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Contents

Abstract

The aim of this dissertation is to contribute to the understanding of the impact of the consumer communication on the market with information asymmetry. We focus in particular on the market for experience good with the vertical differentiation. The dissertation consists of three chapters.

In Chapter 1 we develop a monopoly model close to Akerlof (1970)[3] where we add consumer groups. Consumers do not have information about the product quality, but they do know its expected quality. They share their experience about the product with the other consumers of the group. This information flow affects buying decisions and consequently producer profits. We apply the Bayesian game theory to determine the set of equilibrium prices.

We study the effect of information exchange within a consumer group on price and quality. Firstly, we find that a presence of consumer group enables signalling of quality through price, resulting in a partial or complete resolution of the adverse selection problem. It is important to emphasise that the resolution of the adverse selection problem does not come directly from the fact that consumers are informed about the product quality, but from a transformation of the producer payoffs due to the information sharing. Secondly, although consumers benefit from having an additional member in the group because of information sharing, we find that the optimum group size is finite. Thirdly, if the consumers share the costs of experimentation with the other members of the group, then the positive consumer surplus disappears, the maximum equilibrium price increases even further, and this expands the set of parameter values for which the adverse selection problem is resolved. Finally, a government can resolve the adverse selection problem by introducing minimum quality standards.

In Chapter 1 we find that the communication between the consumers in monopoly enables signalling of the low-quality producer types. In the similar frameworks signalling disappears in oligopoly due to the price (Bertrand) competition. However, Janssen and Roy (2010) develop a Bayesian model and find that high-quality producers may signal their quality in oligopoly. The aim of Chapter 2 is to develop a Bayesian model which combines consumer communication and oligopoly, and check if signalling still occurs. Unlike Janssen and Roy (2010) who find signalling of the high-quality producers, by using a similar oligopoly framework we find that the low-quality producers signal their quality by setting a low price. However, as in Janssen and Roy (2010) if the competition is strong then the signalling disappears. We study a unique signalling equilibrium in oligopoly which satisfies the D1 criterion. We find that despite the fact that the competition decreases the expected social surplus, it increases the expected consumer surplus, so that the loss due to the competition is completely borne by producers. However, if the consumer group size is endogenous, then the competition my increase the expected profit. There is a multiplicity of pooling equilibria in oligopoly, where the consumer group size and the competition increase the maximum possible price, while the product variability decreases it.

In Chapter 3 we apply evolutionary game theory with replicator dynamics to study the interrelation between the consumer communication on the one

hand, and the producer quality choice and its disclosure to the consumers on the other. There is an information asymmetry because the consumers usually do not observe the exact quality before the purchase, but they know the expected quality. Heterogeneous producers do not interact directly, but they affect the payoffs of each other through the market price. We find an interior evolutionary unstable fraction of the high-quality producers, which divides the basins of attraction of two exterior evolutionary stable states, one with high-quality producers only, and the other with low-quality producers only. Larger consumer communication increases the fraction of the initial states which converge to the high-quality equilibrium. An increase in the fraction of the dishonest producers has the opposite effect. A population where both honest and dishonest producers co-exist is evolutionary stable, while homogeneous populations, with exclusively honest or dishonest producers are unstable. The reason is that an increase in the fraction of dishonest producers decreases 'the reputation', and consequently the price, of the high-quality good, which decreases the payoff of the dishonest producer. However, consumer communication and the increase in the low quality may decrease, while an increase in the high quality and in the share of the high-quality producers increases the evolutionary stable fraction of the dishonest producers.

Chapter 1

Price Signalling in a Monopoly with Consumer Experimentation and Group Communication

1.1 Introduction

Social ties in labour markets have a significant effect on economic outcomes. For instance, several studies show that about 50 percent of jobs are found through referrals (see Ioannides and Datcher Loury (2004)[29] for an overview). Although there are far fewer studies about social ties in other markets, here we mention some examples. DiMaggio and Louch (1998)[19] find that in the US between one-quarter and one-half of purchases of goods of which assessment is difficult (e.g. used cars, legal advice and home repair) are made through personal networks. Bandiera and Rasul (2006:899)[8] studied the effects of social networks on the adoption of new technology by farmers in the Northern Mozambique and find that social networks play a more important role in the decision about adoption when farmers have less information about the new crop initially. Baker and Faulkner (2004)[7] show that investors using pre-existing social ties have a much lower likelihood of losing their capital. Similarly, Freedman and Zhe Jin (2008:26-27)[21] explore the effect of social ties among borrowers and lenders on the adverse selection or moral hazard problems by studying the Prosper.com data on on-line lending. An empirical study about the signal quality of video-on-demand service is undertaken by Nam et al (2010:697)[38]. The authors show that the probability that a household adopts the service increases (decreases) if the household has more neighbours with a good (bad) signal quality. The cited empirical works indicate that social ties might be important in markets with asymmetric information. However, a precise way in which social ties affect market price and quality remains unclear.

We explore theoretically if the social ties affect the market price and product quality in a market with information asymmetry. In particular, we question if the communication among consumers resolves the adverse selection problem. For this purpose we develop a monopoly model close to Akerlof[1] (1970)[3] supplemented for the communication among consumers. Quality is a random variable: consumers have a knowledge of its prior distribution, but not of its realisation. They share their experience about the product with the other consumers of the group. This information sharing affects buying decisions and consequently the profit. We study a Bayesian game where the firm observes its quality and sets the price. Then, the experimenter observes the price and makes a buying decision. Finally, the remaining consumers learn about the true quality from the experimenter and decide if to buy the product.

We obtain equilibria where the low- and high-quality producer types set a high price, while those of the moderate quality set a lower price. Such results are driven by the communication between the consumers. Causal observations suggest that there can exist many experience goods which are claimed to be of the high quality, but as consumers learn about their true quality, their sales shrink. On the other hand, there are products which are priced according to its quality. Some highly priced products, such as some medical treatments or technical equipment are introduced as being very useful, and thus they are highly priced, but latter, as the users share their experience with their friends and acquaintance, those products disappear from the market because of their low quality. Usually, the restaurants located in peripheral locations offer a good value for money because they relay on the word-of-mouth, while those located

[1]Quality is chosen by nature.

in the central squares relay on uninformed tourists, and thus sell low-quality goods at high prices.

We find several salient points. Firstly, the communication among the producers creates a signalling game because it induces some producer types to set a lower price than they would otherwise have done. It transforms the producer profit and the outcomes of the incomplete information game. As an implication, it creates a signalling equilibrium which would not exist without the communication.

We find that an increase in the extent of the consumer communication alleviates or completely resolves the adverse selection not because consumers have the information about the quality, but because the communication transforms the payoff structure, it consequently creates the signalling game, and it increases the maximum equilibrium price.

The seminal contributions to the literature about quality signalling by price is given by Milgrom and Roberts (1986)[36] who study introductory pricing and advertising in a monopoly. It is demonstrated that a high price alone, a low price alone, or a combination of price and dissipative advertising may signal the high quality. The costs are constant and independent of the quality. Bagwell and Riordan (1991)[6] study a monopoly with a dynamic pricing where some consumers are informed about the product quality and costs increase with the quality. Over time, the fraction of informed consumers increases and it becomes easier for the high-quality firm to signal its quality. This causes a decline in the price of the high-quality good. Their conclusion is that firms signal new products of high quality by prices that are above full-information profit-maximising prices.

The first one who shows that the word-of-mouth is a source of signalling is Kennedy (1994)[34]. There are two generations of consumers who communicate by the word-of-mouth. The high-quality firm type signals its quality buy charging a low price, so that it incurs loss in the first period. The second-period profits of the low- and high-quality types differ due to the word-of-mouth, so that the high-quality type can, and the low-quality type cannot, compensate the loss from the first period. This signalling mechanism is analogous to costly signalling in one round signalling games. Campbell (2010)[16] extends this for the producers who can target the consumers in the first round. The number and the position of the targeted consumers should ensure sufficiently informed consumers in the second round. In this way the costly signalling in the first round is compensated by the gains in the second period only in case of the high-quality type. This model opens the question if the targeting and accompanying direct information spreading can substitute signalling.

Our approach is different from these two papers because the price is set once and for all the consumers. Thus, we consider a one-period model (there is no introductory pricing). Instead of costly signalling of the high-quality types, the low-quality types in our model decrease the price and signal their quality in order to exploit the volume of trade. Thus, the low price signals the low quality in our model. Furthermore, the costs which increase with quality enable us to provide an answer about the effect of the consumer communication on the adverse selection problem which is not the case in these two papers.

Thus, our setting is different relative to the vast literature on word-of-mouth communication through social networks[2] because it is particularly designed to study the adverse selection problem. Furthermore, the effect of decay is ignored and the model does not allow for introductory pricing (or dissemination of the free samples) and heterogeneous preferences.

While Campbell (2010:24) states that ex ante consumer utility is zero, we find that the consumers may obtain a positive surplus. Owing to the unit demand and the monopoly market structure, consumer surplus without consumer groups should be zero in our model, but with the information sharing we find a positive expected consumer surplus. It increases with the group size because the expected cost of experimentation is shared with more members (information as a public good). However, the optimal group size is finite because an increase in the group size reduces the fraction of the states in which the information about quality is useful. At the same time, the maximum price increases with the group size. With regard to welfare implications, more intensive communication among consumers increases the profit of the high-quality producer type in both, Campbell(2009)[15] and our paper, while the effect on the low-quality producer types is ambiguous.

Furthermore, we show that if the consumers do not free-ride and if the experimentation is a repeated phenomenon, then the consumers can share the costs of the experimentation. In this case, even a lower group size is sufficient to resolve the adverse selection problem. However, the cost-sharing does not affect the emergence of the completely separating equilibrium (complete price differentiation). Finally, the government can completely resolve the adverse selection problem by increasing the minimum quality. This kind of intervention would be ineffective without the consumer communication.

Navarro (2006, 2008)[39][40] also studies a market with asymmetric information, but one in which the quality is chosen by a monopolist, so that this setting corresponds to moral hazard problems. Consumers have initial information about quality, and this in some part corresponds to expected quality in our model. We model consumer networks as groups (defined as completely connected components), and we find that an increase in the group size always raises the expected quality exchanged in the market. On the other hand, Navarro finds that a producer may choose a lower level of quality as consumers become more internally connected. This finding critically depends on the decay. The author shows that if the network is sufficiently dense relative to the marginal costs of quality, then the monopolist provides positive quality. This is in line with our finding that an increase in marginal costs of quality reduces the reservation price and that the group size increases it.

Our results are also consistent with Montgomery (1991)[37] who instead of introducing a social network of consumers, assumes that the same consumer interacts with the producers repeatedly. Note that in his model, consumers are employers and producers are workers. Owing to positive assorting, the wage of the producers who exchange the product through the social network increases. Connections between high-productive workers increase their opportunity costs of participation in the anonymous market. Thus, social networks create conditions for adverse selection to occur.

Janssen and Roy (2010)[32] study the equilibrium price and quality ex-

[2]for further references and more details on word-of-mouth communication see Galeotti and Goyal (2009)[24] and Campbell (2008)[15]

changed in the oligopoly market with asymmetric information. One of the findings is that a producer with low quality decreases his price in order to increase the likelihood of trade. This likelihood actually has the same effect as the group size so that we find that some low-quality producer types prefer to set a lower price than the reservation price in order to exploit trade with the whole consumer group.

1.2 Model

The product is sequentially exchanged between a monopolist (a single producer) and $n \in \mathbb{N}$ consumers with homogeneous preferences. The product's quality is θ where $\theta \sim \mathbb{U}[a,b]$, $f(\theta) = \frac{1}{b-a}$ is probability density function of θ and $0 \leq a < b$. Quality θ is randomly drawn and assigned to the producer and remains unchanged. Consumers buy the product one by one. The producer does observes the quality θ before exchange, while the consumer does not. The consumer observes θ after exchange.

Consumers are divided into groups of N members, $1 \leq N \leq n$. Members of groups share the information, so that when one randomly drawn consumer observes θ, everyone in the group receives that information immediately. We refer to this as consumer *experimentation*. The first consumer in the group to buy the product is called *the experimenter*. Consumers do not receive any other information. Demand is unit and utility function is money-metric: $u = \theta + M$. M is the consumption of other goods in monetary units. Both consumer and producer can refuse to exchange the product. Once the price is set, the producer cannot change it. The costs of production are $C = \theta c$ with $c \in (0,1)$. If the product is not exchanged, the costs are zero.

Since we are studying the effect of information on market outcomes, that information must be useful when it is received. If all consumers buy the product at the same moment, information about quality from the other group member is useless because it is received too late. Therefore, we introduce a sequential framework which is similar to the models where one buyer purchases a good several times. The interaction between the producer and the group can be repeated. Unchanged producer quality θ is crucial for the same reason.

Costs in our model can also be understood as opportunity costs. The fact that costs arise in production or exchange does not affect the results because the firm can predict the size of the trade and adjust production accordingly. The assumption that the producer cannot change the quality and associated cost functions fits the cases where the producer has a certain technology which cannot be changed (in the short term).

A group in our model can be understood as a component of the social network in which all members are internally connected and there is no decay in transmission of information. So, if two consumers are connected they must belong to the same group. Our definition is consistent with the definition in Osborne and Rubinstein (1994:255)[42], where group is defined as any subset of players in a cooperative game. We use this relatively simple social network structure because we believe that more complex network structures would not change the results essentially.

We develop a monopoly model close to Akerlof (1970)[3] where we add consumer groups. The assumptions that we have in common with Akerlof are uniform continuous distribution function of quality, money-metric utility, costs which monotonically increase with quality, absence of fixed costs, and hidden information (the producer knows the quality of the product, but the buyer does not). Apart from consumer groups, there are some more differences relative to Akerlof's paper. In Akerlof's model there are only two agents - a seller and a buyer - both with constant marginal utility, and each unit of good can have different quality. In our model there is one seller (monopolist) and many buyers, but with unit demand. Quality of the goods is constant, but randomly drawn from the uniform continuous distribution. Our costs can be interpreted as both production costs and as opportunity costs. We also assume sequential exchange, which does not affect the results. We set the lower bound of quality as $a \geq 0$, while in Akerlof (1970) it is $a = 0$, which turns out to be a non-trivial modification on which we will comment further in the text. Finally, the details of the game we specify might significantly influence the results. For example, it would be interesting to see the results of a game where the consumer proposes the price and the producer accepts or refuses it.

1.2.1 The Signalling Game

We define here a signalling game and some auxiliary functions used in the next subsection to study the properties of signalling equilibria. We specify a signalling game with three players: nature, producer (sender) and experimenter (receiver).[3] The players make choices in the following order: (1) nature chooses producer type $\theta \in \mathbf{\Theta}$, (2) based on the private information about his quality θ, producer chooses price (signal) $p \in [a, b]$, (3) experimenter observes p and makes a buying decision $x \in \{0, 1\}$. The spaces of mixed prices and buying decisions are $\mathcal{A}_1$ and $\mathcal{A}_2$ with elements α_1 and α_2. The profit is denoted by $\pi(\alpha_1, \alpha_2, \theta)$, and consumer surplus is denoted $S(\alpha_1, \alpha_2, \theta)$. It is common knowledge that the experimenter has a prior belief $f(\theta)$ about the producer type θ. $\sigma_1(\cdot|\theta)$ is a probability distribution over price p for monopolist type θ. $\sigma_2(\cdot|p)$ is a probability distribution over buying decisions x for each price p. $\mu(\cdot|p)$ is experimenter's posterior belief over $\mathbf{\Theta}$. We proceed considering only pure strategies p and x.

The profit is equal to the difference between price and production costs. If the price is higher than the quality, only the experimenter buys the product. Otherwise, the product is sold to the whole group and the profit per consumer in the group is $p - c\theta$.

$$\pi(p, x, \theta) = \begin{cases} x \frac{1}{N} (p - c\theta) & if \quad p > \theta \\ x (p - c\theta) & otherwise \end{cases}$$

[3] In order to simplify the analysis we consider an experimenter and account for the consumers non-experimenters in the payoff function only.

The experimenter surplus is a difference between the actual product quality and the price paid for the product multiplied by the buying decision.

$$S(p,x,\theta) = x\,(\theta - p)$$

We make some observations which will be useful later for the discussion about the signalling equilibria. The profit is increasing in price, but with the discontinuity at $p = \theta$. Hence, depending on the other parameters, some producer types may prefer setting low price and selling to the whole group, over setting the highest possible price. Which producer types do prefer setting a price lower than the quality? We get the answer by comparing two expressions in the profit function. Suppose that the experimenter is ready to buy at any price lower or equal to p. Then the lowest producer type who sets the price below p is such that $\frac{1}{N}(p - c\theta) = \theta - c\theta$. Let's denote his quality by k. It follows that:

$$k = \frac{p}{N - cN + c} \tag{1.1}$$

Note that k increases with p. We introduce additional functions $g(p)$ and $g_a(p)$ which are useful later for study the signalling equilibria. The function $g(p)$ is the experimenter's expected surplus after observing price p if he believes that: all the producer types with $\theta \in [k,p)$ set a price equal to their quality θ; the producer types with $\theta > \bar{\theta}$ do not trade the product[4]; and the remaining types set p.

$$g(p) \equiv \int_a^k \theta \frac{1}{\bar{\theta} - a - p + k} d\theta + \int_p^{\bar{\theta}} \theta \frac{1}{\bar{\theta} - a - p + k} d\theta - p \tag{1.2}$$

where $\bar{\theta} \equiv \frac{p}{c}$ if $p < bc$. Otherwise, $\bar{\theta} = b$. $\bar{p}$ is a maximum price at which $g(p)$ is non-negative. Sometimes through the paper we call it a reservation price.

$$\bar{p} \equiv \{p \in [a,b] : g(p) = 0\} \tag{1.3}$$

A proof that $g(p)$ is positive for prices below $\bar{p}$ and negative at prices above $\bar{p}$, together with a demonstration that $\bar{p}$ admits a unique solution, is provided in the proof of Proposition 4 in Appendix A.2 on page 26. The expressions for $\bar{p}$ are in Appendix A.3, for explicit on page 30, and for implicit on page 35. We also introduce the other function:

$$g_a(p) \equiv \int_a^{\bar{\theta}} \theta \frac{1}{\bar{\theta} - a} d\theta - p$$

$$\bar{p}_a \equiv \{p \in [a,b] | g_a(p) = 0\} \tag{1.4}$$

Function $g_a(p)$ is the experimenter's expected surplus at price p if he believes that all the producer types with quality $\theta \leq \bar{\theta}$ set p, and the rest set some other price. Function $g_a(p)$ is a special case of $g(p)$ because $g(p)$ and $g_a(p)$ coincide when $N = 1$. In this case our model is reduced to the Akerlof framework of adverse selection. One can easily verify this by plugging in $N = 1$ in $\bar{p}$ that $\bar{p} = \bar{p}_a = \frac{ac}{2c-1}$ if $c > \frac{a+b}{2b}$, and $\bar{p} = \bar{p}_a = \frac{a+b}{2}$ otherwise. Note that $\bar{p} > \bar{p}_a$

[4] We can assume that they set some price above p.

whenever $N > 1$ because $\frac{d\bar{p}}{dN} > 0$, which is verified at the end of Appendix A.3 on page 36.

Definition 1. *Producer type* price differentiates *if he sets* $p' = \theta < p_M$.

$p_M \equiv \max\{p : \alpha_2(x = 1|p) = 1\}$. p_M is a maximum price at which consumer buys the product.

If all the producer types with $\theta \in [k, p)$ price differentiate, then the expected consumer surplus at price p is $g(p)$. If no producer type $\theta \in [k, p)$ price differentiates, then the expected consumer surplus at price p is $g_a(p)$. Therefore, all the other functions between these two extremes have the value zero at some price between $\bar{p}_a$ and $\bar{p}$. From now on we define $k = \frac{p_M}{N - cN + c}$. You can perceive $g(p)$, $g_a(p)$ and k as some auxiliary functions.

1.3 Semi-Separating Equilibrium

We defined the signalling game previously and we study here its solution, the signalling equilibrium, for which the definition (adjusted for our need where only the pure strategies are considered) is provided at the beginning of Appendix A.1 on page 23. The essence of this equilibrium concept is that all the players have to play their best responses, the experimenter (receiver) has to establish his best responses based on the belief about the producer (sender) type, and this belief in the equilibrium must result from the Bayesian updating. The out-of-equilibrium belief can be set arbitrarily.

In this section, we find the signalling equilibria of the game, which is stated in Proposition 1. We obtain a multiplicity of equilibria and we check whether they are reasonable. We state that only some of them pass the equilibrium dominance test in Proposition 2. Essentially, by this proposition we select those equilibria where the group increases the maximum reservation price, which is crucial for the resolution of the adverse selection problem. Within this equilibria subset, we choose an equilibrium with the highest price and study its properties in the remainder of the section.

Proposition 1. *There exists a belief μ^* which supports a signalling equilibrium (perfect Bayesian equilibrium in pure strategies) if and only if $p^*_M \leq \bar{p}$.*

We claim in Proposition 1 that every signalling equilibrium must be such that the maximum price at which the experimenter buys in equilibrium must be lower or equal to $\bar{p}$. The proof of Proposition 1 is in Appendix A.1 on 24 which consists of Lemmas 1-3. We demonstrate in Lemma 1 that p^*_M cannot be higher than $\bar{p}$. The reason lies in the fact that even in the best case when all the consumers who can price differentiate do so, the maximum price at which the consumer surplus is non-negative is $\bar{p}$. The reader can refer to Appendix A.1 for details. On the other hand, we show in Lemma 2 that there exists a belief which supports a signalling equilibrium with $p^*_M < bc$. More precisely, if the experimenter observes some price between (not equal to) p^*_M and b, he believes that the type is the lowest one, a. So, the experimenter does not buy at such a price because his expected surplus is negative. Therefore, these beliefs can be set arbitrarily and can be a part of a signalling equilibrium if all the producer types who obtain negative profit set b. The beliefs at b should follow Bayesian updating. However, expected surplus at b is still negative. Finally, the consumer buys at any price $p^* \leq p^*_M$. This can be the best response as long as $p^*_M \leq \bar{p}$ so that $g(p^*) \geq 0$. Furthermore, in Lemma 3 we show that we can always find some beliefs such that $p^*_M \in [bc, \bar{p}]$ is supported as a signalling equilibrium. As long as the experimenter does not buy above p^*_M, no producer type sets $p^* > p^*_M$ so $\mu(\theta|p > p^*_M)$ can be set arbitrarily. This implies that the experimenter belief, that the quality is low at such a price, is admissible as a part of a signalling equilibrium.

After defining the range of possible signalling equilibria, we check how reasonable they are by applying the equilibrium dominance test.

Proposition 2. *The signalling equilibrium with $p^*_M < \bar{p}$ passes the equilibrium dominance test if and only if $\sigma^*_1(\theta|\theta) = 1$ for all $\theta' \in \left[\frac{p^*_M}{N-cN+c}, p^*_M\right)$.*

The prove of Proposition 2 is in Appendix A.1 on page 25 and it consists of Lemmas 4 and 5. While in Lemma 4 we show that the equilibrium where every producer with $k \leq \theta < p^*_M$ price differentiates passes the equilibrium dominance test, we demonstrate the contrary in Lemma 5, that a signalling equilibrium where not all producer types with $\theta \in \left[\frac{p^*_M}{N-cN+c}, p^*_M\right)$ price differentiate, cannot pass the test. The reason is that the maximum profit these producer types can obtain by pricing $p = \theta$ is higher than the profit they obtain by pricing p^*_M.

In Proposition 3 we show that the only signalling which passes the $D1$ criterion is the one where every producer sets the price equal to his quality.

Proposition 3. *Only the signalling equilibrium with $\bar{p} = b$ satisfies $D1$ criterion.*

The proof of Proposition 3 is on page 26. The D1 criterion eliminates the equilibrium multiplicity, but it also eliminates all the signalling equilibria in one part of the parameter space. Divine equilibrium is proposed as a less strong refinement than the $D1$ criterion because it only requires for types who fail the $D1$ criterion at price p that their probability does not increase when p is observed. This is a good argument for selecting the equilibrium with $p^*_M = \bar{p}$. We continue by analysing this equilibrium in order to provide the answer as to the effect of group size N on the market outcomes, and particularly on reservation price, price differentiation and adverse selection. We have already defined $\bar{p}$ such that $g(p) = 0$. As the behaviour of $g(p)$ is different depending on whether the price is above or below bc, we provide two equations for $\bar{p}$ in Appendix A.3 on page 30. In the remainder of the section we study the effect of the exogenous variables on the reservation price. Position of the reservation price relative to bc indicates if the adverse selection problem is resolved.

How do N, c, b and a affect $\bar{p}$, k and $\bar{\theta}$? We determined a threshold c^* such that when $c < c^*$, then adverse selection is completely resolved (Appendix A.3). However, we may have $c > c^*$, but $b > (N - cN + c)a$ and the signalling equilibrium exists such that every producer sets the price equal to the quality (complete price differentiation). The outcome for all the subsets of the parameters relevant for our model is presented in Table 1.1. The definition of c^* is in Appendix A.3, as well as a discussion and the details about the cases. The implicit form of the reservation price when adverse selection is completely resolved is:

$$\bar{p} = b + k - a - \sqrt{2}\sqrt{(b-a)(k-a)}. \tag{1.5}$$

<table>
<tr><th rowspan="2"></th><th rowspan="2">$N = 1$</th><th colspan="2">$N \geq 2$</th></tr>
<tr><th>$\frac{b}{a} \geq N - cN + c$</th><th>$\frac{b}{a} < N - cN + c$</th></tr>
<tr><td>$c \leq \frac{a+b}{2b}$</td><td>BASELINE</td><td rowspan="2">ADVERSE SELECTION COMPLETELY RESOLVED</td><td rowspan="3">COMPLETE PRICE DIFFERENTIATION</td></tr>
<tr><td>$\frac{a+b}{2b} < c < c^*$</td><td rowspan="2">AKERLOF CASE</td></tr>
<tr><td>$c^* < c$</td><td>ADVERSE SELECTION PARTLY RESOLVED</td></tr>
</table>

Table 1.1: Equilibrium where $p_M^* = \bar{p}$: There are five cases. If the marginal cost of quality $c < \frac{a+b}{2a}$, then there is no adverse selection. However, if additionally the consumer group is sufficiently large, so that $\frac{b}{a} < N - cN + c$, then every producer type sets price equal to his quality. If the marginal cost of quality is above the threshold, i.e. if $c > \frac{a+b}{2a}$, then we have the Akerlof case (adverse selection problem). Any communication between consumers alleviates the problem. If the consumer group size is sufficiently large, the problem is completely resolved.

The reservation price increases with the increase of group size N and decreases with the marginal cost of quality c, which is discussed in Appendix A.3.1 on page 36. The model consists actually of three functions: $\bar{p}(a, b, c, N) : \mathbb{R}^4 \to \mathbb{R}^1$, $k(\bar{p}) : \mathbb{R}^1 \to \mathbb{R}^1$ and $\bar{\theta}(\bar{p}) : \mathbb{R}^1 \to \mathbb{R}^1$. Critical points are: $a, b, k, \bar{p}$ and bc. $a < b$ and $k < \bar{p} < \frac{\bar{p}}{c}$. If $b \leq \bar{p}$, then $k \leq a$.

If the marginal cost of quality is small, such that $c < \frac{a+b}{2b}$, then there is no the adverse selection problem, even if there is no communication between consumers. However, the information sharing between consumers still makes some producer types set a lower price than $\bar{p}$ and this increases $\bar{p}$. The extreme case is when $N - cN + c > \frac{a}{b}$. Then, all the producer types price differentiate, so that finally even the producer type with the highest quality can extract the whole consumer surplus. On the other hand, the adverse selection is certainly present without the consumer groups when $c > \frac{a+b}{2b}$. However, any communication between consumers changes the payoffs of the producer, so that adverse selection is resolved if consumer group is large enough. If c is too high, above c^* and consumer group is not large enough to enable complete price differentiation, then adverse selection is present.

Finally, as long as group size N is large enough relative to the product variability $\frac{b}{a}$ and marginal cost of production c, every producer type prices his quality and this is the most robust signalling equilibrium because it satisfies even the $D1$ criterion, which is demonstrated in the proof of Proposition 3 on page 26. However, if the minimum quality is 0, such a signalling equilibrium does not exist no matter what communication takes place between consumers.

Figures 1.1 and 1.2 refer to the situations in which there is no complete

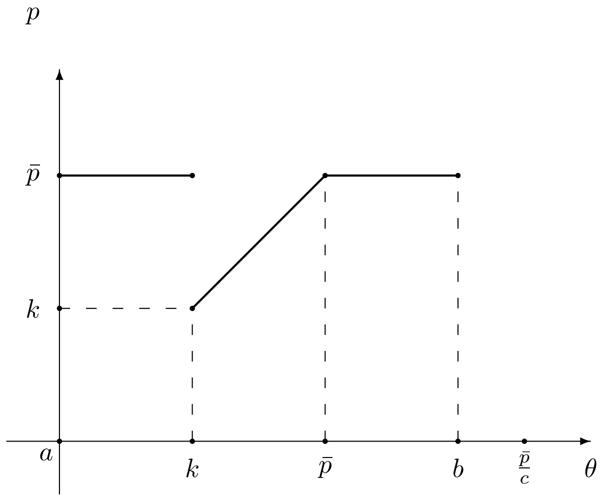


Figure 1.1: Complete absence of adverse selection problem

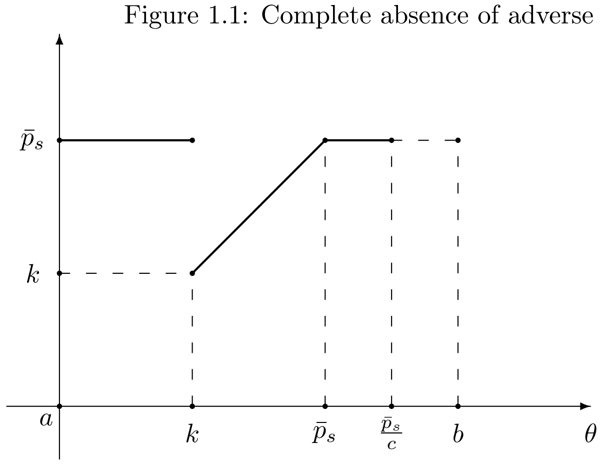


Figure 1.2: Adverse selection partly resolved

price differentiation. Figure 1.1 refers to the case when $bc \leq \bar{p}$ so the signalling equilibrium exists. In that case the monopolist type with $k < \theta < \bar{p}$ sets price equal to his quality $p = \theta$. The other monopolist types set $p = \bar{p}$. Figure 1.2 presents the case when adverse selection is not resolved. Reservation price still cannot cover the production costs of the highest producer type b. Figure 1.3 presents the case when group size is very large so that every producer type sets the price equal to his quality.

