop clusters of size 9 with positive probability. From proposition 3.1.I we know that for $a \geq b$ and a stable cluster of size r_B, playing strategy B, surrounded by cluster of size $r_A = 9$ playing A, it must hold $8a \geq d(r_B - 1) + c(9 - r_B) \geq 7a + b$, from which we obtain the second part of the proposition. For $b > a$, $\frac{\partial \pi_t^A(i)}{\partial \Theta_t} < 0$ and the cluster's maximum pay-off player j is always *external*, if he plays A. For such a cluster, in order to be stable, it must hold $8d \geq a(\mathfrak{r}_A - 1) + b(9 - \mathfrak{r}_A) \geq 7d + c$, for the Pareto inferior strategy. The inequality only holds for $r_A = 1$, whence the second result of the proposition. By proposition 3.1.I, if $b > a$ no cluster of size $r_B < 9$, playing the Pareto dominant strategy, is stable. □

Figure 3.1.B: Examples for clusters of size 4 and 5



Consequently, a minimum necessary pay-off premium for the Pareto dominant strategy exist, to take over the player population and to determine the long-term

$\rho = \frac{3}{5}(a - c)$, both clusters have identical highest pay-off, but only a cluster of size 4 or larger can fully surround a cluster size 1. 11 feasible pairs remain after ruling out the geometrically impossible pairs. These are $(\mathfrak{r}_A, r_B)$: $(1,8)$ if $\rho = 7(a-c)$; $(7,4)$ if $\rho = 3(a-c)$; $(2,7), (8,3)$ if $\rho = 5(a-c)$; $(8,7)$ if $\rho = \frac{(a-c)}{5}$; $(7,6), (4,3)$ if $\rho = \frac{(a-c)}{3}$; $(2,6)$ if $\rho = 2(a-c)$; $(3,6)$ if $\rho = (a-c)$; $(8,6)$ if $\rho = \frac{(a-c)}{2}$; $(5,5)$ $\forall \rho$.

stable convention. Since a completely random initial distribution is unstable, clusters collapse, and some or all clusters will attain a size of 9 for at least one strategy. Whether this is the case for one or both strategies depends on the initial distribution.

Definition: A homogeneous initial distribution is defined as a distribution of the player population, in which the average cluster size for *all* strategies is identical after the first period of interaction. A **heterogeneous initial distribution** is defined by an initial player population, where average cluster size differ strongly for all strategies after the first period of interaction, but the evolution of at least one cluster of size 6 or greater occurs with certainty for any strategy after an initial sequence of interactions.

Hence, a homogeneous initial distribution defines the case, in which all players initially chose one of both strategies completely at random, under the condition that average pay-off for both strategies are sufficiently similar, or alternatively, in which larger agglomerations of players choosing the same strategy exist for *both* strategies. In this case, clusters of size 9 exists for both strategies after the initial interaction period. A heterogeneous initial distribution is the limit situation, in which the entire player population chooses the same strategy except for one minimal mutant cluster of size 6, or in which players choose their strategy at random, but average pay-offs are very different, so that the player population collapses into large clusters of the risk dominant strategy and small clusters of the other strategy with at least one being of size 6. Both distributions are the possible extreme cases that will define the boundary conditions for the evolution of a stable convention.

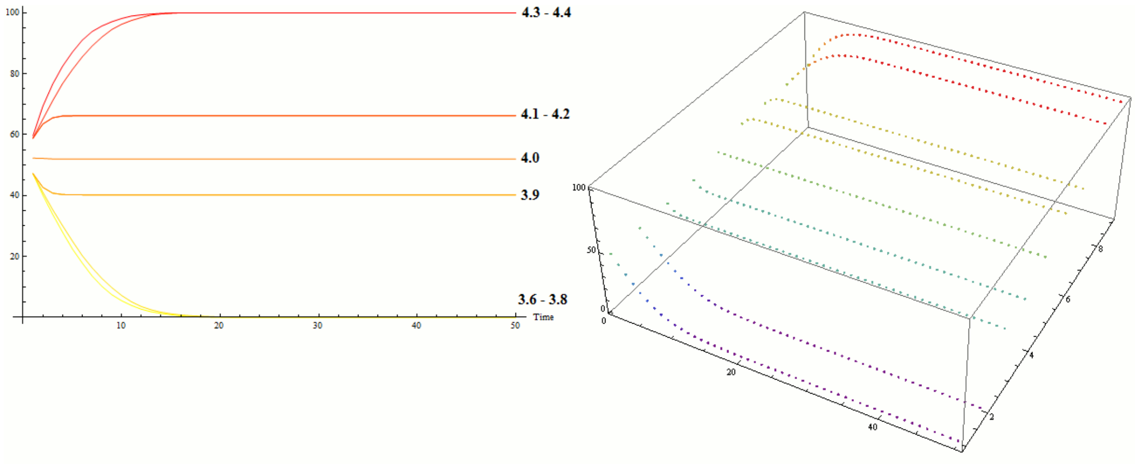
By proposition 3.1.I, for a minimum ρ, minimum cluster size for an invasion is 6. The minimum necessary pay-off premium for highly heterogeneous distributed societies is thus given by $\frac{3}{5}(a-c)$. If this condition holds, the population will be invaded by the Pareto dominant strategy even if there is a sparse distribution of individuals playing the Pareto superior strategy. The second minimum necessary pay-off premium and thus an upper bound can be derived for homogeneously distributed societies. Hence, depending on the degree of homogeneity of the initial distribution, both values define the boundary conditions for the lower limit of ρ such that the Pareto dominant equilibrium defines the convention.

Proposition 3.1.III. *A population with homogeneous initial distribution will converge to the Pareto superior equilibrium h_B, if the pay-off premium ρ of the Pareto dominant equilibrium is greater than $\frac{1}{7}(a-c)$. If the pay-off premium is smaller, but positive, a player population will consist of clusters playing different strategies.*

Proof. The initial distribution is by assumption homogeneous. For an initially homogeneously mixed population, by proposition 3.1.II some clusters will have a size of 9 with positive probability after some initial period of interaction. Consequently, for two such neighbouring clusters C_x and C_y of size 9, and an external player $l \in C_x$, define again two maximum players for each cluster, i.e player k, s.t. $\pi_t(k) = \sup\left(\Pi_t^X(l) : \Pi_t^X(l) = \left\{\bigcup_{h\in\mathfrak{N}(l)\cap C_x} \pi_t(h) \cup \pi_t(l)\right\}\right)$ and player j, s.t. $\pi_t(j) = \sup\left(\Pi_t^Y(l) : \Pi_t^Y(l) = \left\{\bigcup_{g\in\mathfrak{N}(l)\cap C_y} \pi_t(g)\right\}\right)$. By definition $k = internal$ and $j = external$, and for cluster size 9 it must hold either $\pi_t(k) = 8a$, if $s_t(l) = A$ or $\pi_t(k) = 8d$, if $s_t(l) = B$. Since $j = external$, his maximum pay-off is either $\pi_t(j) = c + 7d$, if $s_t(l) = A$ or $\pi_t(j) = 7a + b$, if $s_t(l) = B$. For l to switch strategy $\pi_t(k) < \pi_t(j)$. Since $\rho > 0$ only $c + 7d > 8a$ can occur without contradiction. □

Figure 3.1.C shows the result of a set of simulations identical to those in figure 3.1.A, but $d = 4$, $b = 2$ $c = a - 2$ and a ranging from 3.6 to 4.4. Furthermore each strategy is initially played by 50% of the population and seeding is completely random. Thresholds are again as expected. The population converges to equilibrium (B, B) for a smaller than 3.6 and to (A, A) for values larger or equal to 4.3. The population thus converges to the Pareto optimal convention, except if the pay-off premium is within a marginal perceptible unit.[15] Combining the propositions results in the following proposition:

Figure 3.1.C: Percentage of strategy A players; d=4, b=2, c=a-2, $a \in (3.6, 4.4; 0.1)$ $t \in (1, 50)$



[15] $\frac{1}{7}(a - c)$ defines the marginal perceptible unit, under which no pure equilibrium will occur. Furthermore, the simulation shows that for a small number of periods, the distribution is affected by the relative average pay-off, but stabilises after the initial interaction period.

Proposition 3.1.IV. *A population converges to the Pareto dominant equilibrium h_B, both if it is initially homogeneously distributed and the pay-off premium is $\rho > \frac{1}{7}(a-c)$. The population converges with certainty if $\rho > \frac{3}{5}(a-c)$ and if the population is at least initially heterogeneously distributed.*

Based on these results, the question arises of how strongly risk dominance is offset by Pareto dominance. For this assume as before that $d = a + \rho$ and $b = c + \rho$, but also that $c^* = c - \mu$. Consequently h_B is Pareto dominant by a value of ρ and h_A is risk dominant by μ. Define this value as the "risk premium". Again, the two cases of initial distribution define the boundary trade-off conditions between risk and Pareto dominance.

Proposition 3.1.V. *In a coordination game as in matrix 3.1.1 with two equilibria of which h_B Pareto dominates h_A by a pay-off premium of ρ, and h_A risk dominates h_B by a risk premium of μ, the equilibrium to which the population converges is defined by*

$$\mu < \begin{cases} c - a + 7\rho, & \text{a population with homogeneous initial distribution converges} \\ & \text{to the Pareto dominant equilibrium} \\ c - a + \frac{5}{3}\rho, & \text{any population, with at least heterogeneous initial distribution} \\ & \text{converge to the Pareto dominant equilibrium} \end{cases}$$

If the conditions do not hold, the population will be either in a state of mixed equilibria, or chooses the risk dominant equilibrium, if it is initially heterogeneously distributed and $\mu > \frac{2(c-a)+4\rho}{3}$. If a population is, however, initially sufficiently homogeneously distributed the risk dominant strategy will only overtake a population, if it also Pareto dominates by a value greater than $\frac{a-c}{7}$.

Proof. This is a direct consequence of the former proofs: Given that h_B Pareto dominates h_A, by the former propositions, in the case of a heterogeneous initial distribution the Pareto dominant strategy will overtake if $3c^* + 5d > 8a$ and the risk dominant strategy will prevail if $7a + b > 3c^* + 5d$. For a homogeneous initial distribution the constrains are $c^* + 7d > 8a$ and $7a + b > 8d$. The condition for the risk dominant strategy to prevail in a homogeneous population is thus $\rho < \epsilon$, where $|\epsilon = \frac{c-a}{7}|$ is the marginal perceptible unit for a Pareto dominant strategy to invade a population. □

For symmetric 2×2 coordination games, this contrasts with Young's original approach, which uses best response play. The approach based on imitation does not predict the convention to be solely defined by risk dominance, but dynamics follow a trade-off between risk and Pareto dominance. This stems from clustering owing to local interactions and the subsequent emphasize on the matrix's main diagonal's pay-offs.

Figure 3.1.D: Percentage of strategy A players; $d = 4, b = 3, c^* = 2$, $a = [3.2, 3.3; 0.01]$ $t \in (1, 30)$

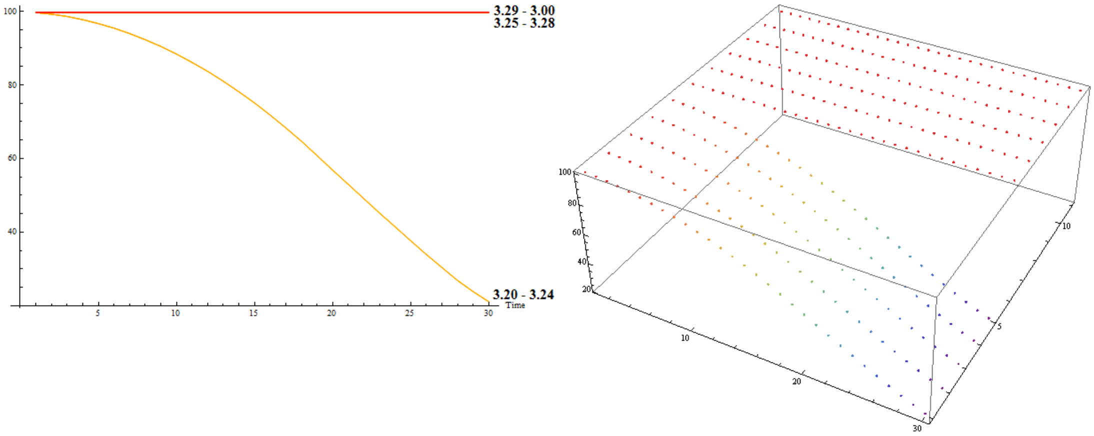


Figure 3.1.D shows the result of a set of simulations identical to those in figure 3.1.A, i.e. given a heterogeneous initial distribution, but c^* is fixed at $c^* = 2$ and a ranges from 3.0 to 3.3. The population converges to the Pareto dominant equilibrium for values of a smaller and equal to 3.24 and converges to the risk dominant equilibrium for $a > 3.29$. For $\frac{1}{4} < \mu < \frac{2}{7}$, the population converges to a mixed equilibrium, with a few square shaped cluster of size 6 or 9 that play the Pareto dominant strategy.

Figure 3.1.E: Percentage of strategy A players; $d = 4, b = 2, c^* = 1$, $a \in (3.0, 4.0; 0.1)$ $t \in (1, 15)$

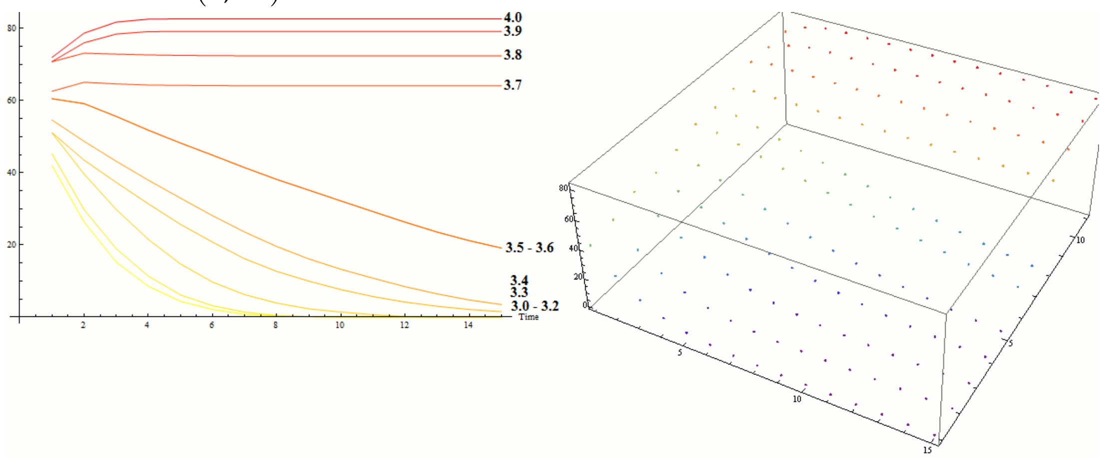


Figure 3.1.E presents the result of a set of simulations identical to those in figure

3.1.C, i.e. given a homogeneous initial distribution, but $c^* = 1$ and a ranges from 3.0 to 4.0. The population converges to the Pareto dominant equilibrium for values of a smaller and equal to 3.6 and remains in a mixed equilibrium for larger values.

Figure 3.1.F: Percentage of strategy A players; $a \in (0, 20; 1)$, Pay-off Premium $(-10, 10; 1)$ and zero risk premium

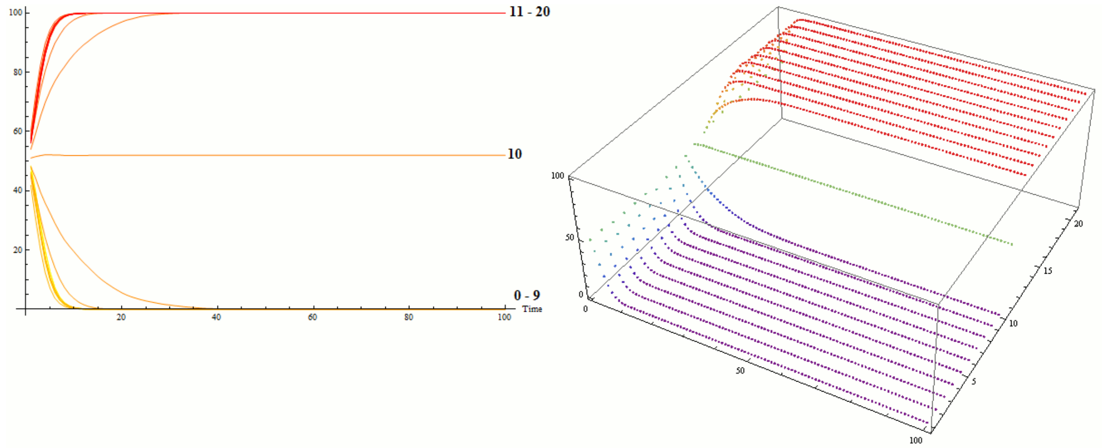


Figure 3.1.F shows the distribution of the player population for $t \in (0; 100)$ and a changing pay-off premium ρ. The pay-off structure is given by matrix 3.1.1, where $b = 6$, $d = 10$, $c^* = a - 4$ and a takes values from 0 to 20 in unit steps. The effect of the size of ρ on convergence speed towards a single equilibrium is negligible. The same holds for the updating probability.

Figure 3.1.G: Percentage of strategy A players; Convergence for different updating probabilities: Probability (1,100; 9)

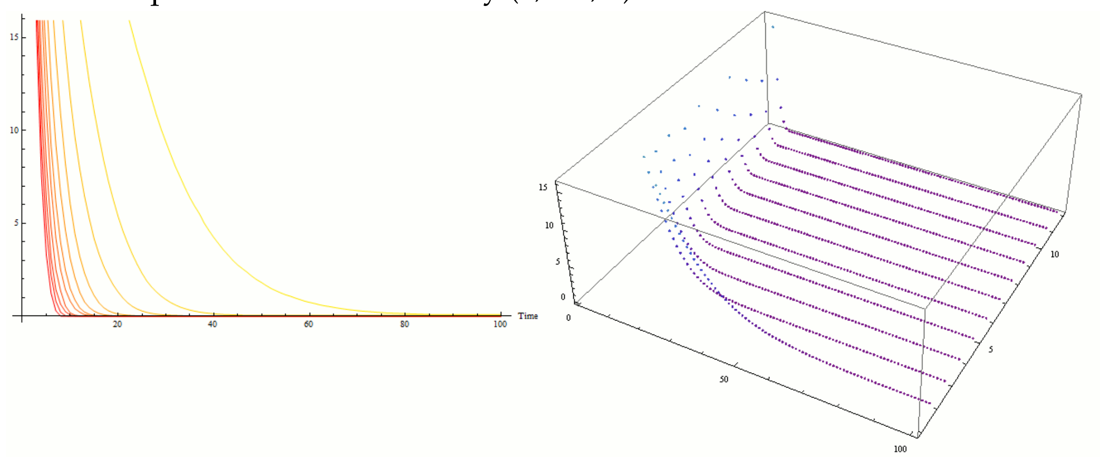


Figure 3.1.G shows the distribution for pay-offs $a = 3, b = 2, c^* = 1$ and $d = 4$

and different updating probabilities. The updating probability ranges from 1 to 100 in steps of 9.

3.2 General 2 x 2 Coordination Game

The following section analyses the dynamics of general 2×2 coordination games, *ceteris paribus*, in which two player types (row and column) interact with each other. Players choose their initial strategy at random. Whether the homogeneous or heterogeneous case applies, depends on the relative average pay-off of each strategy. The only difference with respect to the assumptions in the previous section is that on each patch two players coexist, one of each type. The general pay-off structure is defined by the following pay-off matrix:

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a_1, a_2 & b_1, c_2^* \\ c_1^*, b_2 & d_1, d_2 \end{pmatrix} \end{array} \tag{3.2.1}$$

Define as before $\rho_i = b_i - c_i$ or $\rho_i = d_i - a_i$ and hence $c_i^* = c_i - \mu_i$, for $i = 1, 2$. It must also hold that $a_i > c_i^*$ and $d_i > b_i$.

For two player types x and y with $x, y = 1, 2$ and $x \neq y$, define for a player i of type x the set of neighbours of his own type as $\mathfrak{N}_x(i)$ and the set of neighbours of the other type as $\mathfrak{N}_y(i)$. Notice that for simplicity and reasons of symmetry the other type on the same patch is not a neighbour. Each player has 8 neighbours of the same and other type. Further, let $A_{t,y}$ define the set of players of type y that play strategy A in period t, and accordingly $B_{t,y}$ as the set of players of type y playing B. Correspondingly, define $\Theta^A_{t,y}(i) \equiv |A_{t,y} \cap \mathfrak{N}_y(i)|$ and $\Theta^B_{t,y}(i) \equiv |B_{t,y} \cap \mathfrak{N}_y(i)|$ as the number of strategy A and B playing neighbours of i that are of type y. The pay-off of player i in time t is

$$\pi_{t,x}(i) = \begin{cases} \Theta^A_{t,y}(i)a + \Theta^B_{t,y}(i)b, & \text{if } s_t(i) = A \\ \Theta^A_{t,y}(i)c^* + \Theta^B_{t,y}(i)d, & \text{if } s_t(i) = B \end{cases} \tag{3.2.2}$$

Also define $\Pi_{t,x}(i) \equiv \left\{ \bigcup_{j \in \mathfrak{N}_x(i)} \pi_{t,x}(j) \cup \pi_{t,x}(i) \right\}$ as the joint set of agent i's pay-off and the pay-offs of his neighbours of the same type. The imitation rule then determines player i's future strategy by

$$s_{t+1}(i) = \{ s_t(k) : \pi_{t,x}(k) = sup\left(\Pi_{t,x}(i)\right) \} \tag{3.2.3}$$

The complexity of the analysis increases through the augmentation of possible parameter combinations and the interdependence of the two player types' strategy choices. Yet, the fundamental dynamics are defined by only a few conditions similar to what has been obtained for the single type case. Only 2×6 conditions have to be analysed in the general game. To derive this and as a first step, remember that the conditions for the pay-off structure, in addition to the imitation principle and the local interaction generate three useful characteristics.

I. In general, the player choosing the Pareto dominant strategy with respect to his type benefits from the relative abundance of players in his neighbourhood that belong to the *other* player type and are playing the same strategy. If $a_i > b_i$ and $d_i > c_i^*, \forall i = 1, 2$, this also holds for the strategy that is not Pareto dominant.

II. A strategy change will only occur at the edges of clusters. This property must also hold for mixed clusters, i.e. if both player types play different strategies on the same patches in a neighbourhood.

III. After the initial period of interaction, the strategy distribution on the grid will be determined by the relative average pay-off of each strategy, since it is more likely that a player adopts the strategy that has higher average pay-off if players initially choose their strategy at random with equal probability.

The following proposition greatly simplifies the future analysis:

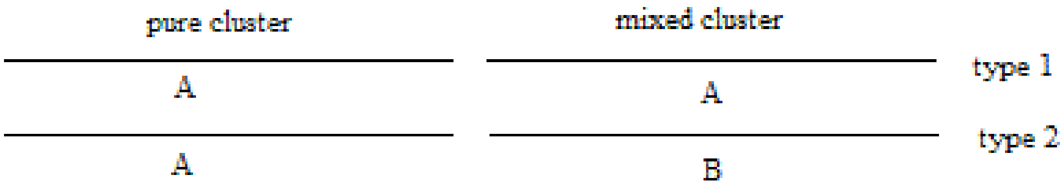
Proposition 3.2.I. *Consider a general* 2×2 *coordination game with asymmetric pay-offs and all other conditions being as before. The strategy distributions of the two player types will coincide after a brief period of interactions.*

Proof. Consider two clusters C_p and C_m. Define the set of players of type x as X and the set of type y players as Y. Assume without loss of generality that $s_t(h) = A, \forall h \in C_p$, and for cluster C_m that $s_t(j) = A, \forall j \in (C_m \cap X)$ and $s_t(k) = B, \forall k \in (C_m \cap Y)$. Hence, C_m is mixed and C_p is pure or uniform. Since strategical change occurs only at borders of clusters assume a player $l = external$ such that $(\mathfrak{N}(l) \cap C_p) \neq \emptyset$ and $(\mathfrak{N}(l) \cap C_m) \neq \emptyset$.

First, assume that $l \in (C_m \cap X)$. Thus $s_t(l) = A$, but also $s_t(i) = A, \forall i \in \mathfrak{N}_x(l)$. Consequently, $s_{t+1}(l) = A$. The same holds if $l \in (C_p \cap X)$. Now assume that $l \in (C_m \cap Y)$. Since, $s_t(i) = A, \forall i \in \mathfrak{N}_x(l)$, it follows that $\pi_t(f) = 8a, \forall f \in (C_p \cap \mathfrak{N}_y(l))$ and $\pi_t(g) = 8c^*, \forall g \in (C_m \cap \mathfrak{N}_y(l))$. As $a > c^*$, it follows $s_{t+1}(l) = A$. The same, for $l \in (C_p \cap Y)$. Equivalent results are obtained for $s_t(h) = B, \forall h \in C_p$, since $\pi_t(f) = 8d > \pi_t(g) = 8b$. Consequently, an *external* will always choose the strategy that is played by the uniform cluster in his neighbourhood if he has not done so before. The proposition is a direct result of this circumstance. □

In order to better understand the idea behind the proof, consider the case in which for both player types the same strategy has higher average pay-off. After the first period of interaction, larger clusters appear, playing this strategy. They surround smaller clusters that are either mixed, i.e. where each type on one patch plays a different strategy, or those that are uniform and play the strategy with lower average pay-off. The

Figure 3.2.H: Dynamics of mixed clusters

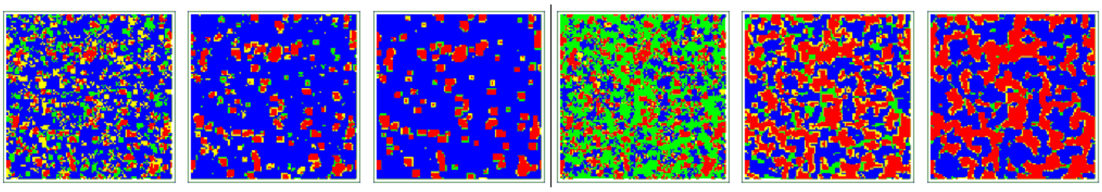


reason why mixed clusters will not sustain is the following: The players on the edge of the mixed cluster will always choose the strategy of the pure cluster in their neighbourhood. The schematic in figure 3.2.H helps to understand the underlying dynamic. It shows two clusters, each with two layers symbolising the inter-acting player types. The pure cluster is stylised on the left side of the figure, the mixed cluster on the right. The higher layer illustrates type 1, the lower type 2. Assume without loss of generality and with respect to the mixed cluster that type 1 plays strategy A and type 2 plays B, and the pure cluster only plays A. By assumption, type 1 players choose the same strategy in both clusters. Since imitation is horizontal, i.e. between players of the same type, this type cannot imitate any other strategy. Consequently, players of type 2 will only interact with players choosing A. Since by definition $a_i > c_i^*$ and $d_i > b_i$, a player of type 2 in the pure cluster has always higher pay-off than a player of the same type in the mixed cluster. Hence, type 2 players at the edges of the mixed cluster will switch to strategy A. The dynamics are independent of what type plays which strategy and whether the strategy is risk or Pareto dominant (since the assumptions have ruled out strictly dominant strategies). Mixed clusters therefore vanish and only small strings of mixed clusters of a maximum width of 3 at the borders of uniform clusters remain.[16]

[16]i.e. the *external* of each cluster and the player in between

Similarly, in the case, where the strategy with higher average pay-off is different for both player types, larger mixed clusters will surround uniform clusters, but any mixed cluster will vanish in the subsequent periods. Hence, the strategy distribution on the grid will coincide for both player types after an initial sequence of interactions, though transition to this state will be faster in the first case than in the second. The following figure 3.2.I illustrates this behaviour for two cases with identical initial distribution and the subsequent 3 periods of interaction.

Figure 3.2.I: Strategic distribution for 3 periods – a.) $a_1 = a_2 = 6$, $b_1 = b_2 = 6$, $c_1^* = c_2^* = 0$, $d_1 = d_2 = 8$; b.) $a_1 = d_2 = 6$, $b_1 = c_2^* = 6$, $c_1^* = b_2 = 0$, $d_1 = a_2 = 8$; using colour coding: blue: $s_i = A$, red: $s_i = B$, green: $s_1 = A, s_2 = B$, yellow: $s_1 = B, s_2 = A$



This leads to the following proposition

Proposition 3.2.II. *For a general* 2×2 *coordination game with a pay-off matrix as in matrix 3.2.1 and* $a_i, d_i > b_i, c_i^*$*, the convergence speed of the player population towards equilibrium* $h_A = (A, A)$ *is determined by the largest integer of* η_{Ai} *that fulfills* $\eta_{Ai} < \frac{-8\rho_i}{a_i - c_i - \rho_i}$ *and the convergence speed towards equilibrium* $h_B = (B, B)$ *is defined by the largest integer of* η_{Bi} *that fulfils* $\eta_{Bi} < \frac{8\rho_i}{a_i - c_i + \rho_i + \mu_i}$ *for each type* $i = 1, 2$.

If the population is initially sufficiently homogeneously distributed, i.e. average pay-offs for both strategies are of similar order and player choose their strategy initially at random with equal probability, the population converges to h_B *if* $\max_i(\eta_{Ai}) < \max_i(\eta_{Bi})$*, and to* h_A *if* $\max_i(\eta_{Ai}) > \max_i(\eta_{Bi})$*. If* $\max_i(\eta_{Ai}) = \max_i(\eta_{Bi})$ *both strategies persist in the long-term.*

Proof. By proposition 3.2.I and proposition 3.1.II, it follows for an initially homogeneously distributed population that, after an initial sequence of interactions, large pure clusters C_A and C_B of size $r_A, r_B = 9$ play strategy A and B, respectively. Further by proposition 3.2.I, player types can be neglected with respect to the dynamics.

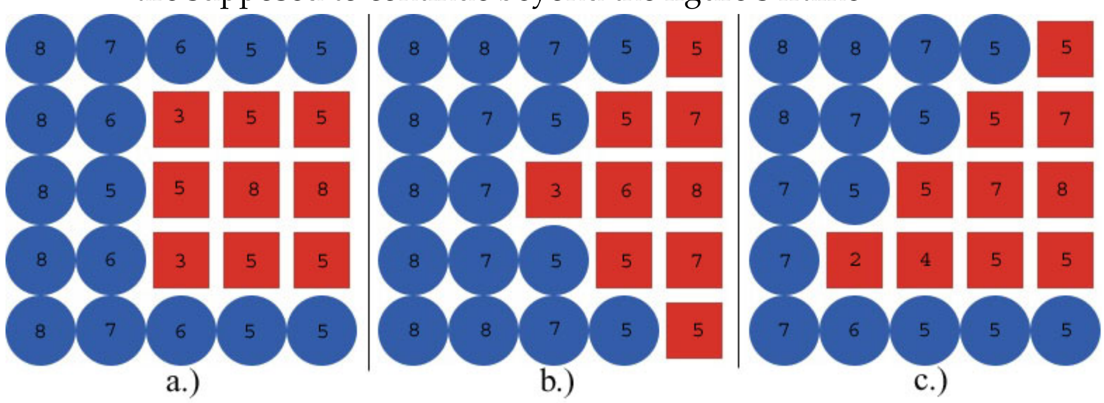
Assume a player $(k = external) \in C_A$ and a player $(l = external) \in C_B$ with $k \sim l$. In order to cause l to change strategy, it must be that $\pi_t(k) > \pi_t(h)$, given player

$(h = internal) \in C_B$ with $h \sim l$. Since $h = internal$, it follows that $\pi_t(h) = 8d$. Define $\eta_A = |B_t \cap \mathfrak{N}(k)|$. In general the pay-off of k is then given by $\pi_t(k) = (8 - \eta_A)a + \eta_A b$, leading to the condition $(8 - \eta_A)a + \eta_A b > 8d$. Similarly, define $\eta_B = |A_t \cap \mathfrak{N}(l)|$. Also $\exists(g = internal) \in C_A$ with $g \sim k$. Consequently $\pi_t(g) = 8a$ and to trigger a strategy switch of player $k \in C_A$ it must hold $(8 - \eta_B)d + \eta_B c^* > 8a$.

For $i = 1, 2$ define $int_{max}(\eta_{Ai})$ and $int_{max}(\eta_{Bi})$ as the largest integer that fulfils each condition, respectively, given the type specific parameters values. Since $int_{max}(\eta_{Ai}) \sim P(\pi_t(k) > \pi_t(h))$ and $int_{max}(\eta_{B}i) \sim P(\pi_t(l) > \pi_t(g))$, both values determine the likelihood by which a cluster expands and thus the speed, at which each type pushes the population towards the corresponding equilibrium. □

As the strategies for both player types are congruently distributed after an initial sequence of interactions, most type specific complexities are eliminated. Subsequent to a transition period, large uniform clusters with cluster size 9 (for most clusters) occur after an initial period of interaction. Each pushes towards a convention and the strongest push will eventually prevail. This is, however, only the case if average pay-off for both strategies are not too different and the distribution is initially sufficiently homogeneous. Even if players choose a strategy at random with equal probability before the first interaction, too diverse average pay-offs will inhibit the evolution of clusters, constituted by players of the risk inferior strategy, that have sufficient size to overtake the player population. In general, the edges of clusters will be either horizontal, vertical or diagonal. Consequently, the clusters' shape can be generalised to the following three types:

Figure 3.2.J: The three variants of cluster edges – numbers indicate the number of players with the same strategy in the individuals neighbourhood, cluster are supposed to continue beyond the figure's frame



From this figure we observe that, though various parameter combinations could lead to strategic changes, only six conditions for each strategy influence the dynamics of the entire population in the long-term:

	For $s_i = A$ to overtake $s_i = B$	For $s_i = B$ to overtake $s_i = A$
I.	$\eta_A = 1: \quad 7a_i + b_i > 8d_i$	$\eta_B = 1: \quad 7d_i + c_i^* > 8a_i$
II.	$\eta_A = 2: \quad 6a_i + 2b_i > 8d_i$	$\eta_B = 2: \quad 6d_i + 2c_i^* > 8a_i$
III.	$\eta_A = 3: \quad 5a_i + 3b_i > 8d_i$	$\eta_B = 3: \quad 5d_i + 3c_i^* > 8a_i$
IV.	$\eta_A = 4: \quad 4a_i + 4b_i > 8d_i$	$\eta_B = 4: \quad 4d_i + 4c_i^* > 8a_i$
V.	$\eta_A = 5: \quad 3a_i + 5b_i > 8d_i$	$\eta_B = 5: \quad 3d_i + 5c_i^* > 8a_i$
VI.	$\eta_A = 6: \quad 2a_i + 6b_i > 8d_i$	$\eta_B = 6: \quad 2d_i + 6c_i^* > 8a_i$

(3.2.4)

for $i = 1, 2$. The condition I. and III. are identical to those found in proposition 3.1.V for the single type case. The first condition implies that clusters can be overtaken along diagonal edges. It will turn an inlying cluster (red) as in a.) into an inlying cluster as in b.). If both conditions in I. are fulfilled, i.e. one for each player type, these corner elements will continuously switch between strategies. Condition III. applies to the horizontal and vertical cluster's edges. It will not affect the players surrounding the corner elements of inlying clusters (see b.) and c.)). Hence, an inlying quadrangular cluster will expand and will incrementally turn the horizontal or vertical edge into a diagonal edge. Since condition III. includes condition I., the cluster will also continue to expand along these diagonal edges. Furthermore, under condition III. any inlying cluster can be invaded. The remaining conditions have a minor effect on the convergence speed than the aforementioned. Condition IV. and greater only concern the growth along the corner elements.

