Three Essays on Institutions, Environmental Quality, and Irreversibility of Pollution Accumulation

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Abstract

This doctoral thesis is aimed at studying two main different aspects of environmental issues. On one hand, it tackles the issue of whether irreversible pollution accumulation may be an optimal strategy for an efficiently planned economy or not, under the assumption of a nonlinear natural rate of decay for the pollution accumulation, which becomes nil when pollution itself grows up to a given quantity, and, on the other hand, it addresses the question of whether different institutional regimes may be responsible for different levels of pollution chosen by a country.

The first issue is addressed in chapter one. This chapter is aimed at proving that when the pollution problem has a global nature, irreversibility of pollution accumulation cannot be optimal, contrary to what argued by Thavonen and Withagen in their local pollution problem (JEDC 1996). This stylised fact is showed through numerical simulations using a dynamic programming algorithm.

The two other chapters of this thesis, instead, investigate the implications of two different institutional regimes, namely, democracy and dictatorship, over the level of environmental quality chosen by a country.

The second chapter investigates this issue from an econometric point of view, by analysing the effect of a regime switch (from dictatorship to democracy and viceversa) over two indicators of pollution (CO2 emissions and annual average PM10 concentrations), controlling for income and income inequality. The econometric tool used is Interrupted Time Series (ITS), which is a widely used instrument of analysis in behavioural science but it is rarely applied in economics. The advantage of this approach is that - contrary to other methods commonly found in economic literature - it allows to prove the result in two ways: i. by showing that democracy is beneficial

to the environment and ii. that dictatorship is detrimental. Studying just one side of the problem is not enough to say that democracy is better than dictatorship for the environment because the effect that is captured by the variable(s) identifying a democratic regime may be caused by other factors, like time per se or technological innovation, and therefore dictatorships may show similar trends. Such an event might make the results lose their importance and robustness, because the goodness of an institution over a variable like the environment must be compared with the performance of the other types of institutions. Briefly, one cannot say that democracies are good for the environment if also dictatorships are. If such things happens, the reasons for such differentials in environmental quality cannot be traced to differences in institutional regimes, but to other reasons, and ITS catches this point in full.

The third chapter is someway linked to the second because it is again aimed at measuring the effect of a democratic or autocratic regime on the level of optimal environmental quality chosen by a country, but this time the problem is analysed by the means of a model of comparative statics. Differences between democracies and autocracies are reflected in differences in wealth between the two decisive political actors, where the autocrat is assumed to be richer than the decisive voter in a democratic regime. Assuming that all the citizens are exposed equally to the source of pollution (environmental equality), democratisation, i.e. a regime shift from autocracy to democracy, may not necessarily be beneficial for the environment since the overall result depends on the size of the income and price effect on the demand for environmental quality associated to a decrease in the decisive political actors wealth. Later, the assumption of equal exposure to pollution of the citizens is removed and a model of social class is introduced. One class, the most numerous, supplies an embodied factor of production and the other class supplies capital. Assuming that the decisive voter belongs to the first class of individuals while the autocrat does not, democratisation is shown to be beneficial for the environment, the better the effect on the environment, the bigger the difference in wealth between the two decisive political actors.

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On the Non-Optimality of Irreversible Pollution Accumulation for an Infinitely Lived Planned Economy.

Abstract

In this paper I address the question of whether irreversible pollution accumulation - in a global pollution problem - may be optimal or not. Based on the Thavonen and Withagen's article (16), I set up a model of economic growth where pollution is a byproduct of production, and its natural decay function follows an inverted-U shape, and becomes irreversible for high levels of pollution. Under some parameter's constellation, the model produces multiplicity of equilibria making local analysis of little relevance. I therefore study the global dynamics of the system using a dynamic programming algorithm, showing that irreversible pollution accumulation cannot be an optimal strategy, unless it is guided by short-term objectives.