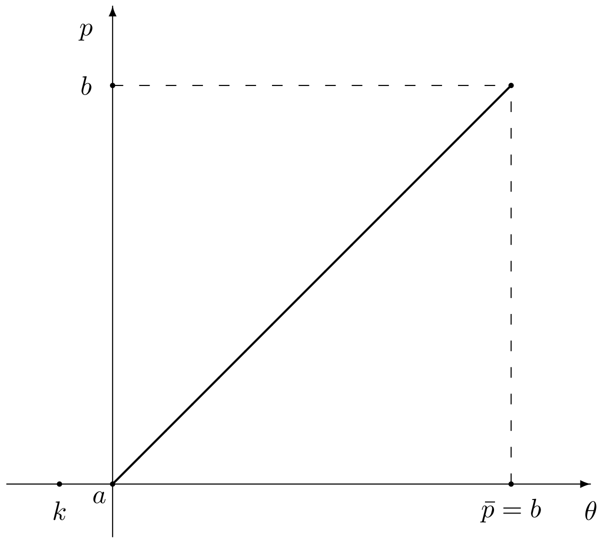


Figure 1.3: Complete price differentiation

1.4 Comparative Statics

The aim of this section is to show that the optimal consumer group is finite. The analysis is based on the maximum equilibrium price equal to $\bar{p}$. The expected consumer surplus[5] of the experimenter is always zero at $\bar{p}$. However, consumer surplus of the non-experimenter can be positive if the non-experimenter can extract the benefits of information about the actual product quality. This is so if the group size is at least two, but not so large that every producer type sets the price equal to his quality. We remind that the *experimenter's expected surplus*, once the group is formed, is represented by $g(p)$ on page 14. Consumers who are not experimenters are informed about the actual product quality before the purchase, so they do not buy the product if its quality is lower than the price. Thus, their surplus is always non-negative and in expectation (once the group is formed) has the following form:

$$\mathbb{E}_N[S|\bar{p}] = \int_p^{\bar{\theta}} (\theta - p) \frac{1}{\bar{\theta} - a - p + k} d\theta$$

If we assume that the experimenter is a randomly drawn consumer in the group, then *the expected consumer surplus* $\mathbb{E}[S|\bar{p}]$ is the sum of the expected surplus of the experimenter multiplied by the probability to be the experimenter $\left(\frac{1}{N}\right)$, and the expected surplus of the non-experimenter multiplied by the proba-

[5]We present here only the analysis of the expected consumer surplus. The findings about profit are in the Appendix A.4.

bility of being a non-experimenter $\left(\frac{N-1}{N}\right)$. After short algebraic manipulations, we obtain the following expression:

$$\mathbb{E}[S|\bar{p}] = h(p) = \frac{1}{N}\int_a^k (\theta-\bar{p})\frac{1}{\bar{\theta}-a-\bar{p}+k}d\theta + \int_{\bar{p}}^{\bar{\theta}} (\theta-\bar{p})\frac{1}{\bar{\theta}-a-\bar{p}+k}d\theta \quad (1.6)$$

The highest quality exchanged $\bar{\theta}$ is equal to b if there is no adverse selection. Otherwise, $\bar{\theta} = \frac{\bar{p}}{c}$.

From equation (A.1) on page 26, and from observation that in the equilibrium it holds[6] that $(k-a)(2p-k-a) = \left(\bar{\theta}-p\right)^2$, it follows that the expected consumer surplus is:

$$\mathbb{E}[S|\bar{p}] = \frac{N-1}{N}A, \quad (1.7)$$

where

$$A = \frac{(k-a)(2\bar{p}-k-a)}{2(\bar{\theta}-a-\bar{p}+k)} = \frac{(\bar{\theta}-\bar{p})^2}{2(\bar{\theta}-a-\bar{p}+k)}. \quad (1.8)$$

We consider the case when there is no the adverse selection problem. The expected consumer surplus is zero when there is no communication between consumers, or if the consumer group is above the threshold $\bar{N}$. When there is no communication between consumers, that is, when $N = 1$, then the reservation price is $\bar{p} = \frac{b+a}{2}$. Thus, $A = \frac{\left(b-\frac{b+a}{2}\right)^2}{b-a} > 0$ at $N = 1$. On the other hand, $\frac{N-1}{N} = 0$ at $N = 1$. Hence, $\mathbb{E}[S|\bar{p}] = 0$ at $N = 1$.

The expected consumer surplus again approaches zero when $N \to \bar{N}$, where $\bar{N}$ denotes the minimum group size at which there is complete price differentiation. By plugging equation (1.5) on page 16 into denominator of A we obtain that

$$A = \frac{(2\bar{p}-k-a)\sqrt{k-a}}{2\sqrt{2}\sqrt{b-a}}.$$

We know that $\lim_{N\to\bar{N}} k = a$ (which is shortly discussed on page 28). Thus, $\lim_{N\to\bar{N}} A = \frac{2\bar{p}\sqrt{0}}{\sqrt{2}\sqrt{b-a}} = 0$ because at such a group size every producer type sets a price equal to his quality. It is evident that $A > 0$ for $1 < N < \bar{N}$. We conclude that the expected consumer surplus is zero, positive, and again zero. The simulations also confirm this and show that $\mathbb{E}[S|\bar{p}]$ has a stable form[7]. It is positive and concave with a maximum at the the group size $N \in \left(1, \bar{N}\right)$. Additional evidence is in Appendix A.5 on page 37.

The expected consumer surplus is presented as a function of a group size in Figure 1.4. It is zero if there is no communication between consumers, it increases with the group size, and it reaches its maximum when the benefits of sharing the information costs with the additional member are surpassed by a decrease in the fraction of producer types with high quality (the quality above $\bar{p}$). If all the producer types set the price equal to the quality, there is no any surplus left to the consumers, so the expected consumer surplus becomes zero again, as when there is no communication at all.

[6]This condition is discussed in the proof of Proposition 4, which starts on page 26.

[7]We ran around 100 iterations.

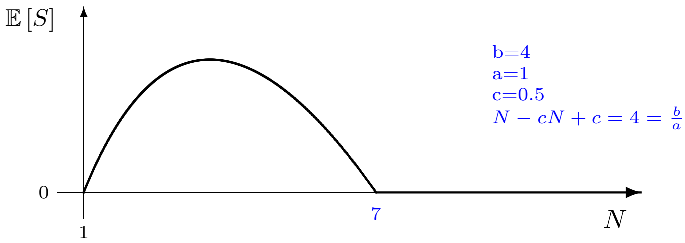


Figure 1.4: The expected consumer surplus: An additional member decreases the costs of experimentation. The decrease is smaller as the group is larger. The fraction of the producer types who signal their quality through price increases with N (so the consumer less often needs his group to conclude about quality). The first effect is stronger than the second effect for small group size. The opposite holds at large group size.

1.4.1 Sharing the Costs of Experimentation

We extend the existing framework by assuming that the eventual negative payoff of the first consumer, called *cost of experimentation*, is shared with other members equally. Sharing costs of experimentation is equivalent to the assumption that the consumers one by one buy the product, in a random manner without replacement. As a consumer who is not drawn first can refuse to buy the product if its quality is lower than the price ($\theta < p$), we can multiply the probability of the consumer's negative expected payoff by $\frac{1}{N}$, which is the probability of being the first one in the group to buy the product. It is reasonable to assume that there is no free-riding if the group faces this situation regularly. Then, $g(p)$ is replaced by $h(p)$ on page 20, which behaves in the same way as $g(p)$, but it is zero at a higher price.

$$\bar{p}_h = \{p \in [a, b] : h(p) = 0\} \tag{1.9}$$

If $\bar{p}_H > bc$, then the solution to (1.9) is:

$$\bar{p}_h = \frac{Nb - a - \sqrt{N(b-a)^2 - (n-1)^2 (Nb^2 - a^2)}}{N - 1 + (n-1)^2}$$

$$\bar{p}_h = b + \frac{(k-a) - \sqrt{(k-a)N[2b - (k+a)] + (k-a)^2}}{N}$$

in implicit form. In the last expression the reservation price $\bar{p}$ is expressed as a function of k in page 35, where k depends on p.

If $\bar{p}_H < bc$, then[8]:

$$\bar{p} = \frac{a^2 c^2}{ac^2 - [N(1-c)^2 + c^2(1-e)^2]}.$$

$\bar{p}_h \geq \bar{p}$ because by sharing the costs of experimentation with the other members of the group, the individual expected costs of experimentation decrease. Therefore, the monopolist can charge a higher price and sell the product. If we express the normalised expected costs in this case by L_h, we could say that L_h

[8] $e \equiv \frac{2d-1}{d^2}$ and $d = N - cN + c$.

increases more slowly than L with the increase of group size because one can easily verify that

$$\frac{\partial L_h}{\partial p} = \frac{1}{N}\frac{\partial L}{\partial p}.$$

Thus, although the collaboration with the other members of the group in general is supposed to be beneficial, in this market framework it reduces the consumer market power, so that the positive expected consumer surplus vanishes.

1.5 Concluding Remarks

We show that communication between consumers increases the price and the expected quality of the products exchanged in the market, and enables signalling of quality through price. Furthermore, the communication enables high-quality producers to increase their profit, while the profit of those with low quality may decrease. Finally, combined with minimum quality laws, the consumer groups make all the producers signal their quality through price. As a further research, it would be interesting to: examine the case when quality is a choice variable of producer (moral hazard case); to check the results with different quality distributions and cost functions; and to obtain the study the oligopolistic case. Also, some comparison with the other quality revealing mechanisms would be useful in order give a sound policy advice.

A

Appendix for Chapter 1

A.1 Signalling Equilibrium

This section of Appendix is devoted to the first part of Section 1.3, from page 15. It consists of the definition of the signalling equilibrium, and of the proofs of Propositions 1, 2 and 3.

We present here a definition of the signalling equilibrium which is adapted from Fudenberg and Tirole (1991:325)[22].

Definition 2. *(Signalling Equilibrium) $\sigma_1{}^*$, $\sigma_2{}^*$ and $\mu^*(p)$ constitute a signalling equilibrium if:*

(i)

$$\forall \theta,\ \ \sigma_1^*(\cdot|\theta) \in \arg\max_p \pi(p, \sigma_2{}^*, \theta)$$

(ii)

$$\forall p,\ \ \sigma_2^*(\cdot|p) \in \arg\max_x \sum_\theta \mu\left(\theta|p\right) S(p, x, \theta)$$

and (iii)

$$\mu(\theta|p) = \frac{f(\theta)\sigma_1^*(p|\theta)}{\sum_{\theta' \in \mathbf{\Theta}} p(\theta)\sigma_1^*(p|\theta)}\ \ if$$

$$\sum_{\theta' \in \mathbf{\Theta}} f(\theta)\sigma_1^*(p|\theta) > 0,$$

and $\mu(a_1)$ is any probability distribution of θ if

$$\sum_{\theta' \in \mathbf{\Theta}} f(\theta)\sigma_1^*(p|\theta) = 0.$$

Furthermore, we also present a definition of the equilibrium dominance:

Definition 3. *(Equilibrium Dominance) Price p can be eliminated for type θ by equilibrium dominance if*

$$\pi^*(\theta) > \max_x \pi(p, x, \theta)$$

Based on these definitions, and the definition of $g(p)$ and $g_a(p)$, we study the signalling equilibria and their refinements. Proposition 1 is on page 15.

Proof of Proposition 1.

$$p_M^* = \max\{p : \sigma_2^*(x = 1|p)\}$$

Lemma 1. *There does not exist* $p_M^* > \bar{p}$.

Proof of Lemma 1. *$\bar{p}$ is the price at which the expected experimenter's surplus is 0 and all the producer types with quality $\theta \in \left[\frac{\bar{p}}{N-cN+c}, \bar{p}\right)$ price differentiate (set $p = \theta$). If the experimenter buys at some price $p_M > \bar{p}$, then the best response for all the producer types with $\theta \in \left[a, \frac{p_M}{N-cN+c}\right]$ is to set the price p_M. But, then, the best response at p_M based on Bayesian updating must be zero*

$$0 = \arg\max_x \sum_\theta \mu(\theta|p_M) S(p_M, x, \theta)$$

because

$$\sum_\theta \mu(\theta|p_M) S(p_M, 1, \theta) \leq g(p_M) < 0 = \sum_\theta \mu(\theta|p_M) S(p_M, 0, \theta).$$

Therefore, there is no an equilibrium belief consistent with $p_M^* > \bar{p}$.

Lemma 2. *There $\exists$ $p_M^* < bc$ such that $p_M^* \leq \bar{p}$.*

Proof of Lemma 2. *Consider some producer type θ'such that $p_M^* < \theta' c \leq bc$. It is easy to verify that his profit is higher by setting a price above p_M^* (denote this price by p') than the profit obtained by setting an equilibrium price* [1]. *$\pi(p', 0, \theta') > \pi(p_M^*, 1, \theta') \geq \pi(p^*, 1, \theta')$ because $0 > p_M^* - \theta' c \geq p^* - \theta' c$.*

Consider a strategy profile such that the experimenter buys at any price lower or equal to p_M^*, and does not buy at prices above p_M^*. Assume that all the producer types with $\theta c > p_M^*$ play $\sigma^*(b|\theta c > p_M^*) = 1$ as a best response. Then, the experimenter belief at price b should be based on the Bayesian updating. That is, $\mu(\theta|b) = \frac{1}{b - \frac{p_M^*}{c}}$ $\forall \theta \in \left(\frac{p_M^*}{c}, b\right]$. If so, then the expected experimenter surplus if not buying is zero, and if buying is $\sum_\theta \mu(\theta|b) S(b, 1, \theta) < 0$. For the prices between p_M^* and b the beliefs can be set arbitrarily, e.g. $\mu(a|p) = 1$. Thus, the best response of the consumer is not to buy at $p \in (p_M^*, b)$. Hence, these beliefs and strategy profiles constitute a signalling equilibrium. We conclude that we can find the beliefs which support the the signalling equilibrium where $p_M^* < bc$.

Lemma 3. *There $\exists$ a belief such that $p_M^* \in [bc, \bar{p}]$.*

Proof of Lemma 3. *Suppose consumer posterior beliefs $\mu(\theta|p)$ are such that the experimenter's best response strategy is to buy at any price $p \leq p_M$ and not to buy at any price $p > p_M$. It follows that there is no producer type who set $p > p_M$. Therefore, the consumer surplus is not-negative at p_M. (See the function $g(p)$)(in other words all the producer types with $\theta \in \left[\frac{p_M}{N-cN+c}, p_M\right)$ are going to set $p = \theta$). Hence, this belief profile μ constitutes a signalling equilibrium.*

[1] Note that $x(p') = 0$ because $p' > p_M$.

It follows from Lemmas 1, 2 and 3 that every signalling equilibrium must be such that $p_M^* \leq \bar{p}$. □

We proceed by testing which signalling equilibria pass the equilibrium dominance test. If some out-of-equilibrium price is equilibrium dominated for some type, then all the strategies which place a positive probability on this type are eliminated. It follows that the experimenter should place belief $\mu(\theta|p) = 0$ for producer type θ which does not pass the equilibrium dominance test. Proposition 16 is on page 16.

Proof of Proposition 2.

Lemma 4. *Any signalling equilibrium where $\theta \in [a, k) \cup [p_M^*, b]$ price p_M^* and $\theta \in [k, p_M^*)$ price θ satisfies the equilibrium dominance test.*

Proof of Lemma 4. $\forall\ p \in [a, k)\ \pi^*(\theta) > \max_x \pi(p, x, \theta)$. *Thus, arbitrary beliefs can be placed over types after observing these prices. Any price $p' \in [k, p_M^*)$ is equilibrium dominated for the producer types $\theta \neq p'$ because $p' - c\theta < p^*(\theta) - c\theta\ \forall \theta > p'$. On the other hand every type $\theta < p'$ can get at least $\frac{(p^* - c\theta)}{N}$ which is more than $\frac{(p' - c\theta)}{N}$. Therefore, $\mu(\theta|\theta) = 1$. Finally, if the experimenter buys at $p'' > p_M^*$, then this is equilibrium dominated only for $\theta \in \left[\frac{p''}{N - cN + c}, \frac{Np_M^*}{c(N-1)}\right)$. The experimenter can still place such beliefs for which the best response at p'' is $x = 0$. Then, the best response for all the producer types is not to set such a price.*

The problem in this Lemma is that both actions $x = 1$ and $x = 0$ bring the same utility at prices $p \in [k, p_M^*]$, so we can treat them both as maxima and minima. However, consider some equilibrium where there is a small fraction in the above set of prices $(p', p'']$ at which the experimenter does not buy. Then, his strict best response is to buy at p'. Now, go back to the equilibrium defined in 4. Consider some price p''' such that $k < p''' < p_M^*$. Set the buying decision which maximises the experimenter surplus as $x = 0$. By the defined equilibrium in Lemma 4, the experimenter only considers the $\theta = p'''$, so there is no type to eliminate at p'''. Furthermore, p''' is a part of the producer equilibrium strategies so we should not treat this as an out-of-equilibrium price. Actually, any equilibria with fractions of prices in the range from k to p_M such that the experimenter does not buy, cannot pass the equilibrium domination test, unless these fractions are atomless. But, if they are atomless, then, such an equilibrium has the same properties as the one in Lemma 4.

Lemma 5. *No signalling equilibrium with $\mu(\theta = p|p) = 0$ for $p \in \left[\frac{p_M^*}{N - cN + c}, p_M^*\right)$ satisfies the equilibrium domination test.*

Proof of Lemma 5. *In case the experimenter does not buy at some price $p' \in \left[\frac{p_M^*}{N - cN + c}, p_M^*\right)$ his beliefs must be that $\mu(\theta < p'|p') > 0$. But for these types p_M^* certainly dominates p'. Then, it follows that the experimenter should buy at such a price. If he buys, then the the producer with $\theta = p'$ charges this price and the posterior beliefs for this type should be positive, $\mu(\theta = p|p) > 0$.*

This completes the proof. □

Proposition 3 is on page 16.

Proof of Proposition 3. Consider two types such that $p_M^* < \theta' < \theta''$ and some price $p' > p_M^*$. Let $\alpha_2(p', \theta)$ be the minimum probability that the experimenter buys such that a type θ is indifferent between pricing p' or p_M^*. It is evident that $\alpha_2(p', \theta'') < \alpha_2(p', \theta')$ (because $c\theta' < c\theta''$), implying that there is a larger set of the experimenter's mixed best responses for which p' is strictly preferred for θ'' than for θ'. This implies that the experimenter should place probability $\mu(\theta'|p') = 0$. But, then the beliefs which remain are such that the experimenter should buy at such a price. Thus, any equilibrium where $\bar{p} < b$ fails the $D1$ criterion. □

A.2 Auxiliary function $g(p)$

In this section we study the behaviour of $g(p)$, and the effects of N and c on it. We demonstrate in Proposition 4 that the reservation price $\bar{p}$ is unique. We use Remarks 2-6 to prove the proposition. Normalised expected losses L continuously increase with the reservation price, while normalised expected benefits B firstly increase and then decrease. At the reservation price the expected losses are equal to expected benefits. There is a single price at which this holds because the difference between B and L is positive at p_2 but it is continuously decreasing after cb, and it is negative or zero at b. We use Lemmas 7 and 8 to prove the proposition. We show by Lemma 7 that if expected consumer surplus is zero at some price lower than bc, then it cannot be again zero at some price equal to or higher than bc . The reverse is demonstrated in Lemma 8. As both normalised losses and benefits have a parabolic shape for the sequence of reservation prices $[a, bc]$ and $[bc, b]$, normalised benefits and losses can be equal only once within the sequence.

Proposition 4. *There exists a unique $p \in [p_2, b]$ such that $g(p) = 0$.*

Proof of Proposition 4.

Lemma 6. *$a < (\geq)k$ iff $\bar{p} < (=)b$.*

Proof of Lemma 6. *If $a < k$ and $\bar{p} = b$, then by it follows by (1.2) that $g(\bar{p}) < 0$ but this is a contradiction.*

If $a \geq k$ and $\bar{p} < b$, then it follows from (1.2) that $g(\bar{p}) > 0$, but this is a contradiction.

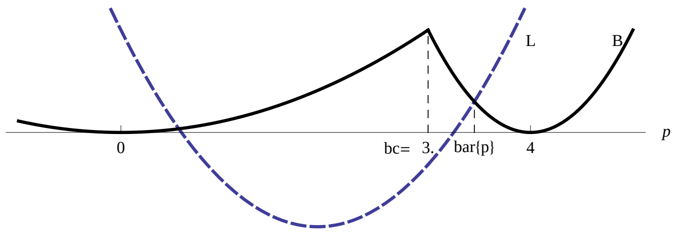
Equation (1.2) on page 14 can be written as:

$$g(p) = -\frac{(k-a)(2p-k-a)}{2(\bar{\theta}-a-p+k)} + \frac{(\bar{\theta}-p)^2}{2(\bar{\theta}-a-p+k)} \tag{A.1}$$

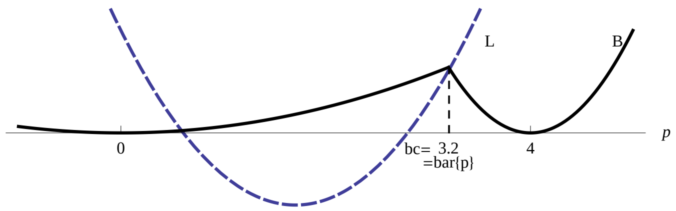
The first part of equation $(A.1)$ represents expected losses due the product testing. It contains consumer surpluses from all the states when the quality is lower than the reservation price. These losses multiplied by the denominator are:

$$L(p) = (k-a)(2p-k-a) \tag{A.2}$$

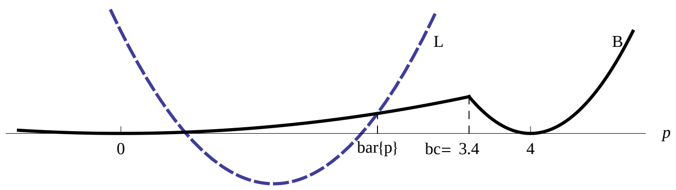
The second part of $(A.1)$ represents expected benefits due to the product testing. It contains consumer surpluses from all the states when the quality is



(a) c=0.75: There is no adverse selection problem, so the reservation price $\bar{p}$ is above bc.



(b) c=0.80: This is a threshold case, such that $\bar{p} = bc$.



(c) c=0.85: The costs are such that the consumer communication does not resolve the adverse selection problem completely, so that $\bar{p} < bc$.

Figure A.1: Normalised benefits benefits B and losses L. The consumer group size N is 10, the minimum quality a is 1, the maximum quality b is 4. The marginal cost c varies from 0.75 to 0.85. The reservation prices $\bar{p}$ is such that $B = L$. We are interested in prices between 1 and 4 (i.e. a and b). B is actually composed of two parabolas with vertices at $p = 0$ and $p = 4$ which intersect at $p = bc$. L is also a parabola. With an increase in c L increases, and the first parabola of B decreases. For this reason the reservation price decreases with c.

higher than the reservation price. Normalised benefits are:

$$B(p) = (\bar{\theta} - p)^2 = \begin{cases} (\frac{p}{c} - p)^2 & if \quad p \leq bc \\ (b - p)^2 & \quad bc \leq p \end{cases} \tag{A.3}$$

Normalised benefits B first increase and then decrease with an increase in the reservation price.

Remark 1. *$g(p) = 0$ in equation (A.1) when $L = B$.*

L is a parabola with $L = 0$ at two reservation prices: $p_1 = \frac{a(N-cN+c)}{2(N-cN+c)-1}$ and $p_2 = a(N - cN + c)$, where $p_1 < a < p_2$. If $p' < p_2$, then $k(p') < a$, so $g(p') > 0$. Therefore, we can restrict the analysis to prices $p \in [p_2, b]$.

Remarks 2-6 emphasise a few properties of L and B which are important for the uniqueness of the reservation price $\bar{p}$.

Remark 2. *For all p such that $[p_2, b]$ the following holds: $L \geq 0$, $\frac{\partial L}{\partial p} > 0$ and $\frac{\partial^2 L}{\partial p^2} > 0$.*

Remark 3. *For all p such that $[p_2, bc]$ the following holds: $B > 0$, $\frac{\partial B}{\partial p} > 0$ and $\frac{\partial^2 B}{\partial p^2} > 0$.*

Remark 4. *For all p such that $[bc, b]$ the following holds: $B \geq 0$, $\frac{\partial B}{\partial p} < 0$ and $\frac{\partial^2 B}{\partial p^2} > 0$.*

Remark 5. *$B > 0$ for all $p \in [p_2, b)$, and $B = 0$ at $p = b$.*

Remark 6. *$B > L = 0$ at $p = p_2$.*

Lemma 7. *If there exists $p \in \{p \in [p_2, bc] : L = B\} \Rightarrow p \in \{p \in (bc, b] : L = B\} = \emptyset$.*

Proof of Lemma 7. *If $\exists\ p = \{p \in [p_2, bc] : L = B\}$, then it follows from Remarks (2,3,5 and 6) that for $p \in [a, b]$*

$$\frac{\partial L}{\partial p} > \frac{\partial B}{\partial p}. \tag{A.4}$$

It follows from (A.4) and Remark 4 that $L > B\ \forall\ p \in (bc, b]$.

Lemma 8. *If $\exists\ p = \{p \in (bc, b] : L = B\} \Rightarrow p \in \{p \in [p_2, bc] : L = B\} = \emptyset$.*

Proof of Lemma 8. *If $\exists\ p = \{p \in (bc, b] : L = B\}$, then it follows from Remark 2 and 4 that $L < B$ at $p = bc$. But, then, by Remarks 2,3 and 6 it follows that $L < B\ \forall\ p \in [p_2, bc)$.*

It follows from Lemmas 7 and 8 that there exists at most one $p \in [p_2, b]$ at which $L = B$.

□

We proceed the section by studying the effect of the consumer group N and the marginal cost of quality c on the normalised costs and benefits.

A.2.1 *The effect of* N *on* L

L is as in equation (A.2) on page 26. The effect of N is composed of the direct effect, and the indirect effect through k.

$$\frac{dL}{dN} = \frac{\partial L}{\partial N} + \frac{\partial L}{\partial p} \cdot \frac{\partial p}{\partial N}$$

The direct effect is negative:

$$\begin{aligned}\frac{\partial L}{\partial N} &= \frac{-p(1-c)}{(N-cN+c)^2}\left(2p - \frac{p}{N-cN+c} - a\right) + \left(\frac{p}{N-cN+c} - a\right)\frac{p(1-c)}{(N-cN+c)^2} \\ &= \frac{-p(1-c)}{(N-cN+c)^2}\left(2p + 2\frac{p}{(N-cN+c)}\right) < 0\end{aligned}$$

The indirect effect is negative as well:

$$\begin{aligned}\frac{\partial L}{\partial p} &= \frac{1}{N-cN+c}(2p-k-a) + (2 - \frac{1}{N-cN+c})(k-a) \\ &= \frac{2}{N-cN+c}(p-k) + 2(k-a) > 0\end{aligned}$$

$\frac{\partial L}{\partial p} > 0$ when $p > k$ and $k > 0$, which corresponds to the parameter space of our interest.

$$\begin{aligned}\frac{\partial^2 L}{\partial p \partial N} &= \frac{-2(1-c)}{(N-cN+c)^2}(p-k) + \frac{2}{N-cN+c} \cdot \frac{p(1-c)}{(N-cN+c)^2} - \frac{2(1-c)p}{(N-cN+c)^2} \\ &= -\frac{2(1-c)}{(N-cN+c)^2}(p-k) - \frac{2(1-c)p}{(N-cN+c)^2}\left(1 - \frac{1}{N-cN+c}\right) < 0\end{aligned}$$

Such an impact of the consumer group size N on L is also visible from the effect of N on the null-points and the maximum of L:

$$\begin{aligned}\frac{\partial p_1}{\partial N} &= \frac{a(1-c)[2(N-cN+c)-1] - a(N-cN+c)2(1-c)}{[2(N-cN+c)-1]^2} \\ &= \frac{-a(1-c)}{[2(N-cN+c)-1]^2} < 0 \\ \frac{\partial p_0}{\partial N} &= \frac{2a(N-cN+c)(1-c)}{2(N-cN+c)-1} > 0 \\ \frac{\partial p_2}{\partial N} &= a(1-c) > 0\end{aligned}$$

A.2.2 *Effect of* c *on* L

$$\begin{aligned}\frac{\partial L}{\partial c} &= \frac{\partial k}{\partial c}(2p-k-a) - \frac{\partial k}{\partial c}(k-a) \\ &= \frac{\partial k}{\partial c}2(p-k) \\ \frac{\partial k}{\partial c} &= \frac{p(N-1)}{(N-cN+c)^2} > 0 \\ \frac{\partial L}{\partial c} &> 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p_0}{\partial c} &= \frac{-2a(N-cN+c)^2(N-1) + 2a(N-cN+c)(N-1)}{[2(N-cN+c)-1]^2} \\ &= \frac{2a(N-cN+c)(N-1)}{[2(N-cN+c)-1]^2}(1-N+cN-c) < 0 \\ \frac{\partial p_1}{\partial c} &= -\frac{a}{\left(2-\frac{1}{N-cN+c}\right)^2}\frac{1}{(N-cN+c)^2}(-)(N-1) > 0 \\ \frac{\partial p_2}{\partial c} &= -(N-1) < 0\end{aligned}$$

A.3 The Analysis of $\bar{p}$

After elaborating $g(p)$ in the previous section, we find the explicit value of $\bar{p}$. We are able to distinguish five cases in the market, according to parameters' values. We conclude the section by studying the effects of parameters on the reservation price $\bar{p}$.

A.3.1 Reservation price $\bar{p}$

The reservation price $\bar{p}$ is a solution to $g(p) = 0$ on page 14. Then (if $\bar{p} > bc$), it follows from equation (A.1) on page 26 that:

$$(b-p)^2 - (k-a)(2p-k-a) = 0$$

We denote $k = np$ and $n \equiv \frac{1}{N-cN+c}$ and rewrite the above expression as

$$(1-n)^2p^2 - 2(b-a)p + b^2 - a^2 = 0.$$

It follows that our reservation price must be:

$$\bar{p} = \frac{b-a-\sqrt{(b-a)^2-(1-n)^2(b^2-a^2)}}{(n-1)^2}$$

We rewrite it as:

$$\bar{p} = \frac{b-a-\sqrt{(b-a)[-2a-(b+a)(n^2-2n)]}}{(n-1)^2}$$

We show that $\bar{p}$ is a real number for parameter values for which there is no complete price differentiation ($\frac{b}{a} < N - cN + c$). We look at the part of the expression under the square root.

It follows from $\frac{b}{a} > N - cN + c$ that:

$$\frac{a}{b} < n \tag{A.5}$$

The expression under the square root is positive if:

$$(b + a)(2n - n^2) - 2a > 0 \tag{A.6}$$

It follows from equation (A.6)

$$\begin{aligned}(1 + \frac{a}{b})(2n - n^2) - 2\frac{a}{b} &> 0\\ 2n - n^2 + \frac{a}{b}2n - \frac{a}{b}n^2 &\geq 2\frac{a}{b}\\ 2\left(n - \frac{a}{b}\right) - n\left(n - \frac{a}{b}\right) + \frac{a}{b}n\,(1 - n) &\geq 0\\ (2 - n)\left(n - \frac{a}{b}\right) + \frac{a}{b}n\,(1 - n) &\geq 0.\end{aligned}$$

This inequality holds because $0 \leq n \leq 1$ and (A.5) holds. Hence, when there is no complete price differentiation, reservation price $\bar{p}$ is a real number.

If $\bar{p} < bc$, then it follows from the expression (A.1) that the reservation price is:

$$\bar{p} = \frac{ac^2 \pm ac\sqrt{(1 - c)^2 + c^2(1 - e)}}{(1 - c)^2 + c^2(1 - e)}$$

where $d = N - cN + c$ and $e = \frac{2d-1}{d^2}$. The solution relevant for our model is $\bar{p} \geq a$, so we choose

$$\bar{p} = \frac{ac^2 + ac\sqrt{(1 - c)^2 + c^2(1 - e)}}{(1 - c)^2 + c^2(1 - e)}$$

which can be writen as:

$$\bar{p} = \frac{ac}{c - \sqrt{(1 - c)^2 + c^2(1 - e)}}. \tag{A.7}$$

A.3.2 *Threshold cost* c^*

We want to see how $\bar{\theta}$ changes with N and c, and particularly, for which N and c all the producer types exchange the product.