The easiest asymmetric game is a game of "common interest". This denotes a pay-off structure, in which the same strategy is Pareto dominant for both types. According to the former notation, either $a_i > d_i$ for both $i = 1$ and $i = 2$ (or the inverse). If at least one player type fulfils at least the first condition given by $\eta_A = 1$ ($\eta_B = 1$), the population either converges to the convention defined by h_A (h_B), or ends up in a mixed equilibrium with interior rectangular shaped clusters playing A that cannot expand (see 3.2.J.a.). This depends on the initial random distribution and the average pay-off of each strategy. Note that condition I. is in this case exactly the same as in proposition 3.1.V, given the homogeneous initial distribution. If at least one type meets condition $\eta_A = 3$ ($\eta_B = 3$), the convention is surely defined by h_A

(h_B). Since both player types either prefer equilibrium (A,A) or equilibrium (B,B), the convergence by one player type towards an equilibrium is not counter-acted by the other player type. Convergence speed is irrelevant for the final distribution and convention, but the more conditions are fulfilled by one or both types, the faster the population converges to its Pareto dominant equilibrium.

If the individuals find themselves in a "conflict game", in which the Pareto dominant equilibria are not identical, the convention is defined by proposition 3.2.II. The strategy that fulfils the higher condition in 3.2.4 will define the convention. In the following I will simulate a player population to confirm that population dynamics behave according to the previous results.

The most convenient way to test for the correctness of these results is to fix pay-off parameters of one player type at the different levels at which the constraints in 3.2.4 can be fulfilled, e.g. ranging from none to all six. The dynamics for each parameter of the other player type are then simulated with respect to each of these levels. As a basis for the analysis assume the following pay-off matrix:

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \quad A \qquad\qquad\qquad B \\ \begin{pmatrix} a_1 = 3.5,\ a_2 = 3.5 & b_1 = 3,\ c_2^* = 2 \\ c_1^* = 2,\ b_2 = 3 & d_1 = 4,\ d_2 = () \end{pmatrix} \end{array} \tag{3.2.5}$$

In order to account for the various variables, at which the conditions can be fulfilled, d_2 is set to one of 6 different values, namely $d_2 = 3.16; 3.22; 3.28; 3.34; 3.41; 3.47$ in each simulation run. This implies values of convergence speed given by $\eta_A = 5; 4; 3; 2; 1; 0$ and that player type 2 converges to $h_A = (A, A)$ except for $d_2 = 3.47$ and thus $\eta_A = 0$. At the last parameter value, player type 2 does not exhibit any convergence.

For each of these values, one parameter of player type 1 is analysed by a set of simulations. In the first set of simulations the parameters of player type 2 are set to the values in 3.2.5 and $d_2 = 3.16$. For the first simulation of this set, one parameter of player type 1 is fixed at its lowest value, at which it does not fulfil any condition (or all, depending on the parameter) of the first column in 3.2.4. His other pay-off parameters are set to the values as in matrix 3.2.5 (if not stated otherwise). The initial distribution is set to 50 : 50 (if not stated otherwise), with completely random seeding. After the system has been simulated for a fixed number of periods, the parameter of player type 1 is changed by an increment and the system is simulated again. Using the same initial distribution renders the results directly comparable. Simulations are repeated until the parameter reached a maximum value, at which all condition (or none) of the first column in 3.2.4 are fulfilled. Hence, player type 1 will progressively converge to equilibrium *(B,B)*, whereas player type 2 will converge to equilibrium *(A,A)* at the speed determined by the value of d_2. After the value of player 1's parameter has reached its maximum value, the set of simulations is repeated for each of the remaining values of d_2 and at the beginning of each set of simulations the population is "seeded" anew. Each remaining parameter of player type 1 is analysed in the same way, obtaining 6 sets of simulations for each parameter of player type 1, thus 24 sets of simulations in total. The figures show the proportion of type 1 players choosing strategy A. Since the distribution for both types concurs after the initial periods, it suffices to graph one player type, as before.

Figure 3.A.O on page 113 shows the result for parameter a_1. In order to maintain the assumption that $a_i > b_i$, the value of b_1 is adapted accordingly and set to $b_1 = 2.3$. This change is made only for the simulations concerning a_1. a_1 takes value from 2.375 to 3.875 in increments of 0.25. If both types have the same convergence speed, the population converges to a mixed equilibrium, where the strategy distribution is determined during the initial periods of interaction, i.e. by the average pay-off and the random initial distribution. In order to compensate this effect (since for $\rho_A = \rho_B$, a_1 is relatively large in the later simulations), the initial distribution was set to 55% strategy B players in the last 4 simulations. The predicted threshold values from proposition 3.2.II for the parameters are as shown in table 3.2.a.

Table 3.2.a: Convergence speed for each player type in the simulations

$\eta_A < \eta_B \rightarrow h_B = (B,B)\ , \eta_A > \eta_B \rightarrow h_A = (A,A)$							
η_A	0	1	2	3	4	5	
d_2	3.47	3.41	3.34	3.28	3.22	3.16	
η_B	0	1	2	3	4	5	6
$a_1 <$	4	3.75	3.5	3.25	3	2.75	2.5
$c_1 >$		0	2	$\frac{8}{3}$	3	3.2	$\frac{10}{3}$
$d_1 >$	3.5	$\frac{26}{7}$	4	4.4	5	6	8

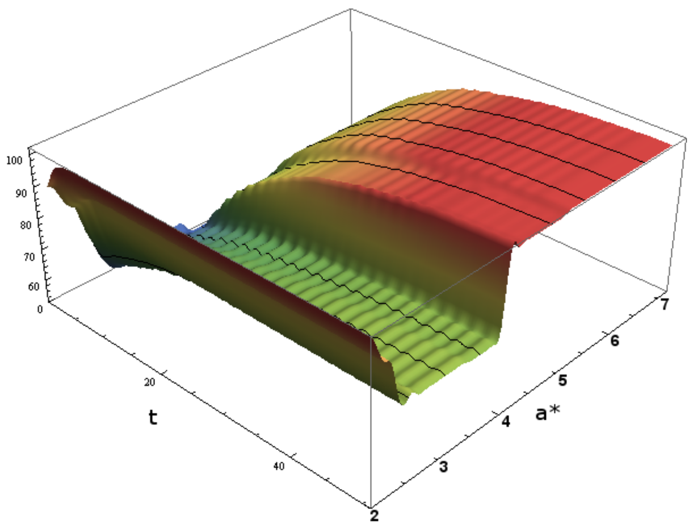
Figure 3.A.P on page 114 shows the results for parameter b_1. It takes value from -4 to 3 in increments of 1. Looking at 3.2.4 shows that b_1 is extraneous in the constrains of the right column. b_1 only affects the average pay-off and thus the number of strategy

A players after the initial sequence of interactions. It thereby has an indirect impact on the time required to converge to an equilibrium, since it modifies the initial sequence after which the distribution of convergence to the convention. $\eta_B = 1$ satisfies $8a_1 = 2c_1^* + 6d_1$ only as equality and it only holds that $8a_1 < c_1^* + 7d_1$. Consequently, convergence to (B, B) is only observable for $d_2 = 3.47$. For $d = 3.41$ the convergence speed for both player types is identical and thus stable mixed equilibria occur. In the case of $d = 3.34$ convergence to (A, A) is slow as condition II. is met with equality by player type 2. Similarly the simulation for $b_1 = 3$ and $d = 3.47$ approaches (B, B) slowly as the number of strategy A players is high after the initial sequence of interactions.

c_1^* adopts value from -0.1 to 3.5 in increments of 0.2 in the simulations in figure 3.A.Q on page 115. Since c_1 can take small critical values, simulations are conducted with an initial distribution of 55% strategy B players in the first two simulations and 60% strategy B players in the later simulation, in order to avoid that cluster size of strategy B is too small after the initial sequence of interactions. The threshold values for this parameter are again to be found in table 3.2.a.

The final set of figures 3.A.R on page 116 shows the dynamics for d_1. d_1 takes values from 3.2 to 8.48 in increments of 0.33. In order to compensate for the "average pay-off effect" the share of initial strategy B player was set to 60% in the last 4 simulations. All simulations behaved according to the predictions made in table 3.2.a. By proposition

Figure 3.2.K: Percentage of strategy A players; Set of simulations with $a_1 \in (2.0, 7.0; 0.1)$ and $d_1 = a_1 + 1$: $|\rho| = 1$ and $|\mu| = 1$



3.2.II, the population can converge to different equilibria, though the level of risk dominance and the level of Pareto dominance are equal. Figure 3.2.K shows a set of simulations for $b_1 = 3$, $c_1^* = 1$, $a_2 = 4$, $b_2 = 1$, $c_2^* = 3$ and $d_2 = 3$. $a_1 \in (2.0, 7.0; 0.1)$ and $d_1 = a_1 + 1$. Hence, for all simulations $|\rho| = 1$ and $|\mu| = 1$, and strategy A is risk superior and strategy B Pareto superior for type 1, and the inverse for type 2. Note that by proposition 3.2.II and equations 3.2.4, the conditions, determining the dynamics, are unaffected by linear transformations of the pay-offs. In addition, such a transformation will have no effect on the relative average pay-off.

3.3 The Effect of Space - Planting Late in Palanpur

So far it has been assumed that only the 8 surrounding neighbours are considered both for the calculation of pay-offs and for imitation. There are several questions, which can be raised: Do the derived properties still hold, if the space (representing the reference group), which affects an individual's decision, increases? What happens, if the space considered for the calculation of pay-offs and for imitation are different? Though the analysis of the former two sections can be expanded to larger spaces, this will not be done in the scope of this chapter. This section will only provide general results without defining the clear conditions for each size of space.

Consider the following illustrative example taken from Samuel Bowles (2006). In Palanpur, a backward village in India, peasants use to sow their crops at a later date than would be maximising their expected yields. This results from a coordination failure. If a single farmer decided to sow early, seeds would be quickly eaten by birds and the harvest would be lost. The more farmers should agree to plant early, the less the loss for an individual "early seeder", since seeds lost to birds would be shared by the entire group of "early planters". Mechanism design, contracting, and implementation theory has dedicated much work to answer, which institutions are necessary to achieve the desired shift towards the Pareto optimal equilibrium. The question of why a population ended up in a Pareto inferior equilibrium is, however, more interesting in this context.

The above analysis has shown that in the case of imitation and local interactions a population converges to the Pareto dominant equilibrium, if the Pareto dominant strategy is played by a minimal number of individuals. Yet, Palanpur is a case, in which a population got trapped at the risk dominant equilibrium. It can be assumed that initially both strategies (*plant early, plant late*) have been played with strictly positive probability. Thus, it seems that the standard solution of this stag-hunt / coordination game (i.e. the 50:50 distribution lies in the basin of attraction of the risk dominant equilibrium) is more appropriate in this case than the answer I have presented so far in subsection 3.1.1. This conclusion is, however, precipitative. To illustrate this, assume the following pay-off matrix (also taken from Bowles, 2006) for the game:

$$\begin{array}{cc} & \textbf{Early} \quad \textbf{Late} \\ \begin{array}{c}\textbf{Early}\\ \textbf{Late}\end{array} & \begin{pmatrix} 4,4 & 0,3 \\ 3,0 & 2,2 \end{pmatrix} \end{array} \tag{3.3.1}$$

A first answer is that in Palanpur the interacting population is very small. As a consequence, a low, but positive probability exists that initial distribution of early and late planters was such that a stable cluster of early planters could not evolve and take over the whole population. Under random initial choice, the probabilities of the Pareto dominant convention or a mixed stable equilibrium are, however, much higher than the risk dominant equilibrium in pure strategies. (In addition, a small mutation rate seems also plausible in that society, and an invading cluster occurs with positive probability). Hence, a possible but less convincing answer is that peasants in Palanpur were simply unlucky.

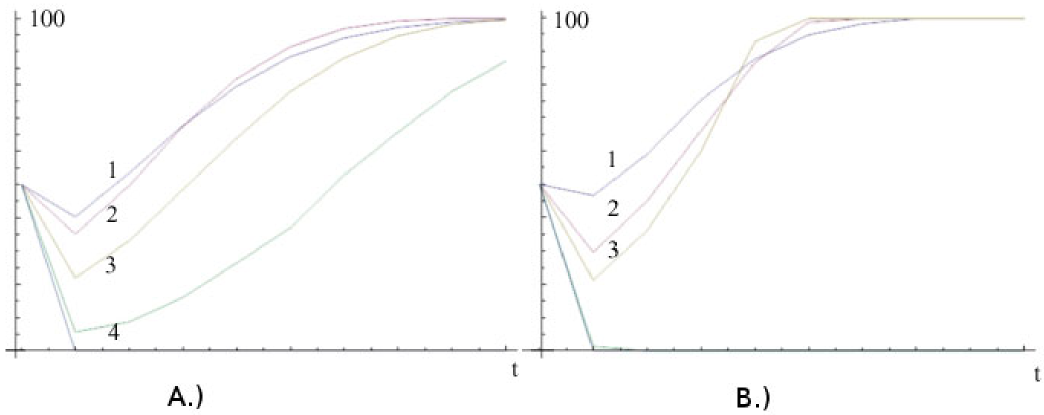
A second answer is that peasants do not only consider neighbours at a distance of 1, i.e. their 8 surrounding neighbours on the grid, but have a much larger space, which defines the peasants they are interacting with.

Definition: The **imitation radius** is defined by the largest Chebyshev distance between a player and a member of the set of observable neighbours who he can imitate. The **pay-off radius** is similarly defined as the largest Chebyshev distance between a player and a member of the set of neighbours that affect his pay-off.

The radii thus define the minimum number of steps a "king" requires to move from the player to his farthest neighbour (in the set of the observable or pay-off affecting players) on the "chess board" grid. In the former sections both radii have been assumed equal to 1, i.e. an individual has only considered the adjacent 8 players. As the radii increase, it is observed that the population converges more likely to the risk dominant equilibrium. This is caused by a rapid decline of those individuals playing the Pareto dominant strategy in the first period. Figure 3.3.L.A.) illustrates this behaviour for a" large" population (10.000 individuals). If the initial fall in the number of Pareto dominant players is higher than a certain threshold, the population will not converge to the Pareto optimal equilibrium, as a cluster of minimal size does not evolve. The reason is that as the pay-off radius increases, pay-offs converge to the expected pay-offs in the first period in a random distributed population, and less weight is placed on the diagonal elements of the pay-off matrix. Since by definition the risk dominant strategy has a higher average pay-off than the Pareto dominant, though risk inferior, strategy, individuals in homogeneously distributed areas, i.e. where no large clusters of one player type exist, will adopt the risk dominant strategy. Only in neighbourhoods, in which a sufficient number of individuals playing the Pareto dominant strategy is agglomerated, individuals will adopt the same strategy. For a random initial distribution, these agglomerations are more likely to occur the larger the population size.

Furthermore, note that minimum sustainable cluster size depends on the imitation radius under consideration. The surrounding clusters, observed after the first interaction period, increase with the imitation radius, and thus, minimum cluster size for the Pareto optimal strategy also has to increase in order to be sustainable.[17] This occurs with decreasing probability. Hence, small societies tend towards the risk dominant equilibrium at smaller radii in comparison to larger societies. Figure 3.3.L.B. shows the dynamics for a small population.

Figure 3.3.L: Convergence for two different societies: both figures show the number of *early seeders* A.) 10.000 B.) 441 - Radius from 1 to 5 - the higher the radius the higher the initial decrease in individuals playing the Pareto dominant strategy



[17]For the given pay-off, minimum sustainable cluster size is: radius 2 = 14, radius 3=30, radius 4=48 in contrast to 6 for radius=1.

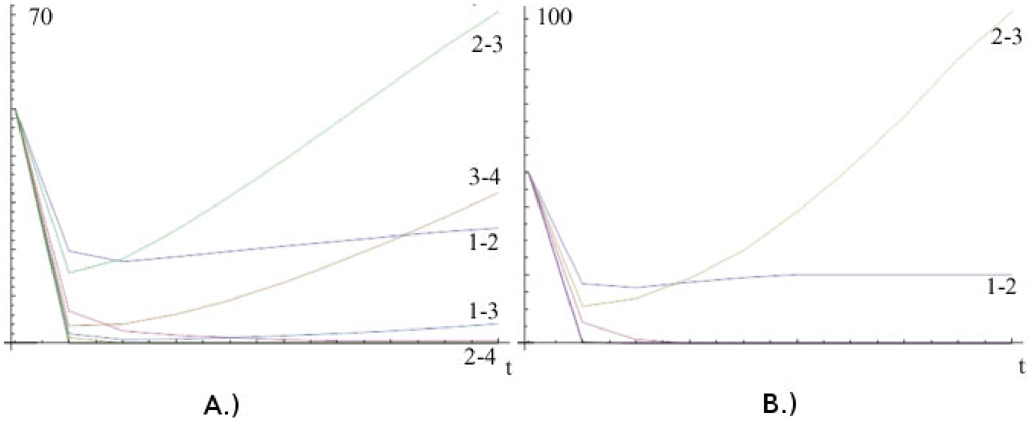
The large population converges to the Pareto dominant equilibrium for radii smaller than 4, the small population converges only for radii smaller than 3. Figure 3.A.S on page 117 shows how both populations appear after the first period of interaction. This implies, however, that in the large population an individual needs to consider 120 neighbours, in the much smaller population still 80 neighbours to cause the population to converge to the risk dominant equilibrium. In addition, changing the initial distribution only slightly in favour of the Pareto dominant equilibrium, necessitates even higher radii.[18]

In the case of the Palanpur peasants, it seems plausible that the imitation space, which peasants observe, is much smaller. If that were not the case, peasants could easily implement the Pareto dominant equilibrium by observing all peasants in the village and collectively impose a fine on anyone sowing late. It is therefore more reasonable to assume that the imitation radius is relatively small with respect to the pay-off radius. The space that defines the individual's next period's strategy is defined by those fields, on which the peasant can observe the last yield. Most probably these are the fields surrounding his own. The pay-off radius is, however, defined by the birds' hunting ground. It is highly probable that this radius is much larger. Consequently, the imitation radius is smaller than the pay-off radius.

Simulation shows that, *ceteris paribus*, the convergence towards the risk dominant equilibrium occurs more likely the higher the discrepancy between the imitation and pay-off radius. The following figure 3.3.M illustrates the simulated results for the same initial distributions as the figure before. Simulations have been conducted for pay-off radii from 1 to 5 and imitation radii from 1 to 3, where the first is always greater than the second. The large population of 10.000 individuals only converges entirely to the Pareto dominant equilibrium for an imitation radius of 2 and a pay-off radii of 3 or 4 (very slowly after 10 periods), or an imitation radius of 3 and a pay-off radius of 4. Imitation radius of only 1 and pay-off radii of 2 and 3 converge to a mixed equilibrium (approximately 95% and 0.5%). All other combinations converge to the risk dominant equilibrium. The small population of 441 individuals converges only to the Pareto dominant equilibrium, if imitation radius is 2 and pay-off radius 3. The pair of imitation radius of 1 and a pay-off radius 2 converges to the mixed equilibrium, at which approximately 20% of the players choose the Pareto dominant strategy. All other pairs converge again to the risk dominant equilibrium.

[18] e.g. changing initial distribution to 58% of Pareto dominant players necessitates a radius of 7 for both populations, in order to converge to the risk dominant equilibrium.

Figure 3.3.M: Convergence for two different societies: A.) 10.000 B.) 441



The effect is explained as follows: Increasing the pay-off radius benefits relatively the risk dominant players. A large imitation radius, conversely, increases the spatial effect of a large agglomeration of Pareto dominant players on the neighbouring players' strategy choice for the next period. If individuals compare pay-offs only highly locally, the agglomeration effect is negligible, leading to the observations:

Observation 3.3.I. *Large populations are more likely to converge to the Pareto dominant equilibrium than smaller populations.*

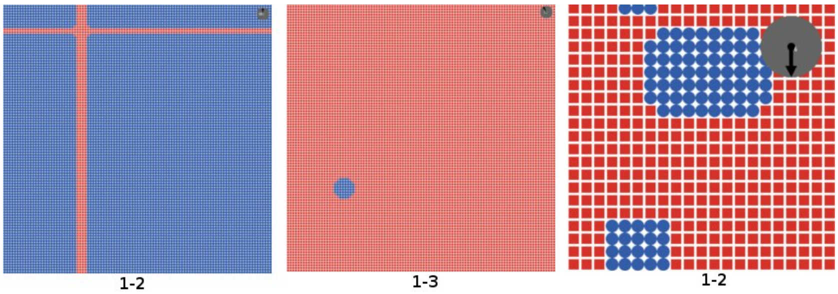
Observation 3.3.II. *For small imitation radii with respect to the pay-off radii, a population will converge to the risk dominant equilibrium with high probability.*

This implies a positive relation between the scope, with which individual choice affects other players' pay-offs, and individual short-sightedness on the one hand, and the probability of convergence towards the risk dominant convention on the other hand. If externalities are far reaching, individuals tend to choose the risk dominant strategy. This is exactly the case of Palanpur. Figure 3.3.N shows the structure of the mixed stable equilibria of the former set of simulations.

3.4 Conclusion

Although the assumptions that have been made at the outset are indeed very reductionist, a straightforward interpretation of the abstract findings displays an intricate and intuitive set of results that governs the evolution of social conventions. In the setting with imitation driven strategy choice and local interactions, a shift in conventions

Figure 3.3.N: Stable Radius Ratios: first number indicates radius of imitation, second number the pay-off radius



can only be triggered by a "minimal group" of individuals that completely adheres to the alternative convention. This must hold for all interacting social levels (strata), defined by the player type, in this group. In contrast to stochastic stability, not a single stratum can cause a conventional change. The sufficient number of individuals in such a group is determined by the size of an individual's reference group and the degree with which a society is connected. The likelihood of the evolution of a certain convention is defined by a non-linear relationship between pay-off (Pareto) dominance and risk dominance. In contrast to stochastic stability this finding holds also for the case with only two potential conventions. In addition, for certain "pay-off" constellations several stable conventions evolve and coexist, yet, each convention is respected by a larger group of individuals.

In the case of very small external effects of an individual's strategic choice, Pareto efficient conventions evolve more likely. In such cases, risk dominant but Pareto inefficient conventions can only persist, if they are incumbent conventions. If a conventional behaviour is not yet defined, a population will either converge to the Pareto dominant convention or, if both conventions offer almost equal pay-off, the population will be in a stable state, in which both conventions co-exist. This will not be the case for large external effects, since there exists a positive correlation between the size of the externalities and the reference group on the one hand, and the likelihood with which a risk dominant convention evolves. If individuals experience large scope external effects, they will almost certainly follow the risk dominant convention. This effect is reinforced in small secluded societies.

In the spatial context of 2×2 coordination games, a population converges either to one of the pure Nash equilibria or a mixed conventional state, in which the population consists of large clusters of players that either play the Pareto or risk dominant strategy. Hence, Young's framework is not contradicted *per se* in the spatial context with imitation driven responses, but,it shows that, in addition, the joint assumption of imitation and local interaction can lead to long-term conventions that are not defined as the *SSS*. The approach described in this chapter thus challenges the assumption that a discriminative criterion can only rely on risk dominance. Here we have observed that Pareto (pay-off) dominant euqilibria are highly probable to define the long-term convention.

Under the conditions of both approaches, a population will never adopt a strategy that is both Pareto and risk inferior. Although this seems intuitive at first sight, there existed and still exist numerous societies that adopted persistent interaction patterns that are inferior (for examples see Edgerton, 2004). The issue at hand is that social conventions and norms mostly exhibit a path dependent strategy set, implying that not only full knowledge but also full accessibility to the individual's strategy set cannot generally be assumed. Behavioural patterns and social customs might dictate a bearing that inhibits the evolution towards other equilibria and thus the adoption of certain strategies. Hence, an evolutionary process might turn out to be a blind alley. As Nelson states: "[...][B]eliefs about what is feasible, and what is appropriate, often play a major role in the evolution of institutions." Since both the spatial and Young's framework assume a conventional game with a fixed strategy set for each player, from which he can chose freely under the constraint of the imitation or the best response principle, these frameworks cannot take account of this circumstance.[19] Nevertheless, I believe that a theory that tries to shed light on the evolution of social conventions should allow for such kind of path dependency that social evolution adheres to.

Although the spatial approach neglects the effect of temporal choice constrains that are imposed by the adoption of a specific strategy profiles on the individual of a society, the approach can be directly expanded to incorporate this effect by adding a third dimension on the spatial grid to the model. This third dimension is not a spatial interaction constraint in this context, but represents the choice constraint that a strategy exhibits on the individual for one period of time. The number of periods is identical to the number of restrictive elements in a row along the third dimension and each player finds himself on a unique 2 dimensional plane in each period. The plane will be defined by the strategy choice, a player has made in the previous per-

[19] Another issue that moves along the same line as the limited accessibility of the individual strategy set, directly refers to *Chapter 2*, but can be more easily implemented into the model described in this chapter: Time does not play a role in the definition of the error rate. Yet, whenever a population stays close to a convention, its becomes increasingly embedded in the behavioural patterns and more difficult to replace. This is only very weakly taken into account by the adaptive play approach, i.e. by the assumption that players remember the history of past play.

iod. Hence, his "choice path" is represented by the spatial depth. In consequence, if a player chooses a certain strategy, he is restricted only to a subset of subsequent alternative strategies. A population or cluster can thus end up in a evolutionary dead end, and an evolutionary path once chosen may lead to such an "inferior convention". This extension might provide valuable material for further research.

In addition, fictitious (or adaptive) play and pure imitation are two extreme representations of the heuristics that individuals apply to choose a strategy. For further research, it is appealing to see which results can be obtained by mixed heuristics, and if a threshold can be found defining which degree of imitation will still maintain a positive probability to access the Pareto dominant though risk inferior equilibrium. Since the group of possible learning algorithms is much larger than only those two described herein, a broader analysis might also be of interest.

Furthermore, an expansion of the approach to more than two strategies seems also promising for future research, not only with respect to the *survival* of strategies given certain parameter combinations, but also in regard to the spatial patterns that can evolve.[20]

[20]e.g. First simulations of an *extended* Prisoner's Dilemma with three strategies, in which the defective strategy can *exploit* the two others, and a semi-defective strategy can only exploit the cooperative strategy, shows very interesting patterns. A structure evolves that is similar to a class society, in which the middle class works as a buffer between the other two.

3.A Figures and Tables

Figure 3.A.O: Simulation for $a_1 \in (2.375, 3.875; 0.25)$ a.) $\eta_A = 5$; b.) $\eta_A = 4$; c.) $\eta_A = 3$; d.) $\eta_A = 2$; e.) $\eta_A = 1$; f.) $\eta_A = 0$; in c.)-f.) initial share of $B_0 = 55\%$

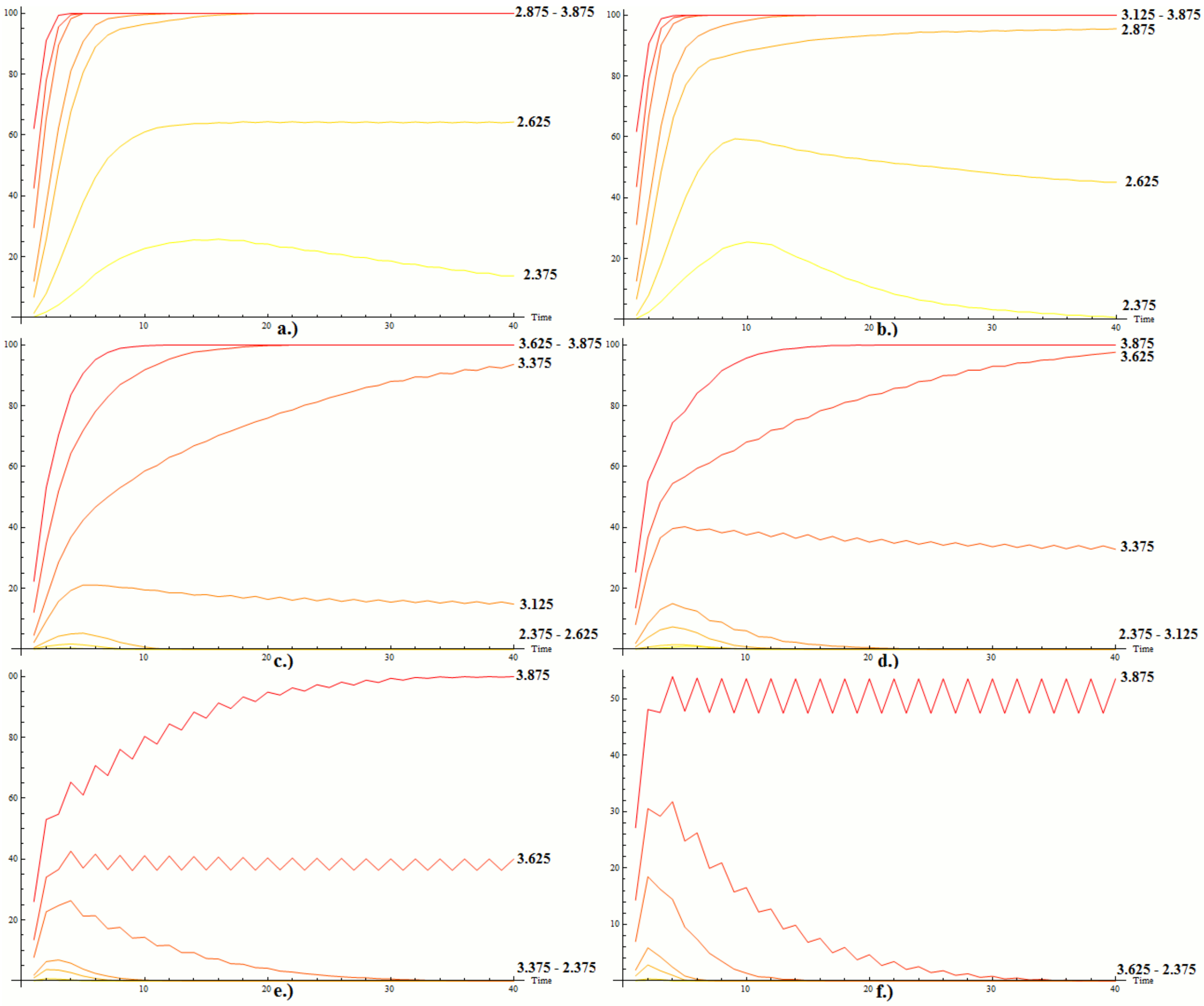


Figure 3.A.P: Simulation for $b_1 \in (-4,3;1)$ a.) $\eta_A = 5$; b.) $\eta_A = 4$; c.) $\eta_A = 3$; d.) $\eta_A = 2$; e.) $\eta_A = 1$; f.) $\eta_A = 0$

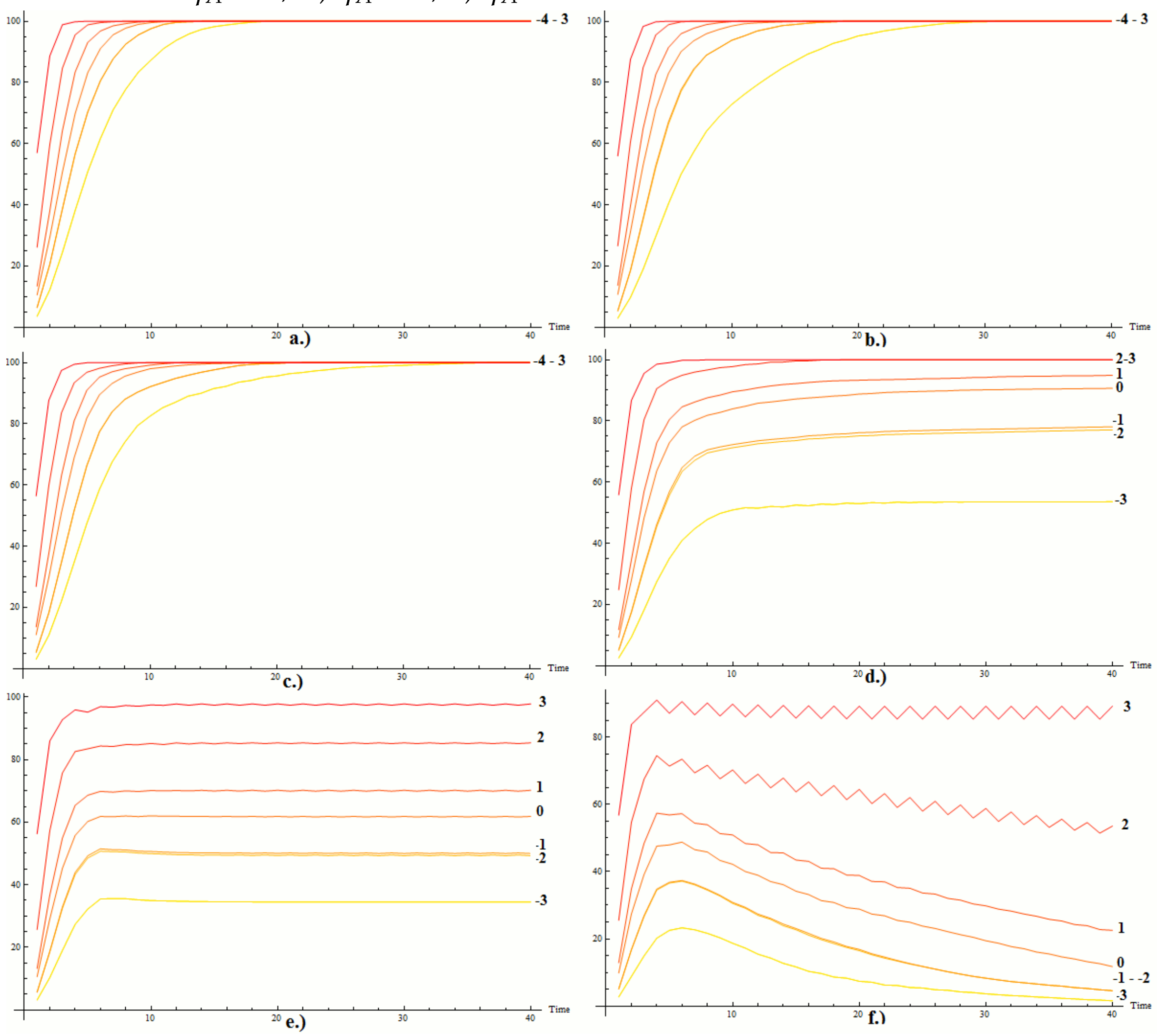


Figure 3.A.Q: Simulation for $c_1^* \in (-0.1, 3.5; 0.2)$ a.) $\eta_A = 5$; b.) $\eta_A = 4$; c.) $\eta_A = 3$; d.) $\eta_A = 2$; e.) $\eta_A = 1$; f.) $\eta_A = 0$; a.)-b.) 55% strategy B players, c.)-f.) 60% strategy B players