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Keywords: Economic growth; Irreversible pollution accumulation; Dynamic programming; Global dynamics, Multiplicity of equilibria.

1. ON THE NON-OPTIMALITY OF IRREVERSIBLE POLLUTION ACCUMULATION FOR AN INFINITELY LIVED PLANNED ECONOMY.

1.1 Introduction

Despite several effects of pollution - like global warming - have a global nature, involving all the countries irrespective of who is responsible for producing wastes, environmental policies are decided in autonomy by the single nations. The coordination problem underlying this "hot" topic is one of the main reasons of the steady growth of greenhouse gases and other toxic substances, which the scientific consensus believes they are the main causes leading to global warming.

Hoping to make a contribution towards the awareness of the necessity of having a unique, global, environmental policy, in this paper I study a model of economic growth with pollution accumulation, where pollution has a nonlinear decay function which follows an inverted-U shape and becomes irreversible when a given stock is reached. In the analysis, I will focus on the case of an efficiently and infinitely lived planned economy, with the aim at responding to the question of whether irreversible pollution accumulation may be an optimal strategy or not.

The model I use in the paper is built on the basis of Thavonen and Withagen's one, published on the JEDC in 1996 (16). I generalise their model by introducing a capital accumulation function and assuming a global pollution problem instead of a local one. Globality of the problem is reflected in the introduction of the hypothesis of a "subsistence" level of consumption. This assumption is crucial in determining the dimensionality of the problem, since in my model the population cannot leave to move in a cleaner and unpolluted area.

Since this problem produces multiplicity of stable solutions, local analysis gives little insights since it does not allow to say what is the dynamics between different equilibria, or far away from them. In order to fill this informational gap, I study the global dynamics of the model using a dynamic programming algorithm (carefully explained in the appendix, with codes also included), and comparing different paths in terms of welfare they produce.

In the literature of economic growth and the environment, little attention has been paid to the fact that the natural decay function of pollution might be endogenously determined by the stock of pollution itself. Linearity has been a commonly assumed hypothesis, and that allowed economists to find an unique stable stationary solution of the system (Keeler E., Spence M., and Zeckhauser R. (9), Nancy Stokey (15)).

The main consequence of this hypothesis was that the insights one could learn from these lessons were that in the long run, all the countries would have converged to that unique equilibria, without realising that the hypothesis they implicitly made was that the more polluted were the environment, the more it was able to clean itself up, hypothesis quite unrealistic. Moreover, this prediction is clearly in sharp contrast with the evidence today, where many countries still find the joining to international protocols not worthwhile, and keep their emissions' level unbounded. The prediction of uniqueness of equilibria is a direct consequence of the choice of the decay function of pollution, because using different function of pollution' s decay may lead to a different solution specifications, even to a multiplicity of stationary solutions.

The choice of such inverted-U shaped decay function for pollution is due mainly to the observation of natural phenomena, which suggest - contrary to what is commonly assumed in the economic literature - that the natural self recreation capacity of the environment certainly isn't always increasing with respect to the stock of pollution. A sort of endogeneity of this ability of the environment was first noticed by Holling (8), who, in an article published in 1972, wrote about the fact that nutrient enrichment of lakes changed its biodiversity permanently, making the lake incapable of recovery its original status even if emissions were to be eliminated. Several authors, subsequently, raised the problem (Dasgupta (4), Forster (7)). Recently, in his highly debated report, Stern (14) predicted future scenarios where, if emissions are kept at the "business as usual" level, global warming may change dramatically the biodiversity of the planet through desertification and the rise of sea levels, putting at serious risk people's health and lowering the probability of survival for some other populations, and therefore suggesting a sort of inability of the environment to absorb pollution, for high pollution levels. Despite all these contributions, this fact has gained very little relevance in the economic literature.

The paper is organised as follows: section 1.2 introduces the theoretical model and describes its properties, section 1.3 introduces the strategies to study the local and global dynamics of this system and presents the results, and section 1.4 concludes.