We ran a few simulations of $\bar{p} - bc$ in order to see for which values of coefficients the adverse selection is present. If we keep N, a and b constant and we increase c, adverse selection appears when c passes some threshold value c^*. However, we notice that this threshold value c^* increases with N. Hence, there are cases when the adverse selection problem would be present when $N = 1$. However, if the group size N is sufficiently large, the adverse selection problem disappears completely. We want to find the threshold value c^* such that the adverse selection is completely resolved if $c < c^*$. c^* is one of the solutions of equation $\bar{p} = bc$. We find that it is a real number if there is no complete price differentiation.

The threshold c^* is c for which $B = L$, that is $p = bc$ which we write more formally in the following definition.

Definition 4. *Let $c^* = \{c \in (0,1) : B = L \text{ at } p = bc\}$.*

$$c^* = \frac{b - a \pm \sqrt{(b-a)\left[e\left(b+a\right) - 2a\right]}}{b\left(1-e\right)}$$

where $d \equiv N - cN + c$, $e \equiv \frac{2d-1}{d^2}$ and $v \equiv \frac{b}{a}$. Thus, $L = \frac{1}{N}ep^2 - 2ap + a^2$. Please, note that if $v > d$, then $0 < e < 1$, $e > \frac{1}{v}$ and $d > \frac{1}{e}$. Intermediate steps, a demonstration that c^* is a real number when there is no complete price differentiation, and that $0 < c^* < 1$ are after the following proposition.

Proposition 5. $c < (>) c^*$ *if and only if* $\bar{p} > (<) bc$.

Proof of Proposition 5.

Lemma 9. $c < (>) c^*$ *if and only if* $B > (<) L$ *at* $p = bc$.

Proof of Lemma 9.

$$\frac{dB}{dc} = \begin{cases} -\frac{1}{c^2} < 0 & if \quad p \leq bc \\ 0 & if \quad bc \leq p \end{cases} \tag{A.8}$$

$$\frac{dL}{dc} = \frac{2p(N-1)}{d^2}\left(p - k\right) > 0 \tag{A.9}$$

Remark 7. $c < (>) c^* \;\Leftrightarrow\; bc < (>) bc^*$.

It follows from equations (A.8) and (A.9) that $c < (>) c^* \;\Leftrightarrow\; B > (<) L$ *at* $p = bc^*$. *It follows from Remarks 2-7 on page 28 that* $B > (<) L$ *at* $p = bc < (>) bc^*$.

Lemma 10. $B > (<) L$ *at* $p = bc$ *if and only if* $L = B$ *at* $p > (<) bc$.

Proof of Lemma 10. *It follows from Remarks 2-6 on page 28.*

It follows from Lemmas 9 and 10 that $c < (>) c^* \;\Leftrightarrow\; \bar{p} > (<) bc$. □

The threshold cost $c = c^*$ is such that $B = L$ at $p = bc$, where $B = (b-p)^2$ and $L = \left(p^2 e - 2ap + a^2\right)$. It follows that:

$$\left(b^2 - 2bp + p^2\right) = p^2 e - 2ap + a^2$$

$$c^* = \frac{b - a \pm \sqrt{(b-a)\left[e\left(b+a\right) - 2a\right]}}{b\left(1-e\right)}$$

By definition $c < 1$, so $c^* < 1$

We want to check if:

$$\frac{b-a}{b(1-e)} > 1$$
$$b - a > b - eb$$
$$eb > a$$

This is true because: $e > \frac{1}{d}$ and if there is no complete price differentiation then $\frac{b}{d} > a$. Therefore, we can write:

$$c^* = \frac{b - a - \sqrt{(b-a)\left[e\,(b+a) - 2a\right]}}{b\,(1-e)}$$

Proposition 6. $\sqrt{(b-a)\left[e\,(b+a) - 2a\right]}$ *is a real number when* $v > d$.

Proof of Proposition 6. The expression under the square root is non-negative if:

$$\begin{aligned} e\,(b+a) - 2a &\geq 0 \\ \frac{2d-1}{d^2}\,(b+a) &\geq 2a \\ b\,(2d-1) + 2da - a &\geq d^2 a \\ (2d-1)\,b &\geq a\,(d-1)^2 \\ \frac{b}{a} &\geq \frac{(d-1)^2}{2d-1} \end{aligned}$$

If there is no complete price differentiation, then $\frac{b}{a} > d$.

$$\begin{aligned} d &\geq \frac{(d-1)^2}{2d-1} \\ d^2 &\geq -d + 1 \end{aligned}$$

This inequality holds because $d > 1$ when $N \geq 2$. Hence,

$$\frac{b}{a} > d \geq \frac{(d-1)^2}{2d-1}.$$

□

A.3.3 Cases

For the sake of the subsequent discussion, we introduce some additional definitions and notation.

Definition 5. *The set of all producer types which price differentiate is* $\mathbf{D} = [k, \bar{p}) \cap \left[a, \overline{\theta}\right]$.

Note that if $N = 1$ then no producer type price differentiates because then $N - cN + c = 1$ and $\bar{p} = k$. Thus, $\mathbf{D} = \emptyset$.

Definition 6. $\mathbf{P}$ *is a set of all producer types who exchange the product and set price* $p = \bar{p}$. $\mathbf{P} = \mathbf{D}^c \cap \left[a, \bar{\theta}\right]$.

Definition 7. $\mathbf{\Theta}$ *is a set of all producer types who exchange the product.* $\mathbf{\Theta} = \mathbf{D} \cup \mathbf{P}$.

Baseline

If $N = 1$ and $c \leq \frac{a+b}{2b}$, then $\bar{p} = \frac{a+b}{2}$, $k = \bar{p}$, $\bar{\theta} = b$ and $p^* = \bar{p}$.

The baseline is the case in which there is only one consumer per group so that information is not shared between the consumers. However, the production costs are sufficiently low, so that there is no adverse selection. All the producer types exchange the product at price $\bar{p} = \frac{a+b}{2}$.

$k = \bar{p}$ if $N = 1$. Hence, it follows from equation (1.2) on page 14 that

$$g(p) = \int_a^b \theta f(\theta) d\theta - p = \int_a^b \frac{\theta}{b-a} d\theta - p = \frac{a+b}{2} - p.$$

$\partial_p \mathbb{E}_E[S] = -1$, so, $\bar{p} = \frac{a+b}{2}$.

It follows that $\mathbf{D} = \emptyset$ and $\mathbf{P} = \mathbf{\Theta} = [a, b]$ if $N = 1$ and $c \leq \frac{a+b}{2b}$.

Akerlof Case (Adverse Selection)

A distinctive characteristic of adverse selection is that the expected quality of products offered in the market is dependent on and lower than the price. Common assumptions satisfying these conditions are the following: uniform quality distribution, cost function $C = c\theta$ (where θ denotes quality and c denotes marginal cost of quality) and $c > \frac{a+b}{2b}$ which is in line with Akerlof (1970).

If $N = 1$ and $c > \frac{a+b}{2b}$, then $\bar{p} = \frac{ac}{2c-1}$, $k = \bar{p}$ and $\bar{\theta} = \frac{a}{2c-1} < b$.

Let's denote by $\bar{\theta}$ a producer type $\theta : \bar{p} - c\theta = 0$. Thus, $\bar{\theta} = \frac{\bar{p}}{c}$. Remember that when $N = 1$, then no producer type price differentiates, that is, every producer type sets a price equal to the reservation price $\bar{p}$. Producer profit is equal to $\pi = \bar{p} - c\theta = \mathbb{E}[\theta|n, \bar{p}] - c\theta$. As $N = 1$, then $\mathbb{E}[\theta|n, \bar{p}] = \frac{\bar{\theta}+a}{2}$. Thus, $\pi = \frac{\bar{\theta}+a}{2} - c\theta$. π is decreasing with θ and it is negative for producer type with quality $\theta = b$ if $c > \frac{a+b}{2b}$. Reservation price $\bar{p}$ must be such that expected consumer surplus $\mathbb{E}[S|p] = 0$ at $\bar{p}$. Hence, it must be that $\frac{\bar{\theta}+a}{2} - \bar{p} = 0$ from which it follows that $\bar{p} = p^* = \frac{ac}{2c-1}$ and $\theta = \frac{a}{2c-1}$. $p > \frac{ac}{2c-1}$ cannot be the equilibrium price because then $E[\theta|p] < p$, so consumers do not buy the product. $p < \frac{ac}{2c-1}$ cannot be equilibrium price because then $E[\theta|p] > p$, so the monopolist does not maximise his profit.

$\mathbf{D} = \emptyset$ and $\mathbf{P} = \mathbf{\Theta} = \left[a, \frac{a}{2c-1}\right]$.

Complete Price Differentiation

If $N > 1$ and $\frac{b}{a} < N - cN + c$, then $\bar{p} = b$, $k < a$ and $\bar{\theta} = b$.

By Proposition 1, if $N > 1$, then $\exists\theta : p = \theta < \bar{p}$.

If $\frac{b}{a} < N - cN + c$, then:

$$k = \frac{\bar{p}}{N - cN + c} \leq \frac{b}{N - cN + c} < a.$$

It follows from Lemma 6 on page 26 that $\bar{p} = b$ if $k < a$. If $\bar{p} = b$, then $bc < \bar{p} = b$, that is, even for the highest-quality producer type the profit maximising behaviour is to sell the product. Hence, we write that the highest-quality producer willing to exchange his product $\bar{\theta} = b$.

$\mathbf{D} = [a, b)$, $\mathbf{P} = [b]$ and $\mathbf{\Theta} = [a, b]$.

Adverse Selection Completely Resolved

If $N > 1$, $\frac{b}{a} > N - cN + c$, and $c < c^*$, then $bc < \bar{p} < b$, $a \leq k < \bar{p}$ and $\bar{\theta} = b$. Equation (1.2) on page 14 must hold for $bc < \bar{p} < b$ when $c < c^*$.
$L = 0$ at $p = ad < \bar{p}$. $kd = \bar{p}$ by definition. Hence, $a < k$.

$\bar{p} > bc$, so even for the highest quality producer it is profitable to exchange product at that price. Hence, there is no adverse selection and $\bar{\theta} = b$.

If $N > 1$, $\frac{b}{a} > N - cN + c$, and $c < c^*$, $\bar{p}$ this is an explicit solution to $g(p) = 0$:

$$\bar{p} = \frac{b - a - \sqrt{(b-a)[-2a - (b+a)(\frac{1}{d^2} - \frac{2}{d})]}}{(\frac{1}{d} - 1)^2} \tag{A.10}$$

However, we use the more tractable implicit form of $\bar{p}$:

$$\bar{p} = b + k - a - \sqrt{2}\sqrt{(b-a)(k-a)} \tag{A.11}$$

We demonstrate that $\bar{p}$ in equation (1.5) on page 16 is between $\frac{a+b}{2}$ and b in Appendix A.3.1.

$\mathbf{D} = [k, \bar{p})$, $\mathbf{P} = [a, k) \cup [\bar{p}, b]$ and $\mathbf{\Theta} = [a, b]$ where $\bar{p}$ is defined in equation (1.5) on page 16.

Adverse selection partly resolved

If $N > 1$, $\frac{b}{a} > N - cN + c$, and $c > c^*$, then $\frac{a+b}{2} < \bar{p} < cb$, $k < \bar{p}$ and $\bar{\theta} < b$. It follows from Proposition 5 on page 32 that equation (1.2) on page 14 must hold for $p_2 < \bar{p} < bc$ when $c > c^*$. $L = 0$ at $p = ad < \bar{p}$. $kd = \bar{p}$ by definition. Hence, $a < k$. $\bar{p} < bc$, so only the producer with $\theta \leq \frac{\bar{p}}{c}$ participate in the market. Hence, $\bar{\theta} = \frac{\bar{p}}{c} < b$.

$\mathbf{D} = [k, \bar{p})$, $\mathbf{P} = [a, k) \cup \left[\bar{p}, \bar{\theta}\right]$ and $\mathbf{\Theta} = \left[a, \bar{\theta}\right]$ where $\bar{p}$ is defined in (A.7).

A.3.4 Effects of N, c, b and a on $\bar{p}$

In this subsection we show the effect of parameters N, c, a and b on the reservation price $\bar{p}$. The reservation price $\bar{p}$ increases with the group size N owing to the decrease of the low-quality producer types who set their price equal to the reservation price. We show that an increase in the group size N increases the reservation price $\bar{p}$ by deriving totally $\bar{p}$ by N (but, for now, only for the case without adverse selection, that is, when $bc < \bar{p}$):

$$\frac{d\bar{p}}{dN} = \frac{\frac{\partial \bar{p}}{\partial N} + \frac{\partial \bar{p}}{\partial k}\frac{\partial k}{\partial N}}{1 - \frac{\partial \bar{p}}{\partial k}\frac{\partial k}{\partial \bar{p}}} > 0 \tag{A.12}$$

Reservation price with no adverse selection is:

$$\bar{p} = b + k - a - \sqrt{2}\sqrt{(b-a)(k-a)}$$

Let us denote $z = \sqrt{2}\sqrt{(b-a)(k-a)}$ and rewrite:

$$\bar{p} = b + k - a - z$$

It follows that

$$z = b - \bar{p} + k - a.$$

$$\frac{\partial \bar{p}}{\partial k} = 1 - \frac{b-a}{z} \tag{A.13}$$

$$= 1 - \frac{b-a}{b-a-(p-k)} < 0 \tag{A.14}$$

$$\frac{\partial k}{\partial N} = -\frac{(1-c)\bar{p}}{(N-cN+c)^2} < 0 \tag{A.15}$$

$$\frac{\partial k}{\partial \bar{p}} = \frac{1}{N-cN+c} > 0 \tag{A.16}$$

An increase in c decreases reservation price $\bar{p}$. $\frac{\partial L}{\partial c} > 0$, $\frac{\partial B}{\partial c} \leq 0$ (see Appendix A.2.2 on page 30). It follows from Remarks 2, 3 and 4 on page 28 that the reservation price $\bar{p}$ decreases with c. In the similar way, one can show that price increases or remains unchanged in the case of adverse selection with increase of b because $\frac{\partial B}{\partial b} = 2(b-p) > 0$. Price is also supposed to increase with an increase in a because $\frac{\partial L}{\partial a} = \frac{12}{N}(p-k) < 0$. Practically, we have a system of two endogenous equations, (1.1) on page 14 and (1.5) on page 16. Endogenous variables are $\bar{p}$ and k. Let us denote some exogenous variable by x. The total effect of x on $\bar{p}$ and k is by implicit function theorem:

$$\begin{bmatrix} \frac{d\bar{p}}{dx} \\ \frac{dk}{dx} \end{bmatrix} = \frac{1}{1 - \frac{\partial \bar{p}}{\partial k}\frac{\partial k}{\partial \bar{p}}} \begin{bmatrix} 1 & \frac{\partial \bar{p}}{\partial k} \\ \frac{\partial k}{\partial \bar{p}} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{p}}{\partial x} \\ \frac{\partial k}{\partial x} \end{bmatrix} \tag{A.17}$$

The effect of N on $\bar{p}$ when the adverse selection is present is obtained by deriving totally expression (A.7) on page 31 by N, which is also positive.

$$\frac{d\bar{p}}{dN} = \frac{-ac\,(1-c)\,2d\,(1-d)}{\left\{c - \left[(1-c)^2 + c^2\,(1-e)\right]\right\}^2} > 0.$$

A.4 The Expected Profit

We can classify producer types into three groups according to their profit function. Producer types with $\theta \in [a, k]$ set the reservation price but sell to one consumer per group only. As their quality is lower than the price, the consumer surplus is negative in this case. Consumer types with $\theta \in [k, \bar{p}]$ charge a price equal to their quality and sell to the whole group. Note that these producer types take all the social surplus, so in this case consumer surplus is zero. The third group are producer types with $\theta \in \left[\bar{p}, \overline{\theta}\right]$ who charge a price lower than the quality, so that both the producer and the consumer receive some part of the social surplus. We denote the profits of these three groups as π_L, π_θ and π_H respectively.

$$\begin{aligned}\pi_L &= \frac{1}{N}\left(\bar{p} - c\theta\right)\ if\ \theta \in [a, k] \\ \pi_H &= \left(p - c\theta\right)\ if\ \theta \in \left[\bar{p}, \overline{\theta}\right] \\ \pi_\theta &= \theta(1-c)\ if\ \theta \in [k, \bar{p}]\end{aligned}$$

If N increases, then the fraction of the producer types with π_L decreases because k decreases with N. Meanwhile the fraction of producer types with π_θ increases because of a decrease in k and an increase in $\bar{p}$. The fraction of producer types with π_H decreases as well because of the increase in $\bar{p}$. However, their profit increases proportionally with $\bar{p}$.

A.5 The Expected Consumer Surplus

We study the effect of the consumer group size on the expected consumer surplus. Firstly, we focus on the cases without the adverse selection problem. By plugging equation (1.5) on page 16 into denominator of A we obtain that:

$$A = \frac{\left(2\bar{p} - k - a\right)\sqrt{k-a}}{2\sqrt{2}\sqrt{b-a}}.$$

We use again equation (1.5) to obtain the following:

$$\begin{aligned}2\bar{p} - k - a &= 2b + 2k - 2a - 2\sqrt{2}\sqrt{b-a}\sqrt{k-a} - k - a \\ &= 2\left(b-a\right) - 2\sqrt{2}\sqrt{b-a}\sqrt{k-a} + k - a \\ &= \left(\sqrt{2}\sqrt{b-a} - \sqrt{k-a}\right)^2\end{aligned}$$

Thus,

$$A = \frac{\left(\sqrt{2}\sqrt{b-a} - \sqrt{k-a}\right)^2\sqrt{k-a}}{2\sqrt{2}\sqrt{b-a}}$$

$$
\begin{aligned}
\frac{dA}{dN} &= \frac{1}{2\sqrt{2}\sqrt{b-a}}\left[\frac{d\sqrt{k-a}}{dN}\left(\sqrt{2}\sqrt{b-a}-\sqrt{k-a}\right)^2 - 2\left(\sqrt{2}\sqrt{b-a}-\sqrt{k-a}\right)\frac{d\sqrt{k-a}}{dN}\sqrt{k-a}\right] \\
&= \frac{1}{2\sqrt{2}\sqrt{b-a}}\cdot\frac{d\sqrt{k-a}}{dN}\left(\sqrt{2}\sqrt{b-a}-\sqrt{k-a}\right)\left(\sqrt{2}\sqrt{b-a}-\sqrt{k-a}-2\sqrt{k-a}\right) \\
&= \frac{\left(\sqrt{2}\sqrt{b-a}-\sqrt{k-a}\right)}{4\sqrt{2}\sqrt{b-a}\sqrt{k-a}}\cdot\frac{dk}{dN}\left(\sqrt{2}\sqrt{b-a}-3\sqrt{k-a}\right).
\end{aligned}
$$

In order to determine the sign of $\dfrac{dA}{dN}$ we study three expressions:

$$\sqrt{2}\sqrt{b-a}-\sqrt{k-a} \tag{A.18}$$

$$\frac{dk}{dN}\ \textit{and} \tag{A.19}$$

$$\sqrt{2}\sqrt{b-a}-3\sqrt{k-a} \tag{A.20}$$

We know that $\dfrac{dk}{dN} < 0$, which is demonstrated on page 36. Next, we study the other two expressions. It is evident that they both increase continuously with an increase in the consumer group N because $\frac{dk}{dN} < 0$. However, their sign is not clear. Therefore, we study their values at two extremes: $N = 1$ and $N = \bar{N}$. When $N = \bar{N}$, then $k = a$, so both expressions are positive. It remains to determine the sign of the expressions at $N = 1$.

If the consumer group size N is 1 and if there is no the adverse selection problem, then the reservation price is $\bar{p} = \frac{a+b}{2}$. Otherwise, if $N = 1$ and there is the adverse selection problem ($c > \frac{a+b}{2b}$), then $\bar{p} = \frac{ac}{2c-1}$.

Firstly, we demonstrate that $\sqrt{2}\sqrt{b-a}-\sqrt{k-a}$ is positive at $N = 1$ in the baseline case:

$$
\begin{aligned}
\sqrt{2}\sqrt{b-a} &> \sqrt{k-a} \\
\sqrt{2}\sqrt{b-a} &> \sqrt{\frac{a+b}{2}-a} \\
\sqrt{2}\sqrt{b-a} &> \frac{1}{\sqrt{2}}\sqrt{a+b-2a} \\
2\sqrt{b-a} &> \sqrt{b-a}
\end{aligned}
$$

We conclude that $\sqrt{2}\sqrt{b-a}-\sqrt{k-a}$ is positive at $N = 1$. We can conclude that this expression is positive for any consumer group N between 1 and $\bar{N}$.

Now, we demonstrate that $\sqrt{2}\sqrt{b-a}-3\sqrt{k-a}$ is negative at $N = 1$ in baseline case.

$$
\begin{aligned}
\sqrt{2}\sqrt{b-a} &< 3\sqrt{k-a} \\
\sqrt{2}\sqrt{b-a} &< 3\sqrt{\frac{b+a}{2}-a} \\
\sqrt{2}\sqrt{b-a} &< \frac{3}{\sqrt{2}}\sqrt{b+a-2a} \\
2\sqrt{b-a} &< 3\sqrt{b-a}
\end{aligned}
$$

To summarise, at $N = 1$, $\sqrt{2}\sqrt{b-a} - \sqrt{k-a}$ is positive and $\sqrt{2}\sqrt{b-a} - 3\sqrt{k-a}$ is negative. At $N = \bar{N}$, both expressions are positive because then $k = a$. $\frac{dk}{dN}$ is always negative. Therefore, $\frac{dA}{dN}$ is positive at $N = 1$ and negative at $N = \bar{N}$.

Secondly, we consider the cases with the adverse selection problem. From equation (1.8), it follows that:

$$A = \frac{p^2(\frac{1-c}{c})^2}{2\left[p(\frac{1-c}{c}) + k - a\right]}.$$

$$
\begin{aligned}
\frac{dA}{dN} = &\frac{1}{2\left[p\left(\frac{1-c}{c}\right)+k-a\right]^2}\cdot \\
&\cdot\left\{2p\frac{dp}{dN}\left(\frac{1-c}{c}\right)^2\left[p\left(\frac{1-c}{c}\right)+k-a\right] - p^2\left(\frac{1-c}{c}\right)^2\frac{dp}{dN}\cdot\frac{1-c}{c} - p^2\left(\frac{1-c}{c}\right)^2\frac{dk}{dN}\right\} \\
&= \frac{1}{2\left[p\left(\frac{1-c}{c}\right)+k-a\right]^2}\left\{p^2\frac{dp}{dN}\left(\frac{1-c}{c}\right)^3 + 2p\frac{dp}{dN}(k-a) - p^2\left(\frac{1-c}{c}\right)^2\frac{dk}{dN}\right\} \\
&> 0
\end{aligned}
$$

The above derivation is positive because $\frac{dk}{dN} < 0$. Thus, the expected consumer surplus increases at least until the minimum (threshold) consumer group size at which the adverse selection problem is resolved. We demonstrate that such a group size is always lower than the group size at which there is complete price differentiation $\bar{N}$. We demonstrate this by showing that $\frac{b}{a} > N - cN + c$ at $\bar{p} = bc$.

We follow the definition of n and the expression for $\bar{p}$ as in Appendix A.3.1 on page 30. It is straightforward to make the following observation: an increase in n decreases $\bar{p}$. By the contradiction, suppose that there is complete price differentiation so that $\frac{a}{b} > n$. Suppose that the consumer group size is such that $\bar{p}(n) = bc$. If in expression for $\bar{p}$ we substitute n by $\frac{a}{b}$, we should obtain that $\bar{p}(\frac{a}{b}) < bc$.

$$bc > \frac{b - a - \sqrt{(b-a)^2 - (1-\frac{a}{b})^2(b^2-a^2)}}{(\frac{a}{b}-1)^2}$$

$$bc > \frac{b^2}{(b-a)^2}\left[b - a - \sqrt{(b-a)^2 - (1-\frac{a}{b})^2(b^2-a^2)}\right]$$

$$c(b-a)^2 > b(b-a) - b\sqrt{(b-a)^2 - (1-\frac{a}{b})^2(b^2-a^2)}$$

$$b\sqrt{(b-a)^2 - (1-\frac{a}{b})^2(b^2-a^2)} > (b-a)\left[b-(b-a)c\right]$$

$$b^2(b-a)^2 - b^2(\frac{b-a}{b})^2(b^2-a^2) > (b-a)^2\left[b-(b-a)c\right]^2$$

$$b^2 - (b^2-a^2) > \left[b-(b-a)c\right]^2$$

$$a^2 > \left[b-(b-a)c\right]^2$$

$$a > b-(b-a)c$$

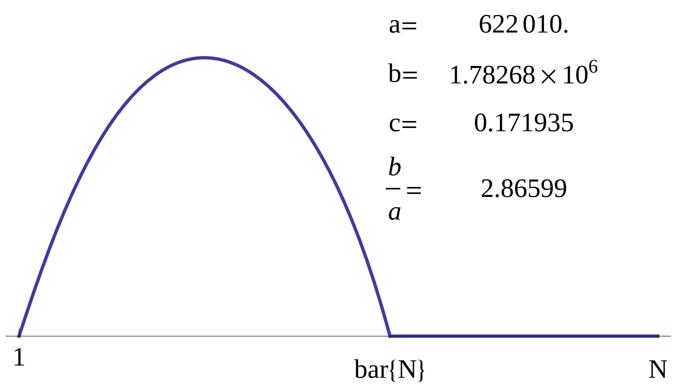
$$(b-a)c > b-a$$

This is a contradiction. Thus, it must be that $\frac{a}{b} < n$ at $\bar{p} = bc$.

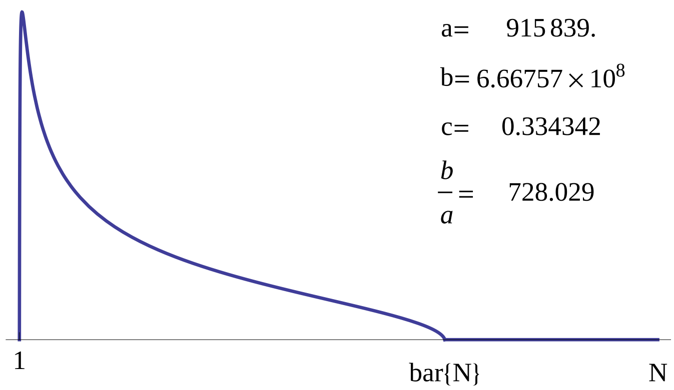
From the above analysis about the expected consumer surplus, with and without the adverse selection problem, we can conclude that the expected consumer surplus is zero at $N = 1$. It is positive and increasing with N, at least until the threshold N at which the adverse selection problem is resolved. It has a maximum at some $1 < N < \bar{N}$, after which it decreases and it is again zero at $\bar{N}$.

We ran around one hundred iterations of the expected consumer surplus, for randomly drawn vector of parameters a, b and c for which there is no the adverse selection problem. The parameters were drawn from the following intervals of real numbers: $0 < c < \frac{a+b}{2b}$, $0 < a < 10^6$ and $a + 10^{-6} < b < a \cdot 10^4$. We find two shapes of the expected consumer surplus presented in Figure A.2. The first one is completely concave, and it is detected in the cases when $b-a$ is relatively small. The second one is first concave and then convex, and it is typical for large $b-a$.

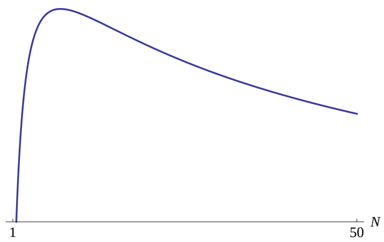
Thus, $\frac{dA}{dN}$ becomes negative at sufficiently large $N < \bar{N}$. An additional member in the group increases the usefulness of information because it increases $\partial_N \frac{N-1}{N} = \frac{1}{N^2} > 0$. The intuition is the following. The information about the product is useful only for the non-experimenter (the experimenter has already faced the loss) because an informed non-experimenter avoids the purchase of the good which quality is lower than its price. An increase in the consumer group size decreases the probability that a consumer is the experimenter, and thus it decreases the probability that he faces the loss. On the other hand, after some N, the usefulness of information starts to decrease because there are less producer types who overprice their product. That is, an increase in the



(a) The difference $b - a$ is relatively small, so that the expected consumer surplus is concave function of N.



(b) The difference $b-a$ is large, so that the expected consumer surplus is first concave, but then convex (and again concave). The maximum is visible in Subfigure A.2(c)



(c) The expected consumer surplus for parameters as in Subfigure A.2(b), but here only until $N = 50$ so that the peak if visible.

Figure A.2: The expected consumer surplus for the consumer groups between 1 and $\bar{N}$. It is zero at $N = 1$ and at $N \geq \bar{N}$.

consumer communication always induces some additional producer types, whose quality is bellow the reservation price, to set price equal to their quality. This reduces the set of the states in which the experimenter's experience is useful for the non-experimenter.

Chapter 2

Signalling Quality through Price in an Oligopoly with Consumer Communication

2.1 Introduction

In some situations there is a variation in quality across sellers and this quality might be exogenously determined. Producers might be better informed about actual quality than consumer. However, consumers may transfer own experience to each other. We study quality signalling by price in the presence of communication among consumer and producer competition.

Some of the first works on the quality signalling by price are Milgrom and Roberts (1986)[36] and Bagwell and Riordan (1991)[6] which we commented on page 10. We proceed by overviewing the papers which study the quality signalling by price in oligopoly (see also Table 2.1). Hertzendorf and Overgaard (2001)[28] is one of the first extensions of the signalling to a duopoly framework. Quality of both goods is known by both competitors, but not by consumers, and consumers have heterogeneous preferences over quality. It is found that advertising compensates for an insufficient difference between the high and the low quality, and enables signalling. Contrary to Bagwell and Riordan (1991)[6], the costs do not increase with the quality. More recently Daughety and Reinganum (2008)[18] show that signalling by price is possible in an oligopoly even without advertising, but if there is enough horizontal differentiation (as well as vertical heterogeneity in preferences). Unlike Hertzendorf and Overgaard (2001)[28], the costs increase with quality, which is a private information, so that each producer only knows his own quality. Very recently Janssen and Roy (2010)[32] show that signalling is possible even without horizontal differentiation (that is, with homogeneous consumers). The source of signalling is a difference in production costs between high and low-quality producers, so that the high-quality producer does not compete by price with the low-quality producer. Instead, the high-quality producer expects to sell the product at high-price if the state with only high-quality producers occurs. The necessary condition for signalling is that the low-quality firm plays a mixed price strategy where the probability of trading decreases with the price. Similar to the last cited, Andriani and Deidda (2010)[5] study as well a market where each buyer demands at most one unit of good, but where each seller produces at most one unit of good. The quality is known only by its own producer and the cost of production increases with the quality. There are two main differences compared to Janssen and Roy (2010)[32]. Firstly, instead of modelling explicitly strategic interactions between the buyers and the seller, only the general mechanism properties are specified (among the others, the incentive compatibility constraint). Second, the model allows for the short-side supply (i.e., there may be an excess of demand). Instead of studying particular equilibria, the authors focus on equilibrium outcomes, where several equilibria can lead to the same outcome. Robust equilibrium outcomes are those which satisfy the D1 criterion, and it is shown that the resulting D1-equilibria yield the same unique outcome in terms of price, quantity and quality. The main result of Andriani and Deidda (2010:7)[5] is that quantities traded are increasing in buyers-sellers ratio θ and in probability that high quality occurs λ (which is somehow intuitive). One can consult the cited papers for further references on this topic.