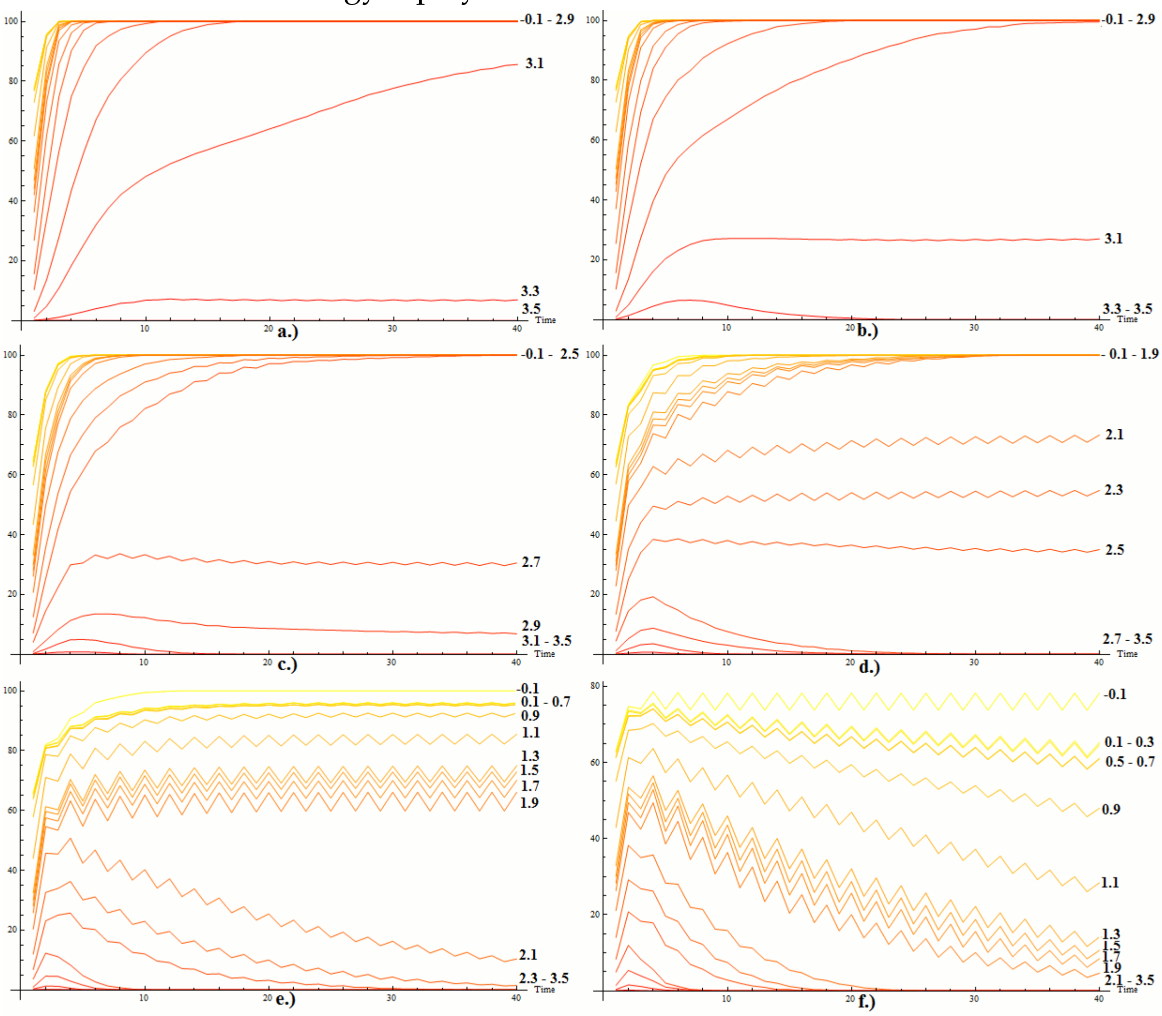


Figure 3.A.R: Simulation for $d_1 \in (3.2, 8.48; 0.33)$ a.) $\eta_A = 5$; b.) $\eta_A = 4$; c.) $\eta_A = 3$; d.) $\eta_A = 2$; e.) $\eta_A = 1$; f.) $\eta_A = 0$; c.)-f.) 60% strategy B players

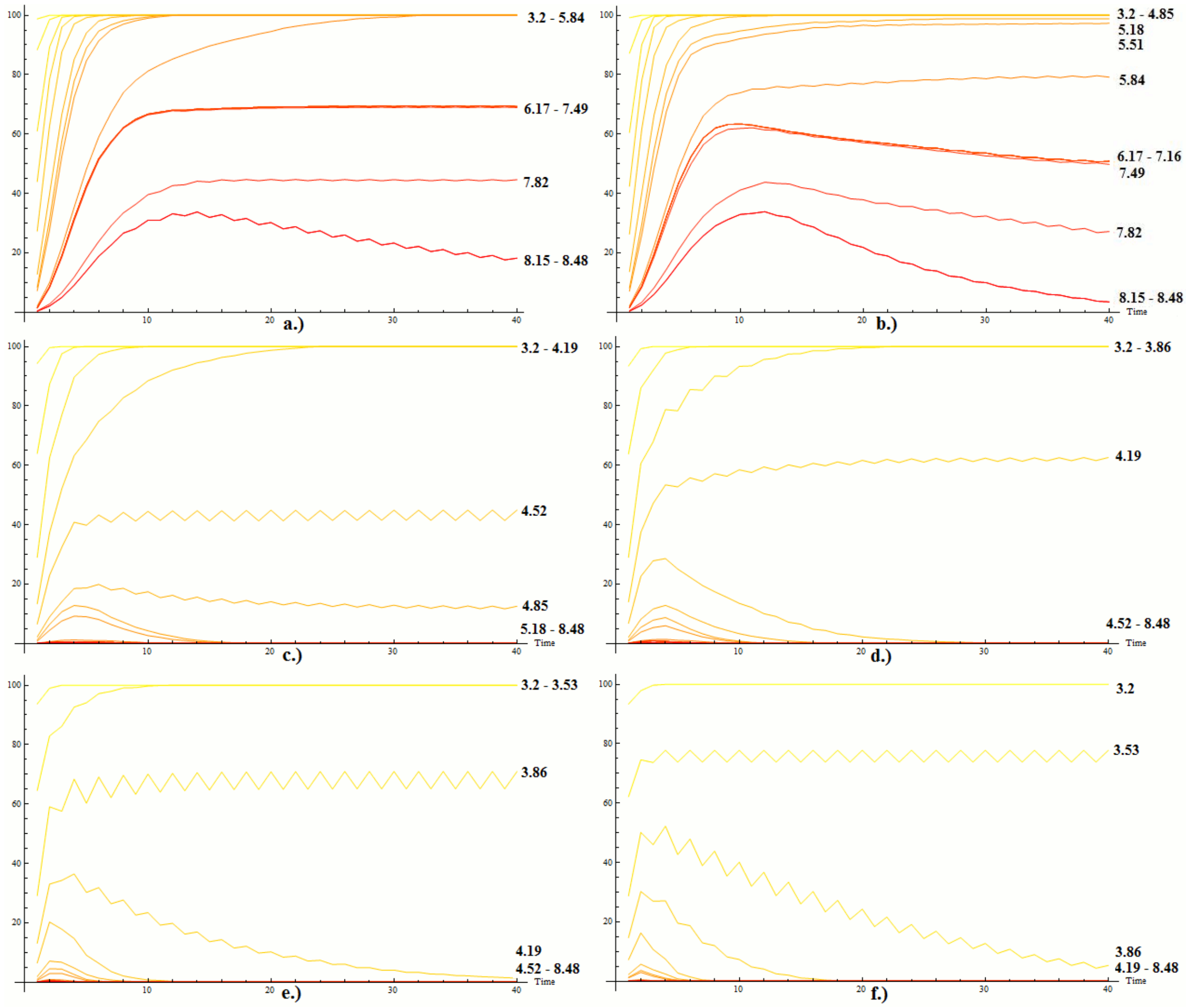


Figure 3.A.S: Two different societies (10.000 and 441 individuals), distribution after one period of interaction

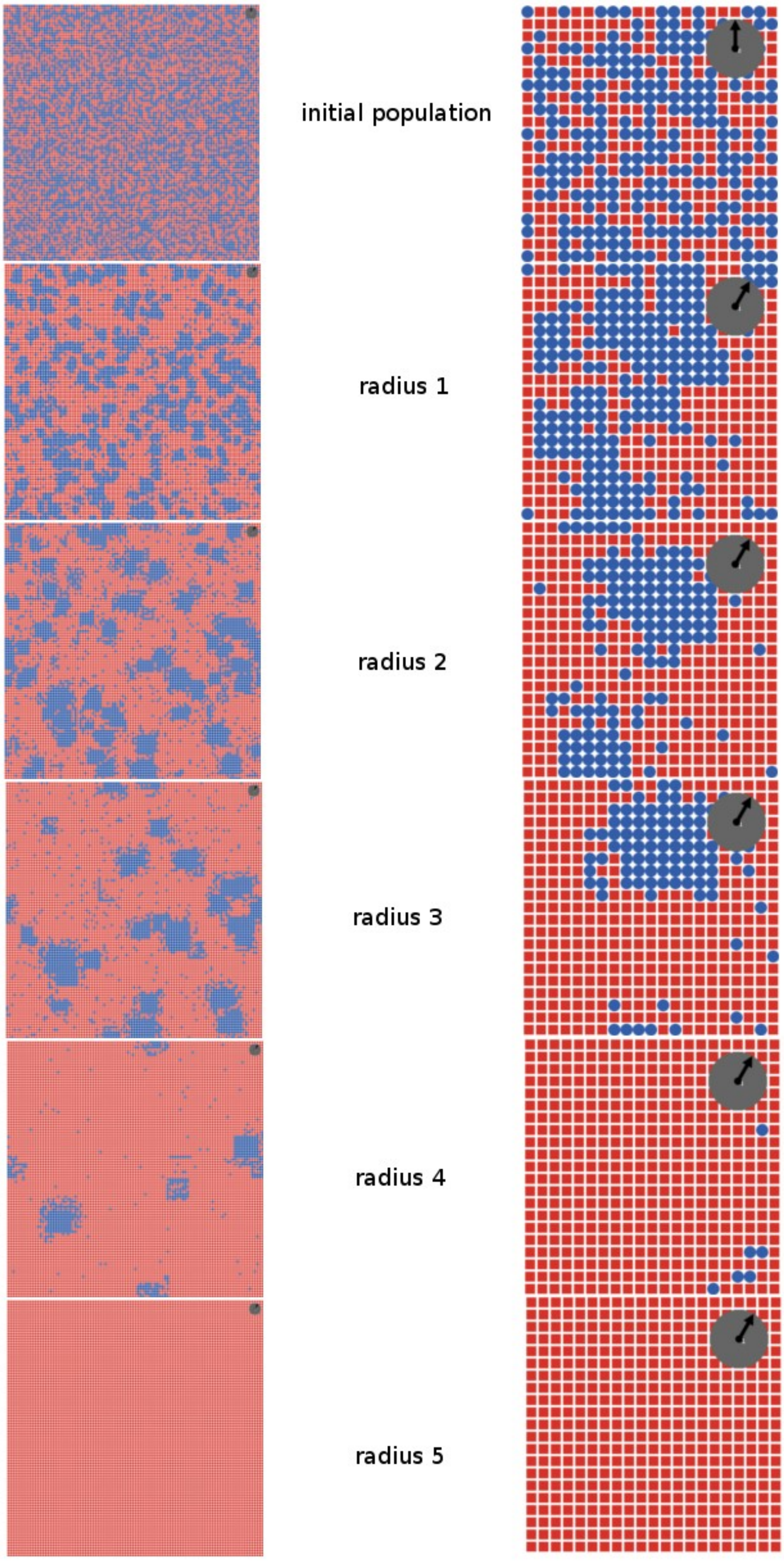
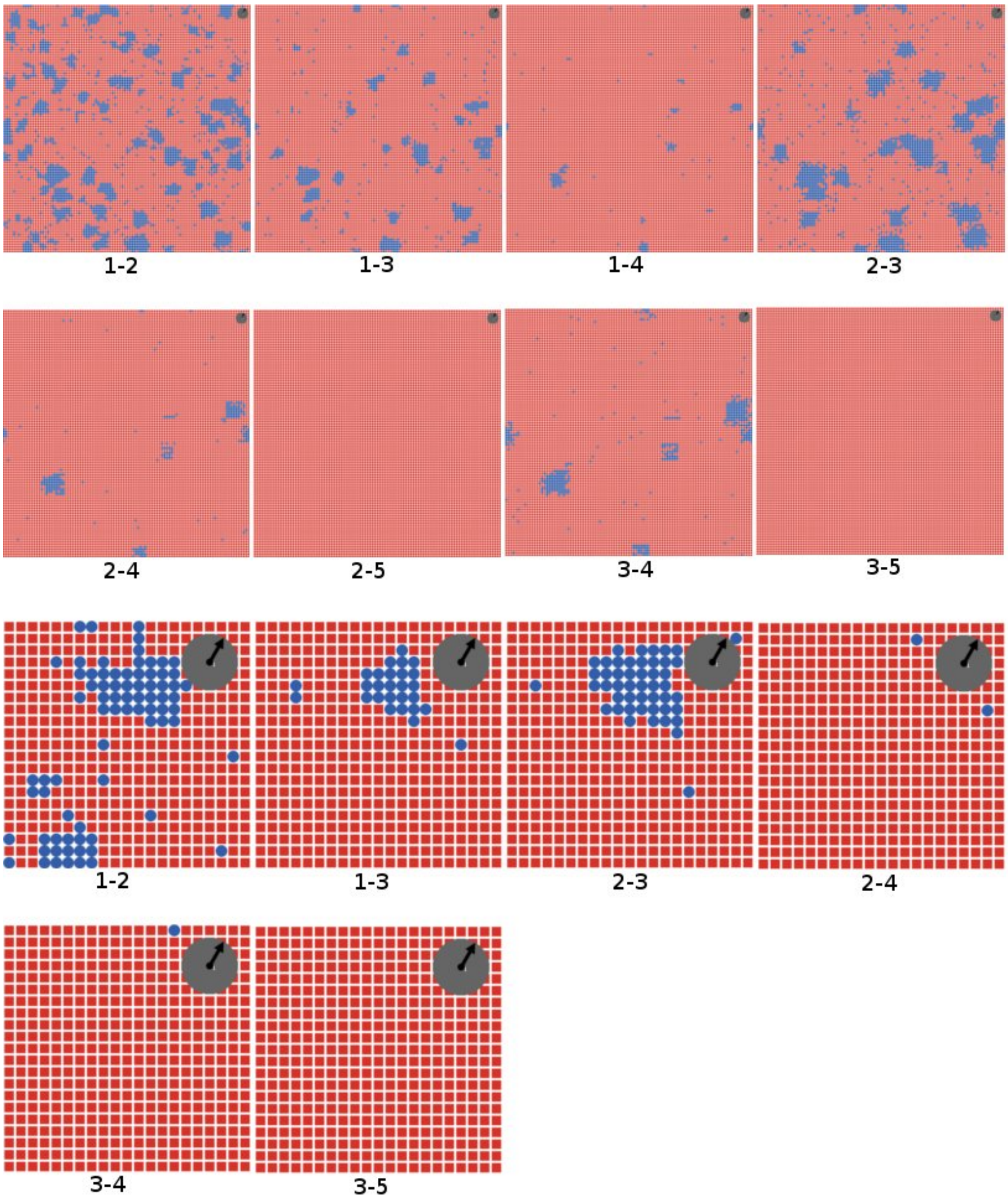


Figure 3.A.T: Two different societies (10.000 and 441 individuals), distribution after one period of interaction - first number indicates radius of imitation, second number pay-off radius



3.B References

1. Ades, Alberto; Di Tella, Rafael (1996):"The Causes and Consequences of Corruption", IDS Bulletin Vol.27, No. 22

2. Bicchieri, Christina (2006): "The Grammar of Society - The Nature and Dynamics of Social Norms", Cambridge University Press, New York

3. Bollinger, Daniel; Hofstede, Geert (1987):"Les Différences culturelles dans le management", Les Editions d'Organisation, Paris

4. Bowles, Samuel (2006):"Microeconomics - Behavior, Institutions, and Evolution", Princeton University Press, Princeton, Oxford

5. Durlauf, Steven N.; Young, Peyton (2001): "Social Dynamics", MIT Press, Cambridge, London

6. Edgerton, Robert B. (2004): "Traditionelle Überzeugungen und Praktiken: Gibt es bessere oder schlechtere?", in "Streit um Werte", ed. Huntington, Samuel P. & Harrison, Lawrence E., Goldman Verlag, München

7. Ellison, Glenn (1993): "Learning, Local Interaction, and Coordination", Econometrica, Vol. 61, No. 5 (Sep.), pp. 1047-1071

8. Ellison, Glenn (2000): "Basins of Attraction, Long-Run Stochastic Stability, and the Speed of Step-by-Step Evolution", Review of Economic Studies, Vol. 67, pp. 17-45

9. Hodgson, Geoffrey M. (1996):"Economics and Evolution - Bringing Life back into Economics", Ann Arbor, The University of Michigan Press

10. Huntington, Samuel P. (1997):"The Clash of Civilizations and the Remaking of World Order", Touchstone Press, New York

11. Huntington, Samuel P.; Harrison, Lawrence E. (2000): "Culture Matters: How Values Shape Human Progress", eds., Basic Books, New York

12. Huntington, Samuel P.; Harrison, Lawrence E. (2004):"Streit um Werte"n, Goldman Verlag, Münche

13. Kandori, Michihiro; Mailath, George J.; Rob, Rafael (1993): "Learning, Mutation, and Long Run Equilibria in Games", Econometrica, Vol. 61, No. 1 (Jan.), pp. 29-56

14. Lee, In Ho and Valentinyi, Ákos (2000): "Noisy Contagion without Mutation", Review of Economic Studies, Vol. 67, pp. 47-56

15. Morris, Stephen (2000): "Contagion", Review of Economic Studies, Vol. 67, pp. 57-78

16. Nelson, Richard R. (2008):"What enables rapid economic progress: What are the needed institutions?", Research Policy, Vol. 37, pp. 111

17. Nowak, Martin A.; May, Robert M. (1992): "Evolutionary games and spatial chaos", Nature, Vol. 359, pp. 826-831

18. Parson, Talcott (1964):"Evolutionary Universals in Society", American Sociological Review, Vol. 29, No. 3, pp. 339-57

19. Page, Scott E. (2007):" The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies", Princeton University Press, Princeton, Oxford

20. Potts, Jason (2000): "The New Evolutionary Microeconomics", Edward Elgar

21. Robson, Arthur and Vega-Redondo, F. (1996): "Efficient Equilibrium Selection in Evolutionary Games with Random Matching", Journal of Economic Theory, Vol. 70, pp. 65-92

22. Weber, Max (1922): Wirtschaft und Gesellschaft, area Verlag, 2007

23. Welzel, Christian; Inglehart, Ronald (1999):"Analyzing Democratic Change and Stability: A Human Development Theory of Democracy", Berlin, Wissenschaftszentrum Berlin für Sozialforschung GmbH (WZB)

24. Young, H. Peyton (1993): "The Evolution of Conventions", Econometrica, Vol. 61, No. 1 (Jan.), pp. 57-84

25. Young, H. Peyton (1998): "Individual Strategy and Social Structure", Princeton University Press, Princeton, Oxford

26. Young, H. Peyton (2005): "Diffusion of innovations in social networks", in "Economy as a Complex Evolving" System, Vol. 3 (ed. L. E. Blume & S. N. Durlauf), Oxford University Press

"Were I Alexander," said Parmenio, "I would accept of these offers made by Darius."
"So would I, too," replied Alexander, "were I Parmenio."

Longinus (400-330 BC)

4

The Theory of Conflict Analysis: A Review of the Approach by Keith W. Hipel & Niall M. Fraser

In the spirit of theoretical pluralism, this chapter critically illustrates an alternative game theoretic approach that expands the Nash equilibrium criterion by assuming that players believe in the empathic ability to anticipate other players' simultaneous and future reactions to their strategic choice. An individual's best response strategy is defined based on this projection, adding additional stability conditions to strategic choice and increasing the set of potential equilibria beyond pure Nash equilibria. Among other interesting properties the approach can thus explain the occurrence of stable outcomes that are not Nash equilibria, such as the cooperative equilibrium in the Prisoner's Dilemma, without the necessity to change the game structure. *Conflict Analysis* further enlarges flexibility as the approach requires only an ordinal preference order. As a basis for future academic debate the assumptions of the *Conflict Analysis* approach are critically analysed and the approach is applied to a set of games, demonstrating its advantages and drawbacks.

Introduction

Conflicts are an essential part of interactions between and within any species and also play an important role in interpersonal relations (Lorenz, 1974).[1] Thus, conflicts constitute a vital part in the description and modelisation of interactions between economic agents. This chapter presents an interesting and efficient alternative game theoretical approach, not only to model conflicts between players, but also to analyse games in general.

This chapter follows Frank Hahn's call for theoretical pluralism, who stated: "...[A]ll these 'certainties' and all these 'schools' which they spawn are a sure sign of ignorance ... we do not possess much certain knowledge about the economic world and ... our best chance of gaining more is to try in all sorts of directions and by all sorts of means." The approach, demonstrated in this chapter, breaks with some of the concepts of rational choice in a way that might not be accepted by all scholars. Yet, the general purpose is to illustrate an alternative perception of rationality that gives rise to academic debate and to call economists' attention to this approach as a potential step towards an augmented game theoretical framework.

The approach, henceforth called "Conflict Analysis" (CA), was originally developed by Keith W. Hipel and Niall M. Fraser(1984) and builds on the initial work of Howard (1971). The approach enjoys some interesting properties that render it especially interesting: It is able to model a higher order reasoning that allows players to anticipate other players' reaction to their strategic choice. It thus expands the original stability concept and obtains equilibria that are plausible but not identified as such by the Nash criterion. Yet, the approach does not require that these "empathic" anticipations of strategy choice and of preferences are correct. The possibility to model a game as a "hypergame" (of higher order) allows for conditions, in which players display a misperception of the rules underlying the game. Furthermore, the approach is capable of tackling larger (non-quantitative) strategy and player sets than most game theoretical approaches solvable in a closed form. Another advantage, though not unique to this approach, lies in the sole requirement of only an ordinal preference order. A cardinal order should demand detailed data on the relative weights that the conflicting parties place on the various outcomes and the strategy profiles. In contrast, this approach stays completely within the concept of preference relations,

[1] Konrad Lorenz is one of the founding fathers of ethology and Nobel prize laureate for physiology or medicine in 1973. In "Das sogenannte Böse" (English title: "On Aggression") he described the root of aggression both for animals and humans and its impact as a primary instinct on social life.

without the necessity of a function that attaches real numbers to each element in the outcome set, thus completely avoiding the standard notion of utility representation. A quantification of the players' preferences, which is because of the lack of precise data mostly arbitrary, can be thus avoided. In addition, the pairwise comparison of preferences over outcomes does not require transitivity of preferences.

After reviewing the theoretical basis of Hipel and Fraser's approach, it is applied to standard one-shot games, to paradoxes and to sequential games. This chapter further analyses the requirements for the "rational" validity of Conflict Analysis and illustrates both the differences and the potentials with respect to standard game theory, discussing the advantages, as well as the drawbacks of this approach. In the current chapter the focus lies more on the static representation. The dynamic representation is only presented in the original formulation of Hipel & Fraser (1984). An extension of the original approach in this respect can be found in *Chapter 5*.

The first section illustrates the general theoretical basis and methodology necessary to analyse games. It constitutes a more concise and analytical representation of the solution algorithm explained in "Conflict Analysis - Models and Resolutions" (Hipel & Fraser,1984). Section 4.2 exemplifies the approach via the Prisoner's Dilemma. It demonstrates how misperception of preferences can be modelled, and introduces a simple dynamic representation. Section 4.3 shows how the approach manages certain popular dilemmas and paradoxes. Section 4.4 applies this approach to various sequential games and shows a way to discriminate between the equilibria obtained. Section 4.5 provides a critical examination of the theoretical background of Conflict Analysis, discussing eventual deficiencies that arise. Section 4.6 constitutes the conclusion. The appendix (section 4.A) contains a short introduction to Metagame Theory, for the interested reader.

4.1 Solution algorithm

In the following the solution algorithm is represented in its easiest form. The general idea is that not all equilibria of a game are captured by the Nash equilibrium concept, but additional stable equilibria exist that fall outside the definition of Nash equilibria. It is assumed that an individual does not only consider those strategies currently chosen by other players, but also takes account of the subsequent and simultaneous potential reactions of other players by applying a certain form of backward induction.[2] Though an individual does not assign specific probabilities to how likely another player chooses a certain subsequent (or simultaneous) response strategy, it is assumed that an individual refrains from playing a strategy that will trigger a response with positive probability, which will change his outcome for the worse. In addition, since the utility ranking is purely ordinal and no pay-offs are assigned to any strategy profile, equilibria are only defined by pure strategies.[3]

An n-person non-cooperative game is defined by $G = (S_1, S_2, ..., S_n; U_1, U_2, ..., U_n)$, with player set $N = (1, 2, 3, ..., n)$. S_i being individual i's strategy set and U_i being defined as i's preference function for each $i \in N$. For the given set of players and the individual strategy sets, a strategy profile is defined by $s = (s_1, s_2, s_3, ..., s_n)$, with $s_i \in S_i$ being the strategy chosen by individual i. The set of all strategy profiles is then defined by $S = S_1 \times S_2 \times S_3 \times ... \times S_n$. There exists a preference function U_i that ranks the strategy profiles according to individual i's preferences over the associated outcomes. U_i is not necessarily a utility or pay-off function, it suffices that it assigns all strategy profiles in the strategy profile set to two subsets with respect to any underlying strategy profile.[4] Hence, U_i is not required to attach a real number to all outcomes in order to present i's preferences. Assume q and p to be two strategy profiles and denote the associated outcome by $O(q)$ and $O(p)$, then

$$\begin{aligned} p \in U_i^+(q), &\text{ iff } O(p) \succ_i O(q) \\ p \in U_i^-(q), &\text{ iff } O(p) \preceq_i O(q) \end{aligned} \tag{4.1.1}$$

with $U_i^+(q) \cap U_i^-(q) = \emptyset$ and $U_i^+(q) \cup U_i^-(q) = S$. Hence, for each strategy profile q, set S is divided into a set of strategy profiles, whose outcomes are strictly preferred to the outcome associated with q and a set of strategy profiles, associated with those outcomes that are not preferred to $O(q)$. Consequently, completeness is the only axiom that must hold for the underlying preferences. Owing to the pairwise comparison

[2]The approach is later on extended to include forward induction. The examples in appendix 4.4 will show that after having obtained the set of equilibria, forward induction can discriminate amongst them, if each equilibrium is defined by a unique path, i.e there is no ambiguity with respect to the position of a player's decision node in the path defined by any equilibrium in the equilibrium set. Once the first mover has chosen a strategy, each subsequent player knows for sure, which equilibrium path is followed.

[3]On the issue of pure strategies, see Harsanyi, 1973 and Morris, 2006. The assumption, by which a minimal loss probability, deriving from a strategy switch, is sufficient to deter a player from choosing this strategy, is similar to the maxmin criterion used in Decision Theory, see Gilboa & Schmeidler, 1988; and Hey et al., 2008.

[4]In other words, U_i is a function from set S to its Boolean $B(S)$.

of outcomes, transitivity is unnecessary. This chapter will, however, concentrate on games, which exhibit transitive preferences.[5] For convenience I will henceforth use the formulation that strategy profile q is preferred to p, meaning that the outcome associated to q is preferred to the outcome associated to p.[6] The preference order thus obtained will be the basis of the analysis. In order to determine equilibrium strategies, first define the set of strategies to which a player has a possible incentive to switch, given the strategies of the other players. These strategies are defined by the set of "dominant profiles". Second, these strategies need to be analysed for their validity, i.e. if the player has still an incentive to switch after taking into account the potential simultaneous and subsequent responses of other players:

I. A given strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ is defined by the strategy $\bar{s}_i$ of player i and the strategy profile $\bar{s}_{-i}$, determined by the strategic choice of all players other than i. Denote a strategy profiles, which can be obtained by a unilateral strategy switch, by $z_i(q) = (s_i, \bar{s}_{-i})$, with any $s_i \in S_i$. Given the set $Z_i(q)$ of all strategy profiles that can be obtained by a unilateral switch of i, the set of "dominant profiles" for q is then defined as

$$\textbf{DP:} u_i^+(q) = Z_i(q) \cap U_i^+(q), \forall s_i \in S_i \tag{4.1.2}$$

In other words, for each underlying strategy profile, the possible better response strategies of player i are defined by a set of strategy profiles *(first)* that can be obtained by a unilateral strategy switch, given the strategy profile of all players other than the player being analysed, and *(second)* that are strictly preferred to the current strategy profile by this player.[7] Hence, we do not only assign a single

[5] It must clearly hold that for any strategy profile o either $o \in U_i^+(q)$ or $o \in U_i^-(q)$, but never both since both sets are disjoint, nor neither, as $U_i^+(q) \cup U_i^-(q)$ cover the entire outcome space. The presentation in 4.1.1 allows for intransitivity. Given $O(o) \succ_i O(p), O(p) \succ_i O(q)$, but $O(q) \succ_i O(o)$, the intransitive preference is represented by $o \in U_i^+(p)$, $p \in U_i^-(o)$, $p \in U_i^+(q)$, $q \in U_i^-(p)$, $q \in U_i^+(o)$, and $o \in U_i^-(q)$. Transitivity should be only required, if it strictly holds that $O(p) \succ_i O(q)$ implies $U_i^-(q) \subset U_i^-(p)$.

[6] Strictly speaking, an individual does not retain a preference over strategy profiles, but over the associated outcomes/consequences. It is assumed that in a given state ω a unique strategy profile is associated to each outcome and, hence, for state ω we can simply speak of an individual preference order over strategy profiles. In another state ω', a player might associate other outcomes to a strategy profiles. Thus the preference order is not unique, but depends on the state. *Chapter 5* exploits and extends this property.

[7] Notice that for complete and transitive preferences and a strict preference order, such a set can only consist of strategy profiles that lie in the direction of preference (here: to the left in the preference order in the later representation). Also notice that in this definition, a *DP* necessitates strict prefe-

strategy profile defined by the best response strategy to a given strategy profile, but all strategy profiles $p = (s_i^*, \bar{s}_{-i})$ for any $s_i^* \in S_i$, such that $O(p) \succ_i O(q)$. In other words, all those strategy profiles defined by all possible "better" response strategies of player i define a dominant profile. - Throughout this chapter a dominant profile is denoted in short as "DP".

II. It needs to be checked whether such a possible better response is still valid, if a player reasons about sequential or simultaneous better response strategies of other players. As a result, in addition to the standard Nash equilibria, Conflict Analysis defines two more criteria for stability: sequential and simultaneous stability. In general, if a strategy is the unique *valid* best response, the strategy is considered to be *stable* for this player as he has no incentive to switch. Though a slight abuse of the standard definition, a *strategy profile* defined by such a stable strategy for player i is also defined as being *stable for player i*. An equilibrium is thus defined by a strategy profile, in which each component is a stable strategy given the other strategies; or using the slightly abusive definition, by a strategy profile that is stable for all players. For any strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ we obtain the following forms of individual stability for any player i:

(a) Rational Stability: Like in the standard Nash approach, an individual has no incentive to change his strategy, if he is already playing the *rational* best response to the strategies chosen by all other players, implying that no other possible better response strategy exists. If strategy profile q characterises the best response strategy for player i to all other players' strategies in the strategy profile, this strategy profile is defined as *rationally stable for player i*. Hence, a strategy profile q is rationally stable for player i, if the set of dominant profiles is empty. Thus for $q = (\bar{s}_i, \bar{s}_{-i})$ to be rationally stable for player i, it must hold:

$$\textbf{Rational Stability: } u_i^+(\bar{s}_i, \bar{s}_{-i}) = \emptyset, \forall s_i \in S_i \tag{4.1.3}$$

(b) Sequential stability: A switch of player i to a better response strategy can entail a subsequent switch in strategies of another player j, since j's strategy is no longer best response. This may result in a strategy profile that is not strictly preferred to the original strategy profile by player i. Consequently,

rence. In later games I will relax this assumption and illustrate the effect of weak preference on the equilibrium set.

player i will refrain from choosing this possible better response strategy, since a switch will not make him better off. If all possible better response strategies will eventually lead to not strictly preferred outcomes, the current strategy defined by the underlying strategy profile is best response; thus this strategy profile is defined as *sequentially stable for player i*.

Assume that player i switches from the strategy defined by strategy profile q to a possible better response strategy, thus changing the outcome to a strategy profile defined in the set of DP's for q. Let this DP be defined as p. Remember that all possible better response strategies for any player are defined by the individual dominant profiles for this player. The set of better response strategies of another player j to the new strategy profile p is then defined by j's DPs for profile p. It is thus sufficient to look at the DPs of player i for strategy profile q and at the DPs of all other players different from i for p. Consequently a DP, or more correctly the strategy of player i defined by the DP, is sanctioned, if there exists some player j, who can choose a *viable* response strategy to the possible better response strategy (defined by p) of player i in such a way that the resulting strategy profile is not strictly preferred by player i to the one from which he originally deviated (q). Viable is thereby defined as a strategy switch that immediately results in a strategy profile strictly preferred by player j (i.e. defined by a DP of player j to strategy profile p).[8]

As a result, in order for a DP to be *sequentially* sanctioned, it is already sufficient that at least one possible better response strategy (i.e. DP) of one other player to player i's strategy choice exists that induces a less preferred strategy profile for i. If all "better response" strategies of player i are sanctioned in such a way, the current strategy is best response.[9]

For any player j define $\hat{u}_j^+(p) = Z_j(p) \cap U_j^+(p)$ as the set of DPs for player j to player i's dominant profile p for q, i.e. the set of strategy profiles obtained

[8]The viability assumption avoids that a player strategically chooses a strategy that deteriorates his utility hoping that the others response strategies will eventually make him better off. The assumption also evades cycles. This is, however, not the case if we assume that equally preferred strategy profiles can be DPs, i.e. if only a weak preference is necessary. Therefore I am in favour of refusing this last assumption of equally preferred strategy profiles serving as DOs. Examples in section 4.3 will elaborate this issue.

[9]One might think of sequential stability as the best response strategy derived from backward induction with high ambiguity aversion. We will, however, observe in section 4.3 that backward induction is not equivalent to sequential stability.

by player j's better response strategies to strategy profile $p = (s_i^*, \bar{s}_{-i})$, with $O(p) \succ_i O(q)$. In order for $q = (\bar{s_i}, \bar{s}_{-i})$ to be sequentially stable for player i, it must hold:

$$\textbf{Sequential stability: } \hat{u}_j^+ (p = (s_i^*, \bar{s}_{-i})) \cap U_i^- (q = (\bar{s_i}, \bar{s}_{-i})) \neq \emptyset, \quad \forall s_i^* \in S_i : p = (s_i^*, \bar{s}_{-i}) \in u_i^+(q) \textit{ and for any } j \neq i \tag{4.1.4}$$

(c) Instability: If i's strategy defined by q is not best response to the other players' strategies defined in q (i.e. if the set of dominant profiles for q is not empty and at least one dominant profile is not sequentially sanctioned by a viable response strategy of at least one of the other players) strategy profile q is termed *unstable for player i*. In other words strategy profile q is unstable, if neither condition 4.1.3 nor 4.1.4 hold. Hence, player i will switch to the strategy defined by the unsanctioned DP, as this will lead with certainty to a strictly preferred strategy profile. As a direct result from the previous definitions, for profiles q and p defined as before, q is unstable, if:[10]

$$\textbf{Instability: } \exists p \in u_i^+(q) : \hat{u}_j^+(p) \cap U_i^-(q) = \emptyset, \forall j \neq i \quad \text{and some } s_i^* \in S_i : p = (s_i^*, \bar{s}_{-i}) \tag{4.1.5}$$

(d) Simultaneous stability: In addition to the previous types of stability, simultaneous stability can occur in games that are not sequential or if the other players' strategy choices are mutually unknown. It is generally a weaker and rarer form of stability and should be checked for plausibility.[11] The main idea is that if more than one player *simultaneously* switch strategies from a current strategy profile, where possible better responses exist for those players, the resulting strategy profile may be not strictly preferred by the player currently analysed. Hence, probable occurrence of such a simultaneous strategy change deters the player from switching his strategy. If for all strategies of player i, which destabilise q through equation 4.1.5, such a simultaneous switch of other players occurs with positive probability and, in addition, can lead to a strategy profile not strictly preferred to q by player i,

[10] Note that the following definition also includes $\exists p \in u_i^+(q) : \hat{u}_j^+(p) = \emptyset, \forall j \neq i$ and some $s_i^* : p = (s_i^*, \bar{s}_{-i})$.