1.2 The model

In this section, I introduce the model of economic growth where citizens' utility depends on both consumption and pollution, and technology is linear. The function for pollution accumulation depends on both the level of production, and its natural rate of decay, which is endogenous to the stock of pollution and follows an inverted-U shape. For simplicity, I describe an optimal solution dictated by a benevolent planner who acts in the interest of the citizens (in the following, I will use interchangeably the words "citizen", "household" and " representative agent" since all these interpretations are correct and do not change the scope of this study). The citizens' instantaneous utility is a separable function of consumption[1], denoted by C, and pollution[2], x, according to the following rule:

$$u(c, x) = v(C) - h(x) \tag{1.1}$$

with v strictly increasing and strictly concave, and h strictly increasing and strictly convex. C is composed by two arguments,

$$C(t) = c(t) + mc \tag{1.2}$$

with c the level of consumption beyond the minimum subsistence level denoted by mc, with $mc > 0$, that is kept constant at all the times. It follows that C is always positive and lower bounded by mc.

These four properties also hold:

$$\begin{aligned}
\lim_{c\to 0} v'(c) &= \overline{MU}_{mc} < \infty &(1.3)\\
\lim_{c\to \infty} v'(c) &= 0 &(1.4)\\
\lim_{x\to 0} h'(x) &= 0 &(1.5)\\
\lim_{x\to \infty} h'(x) &= +\infty &(1.6)
\end{aligned}$$

where equation 1.3 represents the maximum achievable level of marginal utility from consumption, that is to say, the level of marginal utility at the subsistence or minimum

[1] This is a special assumption since this formulation implies that the enjoyment of consumption does not depend on pollution, and that the disutility of pollution is not affected by the level of consumption.

[2] Pollution in this model is a public "bad", so each individual experience its whole amount, while consumption is considered in percapita terms.

level. Condition 1.4 indicates that the marginal utility from consumption decreases as consumption increases, converging to zero for levels of consumption tending to infinity, and conditions 1.5 and 1.6 indicate that at low levels of pollution, the marginal disutility is low, but increases as the stock of pollution increases. The representative agent discounts at the subjective discount rate all the future flows of utility, so his total welfare is

$$U(C,x) = \int_0^\infty u(C,x)e^{-\rho t}dt \tag{1.7}$$

Production is linear in the argument of capital, so capital's productivity is constant and equal to A:

$$y(t) = Ak(t) \tag{1.8}$$

Capital is then a necessary factor of production, and in order to guarantee a minimum subsistence level of consumption, it must be strictly positive, so as production. Assume that $\underline{k}$ is the minimum level of capital which guarantees a production equal to $\underline{y}$. The minimum level of production $\underline{y}$ must be such that the amount of investments is equal to the capital's depreciation, and the amount of consumption in each period is equal to mc. It follows that

$$k \in [\underline{k}, \infty), \qquad \underline{k} > 0 \tag{1.9}$$

$$y \in [\underline{y}, \infty), \qquad \underline{y} > 0 \tag{1.10}$$

The planner decides on behalf of the citizens how much production to consume and to invest to accumulate further capital. The capital accumulation function of the economy is represented by

$$\dot{k}(t) = y(t) - \delta k(t) - C(t) \tag{1.11}$$

with δ representing the constant rate of capital depreciation. Pollution is a byproduct of production, and it is assumed to obey the following equation

$$\dot{x}(t) = y(t) - \eta(x(t) - \frac{\theta}{\eta}x(t)^2) \qquad \text{if } x(t) < \eta/\theta \tag{1.12}$$

$$\dot{x}(t) = y(t) \qquad \text{if } x(t) \geq \eta/\theta \tag{1.13}$$

Equations 1.12 and 1.13 represent the law of pollution accumulation. The term $\eta(x(t) - \frac{\theta}{\eta}x(t)^2)$ is the pollution's natural decay, and follows, as anticipated in the introduction, an inverted-U shape. $\bar{x} = \eta/\theta$ represents the threshold beyond which pollution becomes irreversible (so the decay is zero).