Opposite to Janssen and Roy (2010:194)[32] and Andriani and Deidda (2010:8)[5]

Source of signalling	Hertzendorf and Overgaard (2001)	Daughety and Reinganum (2008)	Andriani et al (2010) Janssen and Roy (2010)	this paper
advertising	√	×	×	×
vertical diff.	√	√	×	×
horizontal diff.	×	√	×	×
diff. costs	×	√	√	×
WOM	×	×	×	√

Table 2.1: Sources of price signalling in oligopoly across the articles

where the source of signalling are costs which are increasing in quality, the source of signalling in Chapter 1 is consumer communication, so that the trade volume may be affected by the quality in the end. Some consumers, called experimenters, buy the product without knowing its quality. They observe it immediately after the exchange and inform other consumers. A low-quality producer may loose the informed consumers if he sets some price above his quality. Thus, this producer may be better off by setting a price equal or lower than his quality and selling to all the consumers. We proceed in spirit of Janssen and Roy (2010)[32] and our model in Chapter 1, questioning if signalling still occurs if consumer communication and competition are combined. We develop a Bayesian model where signalling of the low-quality producers occurs in an oligopoly, like in Chapter 1 in a monopoly, but opposite to Janssen and Roy (2010)[32] where the signalling of the high-quality producers occurs. In line with Janssen and Roy (2010)[32] signalling disappears if the competition is too strong. In the monopoly model the signalling occurs because low-quality producer type decreases the price in order to differentiate from the high-quality producer type. The signalling is not possible in oligopoly because then the Bertrand competition between low-quality producers would dissipate all the profit. However, this is not a problem if a low-quality producer plays a mixed strategy over some price profile. The expected profit is the same along the profile. If a low-quality producer sets the lowest price in the profile, he sells for sure. If it sets the highest price in the profile, it sells only if all the other firms are of the high quality. In this way the Bertrand competition is avoided. On the other hand, the high-quality producer sets some price (pure strategy) above the price profile of the low-quality firm. The high-quality firm sells with some positive probability only if all the other producers are of the high quality.

For signalling equilibrium to exist, the *the single crossing property* must be satisfied. In our model it has to be assured that a high-quality producer does not have an incentive to deviate to a low-quality price profile, and that a low-quality does not have the incentive to deviate to a high price. This is assured if the low quality is between high and low prices because producer's profit is divided by N at prices above l. In that case we can have a situation where the profit of the high-quality producer is higher at a high price than at a low price, and the profit of the low-quality producer is lower at the high price than at the low price. The reason is a sudden fall in the profit (division by N) of the low-quality producer if he sets a price above the low quality.

We study both pooling and separating equilibria in a monopoly and in an oligopoly, and we obtain the following results. First, the signalling occurs in the monopoly if the consumer groups are sufficiently large. Second, when the product variability and consumer groups are endogenous, an increase in the cost of social ties increases the product variability. Third, the consumer communication enables signalling in the oligopoly as well. Although we study similar oligopoly frameworks, Janssen and Roy (2010:202)[32] find that the high-quality producers signal their quality, while we find that the low-quality producers signal their quality. Forth, we study a unique signalling equilibrium in the oligopoly which satisfies the D1 criterion. We find that despite the fact that the competition decreases the expected social surplus, the expected consumer surplus increases with the competition, so that the loss due to the competition is completely borne by producers. However, if the consumer group size is endogenous, then the competition my increase the expected profit. Fifth, there is a multiplicity of pooling equilibria in oligopoly, where the consumer group size and the competition increase the maximum possible price, while the product variability decreases it.

While in both Janssen and Roy (2010:202)[32] and Andriani and Deidda (2010:5)[5] pooling equilibria do not survive the D1 criterion (The reason is that the high-quality type is more likely to deviate to high prices than low quality type), we find the opposite. In line with both papers (Andriani and Deidda (2010:6)[5] in Lemma 3) we find that the D1-robust separating equilibrium satisfies the incentive compatibility constraint with equality. Andriani and Deidda (2010)[5] study two types of equilibria, one where there is an excess of buyers, and the other where there is an excess of producers. The signalling is possible only in the first case, while in the other case the Bertrand competition drives the high-quality producers out of the market. However, Janssen and Roy (2010)[32] obtain D1-robust separating equilibrium although there is an excess of supply. We believe that unconsidered mixed strategies in the former paper cause this difference in the results. Finally, in both papers, Janssen and Roy (2010)[32] and Andriani and Deidda (2010:8)[5], the competition increases the expected consumer surplus, which is in line with our finding on page 61.

Galeotti (2010:6)[23] also studies an oligopoly with incomplete information, but without signalling. The quality of the products is identical and commonly known. However, producer observes only one price, and may be informed about the other. Thus, the common for both models is that the consumer does not have complete information about the products. While in Galeotti (2010:6)[23] the producer cannot compare prices, in our model the producer cannot compare qualities. The consumer communication affects the prices in both models, but in a different way. While we focus on signalling, Galeotti (2010)[23] focuses on simultaneous choice of prices and size of consumer communication. Similarly, Goyal (2010)[26] undertakes an empirical testing of the framework where sellers only know the price offered from one buyer, because there are significant search costs.

Alcala et al (2007)[4] study an endogenous quality formation in an oligopoly with consumer communication. Possibility of the separating equilibria (i.e. signalling) is excluded by an assumption, and the pooling equilibrium is analysed.

The results are sustained by horizontal product differentiation. An increased competition decreases the average quality, and thus, reduces the consumer surplus. This is in contrast to our model where the average quality decreases with the competition, but the accompanying decrease in prices is sufficiently large so that the consumer surplus increases with the competition (However, our result is obtained at the separating equilibrium.) For the sake of tractability, Vettas (1997)[48] also excluded the possibility of signalling in a dynamic monopoly model with the consumer communication. The costs are independent of the quality and normalised to zero, and demand is unit. The producer chooses the price and the quantity in each period. It is found that the consumers may infer about the quality from the fact that they do not receive any information and that they were not able to purchase in the previous period. As an extension, the authors suggest to combine both, learning in equilibrium from both others and prices.

We devote some attention to analysis of both, the pooling and the separating equilibrium, as in a monopoly, so as in an oligopoly. We apply the intuitive criterion and the D1 criterion whenever the model under consideration is sufficiently tractable. We conclude each part by an elaboration of the welfare effects of the parameters.

2.2 Model

The product is sequentially exchanged between $M \in \mathbb{N}$ producers and $n \in \mathbb{N}$ consumers with homogeneous preferences. The product quality θ is a random variable $\theta \in \{l, h\}$, determined by nature and assigned to the producer. h denotes high quality and l denotes low quality, $h > l$. α is a probability that h occurs. All the product units of a given producer are of the same quality. Production costs are zero.

Consumers are divided into groups of N members, $N \in \{1, ..., n\}$. Members of a group share the information, so that when one randomly drawn consumer observes θ, everyone in the group receives that information immediately. We refer to this as consumer *experimentation*. The first consumer in the group who buys the product is called *experimenter*. Consumers do not receive any other information. Demand is unit and utility function is money-metric $u = \theta$. Consumers or the producer can refuse to exchange the product. Once price is set, the producer cannot change it.

2.2.1 The Signalling Game

We specify a signalling game with three players: nature, producer (sender) and experimenter(receiver).[1] The players make choices in the following order: (1) nature chooses producer type θ, (2) based on the private information about his quality θ, the producer chooses the price (the signal) $p \in [a, b]$, (3) the experimenter observes p and makes a buying decision $x \in \{0, 1\}$. The spaces of mixed prices and buying decisions are $\mathcal{A}_1$ and $\mathcal{A}_2$ with elements α_1 and α_2. The producer profit is $\pi(\alpha_1, \alpha_2, \theta)$, and the consumer surplus is $S(\alpha_1, \alpha_2, \theta)$. It is a common knowledge that the experimenter has a prior $f(\theta)$ about the producer type θ. $\sigma_1(\cdot|\theta)$ is a probability distribution over price p for monopolist type θ. $\sigma_2(\cdot|p)$ is a probability distribution over buying decisions x for each price p. $\mu(\cdot|p)$ is experimenter's posterior belief over $\boldsymbol{\Theta}$.

We proceed by considering only the pure strategies of the consumers.

If the price is equal or lower than the quality, the product is sold to the whole group and the profit per consumer in the group is p. Otherwise, only the experimenter buys the product.

$$\pi(p, x, \theta) = \begin{cases} x\frac{p}{N} & if \quad p > \theta \\ xp & else \end{cases} \tag{2.1}$$

Experimenter surplus is a difference between the actual product quality and the price paid for the product multiplied by the probability of buying.

$$S(p, x, \theta) = x\,(\theta - p)$$

We denote the highest and the lowest price of high and low quality firms respectively as: $\overline{p}_H$, $\underline{p}_H$, $\overline{p}_L$ and $\underline{p}_L$. We proceed by searching for signalling equilibria in a monopoly.

[1] In order to simplify the analysis we consider an experimenter here and account for the consumers non-experimenters only in the payoff function.

2.3 A Monopoly

The section consists of three subsections, devoted to separating equilibria, pooling equilibria, and standardisation respectively. We find both, pooling and separating equilibria, in the monopoly. If the consumer communication is sufficiently large, then only the separating equilibrium which satisfies the intuitive criterion exists, where every producer type sets the price equal to his quality. On the other hand, if the consumer communication is small, such that $N < \frac{h}{l}$, but still $N > \frac{\alpha(h-l)+1}{l}$, then there is no an equilibrium that satisfies the intuitive criterion. Finally, when $N < \frac{\alpha(h-l)+1}{l}$, then only pooling equilibria satisfy the intuitive criterion. Thus, the consumer communication enables quality signalling by price. Furthermore, starting from the maximum pooling equilibrium price $\bar{p}$, in the third subsection we develop a framework where the consumer group size and quality variance $(h-l)$ are endogenous. We introduce the costs of social ties and the costs of the product standardisation. We find that an increase in the costs of the social ties increases the product variability.

p_M is the maximum price at which the consumer buys the product, $p_M = \max\{p : x(p) = 1\}$, and p^* is any equilibrium price by p^*. Thus, p_M^* is the maximum equilibrium price at which the consumer buys the product. Consistently with Chapter 1, we denote $\bar{p} = \alpha + (1-\alpha)\,l$.

2.3.1 Separating Equilibria

We demonstrate in the proof of Proposition 7 that the separating equilibrium must be such that the low-quality producer type sets the price equal to l, and the high-quality producer type sets some price lower than lN. Naturally, the high-quality type cannot set some price above h (because the consumers do not buy at prices above h).

Proposition 7. *There exists a belief which supports a separating equilibrium of a signalling game, where the high-quality type sets p_M^* and the low-quality type sets l, if and only if $p_M^* \leq lN$.*

Proof of Proposition 7. We show that $p_M^* \leq lN$ constitutes a separating equilibrium. If the low-quality producer sets price $p = l$, his profit is $\pi_L = l$. If he instead sets $p_M^* \leq lN$, his profit is $\frac{p_M^*}{N}$. Thus, the best response of the low-quality producer is to set price l. The high-quality producer always obtains the profit equal to price, $\pi_H = p$. Thus, his best response is to set p_M^*. Hence, no producer wants to deviate and this is a signalling equilibrium. The only constraint on the beliefs which support this equilibrium is that $\mu(h|p^* \leq p \leq h) < \frac{p-l}{h-l}$, so that the consumer does not buy at prices in the mentioned interval. There is no constraint on the beliefs at the other out-of-equilibrium prices.

It remains to demonstrate that $p_M^* > lN$ cannot constitute the separating equilibrium. If the producer buys at $p_M^* > lN$, then the best response of the low-quality producer is to deviate from l and to p_M^*. Thus, both types set p_M^*, so this cannot be a signalling equilibrium. □

It turns out that the high-quality price must be equal to h, and not lower than this, in order to satisfy the intuitive criterion (see Proposition 8). The reason is that the low-quality producer prefers l to any price between l and lN. Thus, the intuitive criterion requires the exclusion of the low-quality type in the beliefs at the out-of-equilibrium prices in this interval.

Proposition 8. *Only the separating equilibrium where $p_M^* = h$ passes the intuitive criterion.*

Proof of Proposition 8.

Lemma 11. *The separating equilibrium such that $p_M^* < lN$ fails the intuitive criterion.*

Proof of Lemma 11. *Consider some price $p \in (p_M^*, lN)$. Type l can be eliminated in the beliefs at price p by the equilibrium dominance test because $\pi(l, 1, l) = l > \frac{p}{N}\pi(p, 1, l)$. Type h is not eliminated at price p because $\pi(p_M^*, 1, h) = p_M^* < p = \pi(p, 1, h)$. It follows that in the intuitive criterion, the experimenter has to exclude l in the beliefs at price p, that is, $\mu(h|p) = 1$. It follows from this belief that the experimenter best response is to buy at p because his expected surplus is $h - p > 0$. But, then the minimum payoff of h is higher at p than in the equilibrium, $\pi(p_M^*, 1, h) < \pi(p, 1, h)$. Hence, the separating equilibrium where $p_M^* < lN \leq h$ fails the intuitive criterion.*

Lemma 12. *The separating equilibrium such that $p_M^* = h$ passes the intuitive criterion.*

Proof of Lemma 12. *For all the beliefs at price $l < p < h$, there is no producer type who wants to deviate to such an out-of-equilibrium price. Thus, the beliefs at such a price can be set arbitrarily. Also, for all the beliefs at the prices above h, the best response of the consumer is not to buy. We conclude that the separating equilibrium where $p_M^* = h$ satisfies the intuitive criterion.*

□

We conclude that the separating equilibrium exists only if the consumer group is sufficiently large relative to M, and that the separating equilibrium which satisfies the intuitive criterion is unique. The low-quality type sets l, and the high-quality type sets h. Thus, the producer reaps all the social surplus (It is obvious that the consumer surplus is zero). The selected separating equilibrium also satisfies the D1 criterion, but we do not demonstrate it here formally. Instead, we proceed by analysing the pooling equilibria.

2.3.2 Pooling Equilibria

Firstly, we establish necessary conditions for the existence of the pooling equilibrium and we demonstrate that every pooling equilibrium satisfies the intuitive criterion. When the results about separating and pooling equilibria are compared, it comes out that there are some parameter values such that the equilibrium which satisfies the intuitive criterion does not exist. Secondly, we study the expected consumer and social surplus, and the expected profit at the pooling equilibrium. Finally, we treat the consumer communication N and the high-quality probability α as endogenous variables, and we find that they are interrelated. An increase in N increases α, while an increase in α decreases N, so that the final effect remains inconclusive at this stage.

We show in the proof of Proposition 9 that the necessary condition for the existence of pooling equilibrium is:

$$lN < \bar{p}. \tag{2.2}$$

Proposition 9. *The pooling equilibrium exists in the monopoly if and only if* $p_M^* \in (lN, \bar{p}]$.

The proof is in Appendix B.1 on page 78, and it consists of three lemmas. Lemma 14 demonstrates that the pooling equilibrium exists if condition (2.2) holds, while Lemmas 15 and 16 demonstrate that this is not the case when condition (2.2) is not satisfied. On the one hand, the expected quality is lower than the price for the prices above $\bar{p}$. Thus, the experimenter does not buy, so that this cannot be the equilibrium price. On the other hand, for $l < p_M^* < lN$, the best response of the low-quality producer type is to deviate and set l. Thus, this price as well cannot be sustained as the signalling equilibrium. Hence, the necessary condition for the existence of the pooling equilibrium is that:

$$\alpha > \frac{l\,(N-1)}{h-l}.$$

It follows that the existence of the pooling equilibrium is assured if α and h are sufficiently large relative to l and N, since $\bar{p}$ must be larger than lN.

The only restrictions on the beliefs are for the prices between p_M^* and $\bar{p}$. The experimenter beliefs for these prices must be such that the expected quality is lower than the price, so that the experimenter's best response is not to buy the product. Otherwise, the experimenter would buy the product at the out-of-equilibrium price, and consequently, the best response of the producer would be to set the out-of-equilibrium price.

We proceed by discussing the intuitive criterion. As it is stated in Proposition 10, we find that any pooling equilibrium satisfies the intuitive criterion. The intuition is that p_M^* dominates all the prices lower than p_M^*, so that there is no type who deviates to those prices, independently of the beliefs. On the other hand, both types prefer to sell at prices above p_M^*. Thus, the consumer can believe that the type who deviates is low, and in that case his best repose is not to buy at those out-of-equilibrium prices.

Proposition 10. *Any pooling equilibrium satisfies the intuitive criterion.*

Proof of Proposition 10. p_M^* dominates all the prices below p_M^*, so the beliefs can be set arbitrarily even by the equilibrium dominance test. Beliefs at the prices above p_M^* are interesting for our analysis. Consider a price $p > p_M^*$. Neither for l nor for h p is equilibrium dominated. We check further for the intuitive criterion. Translated to our case, the equilibrium fails the intuitive criterion if the equilibrium payoff $\pi(p_M^*, 1, \theta) = p_M^*$ is lower than the minimum possible payoff given the set of beliefs restricted by the equilibrium dominance test $\pi(p_M^*, 0, \theta) = 0$. As $p_M^* > 0$, we conclude that the pooling equilibrium passes the intuitive criterion. □

Intuitively, all the pooling equilibria satisfy the D1 criterion because there is no an experimenter mixed strategy at the out-of-equilibrium price which could make some type deviate and the other not. We continue by comparing the results obtained in this and the previous subsection. We find out that pooling and separating equilibria cannot exists contemporaneously.

Proposition 11. *For any specification of the parameters there exist at most one of the two following types of equilibria: either pooling or separating.*

Proof of Proposition 11. Any pooling equilibrium satisfies the intuitive criterion. The signalling equilibrium exists if $\alpha > \frac{l(N-1)}{h-l}$. It follows that $h > l\left(1 + \frac{N-1}{\alpha}\right)$. The separating equilibrium which passes the intuitive criterion exists iff $h < lN$. From the two conditions follows that: $l\left(1 + \frac{N-1}{\alpha}\right) < h < lN$. $\left(1 + \frac{N-1}{\alpha}\right) < h < N$. It follows that $N - 1 < \alpha\,(N-1)$ and this is a contradiction if $\alpha < 1$. □

For some parameter values (i.e. when $lN < h < l\left(1 + \frac{N-1}{\alpha}\right)$) there does not exist an equilibrium which satisfies the intuitive criterion. This is the case when ratio $\frac{h}{l}$ is sufficiently large relative to N, so that the separating equilibrium is not possible. Moreover, α is still relatively small so that the pooling equilibrium does not exist as well.

We conclude the subsection by undertaking the welfare analysis at the pooling equilibrium prices. We study the expected profit, the expected consumer and the social surplus.[2] Consider the producer who decided about participation in a market, but without knowing a priori the quality assigned by nature. His expected profit is:

$$\mathbf{E}\left[\pi\right] = \alpha p^* + (1-\alpha)\,\frac{1}{N} p^*$$

If the high-quality is assigned to the producer, he enjoys the benefits from the trade with the whole consumer group, otherwise, he manages to sell only to the experimenter. The expected consumer surplus $\mathbf{E}[CS]$ is equal to the weighted sum of the expected surplus of the experimenter and of the expected surplus of the non-experimenter. Note that with an increase of the consumer group, $\mathbf{E}[CS]$ increases (because $l - p^* < 0$):

$$\begin{aligned}\mathbf{E}[CS] &= \frac{1}{N}\left(\mathbf{E}\left[\theta\right] - p^*\right) + \alpha\frac{N-1}{N}\left(h - p^*\right)\\ &= \alpha\left(h - p^*\right) + (1-\alpha)\,\frac{1}{N}\left(l - p^*\right)\end{aligned}$$

The expected social surplus $\mathbf{E}[S]$ is a sum of the expected profit and the expected consumer surplus, which is equivalent to the expected quality exchanged in the market:

$$\mathbf{E}[S] = \alpha h + (1-\alpha)\,\frac{1}{N} l$$

The expected consumer surplus increases with N, while the expected profit and the expected social surplus decreases with N. Hence, the consumers would increase their communication groups just below the level where the pooling equilibrium does not exist anymore. However, this is not socially optimal because in the state when the low-quality is realised there is no exchange with the non-experimenters although even the low-quality production is socially efficient. If

[2]The results refer to the unit of product and in expectation.

the consumer group becomes sufficiently large, such that $h < lN$, then, all the products are exchanges, but the consumer surplus is zero. So, if there is some social planner, he is going to set either no consumer groups (in which case the equilibrium is pooling but the producer reaps all the social surplus), or consumer groups which are sufficiently large, so that the equilibrium is separating. Contrary, if the communication is present, but if it is not sufficiently large, then both producer types set the same price, and the low-quality producer type does not sell to the whole consumer group.

Suppose that every additional member costs k each consumer in the group, and that the cost of α is $q(\alpha)$, where $\frac{\partial q}{\partial \alpha} > 0$ and $\frac{\partial^2 q}{\partial \alpha^2} > 0$. The optimal high-quality probability α^* is obtained by checking where the marginal profit equals the marginal cost of α:

$$\frac{\partial \mathbf{E}[\pi]}{\partial \alpha} = \frac{N-1}{N} p^* - \frac{\partial q}{\partial \alpha}$$

It follows that the optimal high-quality probability is some function of N and $\frac{\partial q}{\partial \alpha}$, that is, $\alpha^* = f\left(\frac{N-1}{N}, \frac{\partial q}{\partial \alpha}\right)$.

The optimal consumer group size N^* is obtained from:

$$\frac{\partial \mathbf{E}[CS]}{\partial N} = \frac{1}{N^2} (1-\alpha)(p^* - l) - k = 0$$

$$N^{*2} = \frac{(1-\alpha)(p^* - l)}{k}$$

The total effect of both marginal costs, k and $\frac{\partial q}{\partial \alpha} > 0$ on α^* is negative. The total effect of k on N^* is negative, and the effect of $\frac{\partial q}{\partial \alpha} > 0$ on N^* is positive. One can question if it is more efficient to direct efforts into a reduction of k or $\frac{\partial q}{\partial \alpha} > 0$, in order to increase α and consequently the expected social surplus. The increase in l decreases α and N, so instead of enabling the separating equilibrium and in this way increasing the expected social surplus, such an intervention may have an adverse effect because the pooling equilibrium may remain present but at a lower α. In the following subsection we proceed by analysing the endogenous consumer group N at the pooling equilibrium, but looking at the effect of k on the product variability $h - l$.

2.3.3 Standardisation and Socialisation

We study the effects of costs of social ties k on the optimal product variability $v = h - l$. Suppose that there are different production techniques, which differ in product variability v, but all these techniques have the same mean value $\bar{p} = \alpha h + (1-\alpha) l$. $c(v)$ is the cost of product variability which decreases with v, $\frac{\partial c(v)}{\partial v} < 0$. k is the cost of a social tie. The expected consumer surplus and the profit are respectively:

$$\mathbb{E}\left[CS\right] = \alpha\frac{N-1}{N}\left(1-\alpha\right)\left(h-l\right) - kN$$
$$\mathbb{E}\left[\pi\right] = \alpha\bar{p} + \left(1-\alpha\right)\frac{1}{N}\bar{p} - c\left(v\right)$$

The players move in the following order. First, the producer chooses the product variability v which maximises the expected profit. Second, the consumers choose the group size N which maximised the expected consumer surplus, taking into account chosen v. Third, nature draws the producer's quality θ.

We study a pooling equilibrium with the highest price $\bar{p}$, and we obtain a result stated in Proposition 12:

Proposition 12. *An increase in the cost of a social tie k increases the product variability v.*

The optimal consumer group size depends positively on both, the cost of social tie k and the product variability v. The producer prefers as smaller group size as possible, and he can foster a decrease in a group size through v, but this is costly. Thus, if k is already small, then the producer also keeps v at the low level.

Proof of Proposition 12. $\frac{\partial CS}{\partial N} = \frac{1}{N^2}\alpha\left(1-\alpha\right)\left(h-l\right) - k$. It follows from this that the optimal consumer group size N^* is:

$$N^* = \sqrt{\frac{\alpha\left(1-\alpha\right)v}{k}}$$

The producer chooses v to maximise his expected profit, taking into account the effect of v on optimal N^* chosen by the consumers. Note that $\frac{d\bar{p}}{dv} = 0$.

$$\frac{d\mathbb{E}\left[\pi\right]}{dv} = \frac{\partial\mathbb{E}\left[\pi\right]}{\partial N^*}\frac{\partial N^*}{\partial v} + \frac{\partial\mathbb{E}\left[\pi\right]}{\partial v}$$

$$\frac{\partial\mathbb{E}\left[\pi\right]}{\partial N^*} = -\left(1-\alpha\right)\frac{1}{N^2}\bar{p}$$

$$\frac{\partial N^*}{\partial v} = \frac{1}{2\sqrt{v}}\sqrt{\frac{\alpha\left(1-\alpha\right)}{k}}$$

$$\frac{\partial\mathbb{E}\left[\pi\right]}{\partial v} = -\frac{\partial c(v)}{\partial v} = c$$

It follows from $\frac{d\mathbb{E}[\pi]}{dv} = 0$ that

$$c = \frac{1}{N}\left(1-\alpha\right)\bar{p}\frac{1}{2v}$$

By taking into account the optimal consumer group size we obtain that:

$$v^* = \sqrt[3]{\frac{(1-\alpha)\,\bar{p}^2 k}{4\alpha c^2}}$$

$$\frac{\partial v^*}{\partial k} = \frac{1}{3}k^{-\frac{2}{3}}\sqrt[3]{\frac{(1-\alpha)\,\bar{p}^2}{4\alpha c^2}} > 0$$

□

An increase in the costs of social ties decreases the optimal group size consumers choose. Therefore, the producers can foster an optimal consumer group size by less standardised products. If the socialisation is easier (which can be a result of different broader social movements and organisation), then the production is more standardised. The consumer communication decreases the producer profit. Easier socialisation increases the consumer communication, but product standardisation decreases it because the communication becomes useless. Therefore, the producers neutralised the positive effect of the socialisation profit on the consumer communication by an increased standardisation. The high variability is characteristic for consumer specific services, or innovations, while standardised products are usually manufactured on the large scale (the typical industrial production). Thus, it may be argued that the transition to the postindustrial production can be fostered by the change in the costs of communication.

2.4 An Oligopoly

In this section we present an oligopoly model which is built by combining the insights from Chapter 1 and Janssen and Roy (2010)[32]. In both mentioned papers signalling occurs, in Chapter 1 due to the presence of the consumer communication, and in Janssen and Roy (2010)[32] due to the competition. We would like to see if the signalling still occurs when these two mechanisms work together.

Compared to the monopoly model in the previous section, the oligopoly structure brings more complexity to the model, which we take into account formally as follows. First, there is an additional stage in the game. The experimenter makes two subsequent choices: (1) the experimenter chooses the producers with the highest expected pay-off, (2) the experimenter chooses if to buy or not. Second, the repeated purchase may create more experiments per group. This can be realised in the pooling equilibrium when price is higher than l. Finally, the experimenter may repeat the purchase after realising that the previous product was bad, which affects the expected profits of the consumers.[3]

[3] Alternatively, one may think of a framework where the consumer never repeats the purchase. The example would be the purchase of a durable equipment (e.g. some electronics for the household) where already significant expenditures have been made, and the equipment satisfies the basic need, although its quality is lower than its price.

We find out that there exists a separating equilibrium where high-quality producers sell only in the state where there are no low-quality producers, or a pooling equilibrium where all producers set the same price. We demonstrate in Appendix B.2.1 on page 79 that the opposite separating equilibrium, where the low-quality producers sell only if there are no high-quality producers, does not exist. The section is composed of two sections, one devoted to the separating equilibria, and another devoted to the pooling equilibria. We remind that the quality of each producer is an independent draw with probability α that h occurs and $1-\alpha$ that l occurs.

2.4.1 Separating Equilibria

In this subsection we determine the necessary conditions for the existence of the separating equilibrium in the oligopoly which satisfies the intuitive criterion. As in the monopoly, the necessary condition for signalling is the consumer communication. However, while in the monopoly the low-quality producer trades-off the high price in order to exploit the trading with the whole consumer group, in the oligopoly the high-quality producer trades-off the probability of trading for the high price. Depending on the parameters, the separating equilibrium which satisfies the intuitive criterion may exist in the monopoly, but not in the oligopoly, and vice versa. However, if the consumer communication is sufficiently large, and if it is combined with a sufficient, but not to strong competition, then the separating equilibrium exists as in the monopoly, so as in the oligopoly. We find that there is a unique signalling equilibrium which satisfies the D1 criterion, which is selected for the welfare analysis. Although both, the expected social surplus and the total expected profit, decrease with the competition M, the consumer surplus increases. The social loss due to the competition is completely borne by the producers. When the consumer group size N is endogenous, the competition may increase profit, although it decreases the profit directly.

We construct a signalling equilibrium where high quality firms charges a common deterministic price $\underline{p}_H$, while the low-quality firms charge the price which follows the common probability function F with a support in $p \in \left[\underline{p}_L, \overline{p}_L\right]$. The equilibrium has the following properties (like in Janssen and Roy (2010:195-197)[32][4]):

1.
$$\overline{p}_L \leq l \leq \underline{p}_H \leq h$$
   If both prices $\underline{p}_H$ and $\overline{p}_L$ are below or both above l, then what is preferred for the low-quality firms is as well preferred for the high-quality firm. So, the separating equilibrium cannot be sustained.

2. The highest price of the low-quality producer $\overline{p}_L$ is such that a consumer is indifferent between buying from the high- or the low-quality producer. Thus:

[4]It is demonstrate in Janssen and Roy (2010)[32] that the signalling equilibrium exists under the properties like these we list here. Thus, we use them to study the signalling equilibrium in our model. This list represents the necessary conditions for the existence of the signalling equilibrium.

$$\bar{p}_L = \underline{p}_H - (h - l) \tag{2.3}$$

3. The high-quality firm sells only if all other firms are of the low-quality and if it is chosen among the other high-quality firms, so that its expected profit is: $\frac{1}{M}\alpha^{M-1}\underline{p}_H$.

4. The low-quality firm which sets $\overline{p}_L$ sells only in the state where all other firms are of the high quality, so that its expected profit is $\alpha^{M-1}\overline{p}_L$.