[11] Section 4.2.3 will discuss a game, where simultaneous stability is of major importance, as it captures an effect similar to risk dominance.

then strategy profile q is termed "simultaneously stable for player i". Other players are only likely to switch, if they also have a valid better response for q. The set of other players is thus defined by all those players for whom q is unstable. Consequently, simultaneous stability needs only to be checked for strategy profiles that were previously defined as unstable and only for those corresponding DPs that are not sequentially sanctioned.

Since a player has no information about the strategy choice of other players, a simultaneous strategy switch can be effected by the entire set of players, who possess a viable, not sequentially sanctioned, better response strategy, but also only by a subset.[12] Let o be a possible strategy profile resulting from a simultaneous switch in strategies of other players for whom there exists a DP for some strategy profile q. Hence, o is defined both by components that are identical to those in q (the players, who did not switch) and by components consisting of strategies defined by a not sequentially sanctioned DP of q for each player that switched. This includes also player i's switch to some strategy s_i^o fulfilling condition 4.1.5. If o is not strictly preferred to q by player i, strategy s_i^o is "simultaneously" sanctioned and will not be chosen by i. If this is the case for all DPs that rendered the strategy profile unstable for i, strategy profile q is *simultaneously stable for* i. The current strategy is best response with respect to the possible simultaneously chosen strategies of the other players, who have an incentive to switch.

For any player j, define $S_j^\varsigma(q)$ as the set of strategies of player j that render a strategy profile q unstable plus the strategy originally played, i.e. all those strategies that are defined by the unsanctioned DPs of q according to equation 4.1.5, as well as the strategy $\bar{s}_j$ corresponding to profile q. Thus, for $q = (\bar{s}_i, \bar{s}_{-i})$ the set of simultaneously attainable strategy profiles is given by $S^\varsigma(q) = S_1^\varsigma(q) \times S_2^\varsigma(q) \times ... \times S_m^\varsigma(q) \times \bar{s}_{-M}$ for a player set $M = (1, 2, \dots, m) \subseteq N : h \in M$, iff $(S_h^\varsigma(q) \setminus \{\bar{s}_h\}) \neq \emptyset$. In other word, set M is defined by those players, who posses a non-sanctioned DP for q, including player i.[13]

[12] e.g. This implies, that seven possible player combinations for each player i have to be analysed $(\sum_i \binom{m-1}{i})$ for a case, in which a strategy profile is unstable for four players of the entire player set.
[13] Hence, simultaneous stability adds the idea of eventual simultaneous switches to the underlying assumption of backward induction.

Simultaneous stability: $\forall s_i^{\varsigma} \in S_i^{\varsigma}(q), \exists \hat{s}_{-i}^{\varsigma} \in S_{-i}^{\varsigma}(q) : (s_i^{\varsigma}, \hat{s}_{-i}^{\varsigma}) \in U_i^{-}(q)$ (4.1.6)

III. The definition of an equilibrium is identical to the standard approach. The set of equilibria of the game is specified by all strategy profiles, in which each component is defined by the best response strategy of each player given the strategies chosen by all other players, i.e. all those strategy profiles that are stable for *all* players (either rational, sequential or simultaneously). Notice, however, that only if a strategy profile is *rationally* stable for all players, it is a Nash equilibrium. All those equilibria that are not rationally stable for one or more players but either sequentially or simultaneously stable would not be defined as an equilibrium in the standard approach.

An example will be given in the following section. The next subsection will elaborate the form of representation used in this and the following chapter.

4.1.1 Representation

Since this approach goes beyond the Nash definition of an equilibrium by adding sequential and simultaneous stability, a representation of a game in normal or extensive form is insufficient. It is therefore necessary to spend a few words on the structure of analysis. Each strategy can define a set of actions, such that an individual strategy consisting of r independent actions is defined as $s_i = (a_{1i}, a_{2i}, \ldots, a_{ri})$. A player has the choice whether or not to take a certain action. Define the set $A_{ki} = (a_{ki}, \neg a_{ki})$, so set A_{ki} consists of two elements, the first meaning that action k is chosen by player i, the second that it is not. Whence we obtain that $S_i \subseteq \times_k A_{ki}$, where the equality holds if none of the actions are mutually exclusive. Hence, each strategy of a player i can be uniquely defined by a binary vector of length equal to the number of actions that player i possesses. Similarly, also each strategy profile can be uniquely identified by a binary vector equal to the sum of all available individual actions. Each element in this vector defines an individual action and its value whether the action is chosen or not.

Assume a game with three players, where the strategy set S_i is defined by the number of actions $x = |\bigcup_k A_{ki}|$, strategy set S_j by the number of actions $y = |\bigcup_k A_{kj}|$, and strategy S_h by the number of actions $z = |\bigcup_k A_{kh}|$, implying that the actions are mutually non-exclusive for player i, j and h, respectively.[14] In such a three player game, each strategy profile $q = (s_i, s_j, s_h)$ can be defined by a binary vector of length $x + y + z$, given by $\hat{q} = (\overbrace{I, I, \ldots, I}^{x}, \overbrace{I, I, \ldots, I}^{y}, \overbrace{I, I, \ldots, I}^{z})^T$. I denotes a binary value of either 0 or 1, where 1 implies that the action is chosen, 0 that it is not. Hence, each player individually defines the sequence of this binary vector for a length equal to the number of available actions. As an example, for a three player game, in which each player has two mutually non-exclusive actions, one strategy profile p is defined by $\hat{p} = (0, 1, 1, 0, 1, 0)^T$. The length of this binary vector can be reduced in the case of mutually exclusive actions. If an action A can only be chosen, if an action B is not and the inverse, but one action has to be chosen, then both action can be described by a single digit in the binary vector. $I = 1$ could be defined as A is chosen by a player i, and thus $I = 0$ would mean that B is chosen.

Each such binary vector can be again uniquely defined by a decimal code, calculated as follows: In general the binary vector has $|\bigcup_{i \in N}(\bigcup_k a_{ki})|$ digits (less the number of those actions reduced by the aforementioned simplification in the case of mutually exclusive actions) that have either the value 1 or 0. Like the binary code of a computer this can be rewritten by taking the sum over the products of the digit times two to the power of the position in the vector. Consequently, the example $\hat{p} = (0, 1, 1, 0, 1, 0)^T$ can be written as $0 * 2^0 + 1 * 2^1 + 1 * 2^2 + 0 * 2^3 + 1 * 2^4 + 0 * 2^5 = 22 = \dot{p}$. The value of 22 does not represent a preference, but is the short representation of a strategy profile.

A preference order can thus be defined as a vector of length equal to the sum of actions ($|\bigcup_i \bigcup_k A_{ki}|$), reduced by the actions that are mutually exclusive, that can be transformed into a natural number defining a strategy profile.

Given the assumptions, the preference function U_i orders the strategy profiles into the preference vector according to the preferences of player i over the associated outcomes. Since preferences are strictly ordinal, it suffices to note down the natural numbers, identifying each a strategy profile, in a vector, where the position of the component indicates the preference. Starting with the most preferred, strategy profiles can be ordered from the left most position to the right. This implies that for strict and transitive preferences each strategy profile can have only one position in the preference vector and it is strictly preferred to all strategy profiles noted further to the right, i.e. for $O(q) \succ_i O(p) \rightarrow U_i = (\ldots, \dot{q}, \ldots, \dot{p}, \ldots)$.

[14] $|.|$ denotes the cardinality of a set, i.e. absolute number of elements in the set.

4.2 Example - A Prisoner's Dilemma

This section will illustrate the approach presented above. First notice that for simplicity, whenever the game representation has been changed to the game form used in the Conflict Analysis approach, I will speak of strategy profile $\dot{x}$, where $\dot{x}$ is in fact the natural number defined by the decimal code that refers to strategy profile x.

For two reasons, the Prisoner's Dilemma (PD) is chosen as an example: First, it is a simple game known to most social scientists and second, it also shows some theoretical intricacies, unapparent in other games (for a detailed discussion of theoretical issues concerning the PD, see section 4.5 beginning on page 158). Suppose a game G with two players $i = A, B$, where each player possesses an action, which he is free to take or not, and thus two strategies $S_i = \{\text{not confess}, \text{confess}\}$. The pay-offs represent the players' preferences over the outcomes, each defined by a strategy profile. Hence, the pay-off of player i is given by $\pi_i(s_i, s_j)$, with $i \neq j$ under strategy profile (s_i, s_j). Furthermore, assume that pay-offs are symmetric, i.e. independent of a player's position. The general symmetric 2x2 game is represented by the following normal form game:

$$\begin{array}{l} \\ \textbf{not confess} \\ \textbf{confess} \end{array} \begin{array}{cc} \textbf{not confess} & \textbf{confess} \\ \left(\begin{array}{c} a, a \\ c, b \end{array} \right. & \left. \begin{array}{c} b, c \\ d, d \end{array} \right) \end{array} \tag{4.2.1}$$

Assuming $c > a > d > b$ and $2a > b + c$ turns the game into a Prisoner's Dilemma. Joint non-confession is welfare maximising, but joint confession is the single Nash equilibrium, since *confess* strictly dominates *not confess*.

In this game, each player has the choice over the single action ($a_i = confess$). Each of a player's two strategies can thus be defined by a single binary value, and a strategy profile can be uniquely defined by a vector with two binary components, one for each player. A vector $(1, 1)^T$ means that both players confess, whereas $(1, 0)^T$ implies that player A confesses, but player B does not. Each of these strategy profiles can be converted to a decimal value based on the strategy composition. Table 4.2.a illustrates the decimal representation.

Following the previous assumption that $c > a > d > b$, the preference order for player A is represented by a vector $(1, 0, 3, 2)$ and for player B the preference order

Table 4.2.a: coding the binary strategy profiles into decimal digits - strategy coding: confession=1, no confession=0

Set of Strategy profiles				
Player A	0	1	0	1
Player B	0	0	1	1
Decimal Code	**0**	**1**	**2**	**3**

is defined by vector $(2, 0, 3, 1)$. By condition 4.1.2 and since *confess* strictly dominates *not confess*, it holds that $u_A^+(1) = \{0\}$, $u_A^+(3) = \{2\}$, $u_B^+(2) = \{0\}$, and $u_B^+(3) = \{1\}$, and all other sets are empty. Player A can unilaterally improve from strategy profile 0 *(not confess, not confess)* to the dominant profile 1 by choosing strategy *confess*. This enables him to increase his pay-off from a to c. Furthermore player A can unilaterally switch from strategy profile 2 to the dominant profile 3. For player B the DP from strategy profile 0 is strategy profile 2, from strategy profile 1 it is strategy profile 3.

By condition 4.1.3 strategy profiles 1 and 3 are rationally stable for player A, and strategy profile 2 and 3 are rationally stable for player B. Since a switch to strategy profile 3 is unsanctioned, strategy profile 2 is unstable for player A according to condition 4.1.5. The same holds for player B with respect to strategy profile 1. A strategy switch from strategy profile 0 to his DP is sanctioned for both players through the subsequent switch of the other player to strategy *confess*. Since $O(0) \succ_i O(3)$ condition 4.1.4 holds for both players. By definition of stability, strategy profiles 0, 1, and 3 are stable for player A, strategy profiles 0, 2, and 3 are stable for B. Consequently, 0 and 1 define the equilibria of the game.

The game can be much easier analysed, especially in the case of more strategies and players, when put into a form similar to the classical normal form. The sequential analysis renders, however, the normal form insufficient. The following presentation is thus a mix between the normal and extensive form. It will be used throughout the remaining parts of the chapter, since it succinctly represents both the game's dynamics and equilibria.

In order to derive table 4.2.b, first, note down the preference order for both players. Those have already been derived above from condition $c > a > d > b$. Second, note down the dominant profiles given by condition 4.1.2 under each strategy profile in order of preference. Notice again that, as in the Prisoner's Dilemma preferences are strict, a DP can only be a strategy profile that appears to the left of the strategy profile, under which the DP is written. Consequently, the strategy profile that is farthest to

Table 4.2.b: Solution to the Prisoner's Dilemma

Stability Analysis						
equilibrium		x	E	E	x
Player A	Stability	r	s	r	u
	Preference Order	1	0	3	2
	DP		1		3
Player B	Stability	r	s	r	u
	Preference Order	2	0	3	1
	DP		2		3

the left is always rational, since a *DP* cannot exist. Based hereupon, the stability of each strategy profile can be analysed according to the conditions described above. The abbreviation for the stability should be read as follows: *u* - unstable, *r* - rationally, *s* - sequentially, and *û* - simultaneously stable.

Starting with player A, for strategy profile 1 and 3, the boxes indicating their *DPs* are empty. Hence, both strategy profiles are rationally stable. For player *A*, the *DP* of 0 is to switch to 1. Yet, the *DP* of 1 for player B is 3. As 3 is further to the left than 1 in the preference order of player A (i.e. it is strictly less preferred), this strategy switch is sequentially sanctioned. Since this is the only available *DP* for 0, this strategy profile is sequentially stable for player *A*. A switch of *A* from 2 to 3, on the contrary, does not affect player B's strategy choice, since 3 has no *DP* for that player. A switch from 2 =*(not confess, confess)* to 3 =*(not confess, not confess)*, will thus not trigger any response by player *B* and thus player *A* can impose unilaterally the preferred strategy profile. The same analysis for player *B* reveals that 1 is unstable, but a switch from 0 to 2 is sanctioned by player *A*'s shift to 3.

It remains to test for simultaneous stability of strategy profile 2. Since player B has no *DP* from 2 condition 4.1.6 does not hold. The same with respect to 1, from which player *A* has no *DP*. Notice that if strategy profile 0 were not already sequentially stable, it would be simultaneously stable.[15] Each strategy profile that is not assigned a *u* defines an equilibrium, illustrated by *E* in the top row.

Thus, for the Prisoner's Dilemma we obtain two equilibria in pure strategies. One is defined by joint non-confession, the other by joint confession. None of the two

[15] Assume that for both players' the *DP*s for strategy profile 0 were not already sequentially sanctioned and simultaneous stability needed to be tested. A simultaneous switch of both player *A* and *B* to confess (which is their preferred strategy according to the *DP*s) will result in strategy profile 3, which is strictly less preferred.

strategies is strictly dominant.[16] This contrasts with the classical analysis. Conflict Analysis keeps the game's structure, and has therefore an advantage over explanations using other regarding preferences, if the original notion of Prisoner's Dilemma and its validity for social interactions should be maintained. By transforming the pay-off matrix in such a way, the game ceases to be a Prisoner's Dilemma. Inference can only be made for this new game as the rules of the game are changed and not the structure of analysis. In the Conflict Analysis approach the class of preference ordering, on the contrary, is not enlarged beyond the original definition of a PD, since preferences stay purely "self-referential". Yet, the stability of the cooperative equilibrium requires at least a supplementary assumption. Furthermore, both equilibria will not occur with equal probability and the cooperative equilibrium will only arise, if additional conditions hold. A discussion of these issues is postponed to subsection 4.2.2 on page 139 and section 4.5, beginning on page 158.

Also notice that the analysis of simultaneous stability can be simplified by using the decimal value that is attributed to each strategy profile. Assume that a strategy profile with decimal value $\dot{q}$ should be tested for simultaneous stability and $\dot{o}_i$ is the decimal value of the corresponding DP for player i. The new possible strategy profile given by value $\hat{q}$ is defined by

$$\hat{q} = \sum_{i=1}^{x} \dot{o}_i - (x-1)\dot{q},\ x = 2, 3, ..., m \tag{4.2.2}$$

where x is equal to the number of players under consideration from the total set of players M, who possesses a DP from q.[17]

4.2.1 Multi-Level Hypergames

A hypergame occurs, whenever some player j is wrong about the true nature of game G and perceives a game that either or both differs with respect to the actual preference order or to the available strategies in the sets S_{-j} of the other players. Define player i's strategy set and preference order by the vector $V_i = \{S_i, U_i\}$. A non-cooperative n-

[16]Those readers familiar with "Metamagical Themas" (Hofstadter,1985) will recognise the similarity between the Conflict Analysis approach and "superrationality".

[17]Consider the example of strategy profile 0. After a simultaneous switch of both players from 0, the new equilibrium would be given by $3 = (1+2) - (2-1)0 = 3$ (since $x = 2$). If there were three players $(A, B, C) \in M$, then x can take value 2 and 3. Simultaneous stability has to be checked for the cases, in which all players choose a DP (hence $x = 3$) and only two players react ($x = 2$).

player game can be represented by $G = (V_1, V_2, .., V_n)$. If one or more players misperceive the underlying rules, game G for player j is given by $G_j = (V_{1j}, V_{2j}, .., V_{nj})$ and hence, a first level hypergame is defined as $H = (G_1, G_2, ..., G_n)$. If other players are aware of the faulty perception of player j, the game turns into a second level hypergame, where the game for player j is defined by an individual first level hypergame $H_q = (G_{1q}, G_{2q}, ..., G_{nq})$. Consequently the second level hypergame is represented by $H^2 = (H_1, H_2, ..., H_n)$. The reasoning can be continued for higher level hypergames. A third level hypergame would occur in the case, where some player erroneously perceive another players misperception, which is again recognised by other players. The third level hypergame will be represented by $H^3 = (H_1^2, H_2^2, ..., H_n^2)$. In the case of two players with $i = A, B$ a first level hypergame is characterised by $H = (G_A, G_B)$. A third level hypergame will have the form

$$H^3 = (H_A^2, H_B^2) = \begin{Bmatrix} H_{AA} & H_{BA} \\ H_{AB} & H_{BB} \end{Bmatrix} = \begin{Bmatrix} (G_{AAA} & G_{BAA}) & (G_{ABA} & G_{BBA}) \\ (G_{AAB} & G_{BAB}) & (G_{ABB} & G_{BBB}) \end{Bmatrix}.$$

The equilibria of a first level hypergame depend on the stability of each player's strategies within their individual games. The set of equilibria is defined by those strategy profiles that are stable according the the individual perception given by the individual stabilities in $H = (G_A, G_B)$, i.e. by the strategy profiles stable both in V_{AA} and V_{BB}. Suppose both players erroneously believe that the other player most prefers none of them in prison and least prefers both to be imprisoned. The game that A believes to be playing is given by $G_A = (V_{AA}, V_{BA}) = (\{1, 0, 3, 2\}, \{0, 2, 1, 3\})$ and B's game will be given by $G_B = (V_{AB}, V_{BB}) = (\{0, 1, 2, 3\}, \{2, 0, 3, 1\})$. The stabilities are derived for each game individually. They are, as well as solution to the first level hypergame, represented in table 4.2.c. The stabilities for V_{AB} and V_{BA} are not written down specifically, since they are irrelevant for determining the set of equilibria of the first level hypergame. They are only of importance, when checking for eventual simultaneous stability. In this game, however, none of the strategy profiles is simultaneously stable.

If both players believe the other player to be more *altruist*, the only possible equilibrium will be strategy profile 3. A switch to the non-cooperative strategy is not sanctioned by a sequential switch of the other player. In the case of higher order hypergames and a two player game, the analysis of higher level hypergames can be reduced to the examination of the two games on the top left and bottom right on the main diagonal of the hypergame matrix (for an example see, Hipel & Fraser, 1983 Ch. 3 & 4). The two games are sufficient to determine the final set of equilibria in the

Table 4.2.c: Solution to the Prisoner's Dilemma Hypergame

Stability Analysis					
equilibrium		x	x	E	x
V_{AA}	Stability	r	u	r	u
	Preference Order	1	0	3	2
	DP		1		3
V_{AB}	Preference Order	0	2	1	3
	DP		0		1
V_{BA}	Preference Order	0	1	2	3
	DP		0		2
V_{BB}	Stability	r	u	r	u
	Preference Order	2	0	3	1
	DP		2		3

higher level hypergame, independent of the order of the underlying hypergame.[18] The higher level hypergame then breaks down into a first level hypergame with $H = (G_{AAA...}, G_{BBB...})$, as the other matrix elements are of no importance. Only strategy profiles that are stable both in $V_{AAA...}$ and $V_{BBB...}$ form elements of the set of equilibria. This is a direct consequence from the simple fact that $H = (G_{AAA...}, G_{BBB...})$ represents the game that each player believes to be playing respectively.[19]

[18] An analysis of the other games is only necessary, if it is of interest what individuals believe about the outcome of their hypergame, i.e. their analysis along the various levels of misperception in the higher level hypergame. If individual perceptions about the equilibria in their game is irrelevant, the analysis can be substantially simplified.

[19] Nevertheless, Hipel & Fraser's analysis can be even further refined, if these individual games are of interest. Consider another approach: First analyse all zero level games of the original n-th level game. Each zero level game (there will be 2^r, with r being the order of the hypergame; but games of higher order than three are unlikely to occur) will result in a set of possible equilibria. These equilibria will define the strategy that a player will choose and hence, will determine the set of equilibria for the next higher level hypergame. The next-level equilibria are determined in the same way that new equilibria are derived, if testing for simultaneous stability (equation 4.2.2). The equilibria obtained should be checked, if it is stable according to the underlying stabilities for both players in the corresponding hypergame (i.e. the top left and bottom right elements). An example will make the idea clear: Assume a second order hypergame of a game as in matrix 4.2.1 given by some arbitrary pay-off configuration, and for simplicity that there is only one equilibrium for each zero level hypergames. Assume them to be $G_{AA} = 2$, $G_{BA} = 1$, $G_{AB} = 1$, and $G_{BB} = 2$. Hence, A assumes that the game without erroneous play would end up in equilibrium 2, but that B will perceive the game to end up in equilibrium 1. B believes the inverse. Consequently, hypergame H_A will lead to equilibrium 0 - player A chooses *not confess* in accordance with equilibrium 2, he expects the same for B in accordance with equilibrium 1, the result being equilibrium 0 ((*not confess, not confess*). The same reasoning will lead to equilibrium 3 ((*confess, confess*) for H_B, which player B believes to be playing. Accordingly, equilibria 0 and 3 will lead to $H^2 = 2$ ((*not confess, confess*). Hence, 2 will define the hypergame's equilibrium. Both approaches will probably lead to identical

4.2.2 Dynamic Analysis

The approach presented so far is purely static, yet most conflicts are dynamic processes. It is necessary to adapt the approach to explain and model the dynamics of repeated games. Fortunately the static analysis is the first of two steps to derive a transition matrix and to represent the game as a Markov chain. The dynamic analysis also has a significance that goes beyond the original purpose intended by Hipel & Fraser: It is, nonetheless, also instructive to understand the *reasoning* of players that led to the stability of the equilibria in one-shot games under the assumptions of CA.

There are two motives that form the basis for the application of the Conflict Analysis approach to one-shot games, first, an empathic rationality of players, or second, an evolutionary incapacity of players to rationalise the singularity of such games. The first supposes that players have the potential to comprehend, experience and predict the feelings of other players that determine their strategic choice. Empathy is thus clearly distinct from sympathy and is not identical with the concept of identification. The second relies on a possible explanation of why individuals are observed to choose non-rational actions in single shot games. Caused by the rare occurrence of non-repeated interactions, evolution did not prepare us for one-shot games, thus creating a lack of apt heuristics for interactions that occur as a single incident. The discussion of both assumptions and the theoretical implications are postponed to section 4.5 on page 158.

The interpretation of the transition matrix of the Markov chain is therefore twofold. The transition matrix enables us to see how a game evolves in each period, if Conflict Analysis is applied to repeated interactions. It also shows the reasoning of players in a one-shot game, if players exhibit an empathic rationality or apply a repeated game solution heuristic.

Consider a game with f possible strategy profiles, let *state* X_{t-1} be defined by the probability distribution of the strategy profiles in time $t-1$ with dimension $f \times 1$. The state transition of such a game can be characterised as the discrete time Markov process $X_t = TX_{t-1}$, where T is the transition matrix of dimension $f \times f$ that describes the transition probability of moving from strategy profile x to y. X_t defines the state in t and has the same dimension as X_{t-1}.[20] Consequently a repeated game

results (as in this example, since $G_{AA} = G_{BB} = 2$), but should they not, the intersection of the two sets of possible equilibria obtained can provide a refinement.

[20]Notice that a Markov process is generally represented in the transposed form. Evidently, this has only an effect on the way the transition occurs. Instead of row defining the current and column the subsequent state in the transition matrix, the inverse holds for the representation chosen here.

$\Gamma = (S_{1t}, S_{2t}, ..., S_{nt}; U_{1t}, U_{2t}, ..., U_{nt})$ played in successive time periods $t = 1, 2, ..., n$ can be represented, given initial distribution (*status quo* state) X_0, as

$$X_t = T^t X_0, \tag{4.2.3}$$

if T is time homogeneous. As a first step, one transition matrix for each player must be derived. In a second step, the final transition matrix T of Γ will result from combining the individual transition matrices T_i.

In the following I will return to the Prisoner's Dilemma to illustrate the way, in which the transition matrix is derived, but also to show how the transition matrix illustrates the reasoning of empathic rationality in one-shot games. Given the preference order of the strategy profiles like in table 4.2.a and based on the DPs for each strategy profile (see table 4.2.b) the individual transition matrices will look as follows:

$$T_A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{array} \text{ and } T_B = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array}.$$

For illustrative simplicity, the decimal code of each strategy profile has been written on top and on the side to represent their corresponding position in the transition matrix. Generally strategy profiles are ordered according to the relative value of their decimal code. Each column in both matrices corresponds to the strategy profile in the last period $t-1$. A row defines the strategy profile in t and its value the transition probability. Consequently, the sum of all values in one column equals to 1. It should be clear that all *stable* strategy profiles can be found with value 1 on the main diagonal and the off-diagonal position is determined by the most preferred non-sanctioned DP (if a profile possesses more than 1 DP). The final transition matrix T is defined by the strategy profiles that occur if both players have chosen their best response strategy. T can be derived from T_A and T_B via an equation similar to equation 4.2.2,

$$\bar{q} = \sum_{i=1}^{x} \dot{o}_i - (x-1)\dot{q}, \tag{4.2.4}$$

where $\bar{q}$ is the new equilibrium value, and $\dot{q}$, $\dot{o}_i$, and x are defined as above. Applying the individual transition matrices to equation 4.2.4 determines the final transition matrix for game Γ as:[21]

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \tag{4.2.5}$$

The absorbing states (i.e. the set of equilibria) are obviously defined by a 1 on the main diagonal. Given any initial condition, in which both players disbelieve in mutual non-confession, i.e. assign probability 0 to strategy profile 0, the single equilibrium of the dynamic game is strategy profile 3 with certainty for all X_t and $t > 1$.

The transition matrix shows the empathic reasoning of players participating in the one-shot Prisoner's Dilemma. We know that if a player believes the other to choose defect, he will also defect. This is defined by a transition of 1 and 2 to 3. In addition, if a player expects that the other player believes that he defects, both will defect. Since he knows that the other player's best response to the belief that he defects is to defect, his own best response is also to defect, represented by T^2. The same logic applies to reasoning of higher order in a similar way. The stabilities are thus defined by the limit distribution of the transition matrix. Since $T = \lim_{n \to +\infty} T^t$ in the Prisoner's Dilemma, the cooperative equilibrium will only occur if both players initially expect the other to cooperate. If one player is expected to defect, the game will result in the defective equilibrium.

Consider that there are two types of players. One player type has the self-regarding preferences as in table 4.2.b on page 135 for A_s and B_s. The other player type are altruist cooperators with preference order $P(A_a) = \{0, 1, 3, 2\}$ and $P(B_a) = \{0, 2, 3, 1\}$. Consequently the possible types of interactions lead to four different games (between two altruists, two self-regarding players, one altruist and one self-regarding and the inverse, though mixed type games are identical, since preference order is "symmetric"). In the case, where simultaneous stability is considered, all games can be represented by a transition matrix identical to T defined in matrix 4.2.5.[22] Hence, we obtain, independent of the player type configuration, that if player i and j meet

[21] Applying equation 4.2.4 gives for 0: $0 + 0 - 0 = 0$, for 1: $1 + 3 - 1 = 3$, for 2: $3 + 2 - 2 = 3$, for 3: $3 + 3 - 3 = 3$.

[22] This result gives an interesting basis for discussion: According to the theory described here, it is irrelevant, whether an individual is self-regarding or altruist in the PD.

$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$ for any i and j. Assume that the proportion of altruists in the population is $p_a = 0.62$ and the proportion of self-regarding individuals is $p_s = 1 - p_a = 0.38$. Hence, the probability that two altruists interact is ${p_a}^2 = 0.3844$, for two self-regarding individuals ${p_b}^2 = 0.1444$, and for two different types to meet $2p_a p_b = 2 \times 0.2356$. Consequently, without sure knowledge on the other player type, only 38% of the individuals will cooperate ($X_0 = ({p_a}^2, p_b p_a, p_a p_b, {p_b}^2)^T$). This is identical to what Kiyonari, Tanida and Yamagishi (2000) found. If the second player is told that the first will always cooperate, a positive probability can be assigned only to equilibria 0 and 2. In this case 62% will cooperate ($X_0 = (p_a, 0, p_b, 0)^T$), which again corresponds to the result found by Kiyonari et al.

4.2.3 A short Excursion to the Stag Hunt Game

The dynamic analysis also shows an interesting property of the Conflict Analysis approach: Given the original pay-off matrix 4.2.1 on page 133 define $a > c > d > b$ and $a + b < d + c$. The original Prisoner's Dilemma turns into a Stag-Hunt game, i.e the special type of coordination game, in which one equilibrium defined by *(not confess, not confess)* pay-off dominates the risk dominant equilibrium determined by *(confess, confess)*. The game based on Rousseau's parable in the "Discourse on the Origin and Foundations of Inequality among Men" is another illustrative example for the underlying rationality of the Conflict Analysis approach. The quantification of a hare or stag seems difficult, yet obviously hunting a stag is more risky than a hare. Conflict Analysis can incorporate the risk issue without the need to quantify in relative terms the pay-off value of both animals.

Define the strategies *Hunt Stag =0* and *Hunt Hare =1*, since the symmetric game consists of two mutually exclusive actions for each player. The Pareto dominant equilibrium has thus decimal code 0, the risk dominant equilibrium decimal code 3, and mixed outcomes are assigned to decimal code 1 for *(stag, hare)* and to decimal code 2 for *(hare, stag)*.

In the static analysis presented in table 4.2.d, two equilibria exist, since Nash equilibria are always rationally stable for all players. The interior mixed equilibrium is neglected since Conflict Analysis only regards pure strategies, owing to the lack of quantification. At the first impression, the static representation has no more explana-

Table 4.2.d: Static Solution to the Stag Hunt

Stability Analysis					
equilibrium		E	x	E	x
Player A	Stability	r	$\hat{u}$	r	u
	Preference Order	0	1	3	2
	DP		0		3
Player B	Stability	r	$\hat{u}$	r	u
	Preference Order	0	2	3	1
	DP		0		3

tory power than the standard approach. Yet, in the mixed strategy profile, where a player chooses hare and the opponent stag, a switch towards the Pareto optimum is simultaneously sanctioned. A look at the transition matrices will make the effect of simultaneous stability more obvious. From table 4.2.d we obtain:

$$T_A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}, T_B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \Rightarrow T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \tag{4.2.6}$$

The final transition matrix in 4.2.6 shows that simultaneous stability captures the effect of risk dominance, though not in the strict sense of $a + b < d + c$. Since Conflict Analysis is based on an ordinal ranking of the equilibria and no pay-off values are assigned, an aggregation of these values *per se* is impossible. The final transition matrix shows that the pay-off dominant equilibrium requires each player to place a high probability on the event that the other player is hunting the stag. If he assigns equal probability to all events, he will finally hunt the hare, as the risk dominant equilibrium occurs in 75% of the time according to his priors. This effect is caused by the relation $c > b$. Any coordination game with two pure Nash equilibria and $c > b$ will show the same dynamics. Hence, game $a > c > d > b$ is equivalent to $a > d > c > b$.[23]

When comparing to the Prisoner's Dilemma, it becomes obvious that both games are dynamically equivalent. The stability of outcome 0 in the PD occurs, however,

[23] We can already observe the $c > b$ relation in *chapter 2, appendix B* (see also Nowak, 2006). If players base their decisions only on their last play, thus setting $m = 2$ in chapter 2 equation 2.B.11., it turns out that evolutionary selection favours the equilibrium that offers the higher pay-off in the case of miscoordination, i.e. which is defined by the strategy granting pay-off value c, instead of b.

through the sequential sanction of the defective strategy. In the stag hunt, a switch towards the cooperative strategy (from 1 to 0 for player A and from 2 to 0 for player B) is sanctioned by the fear the other player might switch, i.e. by a simultaneous sanction. This captures exactly the risk argument in the standard approach. Empirical evidence (Schmidt et al., 2003; Cooper et al., 1992; Van Huyck et al.,1990,) shows that individuals play the Prisoner's Dilemma similar to the Stag Hunt. Notice, however, that the empirical tests observe interactions after presenting a pay-off matrix to their subjects. In real world scenarios such quantification is often infeasible. The Conflict Analysis approach can nevertheless indirectly incorporate the pay-off effect (e.g. the role of the optimisation premium in Battalio et al., 2001). Both, the degree of strong altruism and the pay-off associated to each strategy profile, affect the probabilities of a player's own choice and his expectation about the other player's choice, thereby determining the initial *status quo* distribution in X_0.

4.3 Dilemmas and Paradoxes

The following section analyses the capacity of the Conflict Analysis approach to solve games that pose a challenge for classical game theory.[24] The games analysed here are the Traveller's Dilemma, the surprise test, and the Newcomb's Paradox. Though the sequential stability criterion resembles classical backward induction, this section will also show that the obtained results display no similarities.