5. The low-quality producer sells with certainty at the lowest price $\underline{p}_L$. A condition for mixed strategy is that the expected profit from setting $\overline{p}_L$ must be equal to the expected profit from setting $\underline{p}_L$:

$$\underline{p}_L = \alpha^{M-1}\overline{p}_L$$

6. The low-quality firm obtains the same expected profit by setting any price in the support. We use several relations do derive $F(p)$, the probability that the low-quality firm sets price p. The expected profit of the low quality firm at some price should be the probability to set that price multiplied by the price. It should be always equal to the expected profit when pricing $\overline{p}_L$. The low-quality producer sells at some price only if i out of M producers are of the high quality and if the remaining $M-1-i$ producers set some price higher than p, which is a binomial expression:

$$\sum_{i=0}^{M-1-i} \binom{M-1}{i} \alpha^i (1-\alpha)^{M-1-i} (1-F(p))^{M-1-i}$$

It is equal to write:

$$[\alpha + (1-\alpha)(1-F(p))]^{M-1}$$

Hence, the expected profit when setting $\overline{p}_L$ is equal to the expected profit obtained by setting any other price in the support:

$$\alpha^{M-1}\overline{p}_L = [\alpha + (1-\alpha)(1-F(p))]^{M-1} p$$

It follows that:

$$F(p) = 1 - \frac{\alpha}{1-\alpha}\left(\sqrt[M-1]{\frac{\underline{p}_L}{p}} - 1\right)$$

High quality firm sets a single price, which is already demonstrated in Janssen (2008:24-25)[31] so we do not elaborate on this here. We proceed by elaborating a necessary condition for the existence of the signalling equilibrium. We derive Proposition 13 from the assumption that the low-quality producer does not mix with the high-quality firm and vice versa[5]:

[5]This would correspond to *the single crossing property*, a standard requirement in the signalling games.

Proposition 13. *If and only if $\frac{h}{MN} < l < \frac{hM}{2M-1}$, there exists a belief which supports a separating equilibrium of a signalling game in the oligopoly, such that high-quality firms set $\underline{p}_H$ and low-quality firms set a mixed price with a support in $\left[\underline{p}_L, \overline{p}_L\right]$.*

The proof of Proposition 13 is in Appendix B.2.3 on page 80. We provide the intuition for Proposition 13 by commenting first of all on the incentive compatibility constraints. The low quality firms do not mix with the high-quality firms if the expected profit of the low-quality firm is lower at $\overline{p}_H$ than at $\underline{p}_L$:

$$\alpha^{M-1}\left[\overline{p}_L + (h-l)\right]\frac{1}{MN} < \alpha^{M-1}\overline{p}_L. \tag{2.4}$$

This is the only condition required in the monopoly case where the maximum price at which the experimenter buys in the equilibrium should be lower than lN. Apart from this condition, the profit of the high-quality producer in the oligopoly should be higher at $\underline{p}_H$ than at $\overline{p}_L$:

$$\alpha^{M-1}\left[\overline{p}_L + (h-l)\right]\frac{1}{M} > \alpha^{M-1}\overline{p}_L. \tag{2.5}$$

The expected profit of the low-quality firm at $\underline{p}_H$ is $\pi_L(\underline{p}_H) = \frac{1}{M}\alpha^{M-1}\underline{p}_H$ if $p_H \leq l$, and $\pi_L(\underline{p}_H) = \frac{1}{MN}\alpha^{M-1}\underline{p}_H$ (thus, divided by N) if $p_H > l$. Hence, both inequalities hold only if the low-quality firm sets some price below l and the high-quality firm sets some price above l:

$$\overline{p}_L \leq l < \overline{p}_L + (h-l). \tag{2.6}$$

If the experimenter believes that $\mu(l|\overline{p}_L < p < \underline{p}_H) = 1$, then the separating equilibrium is sustained. With such beliefs, the best response for the experimenter is not to buy at all at price $p \in \left(l, \underline{p}_H\right)$, and consequently no producer deviates to this price. For price $p' \in (\overline{p}_L, l)$ the best response is not to choose this producer because with this belief choosing this producer is dominated by choosing the producer charging $\underline{p}_H$, that is, $h - \underline{p}_H > l - p'$ which immediately follows from equation (2.6).

After determining the parameters for which the separating equilibrium in the oligopoly exists, we test if they satisfy the intuitive criterion, and we find the following:

Proposition 14. *Every separating equilibrium in an oligopoly where $p_H < Ml$ satisfies the intuitive criterion.*

The proof of Proposition 14 is in Appendix B.2.3 on page 81. We provide here a short intuition of the criterion. The intuitive criterion is a standard tool used to test out-of-equilibrium beliefs in the signalling games. The test is composed of two parts. Firstly, if at some out-of-equilibrium price p:

$$\pi^*(\theta) > \max_{x \in BR(\boldsymbol{\Theta})} \pi(p, x, \theta),$$

then we eliminate type θ for the beliefs at price p. T is a set of the eliminated θ-s. $BR(\mathbf{\Theta})$ is the set of x which are the best responses when there is some belief such that every $\theta \in \mathbf{\Theta}$ can happen with the positive probability[6].

Secondly, the equilibrium fails the intuitive criterion if there is some θ for which the following holds:

$$\pi^*(\theta) < \min_{x \in BR(\mathbf{\Theta}/T)} \pi(p, x, \theta)$$

We focus on out-equilibrium prices $\overline{p}_L < p' < p_H$. Our signalling equilibrium (we have discussed it already) is sustained by the belief that the quality is certainly low in this interval of prices. Can such a pessimistic belief pass the intuitive criterion? It does not pass it only when in the first step of the test we reject the low-quality and do not reject the high quality. Then, in the second step, the only quality considered in the beliefs is h. Consequently, the experimenter sets $x \geq \alpha^{M-1}$ at the out-of-equilibrium price.

When both qualities are rejected in the first step, we consider both of them in the second step. Then, the experimenter can believe that quality is l at the out-of-equilibrium price, and he does not buy the product. Thus, the producer's surplus is zero, and it is certainly lower than in the equilibrium. Hence, there is no type whose minimum profit is larger than in the equilibrium, so that the intuitive criterion is satisfied.

We combine the results from Propositions 13 and 14 to obtain Proposition 15:

Proposition 15. *Every separating equilibrium in an oligopoly satisfies the intuitive criterion if*

$$\frac{hN}{MN+N-1} < l < \frac{hM}{2M-1}. \tag{2.7}$$

Proof of Proposition 15. For this proof we use four relations: equations 2.4, 2.5, 2.6, and the finding in Proposition 14. The proof is composed of two parts. We demonstrate that the separating equilibrium in the oligopoly satisfies the intuitive criterion firstly if $\dfrac{hN}{MN+N-1} < l$, and secondly if $l < \dfrac{hM}{2M-1}$.

The first part follows from expression 2.4 on page 58 and Proposition 14. It follows from expression 2.4 that $\overline{p}_L > \frac{h-l}{MN-1}$. It follows from Proposition 14 that $\overline{p}_L < Ml - (h-l)$. There exists some $\overline{p}_L$ which satisfies both conditions if $\frac{h-l}{MN-1} < Ml - (h-l)$. After a few algebraic steps, it follows that the last holds if $h\frac{N}{MN+N-1} < l$. We can say that this condition is obtained from two requirements: firstly, that both the competition and the consumer groups are sufficiently large so that the low-quality producer does not deviate to the high price, and secondly, that the beliefs at the out-of-equilibrium prices satisfy the intuitive criterion.

The second part follows form expressions 2.5 and 2.6. It follows from expression 2.5 that $\bar{p}_L < \frac{h-l}{M-1}$. We know from expression 2.6 that $\bar{p}_L > 2l - h$. From

[6] Formal definition is in Fudenberg et al(1991:448)[22]

these two inequalities it follows that $l < \frac{hM}{2M-1}$. This condition is obtained from the requirement that the low-quality equilibrium price is sufficiently small so that the high-quality producer does not deviate to it, and that the high-quality price is also higher than l.

Finally, we made the following comparison $\frac{hN}{MN+N-1} < \frac{M}{2M-1}$. We find that this always holds since it follows that $-N < M^2N - M - MN$. We can write $-N < M[M(N-1)-1]$. This inequality holds because the expression on the right-hand side is positive. Thus, there exists a separating equilibrium which satisfies the intuitive criterion if $\frac{hN}{MN+N-1} < l < \frac{M}{2M-1}$. □

We can conclude that the separating equilibrium exists when the competition is moderate. While expression 2.4 on page 58 and Proposition 14 which refer to the deviation of the low-quality producer, require the competition to be sufficiently strong, the condition 2.5 requires relatively small competition in order to increase the probability that the high-quality producer sells his product.

We discuss now the results in Proposition 15 and compare them with the results in the monopoly case where the separating equilibrium which satisfies the intuitive criterion exists only $h < lN$. Firstly, we focus on the effect of the consumer group[7]. We compare it with the lower bound in expression (2.7). Expression (2.7) is stronger condition than the monopoly one only if N exceeds the following threshold: $\hat{N} = \frac{M+1+\sqrt{(M+1)^2-4}}{2}$ which is obtained simply from equality $\frac{N}{MN+N-1} = \frac{1}{N}$. Thus, if N is small, then the existence of the separating equilibrium in the monopoly may imply its existence in the oligopoly (i.e., conditional on $l < \frac{hM}{2M-1}$). However, if the separating equilibrium exists in the monopoly even though N is small, this implies that the difference between h and l is small. But, then the upper bound in expression 2.7 may not be satisfied, so that the the separating equilibrium still does not exist in the oligopoly. We proceed by commenting the effect of the number of the competitors. The difference between h and l may be very large, so that the separating equilibrium does not exist in the monopoly. However, this difference can be sufficient, so that a moderate, but not too strong competition may enable signalling in the oligopoly. Thus, we conclude that, in general, the moderate competition and a sufficient consumer communication assures the existence of the separating equilibrium which satisfies the intuitive criterion. We proceed by undertaking the welfare analysis at the separating equilibrium which satisfies the D1 criterion.

The Welfare Effects of the Competition when the Consumer Group Size is Endogenous

We select price $\overline{p}_L = (h-l)\frac{MN}{MN-1}$ by the D1 criterion, which is demonstrated in the proof of Proposition 19 in Appendix B.2.3 on page 82. By undertaking the welfare analysis at the selected price we arrive to the following findings. First, although both, the expected social surplus and the total expected profit, decrease with the competition M, the consumer surplus increases. The social

[7]Note that $\frac{\partial \frac{N}{MN+N-1}}{\partial N} < 0$ and $\frac{\partial \frac{N}{MN+N-1}}{\partial M} < 0$.

loss due to the competition is completely borne by the producers. Second, we suppose that the consumers may choose the group size prior to the exchange. When the consumer group size is endogenous, the competition may increase the profit, although it decreases profit directly. The competition decreases the optimal consumer group size due to the decreased marginal benefit of the communication (Market price is already decreased due to the competition, so that the communication is less effective). The decrease in the consumer group increases the profit. This positive indirect effect of the competition on the profit may overcome the negative direct effect of the competition which works through the decreased price and decreased probability to sell the product.

We start with the exposition of the expected consumer surplus as a difference between the expected social surplus and the total expected profit. Next, we elaborate the marginal effects of N and M on the expected welfare. Then, we study the effect of the competition M on the optimal consumer group size N. Finally, we conclude about the total effect of the competition on the expected profit for different costs of social ties k.

We can focus on a single purchase because The expected social surplus is the weighted sum of the high and the low quality. The high quality is exchanged only if all the producers are of the high quality, so that its weight is α^M. The low quality is exchanged in any other case.

$$\begin{aligned}\mathbb{E}\left[S\right] &= \alpha^M h + \left(1-\alpha^M\right) l \\ &= \alpha^M \left(h-l\right) + l\end{aligned}$$

The total expected profit is the expected profit of a single producer multiplied by the number of the producers. This expression is obtained from the third and the fourth point on page 57:

$$\begin{aligned}M\mathbb{E}\left[\pi\right] &= \alpha^M \underline{p}_H + M\left(1-\alpha\right)\alpha^{M-1}\overline{p}_L \\ &= \alpha^M \overline{p}_L + \alpha^M\left(h-l\right) + M\left(1-\alpha\right)\alpha^{M-1}\overline{p}_L\end{aligned}$$

The expected consumer surplus is a difference between the expected social surplus and the total expected profit:

$$\begin{aligned}\mathbb{E}\left[CS\right] &= \mathbb{E}\left[S\right] - M\mathbb{E}\left[\pi\right] \\ &= l - \overline{p}_L\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right]\end{aligned}$$

It can be noted that the negative effect of the competition on the social surplus is totally contained in the expected profit. The competition decreases the expected social surplus $\mathbb{E}\left[S\right]$, since it decreases the probability that the high-quality product is traded, so that:

$$\frac{\partial \mathbb{E}[S]}{\partial M} = \alpha^M \ln\alpha(h-l)$$

Additionally, the total expected profit $M\mathbb{E}\left[\pi\right]$ decreases with the competition due to a decrease in the market prices.

However, the later effect, although negative for the producers, represents an increase in the expected consumer surplus $\mathbb{E}[CS]$.

$$\frac{\partial \mathbb{E}[CS]}{\partial M} = \frac{\partial \mathbb{E}[S]}{\partial M} - \frac{\partial M\mathbb{E}[\pi]}{\partial M}$$

Since $\frac{\partial M\mathbb{E}[\pi]}{\partial M} < \frac{\partial \mathbb{E}[S]}{\partial M} < 0$, it follows that $\frac{\partial \mathbb{E}[CS]}{\partial M} > 0$.

In the remaining part of the section we study the effect of the competition on the expected profit. First, we suppose that the consumer group size N is exogenous and we obtain a direct effect of the competition. Next, we relax this assumption and study the effect of the competition on the expected consumer surplus when the consumers adjust their communication groups to the competition M.

Direct effect of M: It is evident that the competition decreases the expected social surplus, that is, $\frac{\partial \mathbb{E}[S]}{\partial M} = \alpha^M (\ln \alpha)(h - l) < 0$. The effect on the total expected profit is:

$$\frac{\partial M\mathbb{E}[\pi]}{\partial M} = \alpha^M (\ln \alpha)(\bar{p}_L + h - l) + (1-\alpha)\alpha^{M-1}(1 + M\ln\alpha)\bar{p}_L + \frac{\partial \bar{p}_L}{\partial M}\left[\alpha^M + M(1-\alpha)\alpha^{M-1}\right]$$

The second part of the expression is positive at very small values of M, but it becomes negative with an increase in M. The rest of the expression is negative. Thus, we proceed by assuming that $\frac{\partial M\mathbb{E}[\pi]}{\partial M} < 0$. We suppose for the same reason that $\frac{\partial \mathbb{E}[CS]}{\partial M} > 0$. We can also disentangle the effect of M on the expected profit of a single producer, which is negative:

$$\begin{aligned}
\frac{\partial \mathbb{E}[\pi]}{\partial M} &= \left[-\frac{1}{M^2}\alpha^M + \frac{1}{M}(\ln\alpha)\alpha^M\right][\bar{p}_L + h - l] + \frac{1}{M}\alpha^M\frac{\partial \bar{p}_L}{\partial M} \\
&\quad + (\ln\alpha)(1-\alpha)\alpha^{M-1}\bar{p}_L + \frac{\partial \bar{p}_L}{\partial M}(1-\alpha)\alpha^{M-1} \\
&< 0 \\
\frac{\partial \bar{p}_L}{\partial M} &= -(h-l)\frac{N}{(MN-1)^2} \\
&< 0
\end{aligned}$$

For the moment we ignore the competition effect on price which satisfies the D1 criterion, and we focus on the other competition effects on the individual expected profit. The competition still decreases the individual expected profit. Firstly, it decreases a probability that the state where all the producers are of the high quality occurs. Secondly, once when such a state occurs, it decreases the probability that a particular producer is drawn among the high-quality producers. Thirdly, the competition shifts the probability of trade of the low-quality good to the lower prices. We can observe that these negative effects follow straightly from the equilibrium properties defined on page 56. We proceed by studying the consumer group effects.

Direct effect of N: The consumer group size N works through a decrease in $\overline{p}_L$, which decreases the expected profit and increases the expected consumer surplus, while the social surplus remains unaffected:

$$\frac{\partial \mathbb{E}\left[\pi\right]}{\partial N} = \frac{\partial \overline{p}_L}{\partial N}\left[\frac{1}{M}\alpha^M + \left(1-\alpha\right)\alpha^{M-1}\right] < 0$$

$$\frac{\partial M\mathbb{E}\left[\pi\right]}{\partial N} = \frac{\partial \overline{p}_L}{\partial N}\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right] < 0$$

$$\frac{\partial \mathbb{E}\left[CS\right]}{\partial N} = -\frac{\partial \overline{p}_L}{\partial N}\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right] > 0$$

Indirect effect of M: In what follows we demonstrate that the total effect of the competition on the individual expected profit may be positive. This effect is composed of a direct and an indirect effect. The indirect effect works through the expected consumer surplus, where the consumer chooses the optimal consumer group size, such that the expected consumer surplus is maximised. Thus, we introduce the cost of social ties and we find the optimal consumer group size. Based on this we are able to determine the indirect effect.

Apart from the benefits through the increased expected consumer surplus, suppose that the communication is also costly for the consumer, i.e., there is a constant marginal cost k of an additional member in the group. The expected consumer surplus increases with N but at a decreasing rate, that is:

$$\frac{\partial^2 \mathbb{E}\left[CS\right]}{\partial N^2} = -\frac{2M^2(h-l)}{\left(MN-1\right)^3}\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right] < 0$$

This assures that we have an optimal consumer group size N^* such that $\frac{\partial \mathbb{E}[CS]}{\partial N} = k$. It follows that:

$$-\frac{\partial \overline{p}_L}{\partial N}\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right] = k$$

$$\left(h-l\right)\frac{M}{\left(MN^*-1\right)^2}\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right] = k$$

$$\frac{M\left(h-l\right)\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right]}{k} = \left(MN^*-1\right)^2$$

$$MN^* - 1 = \sqrt{\frac{(h-l)M\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}}{k}}$$

$$N^* = \frac{\sqrt{\frac{(h-l)M[\alpha^M + M(1-\alpha)\alpha^{M-1}]}{k}} + 1}{M}$$

The marginal effect of the competition on the optimal consumer group size is negative:

Denote $x = \sqrt{\frac{(h-l)M[\alpha^M + M(1-\alpha)\alpha^{M-1}]}{k}}$.

$$\frac{\partial N^*}{\partial M} = \frac{\frac{1}{2x} \cdot \frac{(h-l)M}{k}\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right]}{M^2}$$
$$+ \frac{M\frac{1}{2x} \cdot \frac{(h-l)M}{k}\left[(\ln\alpha)\alpha^M + \left(1-\alpha\right)\alpha^{M-1} + M\left(1-\alpha\right)(\ln\alpha)\alpha^{M-1}\right] - x - 1}{M^2}$$
$$= \frac{\frac{1}{2x}\frac{(h-l)M}{k}\{\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right] + M(ln\alpha)\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right] + \left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right]\}}{M^2}$$
$$- \frac{\alpha^M + x + 1}{M^2}$$
$$= \frac{\frac{1}{2x}\frac{(h-l)M}{k}\left\{\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right](2 + M\ln\alpha) - \alpha^M\right\} - x - 1}{M^2}$$
$$= \frac{x + \frac{1}{2}M(\ln\alpha)x - \frac{1}{2x}\frac{M(h-l)}{k}\alpha^M - x - 1}{M^2}$$
$$= \frac{\frac{1}{2}M(\ln\alpha)x - \frac{1}{2x}\frac{M(h-l)}{k}\alpha^M - 1}{M^2}$$

$$\frac{\partial N^*}{\partial M} = \frac{\frac{1}{2}M\ln\alpha\sqrt{\frac{(h-l)M[\alpha^M + M(1-\alpha)\alpha^{M-1}]}{k}} - \frac{M(h-l)\alpha^M}{2k\sqrt{\frac{(h-l)M\left[\alpha^M + M(1-\alpha)\alpha^{M-1}\right]}{k}}} - 1}{M^2} < 0. \tag{2.8}$$

The total effect of M on $\mathbb{E}[\pi]$:

$$\frac{d\mathbb{E}[\pi]}{dM} = \underbrace{\frac{\partial\mathbb{E}[\pi]}{\partial M}}_{-} + \underbrace{\frac{\partial\mathbb{E}[\pi]}{\partial N^*} \cdot \frac{\partial N^*}{\partial M}}_{+}$$

Despite the negative direct effect on the competition on the expected profit, its total effect might be positive. We find a threshold $\hat{k}$, such that the total effect is positive for the costs of social ties which are lower than the threshold, and the total effect is negative above the threshold.

The threshold $\hat{k}$ is such that $\frac{dM\mathbb{E}[\pi]}{dM} = 0$. It follows from this that

$$-\frac{\partial\mathbb{E}[\pi]}{\partial M} = \frac{\partial\mathbb{E}[\pi]}{\partial N^*} \cdot \frac{\partial N^*}{\partial M} \tag{2.9}$$

Lets denote $y = \sqrt{(h-l)M\left[\alpha^M + M\left(1-\alpha\right)\alpha^{M-1}\right]}$

It follows from expression (2.8) that:

$$M^2\frac{\partial N^*}{\partial M} + 1 = \frac{1}{\sqrt{k}}\left[\frac{1}{2}M(\ln\alpha)y - \frac{1}{2} \cdot \frac{1}{y}M(h-l)\alpha^M\right]$$

By combining with equation (2.9), we obtain that the threshold cost of social ties is:

$$\hat{k} = \left(\frac{\frac{1}{2}M(\ln\alpha)y - \frac{1}{2} \cdot \frac{1}{y}M(h-l)\alpha^M}{-\frac{\frac{\partial\mathbb{E}[\pi]}{\partial M}}{\frac{\partial\mathbb{E}[\pi]}{\partial N^*}}M^2 + 1}\right)^2.$$

2.4.2 Pooling Equilibrium

In this subsection we analyse the pooling equilibria in the oligopoly. In order to simplify the analysis, we assume that the experimenter repeats the purchase (but not necessarily the experiment, which is even more restrictive[8]) if the quality in the previous draw was low. This assumption is justified if the actual and the opportunity costs of the transaction are smaller than the benefit: $p^* + l < h$. Thus, we proceed assuming that this holds. We find a multiplicity of pooling equilibria, where all satisfy the intuitive criterion and the D1 criterion. The competition and the consumer communication decrease the maximum pooling equilibrium price, while $h - l$ increases it.

We define Γ as the total probability that the producer sells the product, which is used later to calculate the expected profit. We suppose that the experiments may be repeated in the oligopoly (each time by a different member of a group). The producers are randomly drawn and removed from the pool (so that they do not participate in the next draws). Drawing stops when the high-quality producer is drawn. p_i is a probability to be drawn in round $i \in [1, M]$. Thus, $\Gamma = p_1 + (1 - p_1)((1 - \alpha)p_2 + (1 - p_2)((1 - \alpha)p_3 + (1 - p_3)(...)))$.

$$\Gamma = \frac{1}{M} + \frac{M-1}{M}\left((1-\alpha)\frac{1}{M-1} + \frac{M-2}{M-1}\left((1-\alpha)\frac{1}{M-2} + \frac{M-3}{M-2}\left((1-\alpha)\frac{M-4}{M-3} + \cdots\right)\right)\right)$$

Thus, Γ is a geometric order[9]:

$$\Gamma = \frac{1 - (1-\alpha)^M}{M\alpha}$$

The probability Γ is always lower than 1 and it decreases with the number of the competitors M, which is shown in the proof of Proposition 20 in Appendix B.2.4 on page 84.

The expected equilibrium profit of the producer is:

$$\begin{cases} x\Gamma\frac{p^*}{N} & if \quad p^* > \theta \\ x\Gamma p^* & else \end{cases} \tag{2.10}$$

Our aim is to answer which the pooling equilibria in the oligopoly are. The analysis reduces to checking if p^* is dominated by some out-of-equilibrium price p'. We check this systematically for different p^*, and we focus the analysis on pairs, p^* and p'. The exposition is organised as follows:

1. We define four regions of prices p^* and p', as in Figure 2.1

[8] In that case the implicit assumption is that $p^* + l < \alpha h + (1-\alpha)\,l$.

[9] Calculation of Γ when the number of firms M is 4:

$$\begin{aligned}\Gamma &= \frac{1}{M} + \frac{M-1}{M}\left((1-\alpha)\frac{1}{M-1} + \frac{M-2}{M-1}\left((1-\alpha)\frac{1}{M-2} + \frac{M-3}{M-2}\cdot\frac{1}{M-3}\right)\right) \\ &= \frac{1}{M} + \frac{1}{M}(1-\alpha) + \frac{1}{M}(1-\alpha)^2 + \frac{1}{M}(1-\alpha)^3\end{aligned}$$

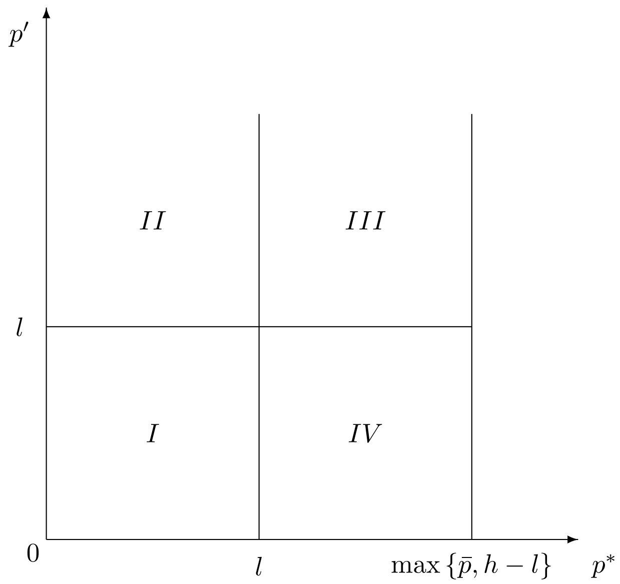


Figure 2.1: Price Regions: The equilibrium price p^* is represented on the horizontal axis, and the out-of-equilibrium price p' on the vertical axis. In region I both equilibrium and out-of-equilibrium prices are lower than l, while the opposite is true in region III. The out-of-equilibrium prices are above l, and the equilibrium prices are below l in region II, and the opposite holds in region IV.

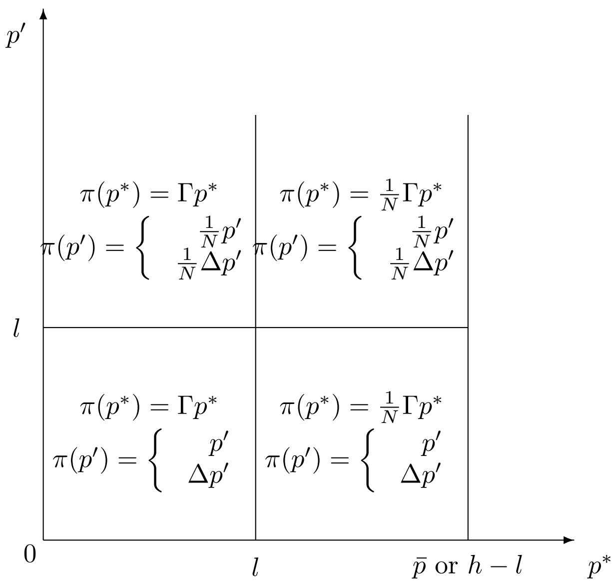


Figure 2.2: The Expected profit of the low-quality producer, at p^* and p', in different regions: The index L is skipped after π in this figure. The expected profit of the low-quality producer at both, the equilibrium and out-of-equilibrium price in region III is divided by N. Only the expected profit at p' is divided by N in region II because p' is higher than l. The converse hold in region IV, where the expected profit at p^* is divided by N because p^* is above l.

(based on the producers' payoffs in expression (2.10)).

2. We show that in some regions (II and III)the payoff dominance of equilibrium price is assured simply by setting very pessimistic beliefs (sufficient condition).

3. We derive the necessary conditions which assured the dominance of the equilibrium price in the remaining regions (I and IV).

4. By applying the necessary conditions under point 3, we analyse the remaining cases, which are not covered by point 2, and we are able to determine the set of pooling equilibria given some parameters.

Price Regions

We consider a two-dimensional space composed of (p^*, p') pairs, which is divided in four regions (Figure 2.1), characterised by different payoffs of the low-quality producer (summarised in Figure 2.2). The horizontal axis represents the equilibrium price p^*, which cannot be higher than $\bar{p}$ or $h - l$ (as commented in the introduction of the subsection). The vertical axis represents the out-of-equilibrium price p'.

Region I **(**$p^* < l$**,** $p' < l$**):** Both, the equilibrium and the out-of-equilibrium price are lower than l. Thus, in any case the low-quality producer sells to the whole consumer group. Thus, his expected profit is equal to the price multiplied by the probability to sell the product, which can be 1, Γ and Δ. Δ is a probability that all the other producers are of the low quality, $\Delta = (1-\alpha)^{M-1}$. It is included in the out-of-equilibrium payoff when the experimenter prefers the equilibrium price, which is formalised as condition (2.11) on page 70.

$$\pi_L(p^*) = \Gamma p^*$$

$$\pi_L(p') = \begin{cases} p' \\ \Delta p' \end{cases}$$

Region *II* **(**$p^* < l,\ p' > l$**):** The equilibrium price p^* is lower than l so that the low-quality producer sells to the whole consumer group at this price. The out-of-equilibrium price p' is higher than l, so that the low-quality producer sells only to the experimenter at this price.

$$\pi_L(p^*) = \Gamma p^*$$

$$\pi_L(p') = \begin{cases} \frac{1}{N} p' \\ \frac{1}{N} \Delta p' \end{cases}$$

Region *III* **(**$p^* > l,\ p' > l$**):** Both, the equilibrium and the out-of-equilibrium price are higher than l. Thus, in any case the low-quality producer sells only to the experimenter. Thus, his expected profit is equal to the price multiplied by the probability to sell the product (1, Γ or Δ), and divided by the group size.

$$\pi_L(p^*) = \frac{1}{N} \Gamma p^*$$

$$\pi_L(p') = \begin{cases} \frac{1}{N} p' \\ \frac{1}{N} \Delta p' \end{cases}$$

Region *IV* **(**$p^* > l,\ p' < l$**):** The equilibrium price p^* is above l so that the low-quality producer sells only to the experimenter. The out-of-equilibrium price p' is lower than l, thus, if the low-quality producer charges this price he sells to the whole consumer group.

$$\pi_L(p^*) = \frac{1}{N} \Gamma p^*$$

$$\pi_L(p') = \begin{cases} p' \\ \Delta p' \end{cases}$$

The payoffs of the high-quality producers do not essentially differ across the price regions:

$$\pi(p^*)_H = \Gamma p^*$$

$$\pi(p')_H = \begin{cases} p' & or \\ \Delta p' \end{cases}$$

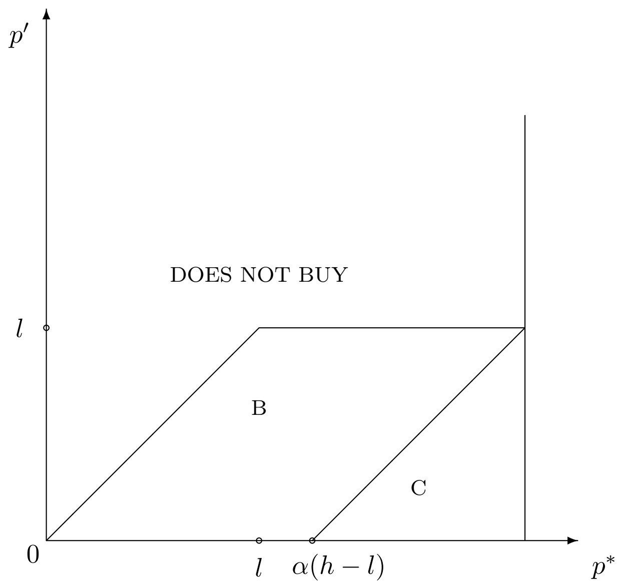


Figure 2.3: Consumer Buying Decision: Consumer decision to buy at p', given p^*. In B consumer buys at p' only if all the other producers are of the low quality. In C the consumer prefer to buy at p', than at p^*.

Thus, the payoff of the low- and high- quality producers are the same in region I, but they differ in region IV, which we take into account in the subsequent analysis. We do not particularly pay attention to regions II and III, which is justified in the following paragraph.

Pessimistic beliefs and equilibrium dominance - regions II and III

$\beta(h|p')$ is a consumer belief that quality is high at out-of-equilibrium price p'. Consider out-of-equilibrium prices $p' > l$ (thus, II and III quadrant). If $\beta(h|p') = 0$, then it is evident the experimenter does not buy at p', even if $p' < p^*$, because $l - p^* < l - p' < 0$. In other words, the consumers refuse to buy at such an out-of-equilibrium price because their expected payoff is negative. Thus, by the pessimistic belief, we exclude any deviation to the out-of-equilibrium prices above l. For this reason, we focus the analysis on regions I and IV only.

The necessary conditions for the equilibrium prices to be payoff dominant - regions I and IV

The producer's expected profit depends on the following factors: (i) region, (ii) own quality, and (iii) experimenter's preferences between p^* and p'.

We derive the thresholds y, z, y' and z' from the producer payoffs in regions I and IV, which are presented in Table 2.2 for the low-quality producer.[10]

[10]If a deviation to the low price is not profitable for the low-quality producer, then it is

		Regions	
		I	IV
Out-of-eq. p'	$p' > v$	$p' > \frac{1}{\Delta}\Gamma p^* = y$	$p' > \frac{1}{N\Delta}\Gamma p^* = y'$
	$p' < v$	$p' > \Gamma p^* = z$	$p' > \frac{1}{N}\Gamma p^* = z'$

Table 2.2: The necessary conditions for equilibrium price p^* to be dominated: If the prices are in region I and if $p' > v$, then the equilibrium price p^* is dominated by out-of-equilibrium price p' if $p' > y$. If the prices are in region I and if $p' < v$, then the equilibrium price p^* is dominated by out-of-equilibrium price p' if $p' > z$. If the prices are in region IV and if $p' > v$, then the equilibrium price p^* is dominated by out-of-equilibrium price p' if $p' > y'$. Finally, if the prices are in region IV and if $p' < v$, then the equilibrium price p^* is dominated by out-of-equilibrium price p' if $p' > z'$.

The thresholds are the out-of-equilibrium prices p' such that the producer is indifferent between choosing the equilibrium price or deviating to p', that is, $\pi(p^*) = \pi(p')$. The producer deviates to the out-of-equilibrium price above the corresponding threshold. The experimenter prefers to buy at p^* when the expected surplus at p^* is larger than the expected surplus at p':

$$\alpha(h-l) + l - p^* > \beta(h-l) + l - p'$$

If we suppose the most pessimistic beliefs $\beta(p') = 0$, the threshold is:

$$v = -\alpha(h-l) + p^* \tag{2.11}$$

Thus, if p' is above v, then the producer sells at p' only if all the other producers are of the low quality, so that the expected profit contains Δ term. We take this into account when assessing if the equilibrium price is dominated by some out-of-equilibrium price. The thresholds obtained from condition $\pi(p^*) = \pi(p')$ are (presented also in Table 2.2):

$$\begin{aligned} y &= \frac{1}{\Delta}\Gamma p^* \\ z &= \Gamma p^* \\ y' &= \frac{1}{N\Delta}\Gamma p^* \\ z' &= \frac{1}{N}\Gamma p^* \end{aligned}$$

not profitable for the high-quality producer as well. The expected profit of the low-quality producer at some price which is equal or lower than l is $x\Gamma p$, the same for high-quality producer. On the other hand, the expected profits at p_H are $\pi(p_H)$ and $\frac{1}{N}\pi(p_H)$ for high and low-quality producer, respectively. $\pi(p_H) > \frac{1}{N}\pi(p_H) > x\Gamma p$. It follows that if deviation is not profitable for the low-quality producer, than it is certainly not profitable for the high-quality producer.

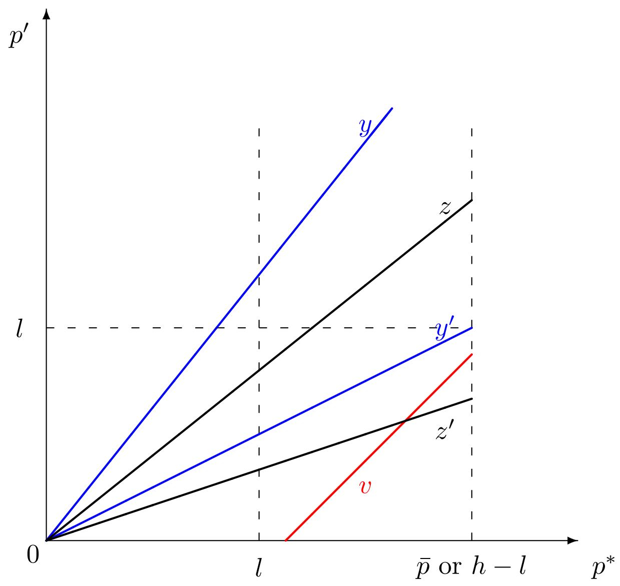


Figure 2.4: Thresholds y, z, y', z' and v: We can make an inference about the existence of the pooling equilibrium by observing these thresholds. The slope of y is above 1, thus the lower bound in region I is 0. $v < z$ for all $p^* < l$, thus the upper bound in region I is l. In this case the pooling equilibrium can be sustained at any price between 0 and l. In region IV there exists p' which is higher than both, y' and v. Thus, the equilibrium does not exists at any price in region IV.

When we summarise the above conditions we can conclude the following:

If p' and p^* are lower then l (region I), then p' dominates p^* if $p^* > p'$, $p' > v$ and $p' > y$, or if $p' < v$ and $p' > z$. If p' is lower lower then l, and p^* is higher than l (region IV), then p' dominates p^* if $p' > v$ and $p' > y'$, or if $p' < v$ and $p' > z'$.

The thresholds y, z, y' and z' are presented graphically in Figure 2.4. Based on these thresholds, we are able to determine the bounds of the sets of the prices at which pooling equilibria exist. The bounds of this set in region I is determined by p_1^* and p_2^*, end in region IV is determined by p_3^* and p_4^*, where:

$$p_1^* = \begin{cases} 0 & if \quad \frac{\Gamma}{\Delta} < 1 \\ l & else \end{cases}$$
$$p_2^* = \{p^* : z = v\}$$
$$p_3^* = \{p^* : y' = l\}$$
$$p_4^* = \{p^* : z' = v\}.$$

We present the thresholds more in details in Figure 2.5 for region I, and in Figure 2.6 for region IV, we formally prove the existence of the pooling equilibria in the proof of Propositions 16 and 17.

The Existence of the Pooling Equilibrium

An insight about the existence of the pooling equilibria can be obtained by combining Figures 2.1, 2.3 and 2.4. However, we formally define the set of pooling equilibria in Propositions 16 and 17.

Based on previously defined p_1^*, p_2^*, p_3^* and p_4^*, we are able to define the following sets, where Q and S simply refer to regions I and IV:

$$\begin{aligned}
\mathbb{P} &= \{p^* : p_1^* \leqslant p^* \leq p_2\} \\
\mathbb{Q} &= \{p^* : 0 \leq p^* \leqslant l\} \\
\mathbb{R} &= \{p^* : p_3 \leq p^* \leqslant p_4\} \\
\mathbb{S} &= \{p^* : l \leq p^* \leq Max\,\{h - l, \bar{p}\}\}
\end{aligned}$$

The subsequent proofs are based on the consumer belief that the quality at the out-of-equilibrium price is certainly low. We first prove the existence of the pooling equilibria in Region I, which is stated in Proposition 16.

Proposition 16. *In region I, any pooling equilibrium price must belong to $\mathbb{P}\cap\mathbb{Q}$.*

Proof of Proposition 16.

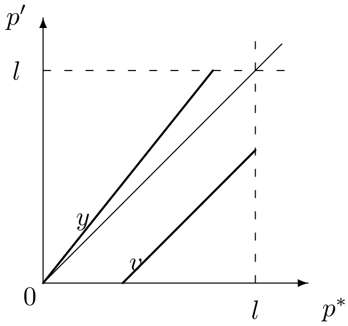
Lemma 13. *There exists a belief such that there is no deviation to out-of-equilibrium price $l > p' > p^*$.*

Proof of Lemma 13. *If $\beta(p' > p^*) = 0$, then the expected consumer surplus at p' is lower than the lowest consumer surplus in the equilibrium: $l - p' < l - p^*$, because $p' < p^*$. But, then the consumer does not buy at all at p', and the expected profit at p' is zero. Thus, when $\beta(p' > p^*) = 0$, there is no deviation to p'.*

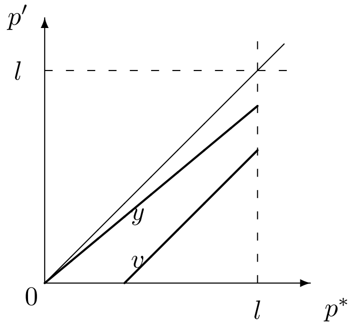
First, we consider the deviation to $p' > v$. If $\frac{\Gamma}{\Delta} > 1$, then $y > p^*$ at any p^* between 0 and l. Thus, the deviation must be to some price $p' > p^*$. But, the consumer does not buy at $p' > p^*$ by Lemma 13. Hence, the deviation to $p' > y$ is not possible. Thus, $p_1^* = 0$, that is, the lower bound of the set of the pooling equilibria is zero.

If $\frac{\Gamma}{\Delta} < 1$, then $y < p^*$ at any p^* between 0 and l. v is always lower than p^*. Thus, at any p^* between 0 and l, there exist some out-of-equilibrium prices such that both holds, $p^* > p' > y$ and $p^* > p' > v$. Thus, any such equilibrium price p^* is dominated by some p'. For this reason we set $p_1^* = l$ in such a case.

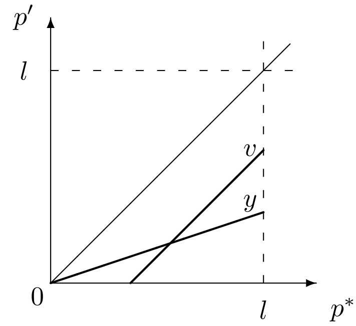
Next, we consider a deviation to $p' < v$. We remind that $v = -\alpha\,(h - l) + p^*$ and $z = \Gamma p^*$. It is evident that: (i) $\frac{\partial v}{\partial p^*} = 1 > \Gamma = \frac{\partial v}{\partial p^*}$, (ii) $v(p^* = 0) < 0$, and that (iii) both v and z are linear functions. (iv) $v = z$ at p_2^* by definition. It follows that $v < z$ at $p^* < p_2^*$ and $v > z$ at $p^* > p_2^*$. If at some p^* $v > z$, then all the out-of-equilibrium prices such that $z < p' < v$ dominate the equilibrium price p^*. Thus, the upper bound of the set of the pooling equilibria is p_2^*.



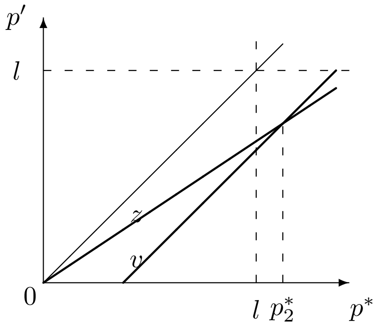
(a) y is above 45° line. Thus, $p_1^* = 0$.



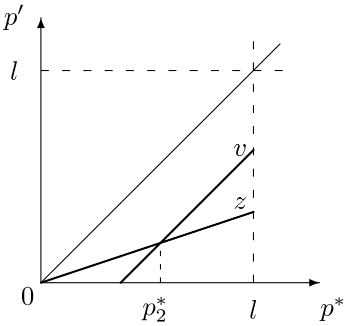
(b) $y > v$ at any $p^* < l$, and y below the 45° line. Thus, $p_1^* = l$.



(c) Although there are some prices $p^* < l$ such that $y < v$, still there is a deviation to out-of-equilibrium prices $l > p' > v > y$. Thus, again $p_1^* = l$.

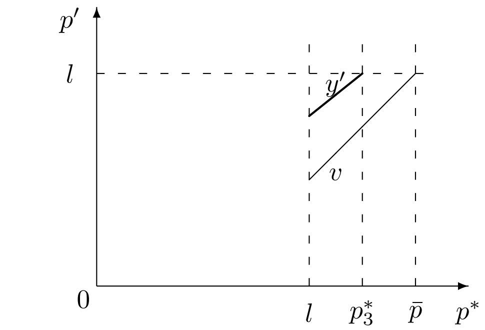


(d) $p_2^* > l$.

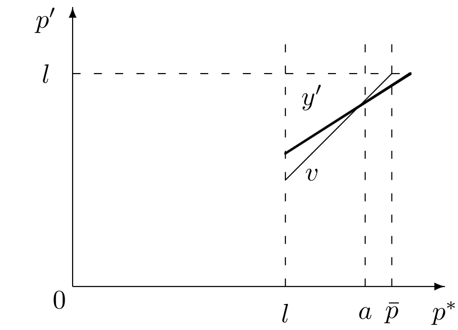


(e) $p_2^* < l$. At $p^* > p_2^*$ there is $v > p' > z$ to which the producer deviates.

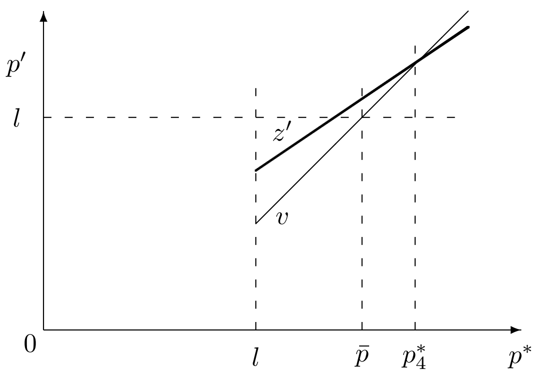
Figure 2.5: Existence of pooling equilibrium in region I (y and z): Possible relationships between v and y are presented in Figures 2.5(a)-2.5(c). This relationship determines the lower bound p_1^* of the set of the pooling equilibria. The possible relationships between v and z are presented in Figures 2.5(d)-2.5(e), which determine the upper bound p_2^* of the set of the pooling equilibria.



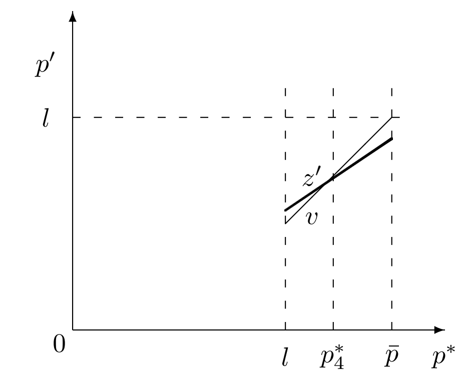
(a) Lower bound p_3^*. At any $p^* < p_3^*$ there exists $p' < l$ which is larger than both, v and y'.



(b) $y' = v$ at $a < \bar{p}$. It must be that $p_4^* < a$, thus the pooling equilibrium does not exists in region IV.



(c) The upper bound p_4^* is larger than $\bar{p}$. $z' > v$ for all the equilibrium prices p^* in region IV.



(d) At prices above p_4^* there are out-of-equilibrium prices $l > v > p' > z'$, so that the low-quality producer deviates.

Figure 2.6: Region IV (y' and z')

It follows that the equilibrium price p^* dominates all out-of-equilibrium prices if $\mathbb{P} \cap \mathbb{Q}$. $\square$

We proceed by showing the set of pooling equilibria in region IV. The proof is composed of demonstrations that the equilibrium prices below p_3^* are dominated and next, that the equilibrium prices above p_4^* are dominated.

Proposition 17. *In region IV, any pooling equilibrium price must be in $\mathbb{R} \cap \mathbb{S}$.*

Proof of Proposition 17. We demonstrate the conditions for the existence of the pooling equilibrium such that $l < p^* < h - l$, and we restrict the analysis to region IV.

We concluded previously that in region IV there exists some p' which dominates p^* if $p' > v$ and $p' > y$, or if $v > p' > z'$.

We look under which conditions $p' > v$ and $p' > y'$. y' is linear and continuously increasing, and it reaches l at p_3^*. Thus, for any $p^* < p_3^*$ we can find some p' such that $p' > y'$. v is also linear and increasing. It reaches l at $\bar{p}$. Thus, for any p^* in region IV, there exists some p' such that $p' > v$. Thus, at any $p^* < p_3^*$ there exists some p' such that $p' > y'$ and $p' > v$, that is, p' dominates p^*.

It remains to demonstrate that at $p^* > p_4^*$ there are some p' which dominate p^*. The demonstration is like in proof of Proposition 16. We remind that $v = -\alpha(h - l) + p^*$ and $z' = \frac{\Gamma}{N} p^*$. It is evident that: (i) $\frac{\partial v}{\partial p^*} = 1 > \frac{\Gamma}{N} = \frac{\partial v}{\partial p^*}$, (ii) $v(p^* = 0) < 0$, and that (iii) both v and z' are linear functions. (iv) $v = z'$ at p_4^* by definition. It follows that $v < z'$ at $p^* < p_4^*$ and $v > z'$ at $p^* > p_4^*$. If at some p^* $v > z'$, then all the out-of-equilibrium prices such that $z' < p' < v$ dominate the equilibrium price p^*. Thus, we show that the pooling equilibrium cannot exist at some price above p_4^*.

Thus, it follows that the pooling equilibrium price in region IV must be between p_3^* and p_4^*, that is, it must belong to $\mathbb{R} \cap \mathbb{S}$.

$\square$

Parameter Values for which the Pooling Equilibrium Exists

We structure the analysis in the following way: Which is the effect of parameters N, M, α, h and l on the thresholds y, y', Z, z' and v? Which is the effect of these thresholds on p_1^*, p_2^*, p_3^* and p_4^*? These effects are summarised in Tables 2.3 and 2.4.

Beside p_1^* which can be zero or l, we can also explicitly determine the values

	y	y'	z	z'	v
N		$-$		$-$	
M	$+$	$+$	$-$	$-$	
α	$+/-$	$+/-$	$+/-$	$+/-$	$-$
$h-l$					$-$

Table 2.3: The effect of the parameters on the thresholds

	p_1^*	p_2^*	p_3^*	p_4^*
y	$-$			
z		$+$		
y'			$-$	
z'				$+$
v		$-$		$-$

Table 2.4: The effect of the thresholds on p_1^*, p_2^*, p_3^* and p_4^*

of the remaining bounds:

$$p_2^* = \frac{\alpha\,(h-l)}{1-\Gamma}$$
$$p_3^* = \frac{lN(1-\alpha)^{M-1}}{\Gamma}$$
$$p_4^* = \frac{\alpha\,(h-l)}{1-\frac{N}{\Gamma}}$$

We can summarise the effect of parameters on p_1^*, p_2^*, p_3^* and p_4^* as follows:

Effect of N: The consumer group size does not affect p_2^*, it increases p_3^* and decreases p_4^*. Thus, in general, the group size decreases the maximum equilibrium price.

Effect of M: The competition decreases the probability to sell the product in the equilibrium (that is represented by Γ). This makes the deviation profitable and decreases the maximum equilibrium price (decreases both p_2^* and p_4^*).

Effect of α: The effect is ambiguous.

Effect of $h-l$: The difference in quality $h-l$ increases the maximum equilibrium price.

2.5 Conclusion

In this paper we question if quality signalling by price occurs when the consumers communicate about the quality in an oligopolistic market. We study both pooling and separating equilibria in a monopoly and in an oligopoly, and we obtain the following results. First, the signalling occurs in the monopoly if the consumer groups are sufficiently large. Second, when the product variability and consumer groups are endogenous, an increase in the cost of social ties decreases the product variability. Third, the consumer communication also enables signalling in the oligopoly. Although we study similar oligopoly frameworks, Janssen and Roy (2010:202)[32] find that the high-quality producers signal their quality, while we find that the low-quality producers signal their quality. Forth, we study a unique signalling equilibrium in oligopoly which satisfies the D1 criterion. We find that despite the fact that the competition decreases the expected social surplus, the expected consumer surplus increases with the competition, so that the loss due to the competition is completely borne by the producers. However, if the consumers group size is endogenous, then the competition my increase the expected profit. Fifth, there is a multiplicity of pooling equilibria in the oligopoly, where the consumer group size and the competition increase the maximum possible price, while the product variability decreases it.

There are methodological and modelling limitations of this study. First, for the sake of tractability, we had to restrict the analysis to two qualities. Thus, in the monopoly case we practically obtain the corner equilibrium solutions from Chapter 1. Second, we had to limit our analysis of the pooling equilibrium in the oligopoly case by the assumption that the experimenter may repeat the purchase, but from the different producer. Third, like Chapter 1, the model is based on the assumption that the the price is set once, thus, it diverges from the two-period models like Kennedy (1994)[34] and Campbell (2010)[16]. Forth, the integer problem may be an issue in this analysis, especially when endogenous effects are taken into consideration. Finally, we undertake welfare analysis on a very abstract model, so the eventual policy implications must be considered with a caution.

The reviewed literature indicates that the signalling role of price is a topic which has not been exhausted yet, in our view mostly due to the intractability of the analysis. The models are restricting, and there is no evidence that the results still hold when some assumptions are relaxed. In our model, it would be interesting to obtain the results with more than two quality types, or with costs which increase with quality. Furthermore, based on endogenous variables, such as consumer groups size, probability that the high quality occurs, or the low and the high quality, additional policy implications can be obtained. Finally, it can be checked if the results significantly change in a game where the consumers propose the price and the producer accepts or refuses it.

B

Appendix for Chapter 2

B.1 A Monopoly

Proof of Proposition 9.

Lemma 14. *$p^*_M \in [lN, \bar{p}]$ constitutes a pooling equilibrium of the signalling game.*

Proof of Lemma 14. *If the price above p^*_M occurs, then the consumer believes that the producer type is certainly of the low quality, $\mu(l|p > p^*_M) = 1$. Therefore, the consumer expected surplus if he buys is $\sum_\theta \mu(\theta|p > p^*_M)S(p, 1, \theta) = l - p < 0$ which is lower than the expected consumer surplus if there is no exchange, which is zero. If the experimenter does not buy at some price above p^*_M, then there is no producer type who sets such a price.*

We now show that both producer types choose p^*_M. Consider the consumer beliefs such that the consumer buys at all the prices equal or lower than p^*_M. The best response of the high-quality producer type is to set p^*_M. The low-quality producer type chooses between profit extracted from the volume of trade $\pi(l, 1, l) = l$, or from trading only with the experimenter at p^*_M, $\pi(p^*_M, 1, h)$. The low-quality producer's best response is to set p^*_M if $\pi(l, 1, l) = l < \frac{p^*_M}{N} = \pi(p^*_M, 1, h)$.

It remains to show that the signalling equilibrium is sustained if $p^*_M \leq \bar{p}$. The consumer beliefs at p^*_M must follow the Bayesian updating, that is, $\mu(h|p^*_M) = \alpha$ and $\mu(h|p^*_M) = 1 - \alpha$, because both producer types choose p^*_M. Hence, the expected quality must be equal to $\bar{p}$ at the equilibrium price. If $p^*_M \leq \bar{p}$, then the expected consumer surplus is positive or zero, so that the best response of the experimenter is to buy. Hence, the pooling equilibrium is sustained.

Lemma 15. *There does not exist a pooling equilibrium such that $p^*_M < lN$.*

Proof of Lemma 15. *Suppose the experimenter buys at any price lower or equal to p^*_M. Then, at any price above l, the profit of the low-quality producer is equal to $\frac{p}{N}$, and at any price equal or lower than l it is equal to p. Furthermore, for every belief about quality at price $p \leq l$, the experimenter's surplus is strictly positive and the best response is to buy (x=1). But, then the best response of the high-quality producer type is to set l because $p^*_M < lN$. Hence, the price p^*_M which is lower than lN cannot constitute the pooling equilibrium.*

Lemma 16. *There does not exist a pooling equilibrium such that $\bar{p} < p^*_M$.*

Proof of Lemma 16. *In the pooling equilibrium both types set* p_M^**, expected consumer surplus must be* $\bar{p} - p_M^*$ *which is negative if* $\bar{p} < p_M^*$*. Hence, the best experimenter's response is not to buy.*

We conclude that there are no beliefs which can sustain the signalling equilibrium where $p_M^* < lN$ in Lemma 15, or $\bar{p} < p_M^*$ in Lemma 16, while there is a belief which can sustain the equilibrium where $lN < p_M^* < \bar{p}$ which is demonstrated in Lemma 14. □

B.2 An Oligopoly

B.2.1 High-quality Producers Play a Mixed Strategy

We study separating equilibria which occurs due to different forces, relative to what we present in Chapter 2. We find that such equilibria do not exist. Consider an equilibrium where the experimenter buys at the low quality only if there is nobody at high quality. We define a mixing strategy of the high-quality firm. The firms sell at $\underline{p}_H$ for sure, that is the expected profit is than $\pi(\underline{p}_H) = \underline{p}$, while at the highest price the high quality firm sells only if all the other competitors are of the low quality $\pi(\overline{p}_H) = (1-\alpha)^{M-1}\overline{p}$. We denote by $H(p)$ a probability that the high-quality firm sets the price lower or equal to p. We now write that the expected profit at some price in the support has to be equal to $\pi(\overline{p}_H)$:

$$[1-\alpha+\alpha(1-H(p))]^{M-1} p = (1-\alpha)^{M-1}\overline{p}_H$$

It follows that

$$H(p) = 1 - \frac{1-\alpha}{\alpha}\left(\sqrt[M-1]{\frac{\overline{p}_H}{p}} - 1\right)$$

We can verify that $H(p)$ behaves correctly because $H(\underline{p}_H) = 0$ and $H(\overline{p}_H) = 1$.

Before specifying all the conditions for the existence of the separating equilibrium, lets show that the consumers prefer to buy from the high-quality firm. Then, the low-quality firms cannot adopt the mixed strategies in the separating equilibrium. At every prices of the mixing strategy of the low-quality firm the following must hold:

$$[1-F(p)]^{N-1} p = \underline{p}_L$$

$$F(p) = 1 - \sqrt[M-1]{\frac{\underline{p}_L}{p}}$$

At any finite price $F(p)$ cannot be equal to 1. Hence, the mixing equilibrium of the low-quality firm in this case is not possible. We proceed by studying the pure strategy of the low-quality producer.

In order that this equilibrium is sustained the following conditions must be satisfied. First, the low-quality producer must gain at least equal expected profit when setting p_l as when imitating the high-quality producer.

$$\pi_L(p_H) = (1-\alpha)^{M-1}\overline{p}_H \leq (1-\alpha)^{M-1}\frac{1}{M}N\overline{p}_L \tag{B.1}$$

The conditions for the existence of the separating equilibrium are:

$$(1-\alpha)^{M-1}\bar{p}_H > (1-\alpha)^{M-1}\frac{1}{M}\bar{p}_L \tag{B.2}$$

$$\frac{1}{N}(1-\alpha)^{M-1}\bar{p}_H < (1-\alpha)^{M-1}\frac{1}{M}\bar{p}_L \tag{B.3}$$

The condition (B.2) is trivial and it says that the high-quality producer should not deviate to the low price. The second condition (B.3) is more interesting, and it says that the low-quality producer should not deviate to the high-quality prices. This happens when the benefit obtained from the volume of trade overcomes the benefits from setting the high price and avoiding the competition. As $\bar{p}_H = \bar{p}_L + (h-l)$, then we write the condition (B.3) as:

$$\bar{p}_L (N-M) > M(h-l)$$

Proposition 18. *There does not exist a belief which supports a separating equilibrium where the low-quality producer plays a pure strategy equilibrium with a price lower than l.*

The intuition is that the experimenter buys at all the prices below l, for any belief about quality at those price. Denote the low-quality producer price by p_L^*. Then, the best response of low-quality producer is to set price $p_L^* - \epsilon$. The low-quality producer sells with probability $\frac{1}{M}$ at the state where all other $M-1$ producers are of the low quality. The expected profit is $\frac{1}{M-1}\alpha^{M-1}p_L^*$. If the producer sets slightly lower price, he certainly sells at this state and obtains the expected profit equal to $\alpha^{M-1}(p_L^* - \epsilon)$. The last expression is strictly higher than the previous one. Hence, the best response of low-quality producer is to deviate to $p_L^* - \epsilon$. Hence, p_L^* cannot be sustained as a signalling equilibrium.

Note that pure strategy of the high-quality producer can be sustained in the equilibrium because we can find the out-of-equilibrium beliefs which prevent the experimenter to buy at a price slightly lower than p_H^*.

B.2.2 High- and Low-quality Producers Play a Pure Strategy

Now consider a pure-strategies separating equilibrium such that all high-quality producers choose p_H^*, the low-quality producers choose p_L^*, and $p_H^* > l > p_L^*$. The experimenter strategy is $x(p > l \textit{ and } p \neq p_H^*) = 0$, $x(p_H^*) = 1$ and $x(p < l) = 1$. This cannot be supported as a signalling equilibrium for the same argument as above.

We conclude that any equilibrium where the low-quality producer plays the pure strategy and sets the price below l cannot be a signalling equilibrium.

B.2.3 The Separating Equilibrium in the Oligopoly

Proof of Proposition 13. We prove the proposition by Lemmas 17 and 18. By Lemma 17 we demonstrate that producers do not deviate to the prices chosen by the producers of the opposite quality. By Lemma 18 we demonstrate that there is a belief which sustains the separating equilibrium.

Lemma 17. *If and only if* $\frac{h}{MN} < l < \frac{Mh}{2M-1}$*, then the low-quality firm does not deviate to the high-quality price, and the high-quality firm does not deviate to the low-quality price.*

Proof of Lemma 17. *Low-quality firm does not deviate to the high price if*

$$\alpha^{M-1}\left[\bar{p}_L + (h-l)\right]\frac{1}{MN} < \alpha^{M-1}\bar{p}_L. \tag{B.4}$$

and the high-quality firm does not deviate to the low price if

$$\alpha^{M-1}\left[\bar{p}_L + (h-l)\right]\frac{1}{M} > \alpha^{M-1}\bar{p}_L. \tag{B.5}$$

The above payoff holds only if:

$$\overline{p}_L \leq l < \overline{p}_L + (h-l) \tag{B.6}$$

It follows from inequality (B.4) that $\bar{p}_L > \frac{h-l}{MN-1}$ *and from inequality (B.6) that* $\bar{p}_L < l$*. Hence,* $\frac{h-l}{MN-1} < \bar{p}_L < l$*. It follows that* $\frac{h}{MN} < l$*.*

It follows from inequality (B.5) that $\bar{p}_L < \frac{h-l}{M-1}$ *and from inequality (B.6) that* $\overline{p}_L > 2l-h$*. By combining them we obtain the next condition:* $M(2l-h) < l$*. It follows that* $l < \frac{Mh}{2M-1}$*.*

Finally, it follows that $\frac{h}{MN} < l < \frac{Mh}{2M-1}$*.*

Lemma 18. *If at any out-of-equilibrium price the experimenter beliefs that quality is low, then there is no producer who wants to deviate to the out-of-equilibrium prices.*

Proof of Lemma 18. *Suppose that the consumer believes that* $\mu(l|p > l, p \neq \underline{p}_H) = 1$*. It follows that the consumer does not buy the product at any price* $p' \in (\bar{p}_L, \underline{p}_H)$*, and rather buys from the producer who sets some equilibrium price, because* $S(p') = l - p' < l - \bar{p}_L = h - \underline{p}_H$*. The consumer does not buy at any price above* $\underline{p}_H$ *because his expected surplus is negative,* $l - \underline{p}_H < 0$*.*

We conclude that no producer wants to deviate. Hence, the equilibrium can be sustained as a signalling equilibrium. □

Next, we prove the existence of the signalling equilibrium which satisfies the intuitive criterion.