The original definition of sequential stability and thus instability implies that a player refrains from all strategy switches that never lead to a strictly preferred strategy profile with certainty. Players do not bear the costs of switching, if it does not offer a benefit. This assumption, however, seems too strong. Assume that a player switches to a new strategy, if he knows that the subsequent switch of other players to their better response strategy will not lead to a less preferred outcome, but that

[24]The more common games, such as the battle of sexes and the chicken / hawk-dove game, have not been analysed in detail, since differences between solutions of standard game theory and those of Conflict Analysis approach are only minor. Conflict Analysis defines an additional equilibrium (both swerve) in the chicken game, if we observe that simultaneous switch may occur. This result is reasonable, but unlikely. If the chicken game is perceived as a sequential game, simultaneous stability does not apply. This exactly illustrates why the application of simultaneous stability requires an a priori justification based the rules of the game. In "The Battle of Sexes" all four outcomes are stable. The mixed are again stable owing to a potential simultaneous switch. This is the case, in which both players wait at different locations. Both players stick to their strategy and keep on waiting, hoping their counterpart is changing place, as both players fear that changing place might occur at the same time, bearing only the costs of moving without the benefit from coordination.

there exists at least one potential better response strategy of another player that leads to a strictly preferred outcome. This means that a player will not switch if it leads him to a not strictly preferred outcome with certainty, but he will choose to do so whenever he has a chance to improve his situation without the risk to worsen it. This requires a stronger condition for sequential stability and weaker condition for instability with respect to the original definition by Hipel & Fraser. Define $q = (\overline{s_i}, \overline{s}_{-i})$, and $o \in U_i^{--}(q)$, if $O(o) \prec_i O(q)$:

Assumption 1. *A player will never switch to a strategy that leads to an equally preferred strategy profile with certainty or a less preferred strategy profile with positive probability. He will, however, change his strategy if a change does not lead to a less preferred strategy profile with certainty and there exists at least one viable response strategy of another player that defines a strictly preferred outcome, i.e. for some* $q = (\overline{s_i}, \overline{s}_{-i})$:
Sequential stability: $\hat{u}_j^{+}(p) \cap U_i^{--}(q) \neq \emptyset$, *for any* $j \neq i$, *or*
$\hat{u}_j^{+}(p) \cap \left(U_i^{+}(q) \cup U_i^{--}(q)\right) = \emptyset, \forall j \neq i$; *both* $\forall s_i^* : p = (s_i^*, \overline{s}_{-1}) \in u_i^{+}(q)$
Instability: $\exists p \in u_i^{+}(q) : \hat{u}_j^{+}(p) \cap U_i^{--}(q) = \emptyset \ \forall j \neq i$ *and there exists at least one*
$\hat{s}_k : (s_i^*, \hat{s}_k, \overline{s}_{-i-k}) \in U_i^{+}(q) \cap \hat{u}_k^{+}(p)$, *for* $k \neq i$ *and some* $s_i^* \in S_i : p = (s_i^*, \overline{s}_{-1}) \in u_i^{+}(q)$

where $(s_i^*, \hat{s}_k, \overline{s}_{-i-k})$ defines the new strategy profile after a switch of a player k other than i to his better response strategy given p. Furthermore, the original definition of Hipel & Fraser assumes that an equally preferred strategy profile is not regarded as a valid DP, meaning that a player will not switch to a strategy not defined by a *strictly* preferred strategy profile (see also footnote 8 on page 128).

The following section thus has the aim to illustrate; first, the results that Conflict Analysis obtains for the given normal form games; second, the eventual issue that arise through the relaxation in assumption 1, and third, the difference in the predicted equilibria if either or not equally preferred strategy profiles qualify as DP's. (In the following analysis of the games the results that would be obtained, if equally preferred strategy profiles qualify as such, are indicated in brackets.)

4.3.1 Traveller's Dilemma

The Traveller's Dilemma by Basu (1994) tells the story of two antiquarians, who bought the same object but did not preserve the receipts. On the flight home, the airline smashes both objects, and the antiquarians ask for a refund. Thus, they are asked independently by the airline manager to state the amount (in integral numbers) they paid, constrained by a minimum amount of 2$. In the case, where they report different amounts, they are compensated by the lower amount stated. In addition, the one, who reported the lower amount, will receive 2$ less (as a punishment for having lied), which will be given to the other as a bonus. If both state the same amount, none will be rewarded nor punished. Backward induction tells us that both players should state 2$, independently of the value of the duty paid. This can be seen from matrix 4.3.1, which represents the game for 2$ to 5$. (2,2) is the only stable Nash equilibrium.

$$\begin{array}{c} \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{cccc} b_2 & b_3 & b_4 & b_5 \end{array} \\ \begin{pmatrix} 2,2 & 4,0 & 4,0 & 4,0 \\ 0,4 & 3,3 & 5,1 & 5,1 \\ 0,4 & 1,5 & 4,4 & 6,2 \\ 0,4 & 1,5 & 2,6 & 5,5 \end{pmatrix} \end{array} \tag{4.3.1}$$

In the following the game will be analysed by using the Conflict Analysis approach. Table 4.3.e shows the encoding of the 16 possible strategy profiles into decimal code in the same way as has been done in the previous section.

Table 4.3.e: Strategy profile Matrix

Travelers' Dilemma: Decimal Coding																	
A	a_2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	a_3	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
	a_4	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
	a_5	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
B	b_2	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
	b_3	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
	b_4	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
	b_5	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
Decimal		17	33	65	129	18	34	66	130	20	36	68	132	24	40	72	136
pay-off		2,2	4,0	4,0	4,0	0,4	3,3	5,1	5,1	0,4	1,5	4,4	6,2	0,4	1,5	2,6	5,5

Consequently, the preference order P_i for player i is gives as follows, where a bar indicates equal preference, i.e. the player assigns equal preference to all strategy profiles under the same line:

$$\begin{aligned} P_A &= \left(132, \overline{66, 130, 136}, \overline{33, 65, 68, 129}, 34, \overline{17, 72}, \overline{36, 40}, \overline{18, 20, 24}\right) \\ P_B &= \left(72, \overline{36, 40, 136}, \overline{18, 20, 26, 68}, 34, \overline{17, 132}, \overline{66, 130}, \overline{33, 65, 129}\right) \end{aligned} \tag{4.3.2}$$

The game has been analysed in table 4.B.q on page 167. *DP* of equal preference are written in brackets. In this analysis all strategy profiles that include a pay-off at least equal to the Nash equilibrium pay-off for both players are stable. The set of equilibria is defined by $E = (17, 34, 68, 136, 72, 132)$. The last two are stable, if equally preferred strategy profiles are not considered to be valid DP's.[25] To see why this is the case assume that the strategy profile is defined by $(\pi - 1; \pi)$, for $\pi > 5$, i.e. player 1 declares the value $\pi - 1$, and player 2 the value π. The second player can improve by switching to a value $\pi^* \in (\pi - 2, \pi - 4)$ (for values of $\pi \leq 5$ the lower bound of π^* is 2), granting him a pay-off of $\pi^* + 2 > \pi - 3$. This would, however, entail a switch of the first player to a value that is even lower. Hence, the second player cannot win by deviating.[26] In a situation given by strategy profile $(\pi - 2; \pi)$ and $\pi > 6$ the second player has again an incentive to underbid the first player with a strategy naming the value $\pi^{**} \in (\pi - 3, \pi - 5)$. Yet, owing to the subsequent switch of the first player, he cannot augment his pay-off and will therefore not switch. In this situation also the first player has an incentive to switch his strategy by declaring $\pi - 1$, which is deterred by the second player's potential switch. The same reasoning applies to larger differences as long as both players obtain at least a value of 2. Below this value a player has an incentive to switch to the Nash strategy that cannot be deterred. If the number of strategies is k, i.e. there are k different amounts that can be stated, the limit distribution of the transition matrix assigns probability $\frac{2(2k-3)+1}{k^2}$ to the Nash equilibrium $(2, 2)$ (see transition matrix 4.B.p on page 166). The other equilibria are played with probability $\frac{1}{k^2}$.

Then why is it more intuitive to state a value higher than 2? A possible explanation is that the game possesses two focal points that are assigned higher probability. It is likely that individuals assume with high probability that their counterpart chooses either the correct amount paid or the maximum possible value. The pay-offs also neglect loss-aversion, which a player will experience when stating a low value. Yet, the Traveller's dilemma illustrates a drawback of the Conflict Analysis approach. Its application to normal form games may result in a large number of potential equilibria.

[25] If equally preferred strategy profiles are considered as DP's, the strategy profiles inside the brackets destabilise both 72 and 132. To see this consider strategy profile 72. A change to 65 leads to 17, which is equally preferred. It may also lead to 33 and 129, both are preferred to 72 by player A. Following Assumption 1 implies that A will choose 65, since this will eventually make him better off without a chance to diminish his expected pay-off. Similarly player B will switch from 132 to 20. Hence, if equally preferred outcomes are considered as valid DP's only strategy profiles that offer a pay-off higher than the Nash equilibrium to both players and the Nash equilibrium itself will be equilibria of the game.

[26] The question that the reader might pose at this point is: Why is the player that acts as second (player 1 in the example) supposed to switch strategy as he himself will also fear a subsequent switch of the other player (the first mover, i.e. player 2), i.e. how credible is the subsequent strategy switch? The answer is connected to the discussion of the "Metagame Fallacy" and the "Newcomb's Paradox". The correlation and the answer to this question will be examined in section 4.5.

4.3.2 The Surprise Test

The Surprise Test is a game, in which a teacher announces to his student(s) that he will write a surprise exam on one day of the following week. Since the exam cannot be written on Friday, because it should not be a surprise any more, this day can be eliminated. Backward induction will then cancel each day of the following week as the student always rejects the last possible day in the remaining list. Finally the student will be sure that no exam will be written the next week and will be surprised, when it happens on one day of the week. Backward induction is no sensible reasoning, since the student ignores that whenever he eliminated one day, the teacher has an incentive to switch to that day.[27] The Conflict Analysis approach takes account of this fact.

If we consider only a week of four days, the coding in table 4.3.e can be used to represent each possible strategy profile. Consider A the teacher and B the student. The subscripts illustrate the days ordered calendrically. Version 1 in table 4.3.f shows a possible pay-off structure. The teacher strictly prefers all strategy profiles, in which the exam is scheduled for a different date than the student expected. The inverse holds true for the student.

Table 4.3.f: Strategy profile Matrix

Surprise Test: pay-offs for Version 1 and 2

	17	33	65	129	18	34	66	130	20	36	68	132	24	40	72	136
1:	0,1	1,0	1,0	1,0	1,0	0,1	1,0	1,0	1,0	1,0	0,1	1,0	1,0	1,0	1,0	0,1
2:	1,5	0,0	0,0	0,0	2,4	1,5	0,0	0,0	3,3	2,4	1,5	0,0	4,2	3,3	2,4	1,5

Conflict Analysis (see table 4.B.r on page 168) defines every strategy profile as an equilibrium and thus potential outcome of the game. The symmetric strategy profiles will be unrealistically defined as unstable only if equally preferred strategy profiles serve as DP's. Any date can be chosen by the teacher and the student will randomly choose one day to study for the exam.

Consider a variant of the preference order as given in version 2 in table 4.3.f. In this version the student least prefers all strategy profiles, in which he studied too late. He prefers most the situations, in which he correctly predicted the exact date of the surprise exam. His preference is diminishing in the number of days he studied before the actual date of the exam. Simply assume that he has to revise each evening in order not to forget what he has studied a day before. The teacher prefers the student to

[27] Assume the student has eliminated Friday from his list of strategy profiles and disregards this day. When contemplating about deleting Thursday the student should realize that he has to consider Friday once more, since this day is now again an option for the teacher as a surprise date.

revise as often as possible, and has no interest in the student's failure, which occurs if he studied too late. Pay-offs are as in table 4.3.f.2 and the solution is given in the lower part of table 4.B.r on page 168.

The set of equilibria is defined by $E = (18, 36, 68, 136)$. Since both are uncertain, to which strategy the other player adheres, the student will learn for any day of the week and the teacher will schedule the exam either on Tuesday, Wednesday or Thursday (remember that in the example the school week is only 4 days long). Strategy profiles 17 and 34 are equilibria in the original definition of Hipel & Fraser, but are destabilised by 40 and 24, respectively, if assumption 1 applies. In this version of the Surprise Exam assumption 1 thus creates a theoretical problem, absent in the original definition of sequential stability and instability.[28]

4.3.3 Newcomb's Paradox

This *paradox* (Nozick, 1969) has been widely discussed in the various social sciences as well as philosophy. It will turn out in the discussion in section 4.5 that the Newcomb Paradox plays a crucial role in understanding the theory behind the Conflict Analysis approach. The game defines a situation, in which one player B has the choice between taking one or two boxes. A second, omniscient player A chooses the value of the first box a priori to player B's choice. The first box may contain either 1.000.000$ or will be empty. The omniscient player will only put one million dollar into the first box, if the other player only chooses this box but neglects the second, which contains 1.000$. The pay-off structure is presented in matrix 4.3.3 and only indicated for the first player, since the pay-offs of the omniscient player are not required.

$$\begin{array}{c c} & \begin{array}{cc} \textit{punish} & \textit{not punish} \end{array} \\ \begin{array}{c} \textit{take both} \\ \textit{take one} \end{array} & \begin{pmatrix} 1.000 & 1.001.000 \\ 0 & 1.000.000 \end{pmatrix} \end{array} \tag{4.3.3}$$

According to Newcomb, this paradox illustrates a conflict between domination of strategies and maximisation of expected pay-off. Conflict Analysis defines both strategies for the first player and the corresponding strategic choice of the omniscient

[28] If the student knows that the teacher will follow a strategy defined by the equilibrium set, he anticipates that the teacher will not write on Monday and he will consequently not study for this day. The teacher will conjecture this and will not schedule for Tuesday and again the student will not study for this day. Finally, only the equilibrium set $\hat{E} = (68, 136)$ should remain.

player as possible equilibria. Table 4.3.g shows the analysis of the game.

Table 4.3.g: Solution to the Newcomb's Paradox

Strategy profiles					
Player A	take both	0	1	0	1
Player B	punish	0	0	1	1
Decimal Code		**0**	**1**	**2**	**3**

Stability Analysis					
equilibrium		x	E	E	x
Player A	Stability	r	s	r	u
	Preference Order	1	0	3	2
	DP		1		3
Player B	Stability	r	r	u	u
	Preference Order	0	3	1	2
	DP			3	0

Assume that the omniscient player strictly prefers not to punish the other player. Classical game theory will define the Nash equilibrium as *(take both, not punish)*. So does Conflict Analysis (see table.4.3.h).

Table 4.3.h: Alternative Version of the Newcomb's Paradox - B prefers never to punish

Stability Analysis					
equilibrium		E	x	x	x
Player A	Stability	r	u	r	u
	Preference Order	1	0	3	2
	DP		1		3
Player B	Stability	r	r	u	u
	Preference Order	0	1	3	2
	DP			1	0

On the contrary, Metagame Theory predicts strategy profile *(take one, not punish)* as the unintuitive equilibrium, but weakly dominant equilibrium; see table 4.3.i (for a formal introduction to Metagame Theory refer to subsection 4.A in the appendix). This shows that Conflict Analysis and Metagame Theory do not necessarily determine the same strategy profiles as equilibria, and provides an example that Conflict Analysis eliminates some of the deficiencies, for which the Metagame framework has been criticized.

Table 4.3.i: Metagame Solution to the alternative version of the Newcomb's Paradox: B - one box; b - both boxes; p - punish; np - not punish, supposing $\pi_o(p, B) = 1, \pi_o(p, b) = 2, \pi_o(np, b) = 3, \pi_o(np, B) = 4$, where $\pi_o(s)$ defines the pay-off of the omniscient player under strategy profile s. Though there are many stable equilibria, (b,B,b,b;p,np) weakly dominates the others, implying equilibrium (np, B).

	p,p	p,np	np,p	np,np
b,b,b,b	1.000,2	1.000,2	1.00.1000,3*	1.00.1000,3*
B,b,b,b	0,1	1.000,2	1.00.1000,3*	1.00.1000,3*
b,B,b,b	1.000,2	1.000.000 , 4*	1.00.1000,3	1.00.1000,3
b,b,B,b	1.000,2	1.000,2	0,1	1.00.1000,3*
b,b,b,B	1.000,2	1.000,2	1.00.1000,3	1.000.000,4
B,B,b,b	0,1	1.000.000,4*	1.00.1000,3	1.00.1000,3
B,b,B,b	0,1	1.000,2	0,1	1.00.1000,3*
B,b,b,B	0,1	1.000,2	1.00.1000,3	1.000.000,4
b,B,B,b	1.000,2	1.000.000,4*	0,1	1.00.1000,3
b,B,b,B	1.000,2	1.000.000,4*	1.00.1000,3	1.000.000,4
b,b,B,B	1.000,2	1.000,2	0,1	1.000.000,4
B,B,B,b	0,1	1.000.000,4*	0,1	1.00.1000,3
B,B,b,B	0,1	1.000.000,4*	1.00.1000,3	1.000.000,4
B,b,B,B	0,1	1.000,2	0,1	1.000.000,4
b,B,B,B	1.000,2	1.000.000,4*	0,1	1.000.000,4
B,B,B,B	0,1	1.000.000,4*	0,1	1.000.000,4

4.4 Sequential Games

In this section, the Conflict Analysis approach is applied to extensive form games. It follows the same structure as the previous section. Though, to my knowledge, Hipel & Fraser have only applied the Conflict Analysis approach to normal form games, it turns out that the approach can cope quite well in finding a solution, since it is able to refine the equilibrium set in a similar way as the local best response criterion (*LBR*) presented by Herbert Gintis (2009), which displays advantages over traditional refinement criteria.[29] In this section I have thus chosen games from Gintis (Chapter Ch. 9, 2009) to illustrate that Conflict Analysis is also a powerful tool to effectively solve sequential games. For notational simplicity and to avoid redundancies, the solution will only be given in the special game form, with whom the reader should be acquainted after having read the earlier sections. Assumption 1 still applies.

Owing to the sequentiality of all games, first, simultaneous stability is inapplicable and second, the first mover can choose an equilibrium as long as it is defined by an unambiguous path. Preplay conjectures are conducted by players according to the

[29]such as subgame perfect, perfect Bayesian, sequential and proper equilibria

rationalities underlying the Conflict Analysis approach, and players will only choose strategies according to an equilibrium profile. This reasoning leads to the following additional assumption that is applied in this section:

Assumption 2. *In a sequential game, if the strategy profile granting highest pay-off to the first-mover, in the set of equilibria, is defined by a unique path in the* ***reduced game tree****, such that no player is ambiguous about the position of his decision node at the time of his decision, then it determines the outcome of the game. The* ***reduced game tree*** *is thereby solely defined by the paths of the strategy profiles in the equilibrium set.*

Table 4.4.j: Incredible Threats

Strategy profile set:

Choice Alice	R	0	1	1
Choice Bob	r	∅	0	1
Decimal		0	1	3
pay-off		1,5	0,0	2,1

Solution:

equilibrium	E*	E	x
stabilities	r	r	u
A. preference	3	0	1
DP			0
stabilities	r	r	u
B. preference	0	3	1
DP			3

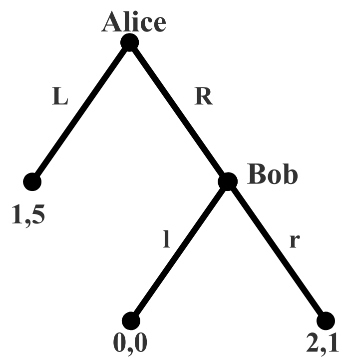


Table 4.4.k: 1. Game

The outcome of each game will be indicated by an E^*. The solution to the first game and its extensive form is shown in table 4.4.j. Notice that ∅ means that the player has no choice at that node. Hence ∅ can take both value 1 and 0, but pay-offs are identical (in order to keep notation as easy as possible, I thus reduced the strategy profile set, where it did not effect the identification of equilibria). The set of possible equilibria for the first game, represented in table 4.4.j and figure 4.4.j, is given by $E = \{0, 3\}$. Each equilibrium defines an unambiguous paths, separated by Alice's initial strategy choice. Alice will act in accordance with the path defined by strategy profile 3, which ranks higher than 0 according to her preference order, and Bob will also choose according to 3. The result is identical to *LBR*.

The second game is shown again to the right in extensive form, and its solution is given in the next table 4.4.l. The bar indicates the set of equally preferred strategy profiles. Strategy profiles that are equally preferred can be interpreted as mutual *DPs* or not. Following the discussion in the previous section (see assumption 1 in section 4.3), it is assumed that a strategy profile will only be a viable *DP* as long as a switch to this strategy profile results in a preferred outcome with positive probability. Like in the previous section, the stabilities occurring if equally preferred strategy profiles qualify as *DP*'s, are indicated by the brackets. Since Bob cannot improve the outcome by switching from 9 to 1 or vice versa, a strategy switch will not happen through assumption 1. The consideration of weak or strict preference for *DP*s is of no importance for the equilibrium set of this game.

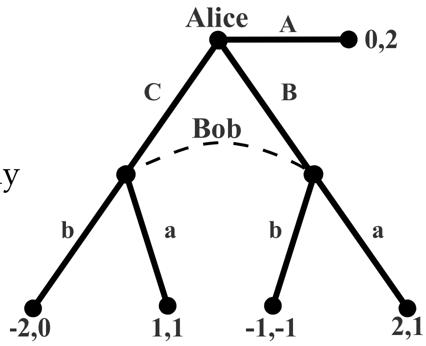


Figure 4.4.A: 2. Game

Table 4.4.l: Picking the Sequential equilibrium I

Strategy profile set:							
Choice Alice	A	1	1	0	0	0	0
	B	0	0	1	1	0	0
	C	0	0	0	0	1	1
Choice Bob	a	0	1	0	1	0	1
Decimal		1	9	2	10	4	12
pay-off		0,2	0,2	-1,-1	2,1	-2,0	1,1
Solution:							
equilibrium		E*	x	E	x	x(x)	x(x)
stabilities		r	u	r	u	u	u
A. preference		10	12	1	9	2	4
DP			10		10 12	1	1 2
stabilities		r(s)	r(s)	r	r	u	u
B. preference		1	9	10	12	4	2
DP		(9)	(1)			12	10

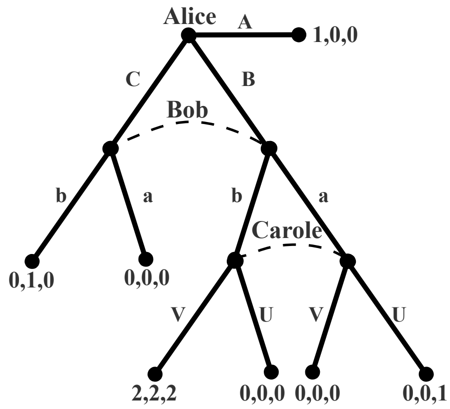


Figure 4.4.B: 3. Game

Hence $E = \{1, 10\}$. Given the set of equilibria, both 1 and 10 define unique paths, since Bob knows for sure whether or not he is on the path defined by 1 or 10, depending on whether he can choose a strategy or not. If Alice plays according to her most preferred equilibrium 10, Bob knows he can choose and takes strategy a, leading to 10, which is identical to the result predicted by *LBR*.

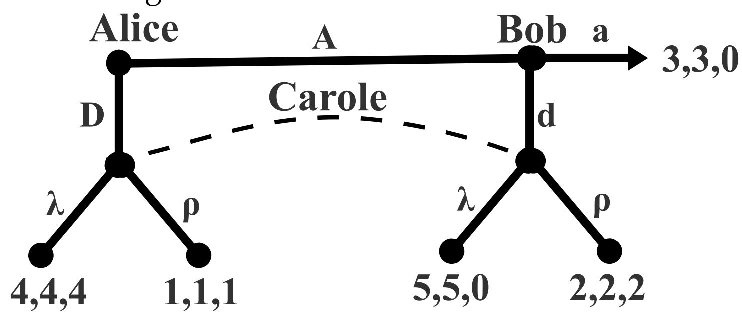
The third game is again illustrated in extensive form to the right in figure 4.4 and in table 4.4.m. A switch of Bob from 2 to 10 will not improve his outcome with positive probability, as no player has a *DP* to a preferred strategy profile. A switch from 10 to 2 can trigger a subsequent change of Carole to 18. By switching, Bob can attain a preferred outcome with positive probability. The set of equilibria is given by $E = \{1, 18\}$, each defining a unique path. Whenever Bob and Carole are free to choose, they will take the strategy defined by 18. Hence, Alice will initially choose strategy B, as predicted by *LBR*.

Table 4.4.m: Picking the Sequential equilibrium II

Strategy profile set:								
Choice Alice	A	1	0	0	0	0	0	0
	B	0	1	1	1	1	0	0
	C	0	0	0	0	0	1	1
Choice Bob	a	∅	0	1	0	1	0	1
Choice Carole	V	∅	0	0	1	1	∅	∅
Decimal		1	2	10	18	26	4	12
pay-off		1,0,0	0,0,0	0,0,1	2,2,2	0,0,0	0,1,0	0,0,0
Solution:								
equilibrium		E*	E*	x	x	x	x	x
stabilities		r	r	u(u)	u(u)	u(u)	u	u
A. preference		18	1	2	10	26	4	12
DP				1 (4)	1 (12)	1 (12)	1	1
stabilities		r	r	r	r/(s)	r/(u)	u	u
B. preference		18	4	1	2	10	26	12
DP					(10)	(2)	18	4
stabilities		r	r	r	u	s/(s)	r	r
C. preference		18	10	1	2	26	4	12
DP					18	10		

The following game, named after Reinhard Selten, is shown below and the solution is given in table 4.4.n. The game is analysed in the same way as the previous

Figure 4.4.C: 4. Game - Selten's Horse



games. Again a strategy profile serves as a DP, only if a switch to this strategy profile results in a preferred outcome with positive probability and never in a less preferred outcome. Notice that a switch of Bob from strategy a to d, or the inverse, is only theoretical if Alice chooses D, since a node, where Bob's choice matters is reached with zero probability. Nevertheless, the notion of subgame-perfection moves along the same line and shows that also nodes of non-positive probability matter. The matrix in 4.4.n thus includes 8 strategy profiles. Following the previous assumption, 0 is rationally or sequentially stable for Bob. In the case, where Bob should choose the equally preferred strategy profile 2, his change would result in 2 with certainty, which is not strictly preferred to 0. Yet, a switch from 2 to 0 entails a switch of Alice to 1, which is strictly preferred by Bob. He will thus have an incentive to change strategies if an equally preferred strategy profile serves as DP. The same argumentation holds for Carole with respect to 3 and 7. A switch by Carole from 7 to 3 will eventually lead to 2 or 1, one is strictly preferred the other is equally preferred. Hence, strategy profile 7 should be unstable for Carole. The equilibrium set is defined by $E = (0, 2, 7)$ or $E = (0)$; the latter is the case if equally preferred strategy profiles qualify as DP's. In both cases, whenever Carole can choose, she will take strategy λ, and whenever Bob has an option to choose, he will select a. Thus, Alice will opt for D and the final outcome is $(4, 4, 4)$, which is identical to what is predicted by LBR, though only strategy profile 2 satisfies the criterion.

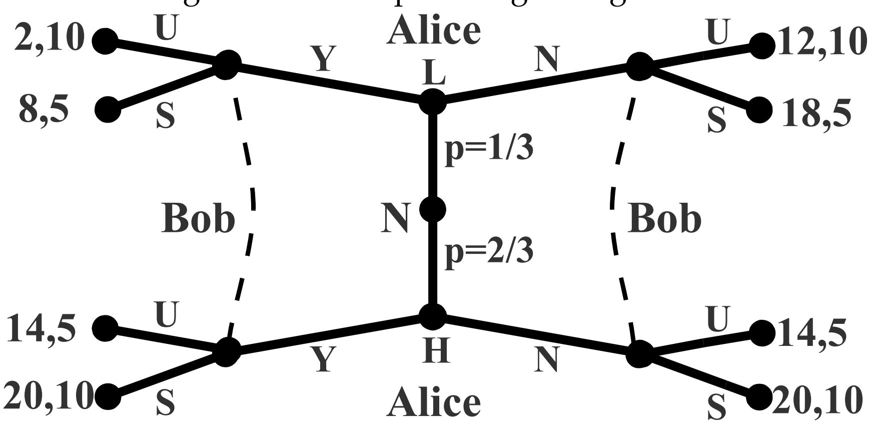
The last game, denoted as the Spence Signalling Model, is shown in extensive form in figure 4.4.D and its solution in table 4.4.o. In this game we add Nature as a player, choosing low ability worker *(L)* with probability $1/3$ and high quality worker *(H)* with probability with probability $2/3$. Bob can observe Alice's strategy choice whe-

Table 4.4.n: Selten's Horse

Strategy profile set:									
Choice Alice	A	0	1	0	1	0	1	0	1
Choice Bob	a	0	0	1	1	0	0	1	1
Choice Carole	ρ	0	0	0	0	1	1	1	1
Decimal		0	1	2	3	4	5	6	7
pay-off		4,4,4	5,5,0	4,4,4	3,3,0	1,1,1	2,2,2	1,1,1	3,3,0
Solution:									
equilibrium	x	E*	E*(x)	x	E(x)	x	x	x	
stabilities	r	s	r	u(u)	r	r	u	u(u)	
A. preference	1	0	2	3	7	5	4	6	
DP		1		2			5	7	
stabilities	r	r/(s)	r/(u)	s	r	u(u)	r/(u)	r/(u)	
B. preference	1	0	2	3	7	5	4	6	
DP		(2)	(0)	1		7	(6)	(4)	
stabilities	r	r	r	s	u(u)	u	r/(s)	r/(u)	
C. preference	0	2	5	4	6	1	3	7	
DP				0	2	5	(7)	(3)	

ther *(Y)* or not *(N)* she invests in education. Bob can then choose whether he pays her as a skilled *(S)* or unskilled *(U)* worker. Yet, choices by Nature entail mixed equili-

Figure 4.4.D: 5. Spence Signalling Model



brium strategies, which the Conflict Analysis approach is unable to predict. It could be assumed that Bob and Alice play two different games and assume that they are placed either in game *(L)* or *(H)* with probability 1/3 or 2/3, respectively. This assumption is, however, unnecessary for the representation, since Nature's choice already defines two separate sets of strategy profiles.

Table 4.4.o: Spence Signalling Model

Strategy profile set:									
Choice Nature	H	0	1	0	1	0	1	0	1
Choice Alice	N	0	0	1	1	0	0	1	1
Choice Bob	S	0	0	0	0	1	1	1	1
Decimal		0	1	2	3	4	5	6	7
pay-off		2,10	14,5	12,10	14,5	8,5	20,10	18,5	20,10
Solution:									
equilibrium		E	E	x	x	x	E	x	x
stabilities		r/(s)	r/(s)	r	r/(u)	r/(u)	r	u	u
A. preference		5	7	6	1	3	2	4	0
DP		(7)	(5)		(3)	(1)		6	2
stabilities		r	r	r	r	u	u	u	u
B. preference		0	2	5	7	1	3	4	6
DP						5	7	0	2

A switch of Alice from 5 to 7 (or the inverse) does not result in a strictly preferred strategy profile, whereas a shift from 1 to 3 (or the inverse) does. Independently of the strict or weak preference assumption for *DP*s, the equilibrium set is defined by $E = (2, 5, 7)$. The equilibria do not define a unique path, since Bob cannot observe Nature's choice. The equilibrium set implies that Alice will never invest in education, if she is a low quality worker. If she is a high quality worker, she will either invest or not. Similarly, Bob will always assign a skilled job to Alice, if she invested in education, but Bob might assign unskilled workers both to skilled and unskilled jobs. We obtain something similar to a semi-pooling equilibrium, where any combination of S,S (assign all workers to skilled jobs) and S,U (assign educated workers to skilled jobs and uneducated workers to unskilled jobs) is best response for Bob. Bob will play S,S/U (assign educated worker to skilled jobs and uneducated worker both to skilled and unskilled jobs). Alice's best response is Y/N,N (if high quality worker, both educate and not educate; if low quality worker, never educate), similar to *LBR*, where the best response of Alice to S,U is Y,N (invest in education if high level worker and not if low level worker) and to S,S it is N,N (never invest). Both (N,N;S,S) and (Y,N;S,U) fulfill the *LBR* criterion. Since no player knows for sure, to which best response the other player adheres, we can also assume that (Y,N;S,S) and (N,N;S,U) occur, which is captured by the equilibrium set obtained by the Conflict Analysis approach.

4.5 Critique and the Metagame Fallacy

We have seen that this approach has several advantages with respect to classical game theory. It has the capacity to find equilibria not predicted as such by other approaches, but which, however, appear intuitive. Though it increases the set of potential equilibria, I have described a means to discriminate between equilibria in sequential games. The approach can handle larger strategy and player sets. Conflict Analysis has also strong advantages over other similar approaches, such as the "Theory of Moves" by Brams (Brams, 1993, Brams & Mattli, 1992).[30] It allows to model situations of misperception via Hypergames. Overall it appears more adapted to real-world scenarios.

The ability to predict additional equilibria required the introduction of two additional stability criteria. This comes at a price. In *sequential games* the stability of a strategy choice, based on the sequential stability criterion, is rational for consistent preferences as defined in the preference ordering, if it is applied backwards towards the root node of the game tree. A player conjectures, which strategy a subsequent player will choose according to his own strategy choices. Players are aware that each player considers the sequence of strategy choices of earlier players and chooses the strategy that is best response to the strategies chosen before. Upon this anticipative conjecture each player chooses his best response. Apparently, a problem, because of the additional stability criterion, does not occur. An issue eventually arises in sequential games with incomplete information and if sequential stability is applied forwards in the direction of the terminal node. A player is bound by the various strategic options of his predecessors, since he knows that a previous player has already made his decision and cannot re-evaluate at a later stage. Hence, a switch *ex post* cannot occur. A similar issue applies to *one-shot games*. Conflict Analysis seems to suffer from what has been termed "Metagame Fallacy". The section discusses this issue and illustrates why the reasoning behind the Metagame Fallacy is illegitimate.

The Prisoner's Dilemma has been chosen deliberately as an introductory example in section 4.2, since it is also the game which best illustrates the Metagame Fallacy. The analysis of the PD has shown that there exists a stable cooperative equilibrium. The argument of the Metagame Fallacy goes as follows: If each player is convinced that his counterpart cooperates, he can still improve by defection. Thus a game of second order reasoning is created by each player that is isomorphic to the original

[30] Theory of Moves requires a strict ordinality of preferences and the solution in normal form games depends on the initial strategy profile, at which the solution algorithm starts. The greatest drawback with respect the Conflict Analysis is that the algorithm is only applicable to 2×2 games.

game. The fallacy, it seems, lies in the assumption that mutual rationality implies symmetric behaviour. Yet, this is obviously not the case and cooperation cannot be a rational strategic choice.

The argument misses an important point. Obviously, a pre-commitment of a player to a strategy will never be credible as long as other players are not entirely convinced that he will stick to his commitment and, further, that the player is also sure that the other players will do the same (see Binmore, p. 179, 1994).[31] Yet, it is not a pre-commitment that stabilises the cooperative equilibrium, but the empathic knowledge of the other player's strategy. This creates a correlation between the strategic choice of each player that the Metagame Fallacy argument neglects. Each player believes that his choice of the defective strategy will impact the probability, with which the other player chooses this strategy. Hence, a player's choice to cooperate is consistent with this belief and *rational*. If p is the probability that a player 1 and 2's strategy choice are correlated, and using the pay-offs as in pay-off matrix 4.2.1 on page 133, the following condition must hold to induce a player to choose the cooperative strategy:

$$\begin{aligned} pa + (1-p)b &> pd + (1-p)c \\ p &> \frac{c-b}{a-d+c-b} \end{aligned} \tag{4.5.1}$$

Since by definition of a PD, $a > d$ and $c > b$, the probability stays within the unit interval. If, in fact, all players have a high level of empathy, such that $p = 1$, the PD turns into a Twins game (see Binmore 1994 & 1998 for more details).

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} a,a & \\ & d,d \end{pmatrix} \end{array} \tag{4.5.2}$$

which is indeed a symmetric Newcomb Paradox. Dominance tells both players that they should defect, but this would only entail a pay-off of d, since both perfectly predict the other player's choice. Maximisation of pay-offs tells them to cooperate.