Proof of Proposition 14. Notes: (i) We focus on the out-of-equilibrium prices $\overline{p}_L < p < p_H$; (ii) We check the sufficient condition for the low-quality firm to deviate to p. (iii) The first step of the intuitive criterion test allows us only to check the most optimistic beliefs: $\mu(h|p) = 1$; (iv) Conditional on this beliefs, the lowest probability that the experimenter buys the product at the out-of-equilibrium price is α^{M-1}.

$p_H - \epsilon$. We follow two steps of the intuitive criterion and we only check the condition necessary for l to deviate.

First, the expected profit of the low-quality producer is $\frac{1}{N}\alpha^{M-1}(p_H - \epsilon)$, thus he deviates to $p_h - \epsilon$ if:

$$\frac{1}{N}\alpha^{M-1}(p_H - \epsilon) > \alpha^{M-1}\overline{p}_L.$$

When is this condition consistent with the condition that low-quality producer does not deviate to the high quality? $\frac{1}{MN}\alpha^{M-1}p_H < \alpha^{M-1}\overline{p}_L$. We combine these two relations and obtain $\frac{1}{MN}\alpha^{M-1}p_H < \alpha^{M-1}\overline{p}_L < \alpha^{M-1}\frac{1}{N}(p_H - \epsilon)$. Both conditions are satisfied if:

$$\frac{1}{M}p_H < (p_H - \epsilon),$$

that is, if

$$p_H < Ml.$$

When the above condition holds, then both types are considered in the beliefs at the second step of the intuitive criterion. We proceed with the second step. Suppose that the experimenter sets belief $\mu(l) = 1$. Then, his best response is not to buy the product at all ($x(p_H - \epsilon) = 0$). Then, both producers obtain zero profit at $p_H - \epsilon$, which is lower than what they obtain in the equilibrium. Therefore, at this price the signalling equilibrium satisfies the intuitive criterion.

$\overline{p}_L < p \leq l$. Note that in that interval the low-quality producer profit is not divided by $\frac{1}{N}$. We ask what are the necessary parameters such that l deviates? We follow the first step of the intuitive criterion and set the most optimistic belief $\mu(h) = 1$ and the lowest probability to buy given this optimistic belief, α^{M-1}. It is obvious that $\alpha^{M-1}p > \alpha^{M-1}\overline{p}_L$. Thus, the low-quality producer certainly deviates to p. It follows that at the optimistic belief low-quality producer always obtains larger profit that in the equilibrium. Hence, we consider the low-quality producer in the second step of the intuitive criterion. But, then the experimenter can believe that at a price below l the quality is certainly low. Then, the experimenter does not buy the product (because the experimenter obtains more by trading at the equilibrium prices), and the producer profit is zero, what is lower than in the equilibrium. We conclude that our equilibrium passes the intuitive criterion for all $\overline{p}_L < p < l$. □

Proposition 19. *Only a separating equilibrium in the oligopoly where* $\overline{p}_L = \frac{h-l}{MN-1}$ *satisfies the D1 criterion.*

The intuition of the D1 criterion: We look at all mixed experimenter's strategies at some out-of-equilibrium price p', $x \in [0, 1]$. If the set of mixed experimenters strategies for which the producer's best response is to deviate to the out-of-equilibrium price is larger for θ than for θ', then we have to place probability zero that θ' deviates, that is, $\mu(\theta'|p') = 0$.

Proof of Proposition 19. The D1 criterion does not impose any restriction on the prices above p_H because the experimenter does not buy at such a price (because in any case he gains strictly higher consumer surplus at the equilibrium prices, as in Janssen and Roy (2008:29)[32]). Thus, we focus on the prices between p_H and $\overline{p}_L$.

The proof is composed of two parts. Firstly, we show that the minimum $x(p)$ for which the producer deviates to the out-of-equilibrium price is lower for h than for l. Based, on this comparison, we can conclude about the $D1$ criterion. However, the comparison is impossible if the parameters are such that the experimenter does not buy at any price even if the beliefs are the most optimistic, $\mu(h|p) = 1$ (that is, $g(p,1) = 0 \ \forall \ p \in \left(l, \underline{p}_H\right)$). Therefore, secondly, we demonstrate that when the beliefs are the most optimistic, then the experimenter buys the product with the probability strictly larger than the h threshold. Finally, we demonstrate that this probability decreases with the decrease in μ, so that at the end we can find μ such that $x_h \leq g(p,\mu) < x_l$.

Additional notation: x_h is the probability to buy the product at p such that the high-quality producer is indifferent between setting p_H or p. x_l is the probability to buy the product at p such that the low-quality producer is indifferent between setting p_L or p.

$g(\mu,p)$ is a probability that the states where the experimenter prefers to by at p than at some equilibrium price occurs, which depends on the out-of-equilibrium price p, as well on the beliefs at that price, $\mu(h|p)$.

$$g(\mu,p) = \left[\alpha + (1-\alpha)\left(1 - F(p_{threshold})\right)\right]^{M-1}$$

We derive $p_{threshold}$ in the following way:

$$\mu(h-l) + l - p = l - p_{threshold}$$

$$p_{threshold} = p - \mu(h-l)$$

$$F(p_{threshold}) = F(p - \mu(h-l)) = 1 - \frac{\alpha}{1-\alpha}\left[\left(\frac{\overline{p}_L}{p-(h-l)}\right)^{M-1} - 1\right]$$

$$\begin{aligned} g(\mu,p) &= \left\{\alpha + (1-\alpha)\left[1 - 1 + \frac{\alpha}{1-\alpha}\left(\left(\frac{\overline{p}_L}{p-(h-l)}\right)^{M-1} - 1\right)\right]\right\}^{M-1} \\ &= \alpha^{M-1}\left[1 + \left(\left(\frac{\overline{p}_L}{p-(h-l)}\right)^{M-1} - 1\right)\right]^{M-1} \\ &= \alpha^{M-1}\frac{\overline{p}_L}{p-(h-l)} \end{aligned}$$

We proceed with the proof:

$x_h < x_l$. The high-quality producer's profit at the equilibrium price is $\frac{1}{M}\alpha^{M-1}p_H$ while his profit at p is equal to xp. The high-quality producer is indifferent between p_H and p when $\frac{1}{M}\alpha^{M-1}p_H = xp$. We denote this threshold value by x_h. Thus, $x_h = \frac{\frac{1}{M}\alpha^{M-1}p_h}{p}$.

The low-quality producer's equilibrium profit is $\alpha^{M-1}\overline{p}_L$ while his profit at p is $\frac{1}{N}xp$. The low-quality producer is indifferent between p and $\overline{p}_L$ when $\alpha^{M-1}\overline{p}_L = \frac{1}{N}xp$. Thus, $x_l = \frac{N\alpha^{M-1}p_h}{p}$.

It is straightforward to verify that $x_h < x_l$, which follows from the equilibrium condition $\frac{1}{MN}p_H < p_L$.

$x_h \leq g(p,\mu) < x_l$. At any p we can find beliefs such that $x_h < g(p,\mu) < x_l$.

At a given price p $g(p,\mu)$ is continuously increasing in μ. When $\mu \to 1$, $x_h < g(p,\mu)$, and when $\mu \to 0$, $x_l > g(p,\mu)$. Hence, $\exists$ μ such that $x_h < g(p,\mu) < x_l$.

Finally, we conclude that any separating equilibrium in the oligopoly where $\frac{1}{MN}p_H < \overline{p}_L$ fails the D1 criterion.

The only equilibrium which satisfies the D1 criterion is where $\frac{1}{MN}p_H = \overline{p}_L$. By combining with $p_H = \overline{p}_L + (h-l)$, it follows that $\overline{p}_L = \frac{h-l}{MN-1}$.

□

The condition for the existence of this equilibrium is that $\overline{p}_L \leq l \Rightarrow lMN < h$. $\frac{1}{MN} < \frac{N}{MN+N-1}$, so this is guaranteed for any separating equilibrium which satisfies the intuitive criterion.

B.2.4 Pooling Equilibrium in the Oligopoly

We demonstrate here that probability to be drawn Γ is lower than 1 and it decreases with M:

Proposition 20. *The total probability to be drawn* Γ *is always lower than* 1 *and decreasing with* M.

Proof of Proposition 20. If $\Gamma = \frac{1-(1-\alpha)^M}{\alpha M} < 1$, then $1-(1-\alpha)^M < M\alpha$. We study the behaviour of two functions in this expression.

$1-(1-\alpha)^M$ is increasing and concave which is verified by checking the first and the second derivative.

$$\frac{d\left[1-(1-\alpha)^M\right]}{dM} = -(1-\alpha)^M \, ln\,(1-\alpha) > 0$$
$$\frac{d^2\left[1-(1-\alpha)^M\right]}{dM^2} = 1 > 0$$

The other function, $M\alpha$ is monotonically increasing in M. Two functions intersect when $M=1$. It follows that $1-(1-\alpha)^M < M\alpha\ \forall M \geq 1$. Hence, the total probability to sell the product is lower or equal to one in our model.

Furthermore, while $M\alpha$ is linear, $1-(1-\alpha)^M$ is increasing at the monotonic and decreasing rate. Hence, Γ decreases with M. □

The effects of the parameters on the thresholds

$$\Gamma = \frac{1-(1-\alpha)^M}{M\alpha}$$
$$\Delta = (1-\alpha)^{M-1}$$

$$\frac{\Gamma}{\Delta} = \frac{1}{M\alpha\,(1-\alpha)^{M-1}} - \frac{1-\alpha}{M\alpha}$$

$$\begin{aligned}
\frac{\partial \frac{\Gamma}{\Delta}}{\partial M} &= -\frac{1}{M^2\alpha^2\,(1-\alpha)^{2(M-1)}} \left\{[\ln(1-\alpha)]\,(1-\alpha)^{M-1} M\alpha + (1-\alpha)^{M-1}\alpha\right\} + \frac{1}{M}\frac{1-\alpha}{\alpha} \\
&= -\frac{(1-\alpha)^{M-1}\alpha}{M^2\alpha^2\,(1-\alpha)^{2(M-1)}} \{[\ln(1-\alpha)]\,M + 1\} + \frac{1}{M}\frac{1-\alpha}{\alpha} \\
&= -\frac{1}{M^2\alpha\,(1-\alpha)^{M-1}} \{[\ln(1-\alpha)]\,M + 1\} + \frac{1}{M}\frac{1-\alpha}{\alpha} \\
&= \frac{1}{M^2}\frac{1-\alpha}{\alpha}\left\{1 - \frac{1}{(1-\alpha)^M}[\ln(1-\alpha)]\,M - 1\right\} \\
&= \frac{1}{M^2}\frac{1-\alpha}{\alpha}\left\{-\frac{1}{(1-\alpha)^M}[\ln(1-\alpha)]\,M\right\} > 0
\end{aligned}$$

The effect of α:

$$\begin{aligned}
\frac{\partial \Gamma}{\partial \alpha} =& \frac{M(1-\alpha)^{M-1} M\alpha - M\left[1-(1-\alpha)^M\right]}{M\alpha} \\
=& \frac{1}{M\alpha^2}\left[(1-\alpha)^{M-1}(M\alpha + 1 - \alpha) - 1\right]
\end{aligned}$$

The effect of α on Γ is ambiguous, but the effect on Δ is positive:

$$\frac{\partial \Delta}{\partial \alpha} = -(M-1)(1-\alpha)^{M-2} < 0$$

The effect of α on $\frac{\Gamma}{\Delta}$ is ambiguous.

Chapter 3

Quality, Cheating, and Consumer Communication: Evolutionary Approach

3.1 Introduction

The scope of this paper is the interrelation between the consumer communication and the trade of unobservable goods. Those are goods which quality is known by the producers, but not by the consumers prior to their purchase (for example, services, innovations and imported goods). There is some evidence in the literature about the growing importance of the unobservable goods and about the importance of the consumer communication for their trade.

In their literature review, Dranove and Zhe Jin (2010:938)[20] suggest that, together with brand development and personal experience, word-of-mouth is the most common quality assurance mechanism. The fraction of services exchanged in the GDP increases over the last years, and they are becoming the subject to international trade. For instance, Crino (2010:595)[17] claims that recently the service off-shoring has become a phenomenon. We might speculate that this is an argument for different processes. It is debatable if this trend is a result of growing long-distance social networks (e.g. Internet communities). In any case, findings of Rauch and Trindade (2002)[45] and Rauch and Casella (2003)[46] are in line with such an interpretation. In the first paper the authors find empirically that the Chinese network increases more the long-distance trade for the differentiated than for the homogeneous products[1]. In the second paper the authors assume that the ties through international information-sharing networks help the producers to solve their matching problems, and they find theoretically that if the share of the tied producers is insufficiently large, the two trading countries become relatively insulated in terms of the factor price correlations. Furthermore, there is some evidence that the consumer communication motivates tourists to visit a new destination; encourages new patient to visit a dentist; and it promotes the trade of fresh food. Already Krbec, D. (2000), Grgona(2002:742)[27], and more recently Persurić et al (2010:10)[44] have emphasised word-of-mouth as the main source of information based on which the tourists choose the Croatian destinations.

Burica (2010)[13] reports that an interviewed dentist tried several marketing tools to attract foreign patients, and he concluded that the best tool was a personal recommendation[2]. Briz et al(2007:158)[11] find that in Turkish fresh-food market the new varieties are mostly communicated between the growers by word-of-mouth, and that the most of the exporters use friends and contacts as an information source about some new market. Ottman et al (2006:34)[43] mention the example of a laundry company who successfully promoted their environmental friendly product by credible messages dispatched through consumers' Internet networks. The mentioned examples show that word-of-mouth affects the trade of the unobservable goods.

Furthermore, a change in the consumer communication affects the motivation of the producers to misrepresent their quality. For example, when the

[1] Differentiated products posses a reference price, while homogeneous do not. The reference price is a price that is quoted without mentioning brand name or other producer identification (Rauch and Trindade (2002:117)[45]).

[2] 'When a patient has experienced a good treatment, competency and a well-equipped ordination, then it is very likely that he is going to visit us again, and that he is going to recommend us to others...'

restaurants in Split were asked about quality of the fish they serve, several owners commented that the times when fresh eyes were put into old fish, or when the octopus arm was put into the shrimp's tail have passed (Galić (2010)[25]). In other words, Split's restaurants could once run their business profitably by cheating uninformed tourists, and still some do that. Instead, nowadays the reputation is built through Internet, so that many tourists have a precise idea about the restaurant quality before arriving to a destination.

We apply evolutionary game theory with the replicator dynamics to answer which the effect of the consumer communication, in terms of the consumer groups size, on the producer quality and honesty is. Berentsen et al (2007)[12] (The evolution of cheating in asymmetric contests) use evolutionary game theory to model cheating behaviour where strategy choice is influenced exclusively by the agent's neighbourhood. They find two absorbing states, one where all agents cheat and another where all agents play fair. Conversely, we find an interior evolutionary stable state where the cheating and the honesty coexist. Both papers apply evolutionary game theory to cheating. While Berentsen et al (2007)[12] include the segregation in the model, our model is driven by 'the reputation' of the high-quality producers, which depends on the fraction of the low-quality producers who misreport their type. When referring to cheating, our model drives a result similar to those obtained from the evolutionary models with the hawk-dove games (as in Bowles (2004:79)[10]). Lozano et al(2010)[35] also use the evolutionary game theory to model the co-evolution of traders with different quality and certifying practices when there is the information asymmetry. The consumers can only imperfectly distinguish between different types of the producers. Similarly like us, they suppose that the fraction of different types affects the market price. However, the authors do not provide explicit mechanisms by which these fractions affect the price. We dedicate a particular attention precisely to this issue.

We comment on the motivation for some basic assumptions used in our model. Our starting premise, which was an issue in the previous chapter, is that the consumer communication is limited. Even if the communication is nowadays cheap due to Internet, there is a problem of information overload and reliability, so that in the end, we can assume that a limited number of consumers exploit a given peace of information. We could imagine sharing the information about a restaurant with 'friends' on Facebook. In a recent research on word-of-mouth and microblogging in Jansen et al(2009)[30] it is found that 20 percent of microblogs mention some brand, and 20 percent of those contain some expression of brand sentiments. It can be debatable which is a reach of information from a microblog. However, we suppose that the information flow is limited to a community. The producer has to take into account this constraint, and to enable the information to reach the consumers who are uninformed. Producer usually does not know which consumer is informed and which not, so that the owner has to find a way to make uninformed consumers come to the restaurant. We argue that this problem is resolved simply through a formation of the reservation price, which should be sufficiently low to attract an uninformed consumer. An additional assumption is that all the economic agents have a common general view about the economy structure, which could be obtained from the media or the general statistics for example.

The information flow is enabled when two consumers interact. Thus, the information flow might be interpreted as an integral part of the broader socialisation process. I.e. the exchange of information is accompanied by sharing the common values, trust and support. Although, all of the issues are important and together can give interesting insights, we focus only on the information flow, for the sake of tractability. In order to come up with a feasible answer we had to introduce an additional aspect. Apart form the quality choice (high or low), the producer also chooses a behavioural trait (honest or dishonest)[3]. The importance of a particular aspect varies depending on the application, so we explore several variants of the model.

We suppose that the experimenters do not know the actual product quality prior to its purchase, but that they know the fractions of different producers, based on which the market price of the high-quality good is formed. Thus, in terms of Cabral (2005:3)[14], this model is about reputation of the producers who choose the high-quality price.

We capture the product reputation by the reservation price of the high-quality a good which is equal to the expected quality of the producer who charges the reservation price. Our results are in line with Tirole (1996)[47] who finds that low-corruption (i.e. high-quality) state only occurs if the information about quality is good. In our model the likelihood of high-quality evolutionary stable state increases with the consumer communication. Simmilarly, Winfree and McCluskey (2005)[50] build a dynamic framework with an endogenous individual choice of quality which affects the collective reputation. In a model without a firm traceability, the individual incentive to producer high-quality decreases with the number of the firms in the market, due to the weaker correlation between individual choice and individual payoff. In our model the number of producers does not matter because the producers are myopic, in sense that they do not consider the effect of their choice on the price in the next period.

The contributions of this paper are the following: (1) We develop a novel framework (not a direct mechanism, but a price formation which "creates a game" between low and high-quality producers), (2) we find that the fraction of the high-quality producers and the consumer communication are positively related in the equilibrium (which should be in line with the results in the first chapter), (3) co-existence of the honest and the dishonest producers is an evolutionary stable equilibrium. First we present a general framework, followed by the findings from the several model variants.

3.2 Model

Players

There are two populations, consumers and producers, who exchange the

[3] Already Nelson (1976)[41] discussed the issue of the honesty in economics.

products. The product quality θ is a random variable $\theta \in \{l, h\}$, determined by nature and assigned to the producer. h denotes high quality and l denotes low quality, $h > l$. All the product units of some producer are of the same quality. The production costs are zero and a producer supply is infinite. The consumers belong to the groups of the same size N. An *experimenter* is a randomly drawn consumer in the group who tests the product. Only if the experimentert́s expectation about the quality is above or equal to the price, them he tests (buys) the product. After testing, the experimenter tells about the quality to the other consumers in his group. The information about the product is exclusively shared within the group. If the product quality is higher than the price, then the whole group buys the product from this producer. Otherwise, only one consumer in the group buys the product from this producer.

There are three types of the producers: high-quality (h) with the fraction α, low quality who mimics high-quality producer (m) with the fraction $(1-\alpha)\gamma$ and low-quality producer who reveals the true quality (l) with the fraction $(1-\alpha)(1-\gamma)$. Thus, γ is a fraction of the produces within the low-quality sub-population who mimic the high-quality.

Matching Process

A producer and an experimenter are matched randomly. If the product is good, then the whole group buys the product. Otherwise, only the experimenter buys it. (We assume that the producers set the price lower or equal to the experimenter's expectation).

(1) The matching is random for the experimenters. (2) Non-experimenters avoid the low-quality producer for whom they have the information. This should decrease the fraction of the low-quality producers in the next draws, and thus, have a positive effect on the reservation price in the next draws. We assume that the populations are sufficiently large so that we can neglect this effect on the reservation price (at least in the short run). (3) The non-experimenters buy with certainty from the high-quality producer if they identified him. (4) All the consumers (experimenters and non-experimenters) who have not identified the high-quality producers participate in the next draws.

Price

A producer chooses the price taking into consideration his behavioural trait and the profit maximisation. Once the producer sets the price, it remains fixed which can be justified by two reasons: (1) For some technical reasons it is not possible to change the price often; (2) The producer cannot identify if the consumer is the experimenter or not. This is an issue because one could argue that the low-quality producer who mimics high-quality producer can simply decrease the price when selling to the non-experimenter (the informed consumer). Alternatively, instead of assuming fixed price, we could assume that the non-experimenters punish this producer by not trading with him even if he subsequently sets the price below the quality.

If the consumer does not know the actual quality, he decides based on the quality expectation. $\bar{p}$ (reservation price) is the maximum price at which the product is sold:

$$\bar{p} = \mu h + (1 - \mu) l$$

where $l \leq \bar{p} \leq h$ and μ is an updated probability that the producer is of the high quality, after observing price $\bar{p}$.

$$\mu = \frac{\alpha}{\alpha + (1 - \alpha)\gamma} \tag{3.1}$$

The experimenter refuses to pay more than his expected quality. We assume that once the price is set, it is the same for all the consumers. We can assume that only one producer is matched with the consumer group and that he has a market power, so that he extracts the whole expected consumer surplus in a one-shot interaction.

Expected Payoffs

While producing the low quality is costless, producing the high quality requires costs c, so that the expected payoff of the high-quality producer is a difference between the reservation price and the production costs:

$$\pi_H = \bar{p} - c$$

The low-quality producer who mimics the high-quality one (m) sells the product only to one consumer in a group so that his expected surplus is divided by the group size.

$$\pi_M = \frac{\bar{p}}{N}$$

The only exception is when $\alpha = 0$. In that case μ is zero and reservation price $\bar{p}$ is l. Thus, even the high-quality producer sets the price equal to the low quality. If the consumers buy from any low-quality producer, they gain a non-negative profit. Therefore, the whole consumer group buys the product. Thus, the profits of mimicking and revealing low-quality producers are equal in this case, $\pi_M = \pi_L = l$.

Low-quality producer who reveals his quality sets the price equal to the quality and he sells to all the consumers in the group. Therefore, his payoff is simply equal to the low quality:

$$\pi_L = l$$

The consumers choose the size of group N where $k(N)$ is a cost of maintaining the relationship. We assume in this paper that $k(N)$ is a linear function and we denote by k a marginal cost of communication due to the additional member in the group. We assume that the consumer population is sufficiently large (or infinite) so that we can neglect the residual consumers who do not enter into the optimal groups. The consumer expected payoff π_C is composed of the weighted payoffs of the purchases from three producer types, decreased for the costs of maintaining the relationships in the group of N size:

$$\pi_C = \alpha (h - \bar{p}) + \frac{1}{N}\gamma (1 - \alpha)(l - \bar{p}) - kN.$$

The price which consumers are ready to pay for the high-quality good is determined by the expected share of the low-quality types who mix with the high-quality types γ. Before we proceed with the analysis, we provide a few notes about evolutionary game theory and the replicator dynamics.

3.2.1 Evolutionary Game Theory

The backgrounds are natural selection (from biology) and classical game theory.

Agents

The basic assumptions of the evolutionary game theory is that the agents bounded-rational. That is, the individuals have limited cognitive capacities (implying that an average human is not able to solve complicated optimisation problems), and that they update their strategies (behaviour) only occasionally buy using imperfectly observed local information.

Two Approaches

Evolutionary dynamics can be modelled by evolutionary stable strategy or by replicator dynamics. The evolutionary process combines two basic elements, *mutation mechanism* that provides variety and a *selection mechanism* that favours some varieties over others. While the criterion of evolutionary stability emphasises the role of mutations, the replicator dynamics emphasises selection. Both give the same conclusions if there are only two traits in the population. That is, among the stationary states obtained by replicator dynamics there are those that correspond to aggregate Nash-equilibrium behaviour. A proposition in Weibull(1996:100) shows the link between two approaches.

Proposition 21. *(Weibull,1996) Every evolutionary stable strategy is asymptotically stable in the replicator dynamics.*

We provide a short exposition about evolutionary stable strategies and then we focus on replicator dynamics which is the main tool used in this paper.

Evolutionary Stable Strategies

We provide the adjusted definition of evolutionary stable strategy taken from Weibull (1996:36)[49]. Lets Δ be a mixed strategy set and $x \in \Delta$ and $y \in \Delta$ be some strategies. Lets $\pi(x, y)$ be a payoff to strategy x when playing against y.

Definition 8. $x \in \Delta$ *is an evolutionary stable strategy if for every strategy* $y \neq x$ *there exists some* $\bar{\epsilon} \in (0, 1)$ *such that*

$$\pi\left[x, \epsilon y + (1 - \epsilon)\, x\right] > \pi\left[y, \epsilon y + (1 - \epsilon)\, x\right]$$

Replicator Dynamics

We apply a replicator dynamic equation in our model. After exchange the producers are matched with the agents from the own population. If they are matched with a different type, and if this type obtained more than they did, they are likely to shift their type to the opponent. The likelihood is proportional to the difference in payoffs (Weibull, 1995:69)[49]. This could be also interpreted in the following way: at the end of the period, the agents get their offspring whose numerosity depends of the difference in payoffs (Bowles, 2004)[10]. We formalise the replicator dynamics in a standard way, that is, as a system of ordinary differential equations that do not include a mutation mechanism. Instead, the robustness against mutations is indirectly taken care of by the way of dynamic stability criteria. In contrast to evolutionary stable strategies approach, the usual replicator dynamics presumes that individuals can only be programmed to pure strategies. There is random pairwise matching in a large population where payoffs represent fitness, measured as the number of offspring, and each offspring inherits its single parent's strategy. If the reproduction takes place continuously over time, then this results in a certain population dynamics in continuous time. This dynamics is called *replicator dynamics*.

Consider a large finite population of agents who are programmed to the behavioural trait $i \in \{1, ..., K\}$ with payoff π_i. Let $n_i(t) \leq 0$ be the *number* of agents who are currently programmed to the trait i, and let $n(t) = \sum_i n_i(t)$ be the total population. The associated population state is the vector $x(t) = (x_1(t), ..., x_k(t))$ where $x_i(t)$ is a population share programmed to the trait i at time t, that is, $x_i(t) = \frac{n_i(t)}{n(t)}$. The expected payoff of an agent with the trait i when the population is in the state x is $\pi_i(x)$. The payoff to an individual drawn at random from the population is

$$\bar{\pi} = \sum_i \pi_i(x)$$

Now, we use biological terminology to arrive to the replicator equation. Suppose that payoffs represent the incremental effect of choosing a trait on an agent's fitness, measured as the number of offspring per time unit. Suppose also that each offspring inherits its single parent's strategy. If reproduction takes place continuously over time, then the birthrate at any time t, of individuals programmed to pure strategy i is $\beta + \pi_i(x)$, where $\beta \geq 0$ is the background fitness of individuals in the population. Let the death rate $\delta \geq 0$ be the same for all individuals. This results in the following population dynamics:

$$\dot{n}_i = [\beta + \pi(x) - \delta] n_i$$

It follows from the definitions that:

$$n(t) x_i(t) = n_i(t)$$

The time derivative of this expression is:

$$\dot{n} x_i + n \dot{x}_i = \dot{n}_i$$

$$n \dot{x}_i = \dot{n}_i - \dot{n} x_i = [\beta + \pi_i(x) - \delta] n_i - [\beta + \bar{\pi}(x) - \delta] n x_i$$

We divide both sides by n and obtain:

$$\dot{x}_i = \left[\pi_i\left(x\right) - \bar{\pi}\left(x\right)\right] x_i$$

Hence, the growth rate of a population share with trait i, $\frac{\dot{x}_i}{x_i}$, is equal to the difference between actual payoff of the agent with trait i and the current average payoff in the population. The growth rate is independent of the background fitness β or of the death rate δ.

Definitions

We have already defined evolutionary stable strategy. The definitions of *Lyapunov stability*, *asymptotic stability* and *basins of attraction* is provided in Weibull(1995:243-245). Intuitively (we proceed from the discussion about evolutionary stable strategy), a state $x \in \Delta$ is Lyapunov stable if no small change in the population composition can lead it away, and x is asymptotically stable if moreover any sufficiently small change results in a movement back toward x. For each evolutionary stable strategy, its basin of attraction is the set of initial conditions leading to the long-time behaviour that approaches that evolutionary steady state.

3.3 Evolution of the High-Quality Producers

Suppose that the fraction of the honest producers γ is given. We study the adoption of the high-quality among the producers. The low-quality producer who mimics the high-quality producer by setting the high price is a kind of dishonest behaviour. It could reflect an individual propensity to disclose the truth if this brings the material costs. We suppose that the honesty is a cultural trait and in the short-run unaffected by the material conditions. The dishonest producers mimic the high-quality producers even when their payoffs are lower than the payoffs of the honest producers. This is justified if we assume that the agents are bounded rational, so that they only consider the benefit from the single transaction, without accounting for the consequences on the future. The informed consumers refuse to trade with the low-quality producer who sets the high price. Thus, the dishonest producers loose some of the trade opportunities.

Following the exposition in Subsection 3.2.1, we develop the replicator dynamics of the high-quality type:

$$\dot{\alpha} = \alpha\left(\pi_H - \bar{\pi}\right)$$

where

$$\bar{\pi} = \alpha\pi_H + \left(1-\alpha\right)\gamma\pi_M + \left(1-\alpha\right)\left(1-\gamma\right)\pi_L$$

It follows that:[4]

$$\dot{\alpha} = \alpha\left(1-\alpha\right)\left[\pi_H - \gamma\pi_M - \left(1-\gamma\right)\pi_L\right]$$

[4]If γ is fixed, we can divide the producers in two subpopulations: γ and $1-\gamma$ and look at the dynamics of each population. The total dynamics would be a sum of the dynamics in subpopulations. $\dot{\alpha} = \gamma\dot{\alpha}_\gamma + \left(1-\gamma\right)\dot{\alpha}_{1-\gamma}$; $\dot{\alpha}_\gamma = \alpha\left(1-\alpha\right)\left(\pi_H - \pi_M\right)$; $\dot{\alpha}_{(1-\gamma)} = \alpha\left(1-\alpha\right)\left(\pi_H - \pi_L\right)$.