Empathy is surely a strong assumption. Yet, even Binmore frequently emphasises the importance of empathy: "*Homo economicus* must be empathic to some degree. By

[31] i.e. one has to assume that both players make a kind of a priori commitment, similar to a categorical imperative that dictates each player to choose the cooperative strategy. This leads to the question of free choice of players. Any player abiding by the axioms of rationality cannot, however, be assumed to have a free will a posteriori; since his choice is predetermined by the rules of the game.

this I mean that his experience of other people must be sufficiently rich that he can put himself in their shoes to see things from their point of view. Otherwise, he would not be able to predict their behavior, and hence would be unable to compute an optimal response." (Binmore, 1994, p. 28) If a player believes he is empathic, then it is rational to assume that other players are also empathic. Furthermore, notice that this approach has found the defective equilibrium also to be stable, and that the cooperative equilibrium requires both players to predict that their counterpart cooperates. A look at the joint transition matrix T shows that if individuals initially assign equal probability to all strategies, the defective strategy profile will occur with a probability of $\frac{3}{4}$. The cooperative strategy profile will only arise with probability $\frac{1}{4}$, since it requires that a player believes the other player to be choosing the cooperative strategy. Furthermore "empathic errors" can occur in this approach. A hypergame, where individuals do not trust their counterpart, will result in the defective equilibrium with certainty. The assumption of empathy alone thus does not pose an issue. Yet, this assumption is only partly sufficient to maintain the cooperative equilibrium.

Before I explain the crux it is necessary to stress again an important point. At first glance Conflict Analysis seems similar to the approaches using other regarding preferences. One might think that the Conflict Analysis approach merely substitutes sympathy by empathy to explain the stability of the cooperative strategy profile. Notice, however, that this form of empathy is immanent in the structure of the Conflict Analysis approach, whereas assuming other-regarding preferences is a change of the game itself. Conflict Analysis takes an entirely different approach by changing the way games are analysed, but keeping the way the problem was originally formulated. In the Conflict Analysis approach preferences are identical to the original specification of the PD. Players are entirely self-referential and only maximise their own pay-offs. It has to be stressed that the stability of the cooperative equilibrium occurs only through the expectations about the consequences a change of strategy will have on the *own* pay-off. Other pay-offs are only considered in order to anticipate other players' strategy choices. Other regarding preferences, on the contrary, expand the assumptions on preferences beyond the original specification of the model. Strictly speaking, the necessity to introduce other regarding preferences into the PD is a sign that the game has been misspecified and the underlying game is in fact no PD. An *ex post* change of preferences creates degrees of freedom that might render the solution of games completely arbitrary.[32] Other-regarding preferences are therefore clearly

[32]The issue, it seems, results from an understanding of pay-offs as a sole welfare measure, such as

distinct from the preference orderings in Conflict Analysis.

In order to induce individuals to cooperate, it is insufficient that they anticipates correctly the choice of the other player with a probability of at least p. Mutual cooperation requires not only that a player believes that the counterpart correctly anticipates his choice to cooperate, but also that he believes the other to believe the same, and so on (common knowledge). The cooperative equilibrium in the PD thus demands a "common knowledge of *empathic* rationality".[33] The commonality of knowledge is obviously a very restrictive assumption, the issue is, however, not unique to Conflict Analysis.[34] In addition, the assumption that individuals anticipate other action's with high probability is a second disputable assumption.

As a last argument, in order to vindicate the approach and to weaken the magnitude of the problem raised in the last paragraph, I emphasize that this line of argument does not hold for repeated or pseudo one-shot games[35] (as has been illustrated

money, years etc. This is caused by the sometimes improper definition of the underlying game. The pay-off type is often not specified, since "[g]ame theorists [...] understand the notion of a pay-off in a sophisticated way that makes it tautologous that players act as though maximizing their pay-offs. Such a sophisticated view makes it hard to measure pay-offs in real-life games, but its advantage in keeping the logic straight is overwhelming."(Binmore, 1994, p.98) The inadmissibility to apply a utility function *ex post* becomes especially obvious for games that are defined as $\Gamma = (S_1, S_2, ..., S_n; U_1, U_2, ..., U_n)$, where U_i defines player i's utility function, and the elements of the pay-off matrix define utility pay-offs. For games defined as such, the *ex post* transformation of pay-offs by applying utility functions, implies that the original utilities are put into utility functions to define new utilities (-the same line of argument applies to pay-offs as incremental fitness). Even for a general game of the form $\Gamma = (S_1, S_2, ..., S_n; \pi_1, \pi_2, ..., \pi_n)$, however, such a transformation by utility functions is inapplicable. A sophisticated interpretation of pay-offs implies that they already include the contextual essence, i.e. all the information necessary to solve a game. In addition, the equivalence of games that are derived from positive affine transformations of the pay-off matrices is no longer maintained if pay-offs are transformable *ex post*. A sufficiently large positive affine transformation of the pay-offs will thus not necessarily maintain the preference relations and strategic choice will be incongruent with the original game. General claims about the Prisoner's dilemma, stag hunt, battles of sexes, or chicken game etc. are infeasible. In contrast, Conflict Analysis has only limited degrees of freedom as equilibria are strictly defined by the preference relation of the original specification and the three stability criteria. In conclusion, whenever it is observed that players do not choose the strict dominant strategy, the only valid deduction in the context of standard Game Theory is that the PD is an incorrect representation of the real interaction.

[33]In fact, the condition is slightly weaker. Mutual cooperation does not require that individuals possess the necessary level of empathy to predict other's strategy choices, but that they believe that others do and further that others believe they do.

[34]Neither common knowledge, nor mutual knowledge of conjectures cannot be supposed *per se*. This is, however, a prerequisite for a rational Nash equilibrium (see Aumann & Brandenburger, 1995).

[35]This is the case in a version of the PD by Wagner, 1983, where both prisoners have friends in the District Attorney's office, who inform them about whether or not the other has confessed. The game is sequential, but there exists no definite first and second mover. The game only ends with certainty, when both confessed (i.e. when one player confessed, since the best reply strategy for the other player is also to confess).

at the beginning of this section), but for one-shot games. According to Larry Samuelson and Andrew Postlewaite such games are an unrealistic representation of life (see Mailath, 1998). In fact, the argument that individuals play Nash equilibria seems to hinge on the assumption of the repetitiveness of games.[36] Thus, there is doubt whether individuals really apply the rationality of one-shot games to the PD. If rational behaviour is explained as a result of an evolutionary selection process[37], it is dubious whether one-shot games regularly occurred in the historical context that shaped our behaviour. The empirical data shows that both equilibria predicted by Conflict Analysis are entirely plausible. Individuals apply heuristics (Page, 2007), such as the believe that others will able to predict ones choice, or the believe in a poetic justice of fate, ultimately leading to a strategy choice predicted by Conflict Analysis.[38]

4.6 Conclusion

This chapter has developed and illustrated a game theoretic approach that provides an interesting alternative to model interactions. This approach offers undeniable advantages over other standard approaches: It can handle larger (non-quantitative)

[36] According to Mailath, 1998, a Nash equilibrium requires mutual knowledge of what other players do, i.e. knowledge, which can only be derived either by preplay communication, self-fulfilling prophecy (what Mailath describes here as self-fulfilling prophecy might be more familiar by the term "Common Knowledge of Logic", see Gintis, 2009, for a critical discussion), focal points, or learning. Laboratory experiments have shown that preplay does not necessarily prevent coordination failures (Cooper et al. 1992), even more so in those cases, where a player benefits from a specific strategy choice of the other player irrespective of his own choice. Such a case is the Stag Hunt game with preference order $a > c > d > b$ for pay-off matrix 4.2.1 on page 133. The argument of self-fulfilling prophecy assumes that if a theory to uniquely predict outcomes is universally known, it will determine the Nash equilibria, i.e. the theory is consistent with the theory used by other individuals. First, the underlying logic is circular: The theory is correct if it is self-fulfilling, and thus if adapted by all agents, who believe it to be correct. Second, it requires a link between what an agent is expected to do and what he does, and further what he assumes others to do. Experiments indicate that individuals do not believe that others respond to pay-offs in symmetric games in the same way as they do (Van Huyck et al.,1990). This does not contradict mutual rationality, but shows that people do not assume that a unique way to predict behaviours exist. Further, seldom a unique "obvious" strategy exists in a game, which defines a focal point. Efficiency, egality, justice, risk are all different approaches of such "obvious" play. Finally, learning requires that (almost) identical games are played repetitively. Thus, the theoretical argument against the Conflict Analysis approach under the classical rationality paradigm is not substantive as it, in fact, applies to a game type, which cast already scepticism on the feasibility of Nash equilibria.

[37] see Bowles & Choi (2007), Gintis (2009), Gintis et al. (2005), Robson(1996a, 1996b)

[38] See the German and English idiom: "You always meet twice." Under the condition that a game is played repetitively the rationality of Conflict Analysis does not seem far-fetched. A first version of the article sketched in *Chapter 5, section 3.1.* shows that an easy learning algorithm enforces behaviour that approaches a reasoning similar to Conflict Analysis.

strategy and/or player sets more easily. It incorporates the higher order reasoning of Metagame theory and allows to model higher level hypergames. It can therefore explain the existence of equilibria not captured by the Nash concept, but which are frequently observed in real-world interactions. It can be efficiently applied to sequential games to discriminate between equilibria. It does not require a transition from a perceivable preference order to a cardinal description of preferences, nor the strict transitivity of preferences.

Conflict Analysis is, however, not as elegant as standard game theoretic approaches, when it comes to two player interactions with limited strategy sets. The addition of two supplementary stability criteria exhibits drawbacks in the axiomatic foundation of this approach and its theoretic basis is by far not as rigorously developed as the "pure" Nash approach. Furthermore, it is also restrictive in some assumptions. Players are required to exhibit a certain level of empathy that enables them to predict others choices, as well as the common knowledge of this capacity. Alternatively they are assumed to misinterpret one-shot games as repeated games. Though players are supposed to enjoy a higher order reasoning that allows them to anticipate the reaction of others to their strategy choice, they are not supposed to deliberately choose a non-best response that makes them immediately worse off if others do not change their strategy, but could improve the outcome to their favour after eventual strategy switches of other players. This is obviously an assumption to keep the approach tractable, but one might wish for the possibility of such "tactical" choice. Furthermore, though the approach handles most games quite well, the determination of mixed equilibria represents an issue, owing to the structure of the Conflict Analysis approach and the absence of specific pay-offs. Yet, this last issue only presents a drawback for the small minority of games that possess quantifiable pay-offs.

Overall Conflict Analysis is an attractive approach, especially, when applied to repeated games. It should not be regarded as an alternative to standard game theory or even as its substitute. It should be more considered as an alternative in perspective. Conflict Analysis is meant as a positive approach that illustrates an argument of why certain equilibria are frequently observable that are indeed not Nash equilibria. This approach is embedded in the spirit of theoretical pluralism, and an eventual step forward in redefining the theoretical development in game theory. It should complement with other alternative attempts and its methodology should thus provide sources for new conceptions that can enrich game theory and offer new starting points for future research.

4.A The Fundamentals of Metagame Analysis

This section will illustrate the basic principles of the Metagame Theory. Most of the basic argument is equivalent to standard game theory. Yet, the higher order reasoning necessitates to depart from standard definitions at various points.

In a game G let there be a set of players N with player $i = 1, 2, ..., n$ and let there be also an individual set of strategies S_i for each player, so that each individual strategy is denoted by $s_i \in S_i$. The entire set of strategy profiles is represented by $S = S_1 \times S_2 \times .. \times S_n$. A strategy profile of a game G in period t is then defined by $s_t \in S_t$, where $s_t = (s_{1t}, s_{2t}, ..., s_{nt}) \in S$. For each player i there exists a preference function U_i, which orders (not necessarily completely) the set of strategy profiles S. Consequently game G can be defined as $G = (S_1, S_2, ..., S_n; U_1, U_2, ..., U_n)$.

A metagame kG is derived from the underlying game G in the following way: Replace the strategy set of player k by a new set of strategies F. Each element $f \in F$ is a function that defines a response strategy to each possible strategy profile s_{-k}.[39] The preference functions for the new game are denoted as $U_i^{'}$ of player i in kG. Consequently, the metagame is defined as $kG = (S_1, S_2, ..., S_{k-1}, F, Sk+1, ..., S_n; U_1^{'}, U_2^{'}, ..., U_n^{'})$. Since the set of strategy profiles of kG is defined by the the Cartesian product $F \times S_{-k}$, we obtain a strategy profile $q = (f, s_{-k} \in F \times S_{-k})$. An rth level metagame L is then represented by $L = k_1 k_2 ... k_r G$. Let there be an operator β that transforms any strategy profile of an rth level metagame L to a strategy profile of the lower (r-1) level metagame, such that $\beta(f, s_{N-k}) = (f(s_{N-k}), s_{N-k})$ and $\beta^r(f, s_{N-k}) = (s_k, s_{N-k}) = q$.[40]

For each strategy profile q, let S be divided into two subsets, where $U_i^+(q)$ denotes the set of strategy profiles that are strictly preferred by player i to strategy profile q and where $U_i^-(q)$ includes all the strategy profiles that are not strictly preferred by player i to q. Hence, for any given strategy profile $\bar{s}_{-i}$ by all players other than i, the set of strategy profiles that can be obtained by a unilateral strategy switch of player i is given by $z_i(q) = (s_i, \bar{s}_{-i})$. An individually attainable dominant profile p with respect to q is then defined as $p \in u_i^+(q) = z_i(q) \cap U_i^+(q)$.[41] That means that a player i can improve on the current strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ given strategy profile $\bar{s}_{-i}$ by

[39] Assume a game with two players and 4 strategies each. F is then a set of $4^4 = 256$ functions, where each f is defined by four response strategies, one for each strategy of the other player.

[40] Suppose that both players have strategies *(u)p, (d)own, (l)eft* and *(r)ight*. For $1G$ let there by an $f = (u/u/d/d)$, meaning that player 1 plays strategy up as a response to both up and down, and down as a response to both left and right. A strategy profile $p = (u/u/d/d, l)$ in $1G$ can than be transformed to a strategy profile in G, since $\beta(u/u/d/d, l) = (f(l), l) = (d, l)$.

[41] An alternative formulation is $\exists s_i : (s_i, \bar{s}_{-i}) \in U_i^+(q)$ such that $p \in \{(s_i, \bar{s}_{-i}) : (s_i, \bar{s}_{-i}) \in U_i^+(q)\}$.

choosing a strategy s_i^* to strategy profile $p = (s_i^*, \bar{s}_{-i})$. Notice that it follows from the paragraph above that the preferences in G determine the preferences in L, i.e. in the case, where q is strictly preferred to another strategy profile p: $p \in U_i^{-'}(q) \Leftrightarrow \beta p \in U_i^{-}(\beta(q))$.[42]

A strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$ is rational for any player i, if $\bar{s}_i$ is the best response strategy to $\bar{s}_{-1}$. Hence, the set of rational strategy profiles for player i is given by $R_i = \{q | \forall s_i, (s_i, s_{-i}) \in U_i^-(q)\}$. The metarational strategy profile for G can then be derived from the underlying metagame L by applying β r-times to the set of rational strategy profiles R. Let $\hat{R}_i(L)$ denote the set of metarational strategy profiles for G, then $\beta^r R_i(L) = \hat{R}_i(L)$. Similarly the set of equilibria $E(L)$ for a metagame L, which is the set of strategy profiles rational for all players in L, connects to the set of equilibria for game G via β and $\beta^r E_i(L) = \hat{E}_i(L)$.[43] The general set of rational strategy profiles R_i^* for player i is then given by the union of all rth level metagames and thus $R_i^* = \bigcup_j \hat{R}(L_j) \cup R_i(G)$.

On the basis of the "Characterization Theorem" by Howard (1971), the determination of the general set of rational strategy profiles does not necessitate the analysis of the infinite metagame tree, but only a three step analysis. The first step is to check for the absence of unilaterally attainable dominant profiles $p = (s_i, \bar{s}_{-i})$, such that $\nexists s_i : (s_i, \bar{s}_{-i}) \in U_i^+(q)$. In this case q is metarational. A *symmetric metarational outcome* is defined if for all such better response strategies s_i^* other players possess a strategy that garantees a new strategy profile not preferred by the particluar player under analysis. Hence, it must hold $\exists s_{-i} \forall s_i^* : (s_i^*, s_{-i}) \in U_i^-(q)$. A *general metarational outcome* occurs if the player under analysis has again a response strategy to improve to a preferred strategy profile. A cycle may evolve and the play can continue indefinitely, which is assumed to be strictly less preferred to q (similar to a deadlock point, see Binmore, 1998). A strategy profile g therefore has to be finally checked for the absence of such an infinite cycle; $\nexists s_i \forall s_{-i} : (s_i, s_{-i}) \in U_i^+(q)$. If this is the case, q is metarational. Thus, if a strategy profile q is metarational for player i, it is also stable for player i and the set of stable strategy profiles for all players defines the set of equilibria of the general (n-th level) metagame.

[42] $(u/u/d/d, l)$ generates the same outcome as (d, l) and thus preference orders with respect to each must be identical.

[43] Suppose that$(u/u/d/d, l)$ is rational for *both* players, then (d, l) is meta-rational for both players and the equilibrium of the underlying game G.

4.B Tables

Table 4.B.p: Dynamics of the Traveller's Dilemma - Individual Transition Matrices

Transition Matrix for A

	17	18	20	24	33	34	36	40	65	66	68	72	129	130	132	136
17	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0
34	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
129	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Transition Matrix for B

	17	18	20	24	33	34	36	40	65	66	68	72	129	130	132	136
17	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
18	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0
20	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
129	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Final Transition Matrix

	17	18	20	24	33	34	36	40	65	66	68	72	129	130	132	136
17	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0
18	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
34	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
129	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
136	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 4.B.q: Traveller's Dilemma - static analysis: double lines indicate the frontiers of each set of equally preferred strategy profiles, *DO*s by equally preferred strategy profiles are written in brackets

Solution of the Travelers' Dilemma																
overall stability	E(x)	x	x	E	x	x	E	x	E	E	E(x)	x	x	x	x	x
stability for *A*	r	r	s	s	r	s	s	s	s	r	s(u)	u	u	u	u	u
As preference order	132	66	130	136	33	65	68	129	34	17	72	36	40	18	20	24
DPs			132	132		66	66	132	33		66	33	33	17	17	17
			(136)	(130)		(68)	(65)	130			65	34	34	(20)	(18)	(18)
								136			68	(40)	(36)	(24)	(24)	(20)
stability for *B*	r	r	s	s	r	s	s	s	s	r	s(u)	u	u	u	u	u
Bs preference order	72	36	40	136	18	20	24	68	34	17	132	66	130	33	65	129
DPs			72	72		36	72	36	18		36	18	18	17	17	17
			(136)	(40)		(68)	40	(20)			20	34	34	(65)	(33)	(33)
							136				68	(130)	(66)	(129)	(129)	(65)

Table 4.B.r: Static Analysis Surprise Test - double lines indicate the frontiers of each set of equally preferred strategy profiles, *DP*s by equally preferred strategy profiles are written in brackets

Solution of Version 1

overall stability	E(E)	E(E)	E(E)	E(E)	E(E)	E(E)	E(E)	E(E)	E(E)	E(E)	E(E)	E(E)	E(x)	E(x)	E(x)	E(x)
stability for *A*	r(s)	r(s)	r(s)	r(s)	r(s)	r(s)	r(s)	r(s)	r(s)	r(s)	r(s)	r(s)	s(u)	s(u)	s(u)	s(u)
*A*s preferences	33	65	129	18	66	130	20	36	132	24	40	72	17	34	68	136
DPs	(36) (40)	(66) (72)	(130) (132)	(20) (24)	(65) (72)	(129) (132)	(18) (24)	(33) (40)	(129) (130)	(18) (20)	(33) (36)	(65) (66)	18 20 24	33 36 40	65 66 72	129 130 132
stability for *B*	r	r	r	r	s(s)	s(s)	s(s)	s(s)	s(s)	s(s)	s(s)	s(s)	s(s)	s(s)	s(s)	s(s)
*B*s preferences	17	34	68	136	33	65	129	18	66	130	20	36	132	24	40	72
DPs					17 (65) (129)	17 (33) (129)	17 (33) (65)	34 (66) (130)	34 (18) (130)	34 (18) (66)	68 (36) (132)	68 (20) (132)	68 (20) (36)	136 (40) (72)	136 (24) (72)	136 (24) (40)

Solution of Version 2

overall stability	x	x	x	x	E	E	x	x	E	E	x(x)	x(x)	x(x)	x(x)	x(x)	x(x)
stability for *A*	r	s	r	r	s	s	u	u	r	s	u	u	u	u	u	u
*A*s preferences	24	20	40	72	18	36	17	34	136	68	33	65	129	66	130	132
DPs		24			24 20	40	24 20 18	40 36		72	40 36 34	72 68 (66)	136 (130) (132)	72 68 (65)	136 (129) (132)	136 (129) (130)
stability for *B*	r	r	r	r	s	s	u	u	u	u	u	u	u	u	u	u
*B*s preferences	17	34	136	68	18	36	72	20	40	24	33	65	129	66	130	132
DPs					34	68	136	68 36	136 72	136 72 40	17 (65) (129)	17 (33) (129)	17 (33) (65)	34 18 (130)	34 18 (66)	68 36 (28)

4.C References

1. Aumann, Richard; Brandenburger, A. (1995):" Epistemic conditions for Nash equilibrium", Econometrica, Vol. 63, pp. 1161-1180

2. Basu, Kaushik (1994): "The Traveler's Dilemma: Paradoxes of Rationality in Game Theory", The American Economic Review, Vol. 84, No. 2, Papers and Proceedings of the Hundred and Sixth Annual Meeting of the American Economic Association (May), pp. 391-395

3. Battalio, Raymond; Samuelson, Larry; Van Huyck, John (2001):"Optimization Incentives and Coordination Failure in Laboratory Stag Hunt Games", Econometrica, Vol. 69, No. 3 (May), pp. 749-764

4. Binmore, Ken (1994): "Game Theory and the Social Contract: Playing Fair", MIT Press, Cambridge, London

5. Binmore, Ken (1998): "Game Theory and the Social Contract: Just Playing", MIT Press, Cambridge, London

6. Bowles, Samuel; Choi, Jung-Kyoo (2007): "The Coevolution of Parochial Altruism and War", Science, Vol. 318, No. 26 (Oct.), pp.636-640

7. Brams, Steven J. (1993):"Theory of Moves", Cambridge University Press

8. Brams, Steven J.; Mattli, Walter (1992):"Theory of Moves: Overview and Examples", Economic Research Report, C.V. Starr Center for Applied Economics, New York University

9. Cooper, Russell; DeJong, Douglas V.; Forsythe, Robert; W. Ross, Thomas (1992): "Communication in Coordination Games", The Quarterly Journal of Economics, Vol. 107, No. 2 (May), pp. 739-771

10. Gilboa, Itzhak and Schmeidler, David (1989): "Maxmin Expected Utility with a Non-Unique Prior", Journal of Mathematical Economics, Vol. 18, pp. 141-153

11. Gintis, Herbert (2009): "The Bounds of Reason: Game Theory and the Unification of the Behavioral Sciences", Princeton University Press, Princeton, Oxford

12. Gintis, Herbert; Bowles, Samuel; Boyd, Robert; Fehr, Ernst (2005): "Moral Sentiments and Material Interests - The Foundation of Cooperation in Economic Life", The MIT Press Cambridge, Cambridge, London

13. Harsanyi, John C. (1973): "Games with Randomly Disturbed payoffs: A New Rationale for Mixed-Strategy Equilibrium Points", International Journal of Game Theory 2, pp.1-23

14. Hey, John D.; Lotito, Gianna and Maffioletti, Anna (2008):"The Descriptive and Predictive Adequacy of Theories of Decision Making Under Uncertainty/Ambiguity", Discussion Papers in Economics, University of York, Vol. 4, http://www.york.ac.uk/depts/econ/documents/dp/0804.pdf

15. Hipel, Keith W.; Fraser, Niall M. (1984): "Conflict Analysis: Models and Resolutions", North-Holland Series in System Science and Engineering, Vol. 11

16. Hofstadter, Douglas R. (1985): "Metamagical Themas: questing for the essence of mind and pattern", Bantam Dell Pub Group

17. Howard, Nigel (1971): "Paradoxes of Rationality: Theory of Metagames and Political Behavior", MIT Press Cambridge, Cambridge, London

18. Van Huyck, John B.; Battalio, Raymond C.; Beil, Richard O. (1990):"Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure", The American Economic Review, Vol. 80, No. 1 (Mar.), pp. 234-248

19. Lorenz, Konrad (1998): "Das sogenannte Böse: Zur Naturgeschichte der Aggression.", Deutscher Taschenbuch Verlag

20. Mailath, George J. (1998): "Do People Play Nash Equilibrium? Lessons from Evolutionary Game Theory", Journal of Economic Literature, Vol. 36, No. 3 (Sep.), pp. 1347-1374

21. Morris, Stephen (2006):"Purification",Princeton University Press,, http://levine.sscnet.ucla.edu/econ504/purification.pdf

22. Nowak, Martin A. (2006): "Evolutionary Dynamics - Exploring the Equations of Life", Harvard University Press, Cambridge, London

23. Nozick, Robert (1969): "Newcomb's problem and two principles of choice", N. Rescher, editor, Essays in Honor of Carl G. Hempel, Reidel, Dordrecht, Netherlands,

24. Page, Scott E. (2007):" The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies", Princeton University Press

25. Robson, Arthur J. (1996a): "A Biological Basis for Expected and Non-expected Utility", Journal of Economic Theory, Vol. 68, pp. 397-424

26. Robson, Arthur J. (1996b): "The Evolution of Attitudes to Risk: Lottery Tickets and Relative Wealth", Games and Economic Behavior, Vol. 14, pp. 190-207

27. Schmidt, David; Shupp, Robert; Walker, James M.; Ostrom, Elinor (2003):"Playing safe in coordination games: the roles of risk dominance, pay-off dominance, and history of play", Games and Economic Behavior, Vol. 42, pp. 281-299

28. Wagner, R. Harrison (1983):"The Theory of Games and the Problem of International Cooperation", The American Political Review, Vol. 77, No. 2 (Jun.), pp. 330-346

Repetition is the reality and
the seriousness of life.
Søren Aabye Kierkegaard(1813-1855)

5
The Dynamics of Conflict Analysis

The Conflict Analysis approach by Hipel and Fraser (1984) is well equipped to model repeated games. Players are assumed to posses a sequential reasoning that allows them to (not necessarily correctly) anticipate the reaction of other players to their strategies. An individual's best response strategy is thus defined based on this projection, adding additional stability conditions to strategic choice and increasing the set of potential equilibria beyond pure Nash equilibria. Yet, the original Conflict Analysis approach lacks the ability to genuinely model dynamic repeated games, in which past play defines the condition for future interactions. This chapter will illustrate how the original model can be adapted to include endogenous individual preferences that are defined by the strategic choice of players during past play, allowing to model the reciprocal connection between preferential change and best response play in repeated games. A dummy game serves as an exemplar and helps to visualise the results obtained from this extension.

Introduction

Conflict Analysis is an alternative approach that assumes the capacity of players to extrapolate the reactions of other players to their strategic choices. The sequential reasoning renders it interesting for the analysis and modelisation of repeated games. *Chapter 4* has illustrated and critically analysed the underlying assumptions, and obtained results by focusing on a static analysis. In the context of repeated games, the approach presented so far is insufficient to effectively model the dynamics of strategic choice over a sequence of interactions. This chapter concentrates on the dynamic analysis of games and expands the initial attempt of Hipel & Fraser (Ch. 14, 1984).

The basic assumptions of the Conflict Analysis approach are briefly reviewed in the following. For a more detailed explanation refer to *Chapter 4*. Given an n-person non-cooperative game defined by $G = (S_1, S_2, ..., S_n; U_1, U_2, ..., U_n)$, with player set $N = (1, 2, 3, ..., n)$, and S_i being individual i's strategy set and $S = \times S_i$ being the set of strategy profiles. Let there be an individual preference function U_i for each $i \in N$ that ranks the strategy profiles according to the individual's preferences over the associated outcomes by assigning all strategy profiles $s = (s_1, s_2, \ldots, s_n) \in S$ to one of two subsets with respect to any underlying strategy profile $q = (\bar{s}_i, \bar{s}_{-i})$.[1]

$$\begin{aligned} p \in U_i^+(q), &\text{ iff } O(p) \succ_i O(q) \\ p \in U_i^-(q), &\text{ iff } O(p) \preceq_i O(q) \end{aligned} \tag{5.0.1}$$

with $U_i^+(q) \cap U_i^-(q) = \emptyset$ and $U_i^+(q) \cup U_i^-(q) = S$,, and $O(q)$ the outcome associated to q. Denote a strategy profiles that can be obtained by a unilateral strategy switch, i.e. a switch of player i given the strategies of all players other than i, by $z_i(q) = (s_i, \bar{s}_{-i})$, with any $s_i \in S_i$. Given the set $Z_i(q)$ of all strategy profiles that can be obtained by a unilateral switch of i, the set of "dominant profiles" for q is then defined as

$$\textbf{DP: } u_i^+(q) = Z_i(q) \cap U_i^+(q), \forall s_i \in S_i \tag{5.0.2}$$

In other words, a dominant profile with respect to q is defined by the strategy profile of a unilateral switch of player i to a *better* response strategy given strategies $\bar{s}_{-i}$ of all players other than i.

[1]Preferences are assumed to be complete and transitive, though the last assumption is not strictly necessary for the approach, see Chapter 4 for details.

Two forms of stability of a player's strategy choice can occur. If a better response strategy is absent, a player is already playing his best response strategy and the underlying strategy profile is termed *rationally stable for player i*.

$$\textbf{Rational Stability: } u_i^+ (\bar{s}_i, \bar{s}_{-i}) = \emptyset, \forall s_i \in S_i \tag{5.0.3}$$

Furthermore, the switch to a better response strategy can entail a probable subsequent switch of another player j to his better response strategy, given the new strategy profile after the switch of player i. This may lead to an outcome, not strictly preferred by player i to the outcome defined by the strategy profile from which player i originally switched. Thus, player i refrains from unilaterally choosing this better response strategy. In that case the strategy switch is *sequentially sanctioned* and the strategy does not qualify as a viable better response strategy. If all better response strategies are sequentially sanctioned, the current strategy is best response and the underlying strategy profile is termed *sequentially stable for player i*. For any player j define $\hat{u}_j^+(p) = Z_j(p) \cap U_j^+(p)$ as the set of DPs for player j to player i's dominant profile p for q,

$$\begin{aligned} \textbf{Sequential stability: } & \hat{u}_j^+ (p = (s_i^*, \bar{s}_{-i})) \cap U_i^- (q = (\bar{s_i}, \bar{s}_{-i})) \neq \emptyset, \\ & \forall s_i^* : p = (s_i^*, \bar{s}_{-i}) \in u_i^+(q) \textit{ and for any } j \neq i \end{aligned} \tag{5.0.4}$$

If a strategy is neither rational nor sequential stable for player i, i.e. neither condition 5.0.3 nor condition 5.0.4 hold, it is unstable.[2] Define the underlying strategy profile as *unstable for player i*. A strategy profile that consists only of components that are best response strategies, i.e. a strategy profile that is *stable for all players*, is considered stable, and defines the game's potential equilibrium. Whether the individual strategies in the profile are sequentially or rationally stable, is irrelevant. Notice, however, that only a strategy profile, in which each component is rationally stable, is a Nash equilibrium.

In the following I will apply the Conflict Analysis approach to a fictitious game, in which two groups with conflicting interests interact. The game will serve as an exemplar to illustrate how the approach is able to model the interaction dynamics of a repeated game. Beginning with the static analysis of the game that constitutes the basis of the dynamic analysis, the approach is extended step by step to provide a more

[2] Simultaneous stability is of no interest in this context, as this chapter will focus solely on repeated games that are played sequentially.

sophisticated description and analysis of the underlying game. Focus will be placed on the dynamic representation. Starting out with a constant time-homogeneous transition matrix, the matrix is changed as such that it is able to model the transitions between various preference orders. Subsequently, the model is transformed into a more realistic dynamic process, where each state defines the interaction basis for the subsequent state, yet with an exogenously determined transition between preference orders. In a last step, the preference orders are directly determined by the strategy distribution of past play, thus previous states both define subsequent states and the rate of transition between preference orders.

5.1 A fictitious Game of Social Conflict

Assume a game with two groups of players C_A and C_B. Further assume that the behaviour of the two conflicting parties can be described by the collective action of all players in each group. This implies that we are not interested in individual decision and the subtle processes inside a group but the aggregate joint action.[3] The game is thus assumed to be sufficiently defined by $\Gamma = (S_{At}, S_{Bt}; U_{At}, U_{Bt})$, where S_{it} defines the strategy set and U_{it} the preference order for player group i, given $i = A, B$, at time t. Suppose both groups have to repeatedly renew a contract (e.g. on the relative monetary pay-off for the joint production of a good, on working conditions or on laws governing the mutual co-existence). Assume that group B is the proposing group that offers a contract and that group A has to decided whether or not to accept. The model is general enough to be interpreted as an abstract representation of a conflict on various social levels, ranging from an interaction between two *classic* social classes (the working class vs. the capitalists) over the conflict inside a single company (advisory board vs. board of directors) down to the individual level.

Suppose that A has only a very limited action set.[4] It can choose whether or not to fulfil B's contract and whether or not to actively demand a change in the (social) contract (e.g. by revolting or striking). B has a larger action set. First, B decides

[3]Notice that this is not equivalent to modelling macroeconomic behaviour on the basis of a representative agent. In this game, no assumptions on the individual preferences and actions exist, i.e. it is not supposed that any action of the group necessarily coincides with individual strategy choice, similar to the negligence of neural processes in the standard economic explanation of individual choices. The model does not start out in a micro level to explain macro dynamics, but assumptions are made on the same level as the general results, thereby evading problems of super- or sub-additivity.

[4]In order to simplify notation I will speak, in the following, of A or B, instead of group A or group B.

about the relative share it offers, i.e. how much the contract should benefit itself at the cost of A's benefit. For simplicity consider this only as the choice between a non-exploitative and an exploitative offer. Second, B considers, whether or not it demands the certification of the terms of the contract, so that group A is legally obliged to fulfil its part of the contract. Third and last, B has to choose whether or not to threaten with drastic additional sanctions, should A not abide to the terms of the contract, in order to pressure A to sign and fulfil the contract. Hence, A can choose between 4 actions (considering that inaction is an action), and B between 6. Consequently, there exist 4 pure strategies for A and 8 pure strategies for B leading to $2^5 = 32$ possible strategy profiles.

To fulfil the contract and to demand a change of the terms is mutually exclusive, as is the inverse. Hence, both the action of fulfilling the contract and of not demanding a change can be represented as a single action, as can be the contrary, resulting in $2^4 = 16$ possible strategy profiles. For notational simplicity the strategy sets can be further reduced by neglecting strategy profiles that are defined by strictly dominated strategies.[5] These are the two strategies, in which B does not certify the terms of the contract, but threatens A by sanctions (a literal incredible threat), namely *(Exploitation, no Certification, Sanction)* and *(no Exploitation, no Certification, Sanction)*. Furthermore, strategies that include the provision of a non-exploitative share of benefit to group A, after it did not accept the contract, are also assumed strictly dominated. This reduces the whole strategy profile set to 9 (=16-2-2-(4-1)).[6] Each binary representation of a strategy profile (where 1 implies that the action is taken and 0 that it is not) in the strategy set can again be symbolised by a decimal code (see table 5.1.a).

The strategy profiles with value 3 and 7 define non-exploitative outcomes, in which both groups have an incentive to abide to a contract that does not advantage one group over another. Strategy profile 0, in contrast, implies the requirement of a contractual change as none of the groups conforms to the terms of the initial contract.[7]

If there exists a bijective relation between strategy profile and outcome, an individual's preference over all outcomes determines a unique preference order of the set of strategy profiles. An outcome is, however, not only defined by a strategy profile,

[5] This assumption requires that the strategies are strictly dominated under the assumption of sequential reasoning. The previous chapter has shown that the cooperative strategy in the Prisoner's Dilemma is indeed not strictly dominated, if the Conflict Analysis approach is applied.