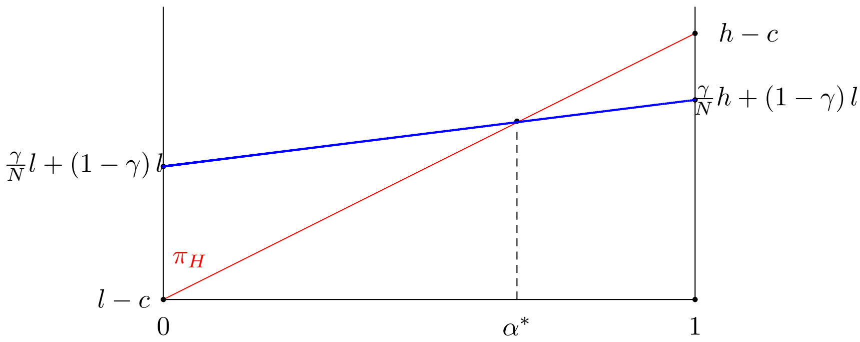


Figure 3.1: The interior stationary fraction of the high-quality producers α^*

As we have already discussed in the previous section, we presume that the agents are bounded rational, in a sense that they only periodically update their behaviour traits and eventually adopt a strategy that brings a higher payoff. If the producer is of the low-quality, he is going to mimic a high-quality producer by setting a reservation price, even if setting price equal to l would be profitable.

It is visible from the replicator dynamics equation that there exist several stationary states: $\alpha = 0$ and $\alpha = 1$, and there may exists an interior stationary state $0 < \alpha^* < 1$. The interior stationary state satisfies the condition:

$$\pi_H = \gamma\pi_M + (1-\gamma)\,\pi_L$$

Thus, the interior stationary state is:

$$\alpha^* = \frac{\gamma\,[\gamma l\,(N-1) - cN]}{(N-1)\,l\gamma^2 + [h-(c+l)\,N]\,\gamma + (c+l-h)\,N}$$

This condition is presented in Figure 3.1.

The horizontal coordinate is a fraction of the high-quality producers α which can range from 0 to 1. The vertical coordinate represent the payoffs: π_H and $\gamma\pi_M + (1-\gamma)\,\pi_L$ (An average low-quality producer payoff). The thin line, which initiates at $(0, l-c)$, is the expected payoff of the high-quality producer, π_H. The less inclined and thick line is the average low-quality producer payoff $\gamma\pi_M + (1-\gamma)\,\pi_L$. Both payoffs increase with the fraction of the high-quality producers. This increase is a result of the increased reservation price, owing to the improved experimenters expectations about the quality when α increases. However, an increase in α affects more the payoff of the high-quality producer because he extracts the benefits from trade with whole consumer group, and not only from the experimenter. Therefore, the interior stationary point exists if $\frac{\gamma}{N} l + (-\gamma)\, l > l - c$ and $\frac{\gamma}{N} h + (-\gamma)\, l < h - c$. Thus, we obtain the following condition:

$$l\gamma \frac{N-1}{N} < c < h\gamma \frac{N-1}{N}$$

The interior equilibrium exists if l is sufficiently small, and h is sufficiently large relative to c. The reason is that the change in price, which is caused by the change in α, has much stronger effect on the profit of the high-quality producer because he trades with the whole consumer group. If the consumer group size N is small relatively to the other parameters, the interior equilibrium may not exist. This is because in that case the average low-quality profit is always larger than the profit of the high-quality producer. On the other hand, the interior equilibrium may not exist as well due to very large N because in that case the profit of the high-quality producer is always larger than the average low-quality producer profit.

We check for the stability and we find that exterior stable states are stable, while the interior is unstable, as it is written in Proposition 22 and demonstrated in Appendix C on page 101.

Proposition 22. *Evolutionary stable states are $\alpha = 0$ and $\alpha = 1$.*

An unstable interior stationary state α^* divides two basins of attraction, one where $\alpha = 0$ is the attractor, and the other where $\alpha = 1$ is the attractor. If the initial fraction of the high-quality producers is above α^*, then it is going to evolve to $\alpha = 1$, and otherwise to $\alpha = 0$. Larger basin of attraction means that larger set of initial states leads to some evolutionary stable state. We are interested in factors which extend the basin of attraction of $\alpha = 1$, and thus, we study the effect of N and γ on α^*. In line with our prior expectation, an increase in the consumer group size N enlarges the basin of attraction of the high quality, which follows from Proposition 23, which proof is in Appendix C on page 102.

Proposition 23. *If consumer group size N increases, the interior equilibrium fraction of the high-quality producers α^* decreases.*

The increase in the consumer group size N decreases the interior stationary fraction of high-quality producers α^* because the average profit of the low-quality producer decreases while the profit of the high-quality producer remains unchanged. The effect of N works through the payoff of the low-quality producer who mimics high-quality producer. Therefore, this effect is stronger, larger is γ. The effect of N should be stronger at higher l and h, and larger c. The effect is strongest if c is very large. This could mean that social networks stimulate socially inefficient production.

On the other hand, an increase in the fraction of the low-quality producers who mimic high-quality producers enlarges the basin of attraction of $\alpha = 0$ (Proposition 24 proved in Appendix C on page 103).

Proposition 24. *The interior stationary point α^* increases with the fraction of low-quality producers who mimic high-quality producers γ.*

At a given reservation price $\bar{p}$, change in γ causes the change in $\bar{p}$, and change in $\bar{p}$ is reflected more in the profit of the high-quality producer than in the profit of the low-quality producer who mimics the high-quality producer. The profit of the low-quality producer who sets low price is unaffected by the change in γ. A decrease in the reservation price at given α means that larger α is needed so that the profit of the high-quality producer reaches the expected profits of the low-quality producers.

3.4 Evolution of Honesty when Fraction of High-quality Producers α and the Group Size N are Given

Suppose that technology represented by α changes very slowly, meanwhile the size of the consumer group and the behavioural trait γ are updated fast. Thus, we treat α as exogenous. We first explore the evolution of γ when α and N are fixed. Next, we investigate the co-evolution between the behaviour trait γ and the consumer group size N. The sub-population of the high-quality producers is composed of two types: honest and dishonest producers. Both types have the same payoff functions, so this distinction is irrelevant for the high-quality producer. On the contrary, the profit of the low-quality producer is affected by its own behavioural trait. Thus, we study the evolution of the dishonest producers within the low-quality sub-population[5], and we denote its fraction by γ. The average payoff of the low-quality subpopulation is:

$$\bar{\pi} = \gamma \pi_M + (1-\gamma)\,\pi_L$$

The replicator dynamics is:

$$\dot{\gamma} = \gamma\,(\pi_M - \bar{\pi})$$

It is zero at: $\gamma^* = 0$, $\gamma^* = 1$, and there may exist an interior stationary point $0 < \gamma^* < 1$, such that $\pi_M = \bar{\pi}$:

$$\begin{aligned}
\pi_M &= \bar{\pi} \\
\pi_M &= \gamma \pi_M + (1-\gamma)\,\pi_L \\
(1-\gamma)\,\pi_M &= (1-\gamma)\,\pi_L \\
\mu\,(h-l)\,\frac{1}{N} + \frac{l}{N} &= l
\end{aligned}$$

It follows that:

$$\gamma^* = \frac{\alpha\,(h - Nl)}{(N-1)\,l\,(1-\alpha)}$$

where $0 < \gamma^* < 1$ if $Nl - h < (h-l)(1-\alpha)$ and $h > Nl$. The interior solution $0 < \gamma^* < 1$ is an asymptotically stable interior equilibrium[6] because the following derivative is negative:

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[5] Suppose γ_α is the fraction of the honest producers in the high-quality sub-population. $\gamma_{(1-\alpha)}$ is the fraction of the honest producers in the low-quality sub-population. $\gamma = \alpha\gamma_\alpha + (1-\alpha)\,\gamma_{(1-\alpha)}$. $\dot{\gamma} = \alpha\dot{\gamma}_\alpha + (1-\alpha)\,\dot{\gamma}_{(1-\alpha)}$. $\dot{\gamma}_\alpha$, so $\dot{\gamma} = (1-\alpha)\,\dot{\gamma}_{(1-\alpha)}$. For the simplicity we suppose that $\dot{\gamma} = \dot{\gamma}_{(1-\alpha)}$.

[6] Note that here we do not check for evolutionary stable strategies when concluding about the stability of the stationary state, but the results should be the same as if we concluded directly from the replicator equation. This was discussed previously.

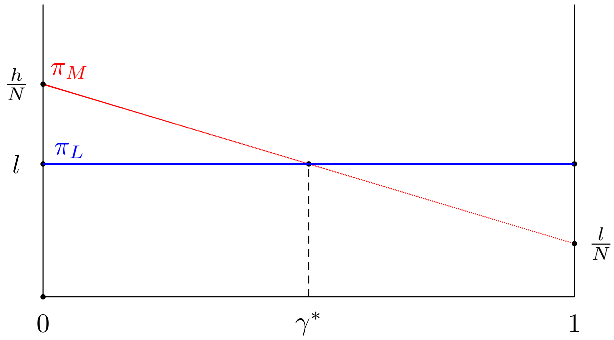

Figure 3.2: The interior stationary fraction of low-quality producers who mimic high-quality producers γ^*

$$\frac{d\left(\pi_M - \pi_L\right)}{d\gamma} = -\frac{\alpha\left(1-\alpha\right)\left(h-l\right)}{N\left[\alpha + \left(1-\alpha\right)\gamma\right]^2} < 0$$

The reason is that π_L is independent while π_M is constantly decreasing with γ. An increase in γ decreases the reservation price $\bar{p}$ (Figure 3.2). Hence, if the sub-population of the low-quality types is lower than γ^*, then their payoff is higher than the payoff of the low-quality producers who reveal their quality. On the other hand, if their fraction is higher than γ^*, then the opposite holds.

Marginal effect of Parameters on γ^*

An increase in the fraction of the high-quality producers α increases the equilibrium fraction of the dishonest low-quality producers γ^* since the increase in α increases the reservation price, and increase in the reservation price increases the profit of m. The effect of α is stronger at the higher values of α:

$$\frac{\partial^2\gamma^*}{\partial\alpha^2} = \frac{2(h-lN)^2}{l^2(N-1)^2(1-\alpha)^3} > 0$$

$$\frac{\partial\gamma^*}{\partial\alpha} = \frac{h-lN}{l(N-1)(1-\alpha)^2} > 0$$

An increase in N shifts π_M down and, thus, decreases the equilibrium fraction of the low-quality producers. An increase in h or α increases the equilibrium fraction of α. An increase in l decreases the fraction of l because the effect of l on payoff of l is much stronger than on the payoff of m.

γ^* decreases with the consumer group size N:

$$\frac{\partial \gamma^*}{\partial N} = -\frac{\alpha}{1-\alpha} \frac{h-l}{l\,(N-1)^2} < 0$$

The intuition is that the profit of the low-quality producer who imitates the high quality decreases because there are less experimenters who test his product (because the consumer population is organised in the larger groups). The intensity of this marginal effect decreases as the consumer group size increases:

$$\frac{\partial^2 \gamma^*}{\partial N^2} = \frac{2\alpha^2 (h-l)^2}{l(N-1)^3(1-\alpha)} > 0$$

While the effect of h is always positive ($\frac{\partial \gamma^*}{\partial h} = \frac{\alpha}{l(N-1)(1-\alpha)}$; h increases $\bar{p}$ and this increases π_M while π_L remains unchanged), the effect of l on γ^* is ambiguous and depends on the sign of $N + h - lN$, that is:

$$\frac{\partial \gamma^*}{\partial l} = -\frac{\alpha}{1-\alpha} \frac{1}{l(N-1)} (N + h - lN)$$

Increase in l shifts both lines in the figure, so depending on h, l and N this shifts will decrease or increase γ^*.

$$\frac{d\dot{\gamma}}{d\gamma} = (1-2\gamma)(\pi_M - \pi_L) + \gamma(1-\gamma) \frac{d(\pi_M - \pi_L)}{d\gamma}$$

3.5 Conclusion

The aim of this paper is to explore the evolution of producers' quality and honesty when there is a communication among consumers, and the producers are bounded-rational. We explore a model where the low-quality producers consider if to cheat by misreporting their type. This approach is justified by observations like in Zak (2011:226)[51] that moral violations are most likely to occur if the incentives to lie, cheat, or steal are sufficiently pronounced, and if the others with whom an individual interacts behave in such a way. The producers affect each other through the market price of the high-quality product. The price increases with an increase in the share of the high-quality or of the honest producers.

We provide a micro-foundation of the high-quality reputation dynamics and we arrive to the following results. Firstly, we study the model where the quality trait is endogenous only. We find that the share of honest producers in the population increases the probability to evolve toward an evolutionary stable equilibrium where all the producers are of the high quality. Furthermore, we explore the model with endogenous honesty only, and we find an evolutionary stable state where honest and dishonest producers co-exist. An increase in the fraction of the honest producers increases the market price of the high-quality good, which increases the payoff of the dishonest producers and increases their fraction, which in turn reduces the market price.

Our model is different from the models initiating with Kandori (1992)[33] for two reasons. Firstly, Kandori (1992)[33] ground his finding on the assumption that there is a sufficient number of punishers. Although being a completely different mechanism, the consumer groups have a similar role like punishers because they do not buy the product after experimentation if its quality is low. Secondly, while in the cited model the agents behave honestly because honesty is rewarded in the future transactions, in our model the agents are backward-looking. At the end of the game, the agents compare their payoffs with the payoffs of the agents who played different strategies, and based on this comparison they decide if to behave honestly in the next period.

Ahn and Suominen (2001)[2] also consider reputation and cheating (in this case the price is fixed and cheating refers to the provision of the low quality). Although they apply classical game theory, they arrive to the finding which is in line with our results. As the communication between consumers increases, the seller's incentive to provide high quality becomes easier to sustain, and this is in line with our finding that an increase in the consumer group decreases the fraction of the low-quality producers who misreport their quality.

Acemouglu (2010)[1] suggests that future researches should answer why all the countries do not choose the optimal institutions (if it is relatively simply to copy them today when the ideas travel around the world very fast), why inefficient institutions do persist, and why many attempts to change them fail. The evolutionary game theory is a promising tool for providing answer to such questions. Bidner and Francois (2010:27)[9] attempts to provides an answer by exploring the effect of institutions on honesty. He finds that the two mutually reinforce, so that some institutional enforcement is needed to sustain honesty. We as well tackle the problem of cheating, but instead of (formal) institutions, we relate it to the quality and we explain why a dishonest behaviour may persist. Thus, this approach opens up a new avenue for explaining the international development differences.

The next step is to develop a model where quality and honesty co-evolve. Furthermore, the consumer group size can be supposed to be endogenous. However, one has to take into consideration the eventual integer problem.

C

Appendix for Chapter 3

Proof of Proposition 22. It is enough to demonstrate that both playing the high-quality or a mix of the low-qualities are Nash-equilibria against the mutant. Strategy here represents a type choice. We define the payoffs of producer type m at $\alpha = 0$ and $\alpha = 1$ as $\pi_{M(\alpha=0)} = l$ and $\pi_{M(\alpha>0)} = \frac{\bar{p}}{N}$ respectively.

First suppose that incumbent strategy is $\gamma \frac{\bar{p}}{N} + (1-\gamma) l$ and mutant strategy is h, with payoffs $\gamma \pi_M + (1-\gamma) \pi_L$ and π_H respectively.

$$\pi \left[\gamma m + (1-\gamma) l, \epsilon h + (1-\epsilon) \left(\gamma m + (1-\gamma) l\right)\right] = \gamma \frac{\bar{p}(\alpha = \epsilon)}{N} + (1-\gamma) l$$
$$= \gamma \frac{\mu(h-l)+l}{N} + (1-\gamma) l > \pi \left[h, \epsilon h + (1-\epsilon) \left(\gamma m + (1-\gamma) l\right)\right] =$$
$$= \bar{p}(\alpha = \epsilon) - c = \mu(h-l) + l - c$$

We write the above statements as

$$\gamma \frac{\mu(h-l)+l}{N} + (1-\gamma) l > \mu(h-l) + l - c$$

The above inequality holds if the following inequality holds:

$$\frac{\gamma}{N} l - \gamma l > \mu \left(h - l\right) - c$$

If we let $\alpha = \epsilon \to 0$, then it should hold $\gamma l \frac{1-N}{N} > -c$. This is satisfied when the condition for the interior stationary state holds, $\gamma l \frac{N-1}{N} < c$. Hence, the incumbent strategy $\gamma m + (1-\gamma) l$ is evolutionary stable strategy. By the proposition from Weibull (1995) it follows that $\gamma m + (1-\gamma) l$ is asymptotically stable in the replicator dynamics.

Next, consider the trait h as an incumbent strategy and $\gamma m + (1-\gamma) l$ as a mutant strategy, where $\alpha = 1$. h is evolutionary stable if $\gamma \frac{\mu(h-l)+l}{N} + (1-\gamma) l < \mu(h-l) + l - c$ when $\epsilon = 1 - \alpha \to 0$, i.e. $\alpha \to 1$. From this inequality it must hold that:

$$c < \gamma h \frac{N-1}{N} + (1-\gamma)$$

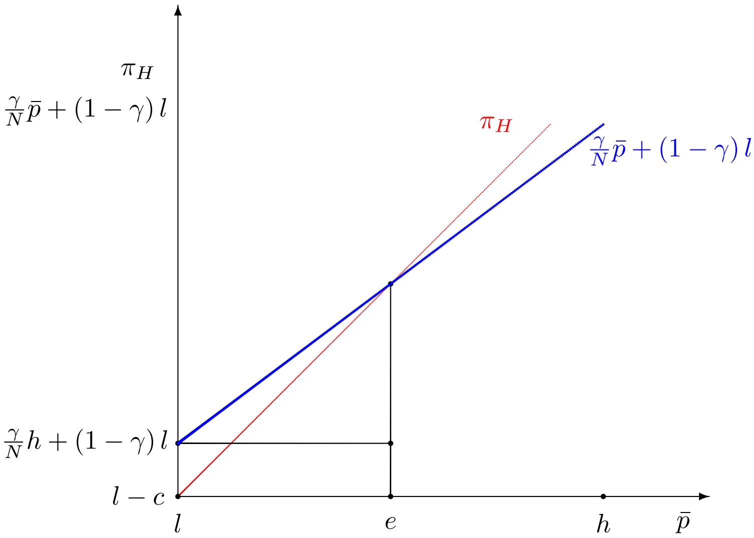

Figure C.1: Equilibrium reservation price e

This is certainly satisfied if the interior equilibrium exists, that is if $c < \gamma h \frac{N-1}{N}$. Hence, the equilibrium α is asymptotically stable.

Finally, we demonstrate that the interior equilibrium is unstable. For the interior equilibrium to be unstable it must be that small deviations from the equilibrium lead to the further deviations. We know that $\bar{\pi}(\alpha^*) = \pi_H(\alpha^*)$. We compare the payoffs if there is small increase ϵ in the fraction of the producers who play h. $\bar{p}(\alpha^*) = \alpha^*(h-l) + l < (\alpha^*+\epsilon)(h-l) + l$. We know from the equilibrium condition of α^* that $(1-\gamma)\,l + c = \bar{p}(\alpha^*)\frac{N-1}{N}$. It follows that $(1-\gamma)\,l + c < \bar{p}(\alpha^*+\epsilon)\frac{N-1}{N}$. We just rearrange this expression and obtain $\bar{\pi}(\alpha^*+\epsilon) = \pi_H(\alpha^*+\epsilon)$. □

Proof of Proposition 23. We define π_H and $\gamma\pi_M + (1-\gamma)\,\pi_L$ as functions of $\bar{p}$. If the reservation price $\bar{p}$ increases by unit, then π_H increases by unit, while $\gamma\pi_M + (1-\gamma)\,\pi_L$ increases by $\frac{\gamma}{N}$. $\pi_H = l - c < \bar{\pi} = \frac{\gamma}{N}l + (1-\gamma)\,l$ at $\bar{p} = l$. However, as π_H increases faster, the two functions become equal at reservation price $\bar{p} = e$. e must be such that the following holds (We conclude from Graph C.1):

$$e - l = \frac{\gamma}{N}(e-l) + \frac{\gamma}{N}l + (1-\gamma)\,l - (l-c)$$

It follows that the equilibrium reservation price e must be:

$$e = \frac{c - \gamma l \frac{N-1}{N}}{1 - \frac{\gamma}{N}} + l = A + l$$

That is, $A = \frac{c-\gamma l \frac{N-1}{N}}{1-\frac{\gamma}{N}}$. e is a reservation price, thus, we can write:

$$\mu\left(h-l\right)+l = A + l \tag{C.1}$$

From this we obtain:

$$\alpha^* = \frac{\gamma \frac{A}{h-l}}{1-\left(1-\gamma\right)\frac{A}{h-l}}$$

$$\frac{\partial A}{\partial N} = -\frac{-\frac{\gamma l}{N^2}\left(1-\frac{\gamma}{N}\right)}{1-\frac{\gamma}{N}} - \frac{\left[c-\frac{N-1}{N}\gamma l\right]\frac{\gamma}{N^2}}{\left(1-\frac{\gamma}{N}\right)^2} = -\frac{c\frac{\gamma}{N^2}}{\left(1-\frac{\gamma}{N}\right)^2} < 0$$

$$\begin{aligned}\frac{\partial \alpha}{\partial A} &= \frac{\frac{\gamma}{h-l}\left[1-\left(1-\gamma\right)\frac{A}{h-l}\right] + \frac{\gamma}{h-l}A\left(1-\gamma\right)\frac{1}{h-l}}{\left[1-\left(1-\gamma\right)\frac{A}{h-l}\right]^2} = \\ &= \frac{1+\left(1-\frac{1}{h-l}\right)\left(1-\gamma\right)A}{\left[1-\left(1-\gamma\right)\frac{A}{h-l}\right]^2}\cdot\frac{\gamma}{h-l} > 0\end{aligned}$$

We conclude that total derivative of the fraction of the high-quality producers α with respect to the consumer group size N is negative.

$$\frac{d\alpha}{dN} = \frac{\partial \alpha}{\partial A}\cdot\frac{\partial A}{\partial N} < 0$$

□

Proof of Proposition 24. The marginal effect of γ on α is:

$$\frac{d\alpha}{d\gamma} = \frac{\partial \alpha}{\partial A}\frac{\partial A}{\partial \gamma} + \frac{\partial \alpha}{\partial \gamma}$$

$$\frac{\partial \alpha}{\partial A} = \frac{1+\left(1-\frac{1}{h-l}\right)\left(1-\gamma\right)A}{\left[1-\left(1-\gamma\right)\frac{A}{h-l}\right]^2}\frac{\gamma}{h-l} > 0$$

$$\frac{\partial A}{\partial \gamma} = \frac{A-l(N-1)}{N\left(1-\frac{\gamma}{N}\right)}$$

$\frac{\partial A}{\partial \gamma}$ has an ambiguous sign.

$$\frac{\partial \alpha}{\partial \gamma} = \frac{\frac{A}{h-l}\left(1-\frac{A}{h-l}\right)}{\left[1-\left(1-\gamma\right)\frac{A}{h-l}\right]^2} > 0$$

because we know from condition C.1 (the one $e = A + l$, then $\mu(h-l) = A$) that $\mu = \frac{A}{h-l}$. μ is by definition between 0 and 1, hence, $0 < \frac{A}{h-l} < 1$. Than it follows that $1 - \frac{A}{h-l} > 0$.

Finally,

$$\frac{d\alpha}{d\gamma}=\frac{\gamma\frac{1}{N\left(1-\frac{\gamma}{N}\right)}\left(l+A\right)\left[1+\left(1-\frac{1}{h-l}\right)\left(1-\gamma\right)A\right]}{\left(h-l\right)\left(1-\frac{1-\gamma}{h-l}\right)^{2}}+\frac{\left(1-\frac{A}{h-l}\right)\left(c-l\gamma\frac{N-1}{N}\right)}{\left(h-l\right)\left(1-\frac{\gamma}{N}\right)\left[1-\frac{1-\gamma}{h-l}A\right]^{2}}$$

This total derivative is positive because α must be positive. $\alpha=\frac{\gamma\frac{A}{h-l}}{1-(1-\gamma)\frac{A}{h-l}}$. The nominator is positive, hence, the denominator must be positive as well. Thus, it must be that $1>(1-\gamma)\frac{A}{h-l}$. Therefore, the second bracket of the first fraction in the above expression is positive as well. Hence, $\frac{d\alpha}{d\gamma}>0$. $\square$

Bibliography

[1] D. Acemouglu. Challenges for social sciences: institutions and economic development. *AEA*, White Papers. Available at http://www.aeaweb.org/econwhitepapers/white$_papers/Daron_Acemoglu.pdf$, 2010.

[2] I. Ahn and M. Suominen. Word-of-mouth communication and community enforcement. *International Economic Review*, 42(2):399–415, 2001.

[3] J. Akerlof. The market for lemons: Quality uncertainty and the market mechanism. *Quartarly Journal of Economics*, 84(3):488–500, 1970.

[4] Gonzales-Maestre M. Alcala, F. and I. Martinez-Pardina. Information and quality with an increasing number of firms. *Working paper, available at http://merlin.fae.ua.es/activities/pdf/GonzalezMaestrewom-nov-07.pdf (downloaded on February 16,2011).*

[5] F. Andriani and L.G. Deidda. Competition and signaling role of prices. *International Journal of Industrial Organization*, 2010.

[6] K. Bagwell and M.H. Riordan. High and declining prices signal product quality. *The American Economic Review*, 81(1):224–239, 1991.

[7] W. Baker and R. Faulkner. Social networks and loss of capital. *Social Networks*, 2004.

[8] O. Bandiera and I. Rasul. Social networks and technology adoption in northern mozambique. *The Economic Journal*, 116(514):869–902, 2006.

[9] C. Bidner and P. Francois. Cultivating trust: Norms, institutions and the implications of scale. *The Economic Journal*, Article in Press, 2010.

[10] S. Bowles. Microeconomics: Behavior, institutions, and evolution. *The MIT Press*, 2004.

[11] Garcia-M. de Felipe I. Briz, J. and N. Poole. Quality control in mediterranean fresh food export products. *Cahiers Options Mediterraneaennes*, 2007.

[12] Berentsen-A. Loertscher S. Bruegger, E. The evolution of cheating in asymmetric contests. *Zurich IEER Working Paper No. 314*, 2007.

[13] I. Burica. Tourists demand the dental service because of price and quality. *Zadarski List*, July 29, 2010.

[14] L. Cabral. The economics of trust and reputation. *New York University and CEPR (a Draft)*, 2005.

[15] A. Campbell. Tell your friends! word of mouth and percolation in social networks. *Working Paper*, 2008.

[16] J.D. Campbell. Signaling to a network of consumers. *(November 17, 2010). Available at SSRN: http://ssrn.com/abstract=1710877*, 2010.

[17] R. Crino. Service offshoring and white-collar employment. *Review of Economic Studies*, 77, 2010.

[18] A. Daughety and J. Reinganum. Imperfect competition and quality signaling. *The RAND Journal of Economics*, 39(1):163–183, 2008.

[19] P. DiMaggio and H. Louch. Social embedded consumer transactions: For what kinds of purchases do people most often use networks? *American Sociological Review*, 63(5):619–637, 1998.

[20] D. Dranove and G. Zhe Jin. Quality disclosure and certification: Theory and practice. *Journal of Economic Literature*, 48(4):935–963, 2010.

[21] S. Freedman and G. Z. Jin. Do social networks solve information problems for peer-to-peer lending? evidence from prosper.com. *NET Institute Working Paper No. 08-43. Available at SSRN: http://ssrn.com/abstract=1304138*, November 14, 2008.

[22] D. Fudenber and J. Tirole. Game theory. *The MIT Press*, 1991.

[23] A. Galeotti. Talking, searching, and pricing. *International Economic Review*, 51(4):1159–1174, 2010.

[24] A. Galeotti and S. Goyal. Influencing the influencers: a theory of strategic diffusion. *RAND Journal of Economics*, 40(3):509–532, 2009.

[25] B. Galić. Once, fresh eyes were put into a fish. *Slobodna Dalmacija*, July 3, 2010.

[26] A. Goyal. Information, direct access to farmers, and rural market performance in central india. *American Economic Journal: Applied Economics*, 2(3):22–45, 2010.

[27] Grgona. Turizam u funkciji gospodarskog razvitka hrvatskih otoka. *Ekonomski Pregled*, 53, 2002.

[28] M.N. Hertzendorf and P.B. Overgaard. Prices as signals of quality in duopoly. *working paper*, 2001.

[29] Y.M. Ioannides and L. Datcher Loury. Job information networks, neighbourhood effects, and inequality. *Journal of Economic Literature*, XLII, 2004.

[30] Zhang-M. Soble K. Jansen, J.B. and A. Chowdury. Twitter power: Tweets as electronic word of mouth. *Journal of the American Society for Information Science and Technology*, 60(11):2169–2188, 2009.

[31] M.C.W Janssen and S. Roy. Signaling quality through prices in an oligopoly. *Working Paper, available at http://faculty.smu.edu/sroy/qualitygebsub.pdf (downloaded on February 15, 2011)*, 2008.

[32] M.C.W Janssen and S. Roy. Signaling quality through prices in an oligopoly. *Games and Economic Behavior*, 68(1):192–207, 2010.

[33] M. Kandori. Social norms and community enforcement. *The Review of Economic Studies*, 59(1):63–80, 1992.

[34] P.W. Kennedy. Word-of-mouth communication and price as a signal of quality. *The Economic Record*, 70(211):373–380, 1994.

[35] Blanco-E. Lozano, J. and J. Rey-Maquieira. Can ecolabels survive in the long run? the role of initial conditions. *Ecological Economics*, 69, 2010.

[36] P. Milgrom and J. Roberts. Price and advertising signals of product quality. *The Journal of Political Economy*, 94(4):796–821, 1986.

[37] J. Montgomery. Social networks and labour-market outcomes: Towards an economic analysis. *American Economic Review*, 81(5):1408–1418, 1991.

[38] Manchanda-P. Nam, S. and P.K. Chintagunta. The effect of signal quality and contiguous word of mouth on customer acquisition for a video-on-demand service. *Marketing Science*, 29(4):690–700, 2010.

[39] N. Navarro. Asymmetric information, word-of-mouth and social networks: from the market for lemons to efficiency. *CORE Discussion Paper*, 2006.

[40] N. Navarro. Quality provision under referral consumption. *CORE Discussion Paper*, 2008.

[41] R. Nelson. The economics of honest trade practices. *The Journal of Industrial Economics*, 24(4):282–293, 1976.

[42] M.J. Osborne and A. Rubinstein. A course in game theory. *The MIT Press*, 1994.

[43] Stafford-E.R. Ottman, J.A. and C.L. Hartman. Avoiding green marketing myopia: Ways to improve consumer appeal for environmentally preferable products. *Environment: Science and Policy for Sustainable Development*, 48(5):22–36, 2006.

[44] Trost-K. Jurakovic L. Mladen R. Persuric, I.A.S. and M. Oplanic. Agrotourism in instria - state and perspectives. 2010.

[45] J.E. Rauch and A. Casella. Overcoming informational barriers to international resource allocation: Prices and ties. *The Economic Journal*, 113(484):21–42, 2002.

[46] J.E. Rauch and V. Trindade. Ethnic chinese networks in international trade. *The Review of Economics and Statistics*, 84(1):116–130, 2003.

[47] J. Tirole. A theory of collective reputations (with applications to the persistence of corruption and to firm quality). *Review of Economic Studies*, 63(1):1–22, 1996.

[48] N. Vettas. Information and quality with an increasing number of firms. *International Economic Review*, 38(4):915–944, 1997.

[49] J. W. Weibull. Evolutionary game theory. *The MIT Press*, 1996.

[50] J.A. Winfree and J.J. McCluskey. Collective reputation and quality. *American Journal of Agricultural Economics*, 87(1):206–213, 2005.

[51] P.J. Zak. Moral markets. *Journal of Economic Behavior and Organization*, 77, 2011.