[6] The last assumption of strict dominance excluded 2^2 strategy profiles, of which one has already been excluded by the previous assumption.

[7] 0 can be interpreted for example as a period of social revolution or the complete renewal of labour contracts in a company.

Table 5.1.a: Strategy profile Set

***A*'s actions:**									
Abide	0	1	1	0	1	1	0	1	1
***B*'s actions:**									
No Exploitation	0	0	1	0	0	1	0	0	1
Certification	0	0	0	1	1	1	1	1	1
Sanction	0	0	0	0	0	0	1	1	1
Decimal code	**0**	**1**	**3**	**4**	**5**	**7**	**12**	**13**	**15**

but depends on the external circumstances defined by the rules of the game. Further, the preferential evaluation of an outcome is defined by the history of play of the other players that can change the bargaining power, the information and belief a player has (for example on the others bargaining power) and also the evaluation of other players (other-regarding preferences). Thus, assume that an outcome is also defined by the relative bargaining power of a groups and each utility/preference value is affected by the sympathy for the other player group. Further suppose that these change exogenously in time. (This assumption is relaxed later in section 5.2.2)

According to the value that is assigned to these variables, the strategy profile associated to an outcome and its underlying utility will change in the course of play. So will the preference order. It is unnecessary to explicitly model these variables and sufficient to implicitly incorporate them into the preference vector. Notice, however, that the decimal number does not describe identical outcomes under different assumptions of sympathy and bargaining power. A decimal number represents indeed identical strategy profiles, but outcomes are defined conditional on the value of bargaining power and sympathy. When sympathy of one player group for another player group decreases, strategy profiles that assign relatively higher utility to the latter will offer less utility to the former and rank lower in the preference order. Bargaining power will affect the feasibility and stability of certain actions (e.g. the effect of strikes or the threat of a sanction). A change in bargaining power will thus alter the relative preference for a strategy profile.[8]

For the first case assume that initially *A* prefers above all those strategy profiles, in which *B* offers a non-exploitative contract. It is indifferent between strategy profile 7 and 3, and slightly dislikes the threat of sanctions. If *B* offers an exploitative contract

[8]Arguably such a model leaves too much room for interpretation of exactly how a change in preference order occurs. Yet, the abstract way, in which complex interactions are analysed, should leave that room of interpretation.

and does not certify the contract, *A* prefers not to abide to the terms of the contract. If the contract is certified, *A* prefers the case of no sanction, since this implies lower costs in the case, where *A* breaks the agreement. In the second case, *A* knows that a non-exploitative contract is no longer feasible, since B has appropriated sufficient market power. As a consequence, *A* loses sympathy for *B*, implying that *A* prefers a change of the contract (e.g. by pressure of strike) above all and no certification or threat of sanction.

The preference order ranks the strategy profiles according to the player's preference over the associated outcomes. The order is from left to right, placing the strategy profile that is associated to the most preferred outcome in the left most position. All strategy profiles to the right are strictly less preferred, except if the profiles are connected by a bar, indicating indifference. The preference order for *A* looks as in table 5.1.b (grey implies that these strategy profiles are considered infeasible). In the first preference order, the outcomes associated with 7 and 3 are equally preferred, and the outcome associated to 12 is least preferred.

Table 5.1.b: Preference Vector I

Preference order of *A*									
Start	$\overline{7\,|}$	$\overline{3}$	15	0	1	5	4	13	12
End	3	7	15	0	4	12	1	5	13

The preference order of *B* is separated into three different cases. Like for *A*, the first case represents the initial situation of the model. *B* prefers to offer a non-exploitative contract. Certifying the contract beforehand is slightly preferred to just providing the non-exploitative share to *A* without contractual certification. The threat of sanctioning *A* is even less preferred, since *B* considers this unnecessary if *A* approves the contract (both groups are aware of their mutual benevolence). The second case is the situation, in which *B* gained in relative bargaining power. *B* believes that not fulfilling the contract by *A* will have no fundamental repercussions, e.g. that *A* reacts only with a limited violent aggression and the effect of strike is negligible. *B* also believes that an *a priori* threat of sanction will push *A* to abide to the terms of the contract and will limit the violence of *A*'s reaction. Hence, *B* prefers above all to exploit *A* and always to threaten with a sanction to only certifying, which is again preferred to not certifying. *B* prefers 12 to 15, since the possible loss from non-fulfilment is expected to be lower than the expected loss from offering a non-exploitative contract. Since *B* has an interest to maintain his *status quo* power, strategy profile 0 is least preferred.

In the third stage, *B* is aware that *A* has no sympathy for *B*. If *A* does not abide to the contract, the subsequent reaction (i.e. a violent general strike or a social revolution) will question and endanger *B*'s *status quo* position, since *A* is likely to win the conflict. Therefore *B* prefers all those strategy profiles, in which *A* fulfils the contract, and desires most those, in which *A* is still exploited. In the case, where *A* does not intend to abide to the terms of the contract, no certification is preferred to just certifying, which is again preferred to threaten with sanctions. The idea is that *B* fears that the violence of *A*'s reaction will depend on how much *B* abused his bargaining power.[9] Consequently *B*'s preference order is assumed as illustrated in table 5.1.c.

Table 5.1.c: Preference Vector II

Preference order of *B*									
Start	7	3	15	5	1	13	12	4	0
Intermediate	13	5	1	12	15	7	3	4	0
End	13	5	1	15	7	3	0	4	12

5.1.1 Stability Analysis without Mis-perception

It is required to carry out a static analysis of the game before proceeding to a dynamic representation. Based on the given assumptions, 4 different games can be derived for the static model. The first is the case, in which both parties have preferences given by "start". The second is defined by *A*'s preference order in "start" and *B*'s preference order as in "intermediate". This implies a situation, in which *B* has sufficient bargaining power to exploit, but *A* has insufficient power to successfully demand a change of the contract. Similarly, the next two stages are then given by preferences "end-intermediate" and "end-end". The solution of every static game is represented for efficiency in the tabular form (for a detailed description of the derivation of this form, refer to chapter 4). The first static game and its analysis are given in table 5.1.d below.

The *DP*s are derived as described from condition 5.0.2. The *DP*s below a certain strategy profile are vertically ordered according to their position in the preference vector, i.e. the highest *DP* is most preferred by this player group. Strategy profiles lacking a *DP* are rationally stable according to condition 5.0.3. A change from 1 to

[9] A non-violent change of the contract is preferred to an act of violence that will cause additional costs for *B*.

Table 5.1.d: First static game

start-start									
overall stability	E	x	x	x	x	x	x	x	x
stability for *A*	r	r	r	r	s	r	u	r	u
A's preference order	7	3	15	0	1	5	4	13	12
DPs					0		5		13
stability for *B*	r	u	u	u	u	u	r	u	u
B's preference order	7	3	15	5	1	13	12	4	0
DPs		7	7 3	7 3 15	7 3 15 5	7 3 15 5 1		12	12 4

0 of A, i.e. a switch from strategy *Abide* to *not Abide* given *B*'s strategy *Exploitation, no Certification, no Sanction*, entails a subsequent shift of *B* to 12 or 4. Both are less preferred than the outcome associated to the original strategy profile 1. Condition 5.0.4 is fulfilled for 1 and player *A*. A shift of *A* from 4 to 5 causes a subsequent shift of *B* to 7, 3 or 15, of which all rank higher in the preference order. Neither condition 5.0.3 nor 5.0.4 are satisfied for 4. In the same fashion the remaining stabilities are calculated for *A* and *B*. In the first static game only strategy profile 7=*(Abide; no Exploitation,Certification)* is stable for both groups. Hence, the contract is defined by a fair cooperation. The solution to the second static game is shown in table 5.1.e.

Table 5.1.e: Second static game

start-intermediate									
overall stability	x	x	x	x	x	x	x	E	x
stability for *A*	r	r	r	r	s	r	s	r	u
A's preference order	7	3	15	0	1	5	4	13	12
DPs					0		5		13
stability for *B*	r	u	u	r	u	u	u	u	u
B's preference order	13	5	1	12	15	7	3	4	0
DPs		13	13 5		13 5 1	13 5 1 15	13 5 1 15 7	12	12 4

The stable equilibrium of the second game in table 5.1.e is defined by 13=*(Abide; Exploitation, Certification, Sanction)*. Table 5.1.f describes the third static game, in which *A* prefers a social change to the *status quo* situation, but *B* believes to have sufficient bargaining power to prevent such a change.[10] The equilibrium of the third game in table 5.1.f is defined by 12=*(Not Abide; Exploitation, Certification, Sanction)*. *A* asks for a social change, but *B* is unwilling to give up his *status quo* position. The last game, in which the probability of a successful social change is high, is depicted in table 5.1.g.

Table 5.1.f: Third static game

end-intermediate									
overall stability				x	x	E	x	x	x
stability for *A*				r	r	r	u	u	u
A's preference order	3	7	15	0	4	12	1	5	13
DPs							0	4	12
stability for *B*	r	s	s	r	u	u	u	u	u
B's preference order	13	5	1	12	15	7	3	4	0
DPs		13	13 5		13 5 1	13 5 1 15	13 5 1 15 7	12	12 4

Table 5.1.g: Fourth static game

end-end									
overall stability			E	E	x	x	x	x	x
stability for *A*				r	r	r	u	u	u
A's preference order	3	7	15	0	4	12	1	5	13
DPs							0	4	12
stability for *B*	r	s	s	s	u	u	r	u	u
B's preference order	13	5	1	15	7	3	0	4	12
DPs		13	13 5	13 5 1	13 5 1 15	13 5 1 15 7		0	0 4

[10] In the game context, this period could be interpreted as an attempt of social turnover.

The equilibrium is defined by 0=*(Not Abide; Exploitation)* or 15=*(Abide, No Exploitation, Certification, Sanction)*. The former is the situation, in which A and B do not reach a mutual consent regarding their contract. The second defines a situation similar to the previous game in table 5.1.f, but B prefers to offer a non-exploitative contract fearing the eventual repercussion. Based on the original interpretation of the game, the strategy profile symbolised by 0 is regarded as a period, during which the social or work contract is rewritten. Since A deems 15 infeasible, it will not choose action *Abide*. The static model describes a player population moving through the following states: $(7 \rightarrow 13 \rightarrow 12 \rightarrow 0)$.

5.1.2 Hypergames

Before developing the dynamic representation, the following short subsection will consider a first level hypergame. This will be of interest, when the dynamic representation with state dependent transition probabilities will be developed. In comparison with the results obtained here, the state dependent approach can dynamically model hypergames.[11] Suppose that B overestimates the benevolence of A and its general willingness to accept any contract. On the other hand, A wrongly estimates B's bargaining power and intentions. Both groups believe to be playing entirely different games (see table 5.1.h), illustrating the situation, in which B believes to be still playing "start-start", and A to be playing "end-end". The stabilities for the individual strategy profiles in the two games can be directly taken from the tables 5.1.d and 5.1.g, and are stated again in the upper half of table 5.1.h for simplicity. The equilibria of this hypergame are given by the stabilities of each group according to its individual game. This is represented in the lower part of the table. Consequently, in this case the strategy profile would be either 7 or 12, unlike only 7 or 0 as expected by B or by A, respectively. If A assumes 3, 7, and 15 infeasible and will not choose action *Abide*, the final and only equilibrium of the game is defined by 12.

5.1.3 Dynamic representation

Though the analysis of the game in its static form already indicates the possible equilibria that might occur during various stages of interaction, a dynamic representation, which can be controlled to continuously move between the various preference orders,

[11] This refers also to an n-level hypergame, since, as illustrated in *Chapter 4*, a hypergame of any level can be represented by a first level hypergame.

Table 5.1.h: First Level Hypergame: Mutual Mis-perception of the Game

"end-end" - A's perception									
stability for A				r	r	r	u	u	u
A's preference order	3	7	15	0	4	12	1	5	13
stability for B	r	s	s	s	u	u	r	u	u
B's preference order	13	5	1	15	7	3	0	4	12
"start-start" - B's perception									
stability for A	r	r	r	r	s	r	$\hat{u}$	r	u
A's preference order	$\overline{7 \mid 3}$		15	0	1	5	4	13	12
stability for B	r	u	u	u	u	u	r	u	u
B's preference order	7	3	15	5	1	13	12	4	0
Combining A's and B's Stability									
overall stability		E		x	x	E	x	x	x
stability for A				r	r	r	u	u	u
As preference order	3	7	15	0	4	12	1	5	13
stability for B	r	u	u	u	u	u	r	u	u
B's preference order	7	3	15	5	1	13	12	4	0

is preferable. As a first step, the individual transition matrix for each group must be derived on the basis of the static analysis. The individual transition matrices will be used to form the transition matrix T for the entire game. A definition of the individual transition matrices, solely on the basis of the initial (*start-start*) and the final (*end-end*) game, should suffice as an example.

For a games with f strategy profiles, let a *state* X_t be defined by the probability distribution of the strategy profiles in time t with dimension $f \times 1$, i.e. by a vector, where each component indicates the likelihood of a strategy profile at time t over the entire set S. The Markov process is determined by

$$X_t = T\,X_{t-1}, \tag{5.1.1}$$

and T is the transition matrix of dimension $f \times f$ that describes the transition probability of moving from strategy profile x in period $t-1$ to y in t. Under the condition that the transition matrix T is time homogeneous the Markov process is defined by $X_t = T^t\,X_0$. Two variants can be used as a basis to model preferential change:

Variant 1: As a first assumption, consider that players exogenously change their expectations about the game they are playing. Since each individual transition matrix should reflect the preference order in both games, the original approach in *Chapter 4* needs some adaptation. Define a "transition probability" α and γ. The first refers to the preference order of A, the latter to preference order of B. Define the transition probability for A in such a way that $1 - \alpha$ gives the probability of being in a state defined by game *(start-start)* and hence, α is the probability of being in state defined by game (end-end). A continuum of states, defined by a specific value of the transition probability, can be thus described, ranging from the stabilities of A as in *start, start* ($\alpha = 0$) to its stabilities as in *end, end* ($\alpha = 1$). The transition probabilities hence enable us to shift between the preference orders of each group. The transition matrix T_A for group A is given in table 5.1.i.

Table 5.1.i: Individual Transition Matrix with identical Perception

Transition Matrix for A

	0	**1**	**3**	**4**	**5**	**7**	**12**	**13**	**15**
0	1	α	0	0	0	0	0	0	0
1	0	1-α	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0
4	0	0	0	1	α	0	0	0	0
5	0	0	0	0	1-α	0	0	0	0
7	0	0	0	0	0	1	0	0	0
12	0	0	0	0	0	0	α	α	0
13	0	0	0	0	0	0	1-α	1-α	0
15	0	0	0	0	0	0	0	0	1

Though derivation of this matrix is intuitive, it is useful to look at the original preference order and the stabilities of tables 5.1.d and 5.1.g, shown again in table 5.1.j. For simplicity, it is generally assumed that a player switches to his *most preferred*

Table 5.1.j: Comparison Initial & Final Stage

Preference order of A at the initial and final stage									
stability for A in initial stage	r	r	r	r	s	r	*u*	r	u
As preference order	$\overline{7}$	$\overline{3}$	15	0	1	5	4	13	12
DPs					0		5		13
stability for A in final stage				r	r	r	u	u	u
A's preference order	3	7	15	0	4	12	1	5	13
DPs							0	4	12

non-sanctioned DP. In both games, strategy profiles 0, 3, 4, 7, and 15 are stable. Hence, a value of 1 is written along the main diagonal for these strategy profiles. Strategy profile 1 is stable in the initial game, but has a *DP* and therefore a transition to 0 in the final game. Thus, with probability $1 - \alpha$, A remains in strategy profile 1, with probability α it switches to strategy profile 0. The same logic provides the rest of the matrix for the remaining strategy profiles. In the same manner the transition matrix T_B in table 5.1.k for B is derived, where γ gives the transition probability from the initial preference order to the one in the final state of the game. Hence, $\gamma = 0$ describes B's stabilities in *(start, start)* and $\gamma = 1$ its stabilities in *(end, end)*. Keep in mind that a player can only switch to a non-sanctioned DP. The most preferred non-sanctioned DP of B for 3 and 7 is 15.

Table 5.1.k: Individual Transition Matrix with identical Perception

Transition Matrix for B

	0	1	3	4	5	7	12	13	15
0	γ	0	0	γ	0	0	γ	0	0
1	0	γ	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	γ	0	0	0	0
7	0	1 - γ	1 - γ	0	1 - γ	1 - γ	0	1 - γ	1 - γ
12	1 - γ	0	0	1 - γ	0	0	1 - γ	0	0
13	0	0	0	0	0	0	0	γ	0
15	0	0	γ	0	0	γ	0	0	γ

Subsequently, these two matrices are combined into a single transition matrix T (see table 5.1.l). The derivation of this matrix is more complicated than the derivation of the individual transition matrices. By looking at the transition matrix for A, it

Table 5.1.l: Final Transition Matrix

Combined Transition Matrix for the Game

	0	1	3	4	5	7	12	13	15
0	γ	$\gamma\ \alpha$	0	$\gamma\ \alpha$	0	0	$\alpha\ \gamma$	0	0
1	0	(1-α) γ	0	(1-α) γ	0	0	(1-α) γ	0	0
3	0	0	0	0	0	0	0	0	0
4	0	$\alpha\ (1 - \gamma)$	0	0	α	0	0	α (1-γ)	0
5	0	0	0	0	(1-α) γ	0	0	0	0
7	0	(1-α) (1-γ)	1 - γ	0	(1-α) (1-γ)	1 - γ	0	(1-α) (1-γ)	1 - γ
12	1 - γ	0	0	α (1-γ)	0	0	α (1-γ)	$\alpha\ \gamma$	0
13	0	0	0	(1-α) (1-γ)	0	0	(1-α) (1-γ)	(1-α) γ	0
15	0	0	γ	0	0	γ	0	0	γ

can be seen that A stays at strategy profile 0 with certainty. Therefore group A will choose $(0, -, -, -)^T$ with probability 1.[12] B stays in 0 with probability γ and switches to 12 with probability 1-γ. Accordingly, group B is expected to choose $(-, 0, 0, 0)^T$ or

[12]Remember that the strategy profile can also be written as a vector in binary code. Here A is only able to change the first value of the vector. A slash indicates that the group cannot affect these values.

$(-,0,1,1)^T$. The expected strategy profiles for the final transition matrix are given by $(0,0,0,0)^T$ with probability $1 \times \gamma$, and $(0,0,1,1)^T$ with probability $1 \times (1-\gamma)$. Strategy profile 12 stays with probability α in 12 and with probability $(1-\alpha)$ switches to 13. Hence, A chooses to accept the contract with probability $(1-\alpha)$ and not to with probability α. B changes to strategy profile 0, i.e. it chooses $(-,0,0,0)^T$, with probability γ, and stays in 12, defined by $(-,0,1,1)^T$, with probability (1- γ). Hence, strategy profile 12 is followed by 0 with probability $\alpha \times \gamma$, by 1 with probability $(1-\alpha) \times \gamma$, by 12 with probability $\alpha \times (1-\gamma)$, and by 13 with probability $(1-\alpha) \times (1-\gamma)$. The calculation used to find the equilibria in the joint transition matrix is simplified by the following equation:

$$\bar{q} = \sum_{i=1}^{x} \dot{o}_i - (x-1)\dot{q}, \tag{5.1.2}$$

where $\bar{q}$ defines the value of the new equilibrium, x the number of player groups (here $x = 2$), and $\dot{o}_i$ the value of the DP of group i from strategy profile q given by value $\dot{q}$.

Some transitions would lead to strictly dominated strategy profiles, which have been ruled out before. Equation 5.1.2 is thus inapplicable in these cases. Strategy profile 1, for example, is followed by strategy profile $(0,1,1,0)^T$ with probability $\alpha \times (1-\gamma)$. Yet, this strategy profile is considered infeasible. There is, however, a logical solution. The reason for ruling out strategy profiles $(0,1,-,-)^T$ in the first place, has been that if A does not abide, a strategy including a non-exploitative contract was assumed strictly dominated. Hence, it is reasonable to suppose that the group will cause the strategy profile to switch to $(0,0,1,0)^T$, namely 4. Therefore the probabilities are added to strategy profile 4.[13]

The two games and the transition between them can be represented by an adapted

[13]Though contrary to the initial assumption, it can be argued that B responds to A. In that case, strategy profile 6 can be substituted by 7. The matrix then looks as:

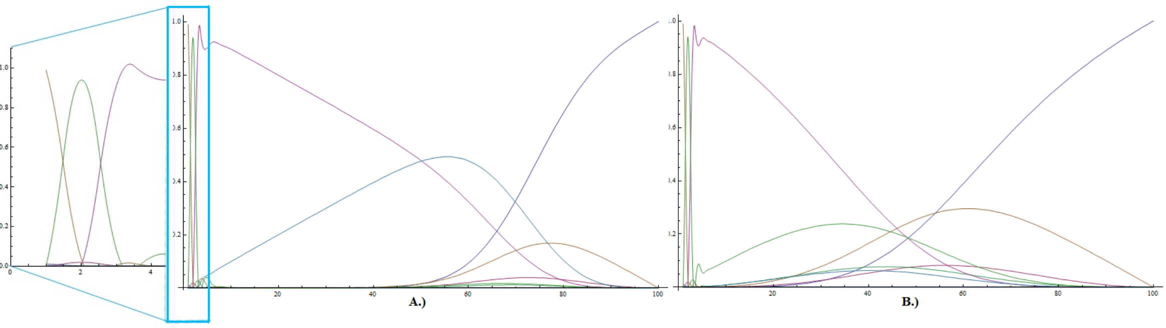
Combined individual Transition Matrices of each group

	0	1	3	4	5	7	12	13	15
0	γ	$\alpha\ \gamma$	0	$\gamma\ \alpha$	0	0	$\alpha\ \gamma$	0	0
1	0	(1-α) γ	0	(1-α) γ	0	0	(1-α) γ	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	$\alpha\ \gamma$	0	0	0	0
5	0	0	0	0	(1-α) γ	0	0	0	0
7	0	(1-γ)	1 - γ	0	(1-γ)	1 - γ	0	(1-γ)	1 - γ
12	1 - γ	0	0	α (1-γ)	0	0	α (1-γ)	$\alpha\ \gamma$	0
13	0	0	0	(1-α) (1-γ)	0	0	(1-α) (1-γ)	(1-α) γ	0
15	0	0	γ	0	0	γ	0	0	γ

Simulations results are similar to what is obtained from applying transition matrix 5.1.1.

version of equation 5.1.1. Each state is determined by a unique transition matrix $T_{\alpha,\beta}$; a variant of the original transition matrix T, given the unique values of the transition probabilities $\alpha(t)$ and $\gamma(t)$ in each period t. Hence, a state in period t is defined by $X_t = T^t_{\alpha,\beta}X_0$. Notice that the transition probabilities disregard the probability a player assigns to certain strategies in different states. Probabilities are only defined by the composition of X_0. Equilibrium 15 is therefore observable, though it has been exogenously assigned zero probability in the analysis, presented in table 5.1.g. The vector X_0, indicating the *status quo*, needs to be defined *a priori* for the simulation. Strategy profile 0 is a reasonable assumption to describe a situation at the initial stage of interaction. No form of contract has yet been offered. The player population could also be considered at a turning point. Hence $X_0 = (1,0,0,0,0,0,0,0,0)^T$. Notice that the initial vector can also describe a state with mixed strategy profiles, such as $X_0' = (0,0.5,0.5,0,0,0,0,0,0)^T$, implying society starting at strategy profile 1 and 3 with equal probability of 0.5.

Figure 5.1.A: Game with 2 Groups



The simulation was conducted for transition probabilities $\alpha(t) = \gamma(t) = \frac{t}{100}$. Figure 5.1.A.*A.)* shows the probability of being in one of the nine possible strategy profiles for $t \in (0,5)$ and $t \in (0,100)$. Table 5.A.s on page 203 provides the rounded probabilities from the first period to the last, in steps of five periods, in order to interpret the figure more easily. Though the two "intermediate games" (i.e. *start-intermediate* and *end-intermediate*) were neglected, the dynamic representation already shows a sequence that is alike to the static representation. The player population starts out at 0; a point, where initial bargaining begins. It goes immediately to 12 and then to 13 and finally stays at 7 for a longer time. It takes three periods for our player population to reach the first (initially stable) equilibrium. Both groups reach a preliminary agreement on a non-exploitative contract. Strategy profile 15 grows steadily until the middle of the simulation, whereas 7 slowly diminishes. Strategy profile 12 lags behind and obtains a low maximum probability of 11% in period 80 of the simulation. Finally at the end of the cycle, the society returns to strategy profile 0; a social change or reform of the underlying contract.[14] We thus obtain the sequence $7 \rightarrow 15(\rightarrow 12) \rightarrow 0$. Also notice that the dynamic representation includes first level hypergames. The example of section 3.2 can be obtained for setting $\alpha = 1$ and $\gamma = 0$. Since a higher level hypergame can be simplified to a first level hypergame as shown above, the dynamic representation is able to incorporate all the properties developed for the static game form. Evidently, in this case 7 and 12 are the only equilibria with value 1 on the main diagonal.

Variant 2: The first variant considered the case, in which players believe that the other players in the game update their preference order in an identical way.[15] Yet previous play, information and expectation can have a different effect on the players' preference orders. As a consequence, players perceive that they shift *independently* their preference order. For the given example, we thus obtain 4 different games, instead of 2; the same as before, but in addition a game, in which A has preference order "start" and B has preference order "end", and a fourth that represents the inverted case. Table 5.1.m on page 189 illustrates the stabilities of these 4 games. Following the same logic as variant 1, the first game with preference order "start-start" occurs with probability $(1-\alpha)(1-\beta)$, the second defined by "end-end" with probability $\alpha\beta$, the third game defined by "start-end" with probability $(1-\alpha)\beta$, and the fourth defined by "end-start" with probability $\alpha(1-\beta)$.

Based on the transitions in table 5.1.m, the individual transition matrix for A has the shape as in table 5.1.n. The transition probabilities are determined by the *DP*s in table 5.1.m and the corresponding probability of the game, e.g 0 stays at 0 in all 4 games and thus transition occurs to 0 with probability 1, strategy profile 1 stays in 1 only in the first game but shifts to 0 in the remaining three, thus transition occurs to 1 with probability $(1-\alpha)(1-\beta)$, and to 0 with probability $1-(1-\alpha)(1-\beta)$. In the same way the transition matrix for B is obtained and represented in table 5.1.o The final transition matrix is obtain in the same way as before, i.e. by combining the two individual transition matrices. Again the transition to strategy profile 6 can occur if equation 5.1.2 is applied to calculate the final transitions. As in the earlier variant, the transition probabilities have been attributed to strategy profile 4 (see

[14]Notice that the rise of 15 occurs through the shift of B to its most preferred **unsanctioned** DP. If it switches from 15 to 13, the dynamics should exactly replicate the results of the static analysis.This assumption contradicts, however, the requirement that a player never chooses a sanctioned DP.

[15]e.g. if A has preferences as in *start*, it believes that B has also preferences as in *start*.

Table 5.1.m: Reduced Stability Analysis- only most preferred unsanctioned DP is shown

Preference order of A and B as in "start" $P = (1-\alpha)(1-\beta)$

stability for A	r	r	r	r	s	r	u	r	u
A preference order	7|3		15	0	1	5	4	13	12
DPs							5		13
stability for B	r	u	u	u	u	u	r	u	u
B's preference order	7	3	15	5	1	13	12	4	0
DPs		7	7	7	7	7		12	12

Preference order of A and B as in "end" $P = \alpha\beta$

stability for A	r	r	r	r	r	r	u	u	u
A's preference order	3	7	15	0	4	12	1	5	13
DPs							0	4	12
stability for B	r	s	s	s	u	u	r	u	u
B's preference order	13	5	1	15	7	3	0	4	12
DPs					15	15		0	0

Pr. order of A as in "start" and B as in "end" $P = (1-\alpha)\beta$

stability for A	r	r	r	r	u	r	s	r	u
A's preference order	7|3		15	0	1	5	4	13	12
DPs					0		5		13
stability for B	r	u	u	u	u	u	r	u	u
B's preference order	13	5	1	15	7	3	0	4	12
DPs		13	13	13	13	13		0	0

Pr. order of A as in "end" and B as in "start" $P = \alpha(1-\beta)$

stability for A	r	r	r	r	r	r	u	u	u
A's preference order	3	7	15	0	4	12	1	5	13
DPs							0	4	12
stability for B	r	u	u	u	u	u	r	u	u
B's preference order	7	3	15	5	1	13	12	4	0
DPs		7	7	7	7	7		12	12

table 5.1.p). A simulation as in variant 1, with $\alpha(t) = \gamma(t) = \frac{t}{100}$, is shown in figure 5.1.A B.) making it directly comparable to the results of variant 1. Though the initial sequence is identical, as well as the most predominant strategy profiles (i.e. 7 =violet and 0 =blue), transition occurs to 13 =green and 12 =brown in the intermediate time periods. We obtain a transition that is akin

Table 5.1.n: Individual Transition Matrix of A: correct perception

Transition Matrix for A									
	0	**1**	**3**	**4**	**5**	**7**	**12**	**13**	**15**
0	1	1-(1-α) (1-γ)	0	0	0	0	0	0	0
1	0	(1-α) (1-γ)	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0
4	0	0	0	1-(1-α) (1-γ)	α	0	0	0	0
5	0	0	0	(1-α) (1-γ)	(1-α)	0	0	0	0
7	0	0	0	0	0	1	0	0	0
12	0	0	0	0	0	0	α	α	0
13	0	0	0	0	0	0	1-α	1-α	0
15	0	0	0	0	0	0	0	0	1

to the static analysis (7 $(\rightarrow 13) \rightarrow 12 \rightarrow 0$).

Both variants suffer, however, from a decisive shortcoming. They model only a "pseudo" dynamic process, since each state is independent of the previous states, and does not add informational value to the static model, though the form of representation is more efficient. Each state X_t is defined by $X_t = T^t_{\alpha,\beta} X_0$, i.e. a state in period t is solely determined by the initial state of the world X_0 and its unique transition matrix $T_{\alpha,\beta}$. Hence, states only differ in the variation of the transition matrix and its exponent. Although, this approach allows to describe a transition between the individual static games and hypergames, and offers the convenient determination of a state without the requirement to calculate previous states, a *true* dynamic process is defined by $X_t = T_{\alpha,\beta} X_{t-1}$. This definition implies that each state defines the "playing ground", i.e. probability distribution of strategy profiles, at the beginning of the next interaction period, based on which players define their best response strategies. Since each state has a unique transition matrix, which shapes the path between states, it obviously holds that $T_{\alpha,\beta} X_{t-1} \neq T^t_{\alpha,\beta} X_0$, as T is not time homogeneous.

The following analysis is independent of which variant is chosen, since each of the following extensions can be equivalently applied to any variant. Though variant 2

Table 5.1.o: Individual Transition Matrix of B: correct perception

Transition Matrix for B									
	0	**1**	**3**	**4**	**5**	**7**	**12**	**13**	**15**
0	γ	0	0	γ	0	0	γ	0	0
1	0	α γ	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	α γ	0	0	0	0
7	0	1-γ	1-γ	0	1-γ	1-γ	0	1-γ	1-γ
12	1-γ	0	0	1-γ	0	0	1-γ	0	0
13	0	(1-α) γ	(1-α) γ	0	(1-α) γ	(1-α) γ	0	γ	(1-α) γ
15	0	0	α γ	0	0	α γ	0	0	α γ

Table 5.1.p: Game Transition Matrix: correct perception

Final Transition Matrix

	0	**1**	**3**	**4**	**5**	**7**	**12**	**13**	**15**
0	γ	$(1-(1-\alpha)(1-\gamma))\alpha\gamma$	0	$(1-(1-\alpha)(1-\gamma))\gamma$	0	0	$\alpha\gamma$	0	0
1	0	$(1-\alpha)(1-\gamma)(\alpha\gamma)$	0	$(1-\alpha)(1-\gamma)\gamma$	0	0	$(1-\alpha)\gamma$	0	0
3	0	0	0	0	0	0	0	0	0
4	0	$(1-(1-\alpha)(1-\gamma))(1-\gamma)$	0	0	$\alpha(\alpha\gamma+(1-\gamma))$	0	0	$\alpha(1-\gamma)$	0
5	0	0	0	0	$(1-\alpha)\alpha\gamma$	0	0	0	0
7	0	$(1-\alpha)(1-\gamma)(1-\gamma)$	$1-\gamma$	0	$(1-\alpha)(1-\gamma)$	$1-\gamma$	0	$(1-\alpha)(1-\gamma)$	$1-\gamma$
12	$1-\gamma$	$(1-(1-\alpha)(1-\gamma))(1-\alpha)\gamma$	0	$(1-(1-\alpha)(1-\gamma))(1-\gamma)$	$\alpha(1-\alpha)\gamma$	0	$\alpha(1-\gamma)$	$\alpha\gamma$	0
13	0	$(1-\alpha)(1-\gamma)(1-\alpha)\gamma$	$(1-\alpha)\gamma$	$(1-\alpha)(1-\gamma)(1-\gamma)$	$(1-\alpha)(1-\alpha)\gamma$	$(1-\alpha)\gamma$	$(1-\alpha)(1-\gamma)$	$(1-\alpha)\gamma$	$(1-\alpha)\gamma$
15	0	0	$\alpha\gamma$	0	0	$\alpha\gamma$	0	0	$\alpha\gamma$

is more apt to model most dynamic interactions, variant 1 offers a less demanding representation, rendering it more accessible to the reader. Consequently, variant 1 is chosen for the subsequent analysis.

5.2 Interaction between three parties with endogenous preferences

This section will take account of the issue raised in the previous section, but will also add some complexity to the underlying group structure. The Conflict Analysis approach has modelled the interaction of two distinct groups. Since the approach is capable of modelling a larger number of player groups, the following subsection will analyse a non-homogeneous group, i.e. it is assumed that the group A consists of two sub-groups. One sub-group is still defined as A, the second sub-group is named C. In game $\Gamma = (S_{At}, S_{Bt}, S_{Ct}; U_{At}, U_{Bt}, U_{Ct})$, both sub-groups, C and A, have the same relation towards B and decide, whether or not to abide to the rules of the contract.

5.2.1 Non-homogeneous group members

Since there are now three interacting groups, the strategy profile set increases correspondingly with the added strategy set. Following the same logic in ruling out strictly dominated strategies, i.e. strategy profiles $((0,-,1,-,-)^T$, $(-,0,1,-,-)^T$, and $(-,-,-,0,1)^T$), there are 15 possible strategy profiles presented in table 5.2.q. The same reasoning for the preference order like in the previous section apply. The previous decimal code for the strategy profiles is rewritten as:

original value→new value	0→0, 2	1→1, 3	3→7	4→8, 10	5→9, 11	7→15	12→24, 26	13→25, 27	15→31

Table 5.2.q: Strategy profile Set

***A*'s options:**															
Abide	0	1	0	1	1	0	1	0	1	1	0	1	0	1	1
***C*'s options:**															
Abide	0	0	1	1	1	0	0	1	1	1	0	0	1	1	1
***B*'s options:**															
No Exploitation	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1
Certification	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
Sanction	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
Decimal code	**0**	**1**	**2**	**3**	**7**	**8**	**9**	**10**	**11**	**15**	**24**	**25**	**26**	**27**	**31**

Furthermore *A* and *C* are assumed, for simplicity, to have the same preference order (since they have the same relation to *B*); the corresponding strategy profiles are only mirrored according to the affected group. Hence, for example strategy profile 1 and 2 exchange places in the preference order of *C* with respect to *A*. Some additional assumptions are necessary for the asymmetric strategy profiles, in which *A* and *C* choose a different strategy. Generally assume that group *A* and *C* prefer to take identical decisions, and for the first preference order assume that both prefer to sign, but not being the only group exploited. For the second preference order, suppose that both favour refusing an exploitative contract at all costs. Hence, the preference order for the three groups can be written as in table 5.2.r (since the focus lies on the dynamic representation, only games "start-start-start" and "end-end-end" are of interest, thus *B*'s "intermediate" preference order is neglected).

Table 5.2.r: Preference Vectors

Preference order of *A*															
Start	7	15	31	0	2	3	1	11	8	9	10	27	25	24	26
End	7	15	31	0	8	24	2	10	26	1	9	25	3	11	27
Preference order of *C*															
Start	7	15	31	0	1	3	2	11	8	10	9	27	26	24	25
End	7	15	31	0	8	24	1	9	25	2	10	26	3	11	27
Preference order of *B*															
Start	15	7	31	11	3	27	9 | 10		1 | 2		25 | 26		24	8	0
End	27	11	3	31	15	7	9 | 10		1 | 2		25 | 26		0	8	24

The static solution, both of the initial game with the related preference order "start-start-start" and the final game with preference order "end-end-end" is presented in the table 5.A.t on page 204. The set of equilibria is defined by $E = (1, 2, 15)$ for the former game, and by $E = (0, 31)$ for the latter game. All equilibria are reasonable, but not equally likely. By using the same procedure as before and based on the obtained stabilities, the individual transition matrices can be derived as presented in table 5.A.u on page 205. Combining the three individual transition matrices results in the final transition matrix T in table 5.A.v on page 206.

The structure of this matrix provides interesting information about the dynamics of the underlying game. None of the strategy profiles is an absorbing state for all values of α, β, and γ. Notice that the number of potential equilibria exceeds the five equilibria that have already been determined in the static analysis. The original five equilibria are, however, defined by the transition probability of a single group.[16] The additional potential equilibria can only be stable if two or more groups jointly show the necessary transition probability. If it is assumed that group A and C have similar interests and, hence, approximately the same values for their transition probabilities, the additional equilibria are defined by $3, 11, 24, 27$.

Running the model under the same conditions as before, yields the expected results. Yet, owing to the issue that has been raised in the previous subsection the simulation will take account of the path dependency of each state, i.e. $X_t = T_\omega X_{t-1}$ (where T_ω is the short notation for $T_{\alpha,\beta,\gamma}$). Since this requires the calculation of each state, I programmed a loop in Mathematica that iterates the calculation process in each period. In addition, I assumed that transition probabilities do not change at the same rate for all groups. For the example, the transition probabilities were defined as $\alpha(t) = 0.5 \sin\left[\frac{t-50}{10\pi}\right] + 0.5$, $\beta(t) = 0.5 \sin\left[\frac{t-75}{15\pi}\right] + 0.5$ and $\gamma(t) = 0.5 \sin\left[\frac{t-100}{20\pi}\right] + 0.5$.[17] The simulation was run for 800 periods. The distribution of the strategy profiles from $t = 0$ to $t = 800$ are presented in figure 5.2.B A.) (values are given in table 5.A.w on page 207) and transition probabilities are illustrated below in 5.2.B.*B.)*.[18]

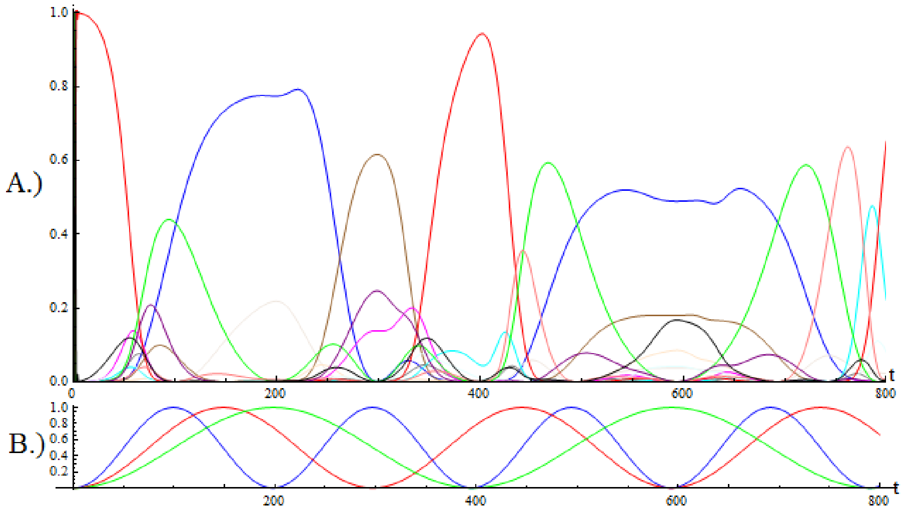
The non-homogeneous model shows periods of cooperation, exploitation and conflict; represented by the simplified sequence of predominant profiles: $15(red) \to 24(green) \to 0(blue) \to 2(brown) \to 15(red) \to 24(green) \to 0(blue) \to 24(green) \to 25(pink) \to$

[16] Group A determines the stability of 1, group C the stability of 2, and group B the stability of 0, 15 and 31.

[17] I have not chosen a nested simulation with linearly increasing and decreasing transition probabilities, since Mathematica showed problems with nested function. A simulation of the form $\alpha(t)$: $\alpha(t) = \frac{t}{n}$ for $t \in (0, 100) \cup (200, 300) \cup (400, 500)$ and $\alpha(t) = 1 - \frac{t}{n}$ for $t \in (100, 200) \cup (300, 400) \cup (500, 600)$, where n is the upper bound of the corresponding interval was therefore impossible.

[18] This and all following figures use the same colour code: 0- Blue,1- LightBrown, 2-Brown, 3- LightOrange, 7-Yellow, 8-Gray, 9-Cyan, 10-Magenta, 11-LightCyan, 15-Red, 24-Green, 25-Pink, 26-Purple, 27-Black, 31-Orange

Figure 5.2.B: Game with 3 Groups:A.) shows the distribution for each state in $t \in (0, 800)$; B.) shows the corresponding values for $\alpha(t)$ (blue), $\beta(t)$ (red), and $\gamma(t)$ (green).



$9(cyan) \rightarrow 15(red)$. This sequence shows that the group, which has a longer cycle (i.e. group C), accepts the offered contract more quickly and is more prone to exploitation in the initial periods. Yet, the validity of these results is impaired by the exogenous and entirely arbitrary definition of the transition probabilities, which do not exhibit any underlying dependencies. A model that allows for more significant results, requires to *endogenise* the transition variable. Hence assume that individuals update their preference order based on their experience, when interacting with other players. The strategies, which have been played in previous encounters, will directly affect other-regarding preferences and the bargaining power in future interactions.

5.2.2 State Dependent Transition Probabilities

This section will thus address the issue of *endogenising* the state dependent individual preference order. Until this point the transition probabilities have not been affected by the outcomes of previous play, but have been defined by an arbitrary relation. Yet, past play will actively influence a player's affections towards another player and the success and the gain from previous play will also determine future bargaining power. An approach that takes account of the effect of past play on the current preference order of a player, is to define a direct relation between the frequency, with which strategy profiles occur, and the transition probability values.[19] For the given case of two different preference orders for an individual group, a higher value of the transition probability illustrates that this group tends towards the second preference order. Lower transition probabilities refer to the first preference order. Thus, the value of each individual transition probability can be assumed to increase in the case, where strategy profiles, which support the conditions underlying the second preference vector of this group/player, occur with higher probability and frequency. On the contrary, transition probability decreases if strategy profiles, which are likely to "shift" individual preferences towards the first preference order, are played with higher probability in the current play and appear more often.

Assume a finite number of f different strategy profiles, so that for $\tilde{s}_k \in S$, with $k = 1, \dots, f$, a state X_t is defined by a vector $X_t = (x_1(t), x_2(t), ..., x_f(t))$, with $\sum_{i=1}^{f} x_i(t) = 1$, where each $x_k(t)$ defines the probability, with which strategy profile $\tilde{s}_k$ occurs in t. A transition probability $\varphi(t)$, with $\varphi(t) = \alpha(t), \beta(t), \gamma(t)$, can be represented as a function of X_t. Following the previous line of argument, define a set of strategy profiles O^+_φ, which consists of all the strategy profiles that increase the transition probability φ, since these are expected to lead to preferences as described in the later preference order. Define further another set O^-_φ, which consists of all the strategy profiles that diminish the value of φ, as they induce an individual preference according to the first preference order, implying $O^+_\varphi \cap O^-_\varphi = \emptyset$ and $O^+_\varphi \cup O^-_\varphi \subseteq S$. Hence, each individual transition probability can be represented as:

$$\begin{gathered}\varphi(t) = \varphi(t-1) + \epsilon^+_\varphi \sum_j x_j(t-1) - \epsilon^-_\varphi \sum_h x_h(t-1), \textit{ where} \\ \varphi(t) \text{ is bound to } \varphi(t) \in (0,1), \text{and } \tilde{s}_j \in O^+_\varphi \text{ and } \tilde{s}_h \in O^-_\varphi\end{gathered} \tag{5.2.1}$$

so that $\sum x_j(t-1) + \sum x_h(t-1) \leq 1$. ϵ^+_φ and ϵ^-_φ can be constant or stochastic variables that define the impact of the relative occurrences of a strategy profiles on the transitional change. The game with three groups will again serve as an example.

[19]Notice, however, that there lies an issue here. Strategy profiles determine outcomes based on the current state, i.e. if the transition probability is high, the outcome that a player associates to a strategy profile is different from the one, he associates in the case of a low transition probability. Hence, on the one hand, the effect of a strategy profile on the transition probability will be determined by the current state. On the other hand, the relative frequency of the strategy profile defines the state. We therefore obtain a circular relation. I will neglect this issue here, since I believe it to be only of minor importance. The strategy profiles that either raise or lower the probability are simply expected to do so under every state. A non-exploitative contract will generally lead to A's and C's preferences as in "start-start-start". Similarly an exploitative contract with a threat of sanction will generally have the opposite effect.

The assumptions on how to define the sets O_φ^+ and O_φ^- for each transition probability are manifold and so are the resulting dynamics. The following results are only meant as an illustrative example for the approach. The definition of O_φ^+ and O_φ^- are, however, chosen in a plausible way with respect to the context of the game. Remember that the transition probability of A is given by $\alpha(t)$, those of C by $\beta(t)$ and those of B by $\gamma(t)$. For simplicity assume as before that A and C have symmetric preferences. Consequently, sets O_α^+ and O_β^+, as well as sets O_α^- and O_β^- will have a similar structure. They will only differ in those strategy profiles, in which the strategic choices of both groups are different. In such cases the strategy profiles are defined by the *mirror image* of the corresponding profile for the other group (e.g. the strategy profile denoted by $9 = (1,0,0,1,0)^T$ in O_α^+ corresponds to the strategy profile denoted by $10 = (0,1,0,1,0)^T$ in O_β^+).

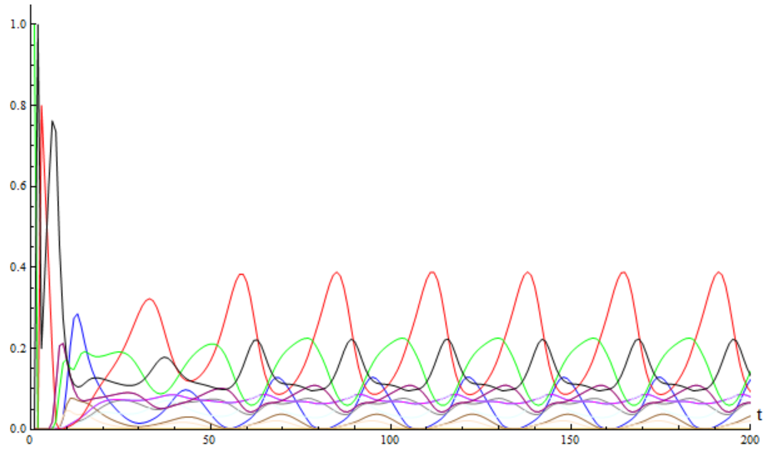
Assume that the sympathy for B is reduced in the case, where exploitative and certified contracts occur with higher probability. In the case, where the exploitative contract is not certified, A and C are not legally required to stick to their initial commitment, and these contracts will thus have no effect on the transition probability. An increase in sympathy for B arises only for non-exploitative contracts, where both groups accept or refuse[20], and where B does not threaten with a sanction in the case of rejection.

The bargaining power of group B is weakened in all those cases, in which both A and C refuse the contract, or in which one group refuses, though B has threatened to sanction the group that does not accept. In these cases, B realises that a sanction is no effective intimidation, thus re-evaluating its bargaining power. Congruently, the bargaining power of B increases in those situation, in which A and C accept the offered contract under any terms. Following this reasoning, the sets are defined as follows: $O_\alpha^+ = (9,11,25,27)$, $O_\alpha^- = (0,7,8,15) = O_\beta^-$, $O_\beta^+ = (10,11,26,27)$, $O_\gamma^+ = (3,7,11,15,27,31)$, and $O_\gamma^- = (0,8,24,25,26)$. Assume that ϵ_φ^- and ϵ_φ^+ are equal to 0.2, which implies that a complete transition from one preference order to the other requires at least 5 interaction periods. Again letting Mathematica simulate the game for 200 periods, shows the result as given in figure 5.2.C

The game is cyclic after about 10 periods. Non-exploitative contracts $15 = (1,1,1,1,0)^T$ (red) occur at approximately 10-40%, exploitative contracts with sanctions is accepted, $27 = (1,1,0,1,1)^T$ (black), with approximately 5-25% and refused, $24 = (0,0,0,1,1)^T$ (green), with approximately 10-20% and no contracting,

[20]Collective rejection is considered as the requirement for re-negotiating the current contract.

Figure 5.2.C: Dynamic Representation of Game with 3 Groups - constant ϵ^- and ϵ^+

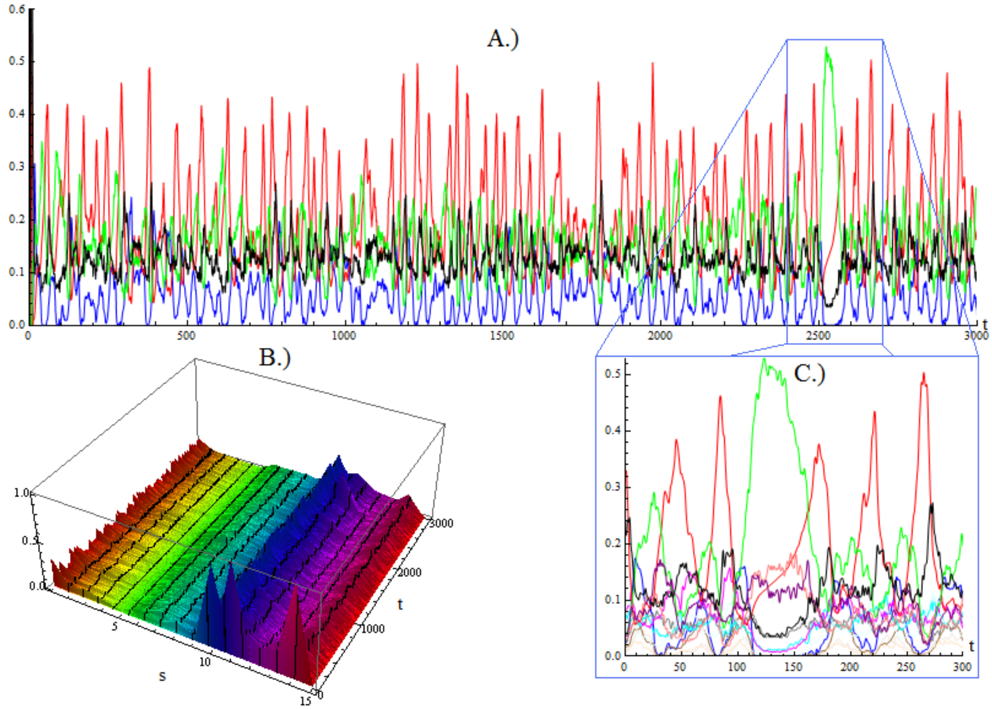


$0 = (0,0,0,0,0)^T$ (blue), appears with approximately 0-12%.

The regularity, with which these strategy profiles occur does not represent a realistic image of what we would expect the game to look like. The issue here is the constant impact that strategy profiles / outcomes, have on the transition probability, as well as the strict symmetry between group A and C. All three groups represent a larger number of players. Since some players will react more violently to certain outcomes than others, idiosyncratic reactions will add noise to the effect of outcomes on the preference order. A similar argument holds for the assumption of strict symmetry between both groups. The following simulation, however, only concentrates on the first aspect. Both ϵ_φ^- and ϵ_φ^+ are assumed to be continuous and uniformly distributed over 0 to 0.2 for all groups, thus having half the same expected value as in the previous simulation. With the addition of added noise in ϵ_φ^- and ϵ_φ^+, the simulation has been conducted in the same way as before. The result in figure 5.2.D shows a fairly different and more intuitive result, but the general dynamics are maintained.

Figure A.) illustrates the simulation for 3000 periods. Since most strategy profiles only occur with very low frequency (on the average below 5%), I have reduced the presentation to the predominant strategy profiles in figure 5.2.C, namely 0 (blue), 15 (red), 24 (green), and 27 (black). Non-exploitative contracts still occur with highest probability. In the stochastic case the exploitative contract with sanction is on average more often rejected (represented by the green line) than accepted (represented by the black line). The relative frequencies of all strategy profiles is better visualised in figure B.). The axis labelled s shows the strategy profiles, as they are simply ranked

Figure 5.2.D: Dynamic Representation of Game with 3 Groups - $\epsilon^-_\varphi, \epsilon^+_\varphi \in (0, 0.2)$: A.) Simulation for 3000 periods, showing only predominant strategy profiles, B.) three dimensional representation- strategy profiles are ordered according to their relative decimal code (from 1 to 15), C.) detailed view of period 2400-2700 with all strategy profiles



according to their position in the transition matrix T.[21] Notice that the colours do not correspond with figure A.). The "combs" at position 1, 10,11,14, refer to strategy profile 0,15,24 and 27, respectively. The highest elevation is at the blue ridge (15).

The simulation generated a sequence of periods, in which the exploitative contract has been frequently refused by both A and C, though B has threatened with a sanction (see Figure C.)). Although it seems that a direct correlation between 15 (black) and 24 (green) exists, a look at the transition matrix 5.A.v on page 206 shows that this is not the case. Notice that 25 (pink) and 26(purple) occur more frequent. Looking at the transition matrix 5.A.v on page 206 shows that there are two large blocks of highly connected strategy profiles. The first block is defined by 9, 10, 11, 15, the second by 24, 25, 26. The second block is directly connected to the first by 27 and the first to the second by 8. Consequently during this period, the game switched from the first to the second block of highly correlated strategy profiles. This switch

[21] Position/Strategy Profile: 1/0, 2/1, 3/2, 4/3, 5/7, 6/8, 7/9, 8/10, 9/11, 10/15, 11/24, 12/25, 13/26, 14/27, 15/31

has occurred through a rise in all transition probabilities leading the system to 8 and 0. Both strategy profiles are an elements of O_{γ}^{-}. Hence, the subsequent slump in γ pushed the system towards 24. Due to high values of α and β and zero value of γ the system kept high probability values for this strategy profile, thus creating the short period of "social discontentment".

The values for ϵ_{φ}^{+} and ϵ_{φ}^{-}, and the composition of the sets O_{φ}^{+} and O_{φ}^{-} leave much room for further analysis. Different values of ϵ_{φ}^{+} and ϵ_{φ}^{-} for the transition values change the dynamics.[22] Different strategy profiles can be expected to have a different degree of impact. This can be done by splitting O_{φ}^{+} and O_{φ}^{-} into various disjoint subsets, and by assigning to each subset a different value of ϵ_{φ}^{+} and ϵ_{φ}^{-}. In addition, the robustness of the results can be tested for changes in the composition of $O_{\varphi}^{+/-}$. The strategy profiles, played in each period, can also be the outcome of a sequence of negotiations. This can be simply included by defining $X_t = (T_{\alpha,\beta})^r X_{t-1}$, where the exponent r simply defines the number of plays that determine the strategy outcome in a given period, i.e. the number of interactions until a conclusion is reached.[23] In short, there are many directions, in which the approach presented in this chapter can be adapted to various purposes and requirements. Since these changes constitute only a variation of equation 5.2.1, they will not be discussed in the scope of this chapter.

5.3 Possible Extension and Conclusion

5.3.1 Ideas for an Agent-based model

As Jason Potts (2000) illustrated, it is more reasonable to model on the basis of what he called "a non-integral framework". We should take account of the incomplete links between agents, incomplete knowledge and necessity to explore technologies and thus the endogeneity of preferences, as equivalent internal constructs to the underlying technologies.

Technologies is synonymous with strategy sets in the context of game theory. An extension of the presented approach should thus include the following: Strategy sets as well as preferences should be able to evolve over time as players interact. Furthermore, the form of inter-group and intra-group links between players should have

[22] It can be observed, for example, that a group, which exhibits a relatively low impact of the strategy profiles on the transition probabilities, is more prone to exploitation than the other group.

[23] Think of r as the time span of a conference or congress, that determines a treaty or contract for the future period, such as a climate summit or plant bargaining.

significant impact on the dynamics of the entire system. Yet, if the effect of connections on the external level (between players and between groups on various levels of aggregation) and on the internal level (in form of changing preferences and strategy options) is accounted for, the model will be unsolvable in closed form. In addition, heterogeneous and boundedly rational agents make it inevitable to use non-arithmetic modelling techniques and complex systems. Weakening the assumption in the described way thus will make it necessary to model by means of agent-based simulations.

In this context the model has been extended in the following way (working paper CEU –Complex Systems– 2009, *in progress*): The model acts on a macro basis (network structure of interactions between agents, groups and classes) and on a micro basis that evolves internally to the individual agent (preferences and resources). Each agent is initially endowed with a constrained set of actions, which evolves over time. They are primary unaware of the results caused by a specific action and also their possible combination into more sophisticated strategies. Thus, new strategies are a function both of previous combinations of actions on the basis of a currently existing action set, and of previous interactions and experiences with other players. The strategy set evolves according to endogenous search heuristics, which have the form of genetically evolving operators (Cross-Over, specification, point mutation - see for example Dosi et al., 1999 & 2003 and Fagiolo et al., 2003).

Interactions takes place in pairs, matching is random. Strategies and counter-strategies are played until an equilibrium is reached or cyclic behaviour occurs, providing only the fall-back position to each player. With each interaction, a player is increasingly able to associate his and his counterpart's strategies with an outcome, and to form expectations about the sequence of strategies that will be played. Hence, he becomes increasingly aware of his and the other agents' strategy sets (i.e. resources). Furthermore, preferences are incomplete, but adaptive. The pay-off, an individual receives conditional on his chosen strategy, affects his future preference order. Consequently, the evolution of his preference vector depends on past experiences and chosen strategies. Since the same holds for his counterpart, an individual cannot guess his partners preferences during early interaction periods. Hence all players are initially unaware of the rules of the game, but understand them better with each interaction. Furthermore, they are also able to learn new strategies and associate outcomes by observing the interactions of their neighbours. A new *mutant* action and its combination with other actions into a strategy can then spread locally to the neighbours of the individual, who will use this strategy, if successful.

5.3.2 Conclusion

Since the rationality underlying the Conflict Analysis approach is grounded on a sequential reasoning of players, the approach is especially powerful if applied to games that are repeated for an undefined time period, which is obviously the most predominant game type in real-world interactions. Yet, the original approach is very limited in its ability to model and to illustrate the dynamics of such games.

This chapter targeted the issue by illustrating how the static solution form can be redefined as a time inhomogeneous Markov chain. This approach offers thus a more realistic representation of repeated games. The last state affects the future state in two ways; directly via its composition but also by its impact on the players' preference order, allowing for endogenous preference changes. This impact can be flexibly modelled and easily be adopted to a variety of repeated games. Its integration and complementation with other approaches should provide a valuable source for future research.

5.A Tables

Table 5.A.s: 2 groups - Rounded probabilities per Strategy profile

From period 0 to 100 in steps of 5 periods

	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0	1.00	0	0	0	0	0	0	0	0	0	0	0	0.01	0.02	0.07	0.18	0.37	0.6	0.81	0.94	1.00
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0.02	0.02	0.01	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0.95	0.90	0.85	0.8	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.34	0.27	0.19	0.11	0.04	0.01	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0.03	0.06	0.09	0.11	0.09	0.05	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.59	0.62	0.62	0.55	0.41	0.23	0.08	0.01	0

Table 5.A.t: Static game - 3 groups

start-start-start

overall stability	x	E	x	x	E	x	E	x	x	x	x	x	x	x	x
stability for *A*	r	r	r	r	r	s	s	r	r	s	u	r	r	u	u
A's preference order	7	15	31	0	2	3	1	11	8	9	10	27	25	24	26
DPs						2	0			8	11			25	27
stability for *C*	r	r	r	r	r	s	s	r	r	s	u	r	r	u	u
C's preference order	7	15	31	0	1	3	2	11	8	10	9	27	26	24	25
DPs						1	0			8	11			26	27
stability for *B*	r	u	u	u	u	u	r	r	s	s	s	s	r	u	u
B's preference order	15	7	31	11	3	27	9	10	1	2	25	26	24	8	0
DPs		15	15	15	15	15			9	10	9	10		24	24
			7	7	7	7					1	2			8
				31	31	31									
					11	11									
						3									

end-end-end

overall stability	x	x	E	E	x	x	x	x	x	x	x	x	x	x	x
stability for *A*	r	r	r	r	r	r	r	r	r	u	u	u	u	u	u
A's preference order	7	15	31	0	8	24	2	10	26	1	9	25	3	11	27
DPs										0	8	24	2	10	26
stability for *C*	r	r	r	r	r	r	r	r	r	u	u	u	u	u	u
C's preference order	7	15	31	0	8	24	1	9	25	2	10	26	3	11	27
DPs										0	8	24	1	9	25
stability for *B*	r	s	s	s	u	u	r	r	s	s	s	s	r	u	u
B's preference order	27	11	3	31	15	7	9	10	1	2	25	26	0	8	24
DPs		27	27	27	27	27			9	10	9	10		0	0
			11	11	11	11					1	2			8
				3	3	3									
					31	31									
						15									

* no strategy *DP* is simultaneously sanctioned in both games

Table 5.A.u: Individual Transition Matrices - 3 groups

Transition Matrix for A

	0	**1**	**2**	**3**	**7**	**8**	**9**	**10**	**11**	**15**	**24**	**25**	**26**	**27**	**31**
0	1	α	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1-α	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	α	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1-α	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	1	α	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	1-α	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	α	α	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1-α	1-α	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	α	α	0	0	0
25	0	0	0	0	0	0	0	0	0	0	1-α	1-α	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	α	α	0
27	0	0	0	0	0	0	0	0	0	0	0	0	1-α	1-α	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Transition Matrix for C

	0	**1**	**2**	**3**	**7**	**8**	**9**	**10**	**11**	**15**	**24**	**25**	**26**	**27**	**31**
0	1	0	β	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	β	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1-β	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1-β	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	1	0	β	0	0	0	0	0	0	0
9	0	0	0	0	0	0	β	0	β	0	0	0	0	0	0
10	0	0	0	0	0	0	0	1-β	0	0	0	0	0	0	0
11	0	0	0	0	0	0	1-β	0	1-β	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	β	0	β	0	0
25	0	0	0	0	0	0	0	0	0	0	0	β	0	β	0
26	0	0	0	0	0	0	0	0	0	0	1-β	0	1-β	0	0
27	0	0	0	0	0	0	0	0	0	0	0	1-β	0	1-β	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Transition Matrix for B

	0	**1**	**2**	**3**	**7**	**8**	**9**	**10**	**11**	**15**	**24**	**25**	**26**	**27**	**31**
0	γ	0	0	0	0	γ	0	0	0	0	γ	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	γ	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	γ	0	0	0	0	0	0
15	0	0	0	1-γ	1-γ	0	0	0	1-γ	1-γ	0	0	0	1-γ	1-γ
24	1-γ	0	0	0	0	1-γ	0	0	0	0	1-γ	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
27	0	0	0	0	γ	0	0	0	0	γ	0	0	0	γ	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	γ

Table 5.A.v: Game Transition Matrix

Final Transition Matrix for the Game - 3 groups

	0	1	2	3	7	8	9	10	11	15	24	25	26	27	31
0	γ	α	β	$\alpha\beta\gamma$	0	γ	0	0	0	0	$\alpha\beta\gamma$	0	0	0	0
1	0	$1-\alpha$	0	$(1-\alpha)\beta\gamma$	0	0	0	0	0	0	$(1-\alpha)\beta\gamma$	0	0	0	0
2	0	0	$1-\beta$	$\alpha(1-\beta)\gamma$	0	0	0	0	0	0	$\alpha(1-\beta)\gamma$	0	0	0	0
3	0	0	0	$(1-\alpha)(1-\beta)\gamma$	0	0	0	0	0	0	$(1-\alpha)(1-\beta)\gamma$	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	$\alpha\beta(1-\gamma)$	0	0	$\alpha\beta$	$\alpha\beta$	$\alpha\beta$	0	0	0	0	$\alpha\beta(1-\gamma)$	0
9	0	0	0	$(1-\alpha)\beta(1-\gamma)$	0	0	$(1-\alpha)\beta$	$(1-\alpha)\beta$	$(1-\alpha)\beta$	0	0	0	0	$(1-\alpha)\beta(1-\gamma)$	0
10	0	0	0	$\alpha(1-\beta)(1-\gamma)$	0	0	$\alpha(1-\beta)$	$\alpha(1-\beta)$	$\alpha(1-\beta)$	0	0	0	0	$\alpha(1-\beta)(1-\gamma)$	0
11	0	0	0	0	0	0	$(1-\alpha)(1-\beta)$	$(1-\alpha)(1-\beta)$	$(1-\alpha)(1-\beta)\gamma$	0	0	0	0	0	0
15	0	0	0	$(1-\alpha)(1-\beta)(1-\gamma)$	$1-\gamma$	0	0	0	$(1-\alpha)(1-\beta)(1-\gamma)$	1-γ	0	0	0	$(1-\alpha)(1-\beta)(1-\gamma)$	1-γ
24	$1-\gamma$	0	0	0	0	$1-\gamma$	0	0	0	0	$\alpha\beta(1-\gamma)$	$\alpha\beta$	$\alpha\beta$	$\alpha\beta\gamma$	0
25	0	0	0	0	0	0	0	0	0	0	$(1-\alpha)\beta(1-\gamma)$	$(1-\alpha)\beta$	$(1-\alpha)\beta$	$(1-\alpha)\beta\gamma$	0
26	0	0	0	0	0	0	0	0	0	0	$\alpha(1-\beta)(1-\gamma)$	$\alpha(1-\beta)$	$\alpha(1-\beta)$	$\alpha(1-\beta)\gamma$	0
27	0	0	0	0	γ	0	0	0	0	γ	$(1-\alpha)(1-\beta)(1-\gamma)$	$(1-\alpha)(1-\beta)$	$(1-\alpha)(1-\beta)$	$(1-\alpha)(1-\beta)\gamma$	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	γ

Table 5.A.w: Transition Probabilities for simulation - 3 groups

t	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625	650	675	700	725	750	775	800
0	0	0.01	0.02	0.03	0.02	0.02	0.01	0	0.01	0.02	0.02	0.02	0.02	0.01	0	0	0	0.01	0.02	0.03	0.02	0.01	0.01	0.01	0.01	0.02	0.03	0.03	0.02	0.01	0	0
1	0.02	0.01	0	0	0	0.01	0.02	0.03	0.02	0.01	0	0	0	0.01	0.02	0.02	0.02	0.01	0	0	0.01	0.01	0.02	0.02	0.02	0.01	0	0	0.01	0.01	0.02	0.02
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0.04	0.13	0.23	0.24	0.15	0.04	0	0.03	0.04	0.01	0	0.02	0.04	0.02	0	0.06	0.18	0.25	0.21	0.1	0.02	0	0	0.01	0.06	0.17	0.25	0.22	0.11	0.02	0
9	0.02	0.04	0.02	0	0.05	0.16	0.25	0.22	0.12	0.03	0	0	0	0.05	0.15	0.24	0.24	0.13	0.03	0	0.03	0.04	0.01	0	0.03	0.04	0.01	0	0.08	0.2	0.26	0.2
10	0.04	0.12	0.13	0.08	0.02	0	0	0	0.03	0.13	0.26	0.31	0.23	0.1	0.01	0	0	0	0.03	0.09	0.13	0.1	0.02	0	0.07	0.13	0.12	0.05	0.01	0	0	0
11	0.18	0.09	0.02	0	0	0	0.02	0.08	0.13	0.1	0.03	0	0.04	0.09	0.09	0.05	0.01	0	0	0	0.04	0.14	0.27	0.3	0.2	0.07	0.01	0	0	0	0.03	0.07
15	0.7	0.59	0.46	0.33	0.2	0.09	0.02	0	0.03	0.11	0.22	0.34	0.48	0.6	0.67	0.67	0.62	0.54	0.43	0.3	0.18	0.08	0.02	0	0.04	0.13	0.24	0.37	0.49	0.59	0.65	0.68
24	0	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0	0	0	0.01	0.01	0	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0	0.01	0.01	0.01	0.01	0.01	0
25	0	0	0	0	0	0.01	0.01	0.01	0	0	0	0	0	0	0.01	0.01	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0.01
26	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0.01	0	0	0	0	0	0	0	0.01	0	0	0	0	0.01	0	0	0	0	0	0
27	0.03	0.1	0.2	0.33	0.45	0.56	0.63	0.65	0.63	0.55	0.44	0.31	0.19	0.09	0.02	0	0.03	0.11	0.23	0.35	0.48	0.58	0.65	0.66	0.62	0.53	0.41	0.29	0.16	0.07	0.01	0.01
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

5.B References

1. Dosi Giovanni; Fagiolo, Giorgio; Aversi, Roberta; Meacci, Mara; Olivetti, Claudia (1999): "Cognitive Processes, Social Adaptation and Innovation in Consumption Patterns: from Stylized Facts to Demand Theory", in S. C. Dow, P. E. Earl (eds.), Economic Organizations and Economic Knowledge. Essay in Honour of Brian Loasby, Vol.1, Cheltenham, Edward Elgar

2. Dosi, Giovanni; Hobday, Mike; Marengo, Luigi (2003): "Problem-solving Behaviors, Organizational Forms, and the Complexity of Tasks", in C. E. Helfat (ed.) The SMS Blackwell Handbook of Organizational Capabilities Emergence, Development, and Change, London, Blackwell Publishing

3. Fagiolo Giorgio; Dosi, Giovanni (2003): "Exploitation, exploration and innovation in a model of endogenous growth with locally interacting agents", Structural change and Economic Dynamics, Vol. 14

4. Hipel, Keith W.; Fraser, Niall M. (1984): "Conflict Analysis: Models and Resolutions", North-Holland Series in System Science and Engineering, Vol. 11

5. Potts, Jason (2000): "The New Evolutionary Microeconomics", Edward Elgar