In order to write down the conditions for maximization, I will use the same utility function used by Stokey (15) so the specification of the welfare function becomes:

$$v(C) = \frac{C(t)^{1-\sigma} - 1}{1-\sigma} \tag{1.14}$$

$$h(x) = \frac{Bx(t)^{\gamma}}{\gamma} \tag{1.15}$$

with $\sigma > 0$, $B > 0$ and $\gamma > 1$. Moreover, I will assume in the following:

HP 2.1. The marginal product of capital net of the depreciation is positive, $A - \delta > 0$.

HP 2.2. The marginal product of capital, net of the depreciation is greater that the intertemporal rate of preferences, $A - \delta > \rho$

HP 2.3. The sum of the intertemporal rate of preferences and the marginal rate of decay of pollution is positive, $\rho + \eta(1 - 2 \cdot \frac{\theta}{\eta} x) > 0$. As long as the marginal decay function is positive, this hypothesis is always satisfied, but when it is negative, this implies that the rate of impatience is greater than the marginal loss in the self purification capacity of the environment.

Later, I will compare two possible outcomes of this model: a reversible solution and an irreversible one. In the first case, I will study the reversible solution, assuming that the planner will maximise utility in infinite time letting pollution to stay below its threshold level forever. In the second case, I will study an irreversible solution, and since the solution admits a point of non-differentiability, I will follow the same approach used by Thavonen and Withagen and I will split the problem into two subproblems: a first period problem, where the planner maximises utility from zero to T (finite time) letting pollution to reach the irreversibility threshold at T, followed by a second period problem where the planner maximises utility from T to infinity when the natural decay function for pollution is nil.

1.2.1 Reversible pollution accumulation

Let us assume that the planner wants to maximise the representative citizen's welfare having an infinite time horizon plan, and letting pollution not to reach $\bar{x}$. In this case, the problem faced by the planner is:

$$\max_{c(t)} W^{\infty} = \int_0^{\infty} e^{-\rho t} \left[\frac{C(t)^{1-\sigma} - 1}{1-\sigma} - \frac{Bx(t)^{\gamma}}{\gamma} \right] dt \tag{1.16}$$

subject to

$$\dot{k}(t) = (A-\delta)k(t) - C(t) \tag{1.17}$$

$$\dot{x}(t) = Ak(t) - \eta(x(t) + \frac{\theta}{\eta}x(t)^2) \tag{1.18}$$

$$\lim_{t\to\infty} x(t) < \bar{x} \tag{1.19}$$

Denote the solution of this first period problem by $(c^{\infty}, k^{\infty}, x^{\infty})$ and the respective costate variables by λ_1^{∞} and λ_2^{∞}. Denote also the flow of utility yield by this optimal plan W^{∞}. The Hamiltonian for this problem is

$$\begin{aligned} \mathcal{H}(t, k(t), x(t), c(t), \Lambda; \Theta) &\stackrel{def}{=} \lambda_0 \cdot \left[\frac{C(t)^{1-\sigma} - 1}{1-\sigma} - \frac{Bx(t)^{\gamma}}{\gamma} \right] + \\ +\lambda_1(t)\Big[(A-\delta)k(t) - c(t)\Big] &+ \lambda_2(t)\Big[Ak(t) - \eta(x(t) + \frac{\theta}{\eta}x(t)^2)\Big] \end{aligned} \tag{1.20}$$

where Λ is the set of shadow prices, $\Lambda = \{\lambda_0(t), \lambda_1(t), \lambda_2(t)\}$ and Θ represents the set of exogenous parameters of the model, $\Theta = \{A, B, \sigma, \rho, \delta, \eta, \theta, \gamma, mc\}$ and, in more detail, $\lambda_1(t)$ represents the shadow price of capital, and $\lambda_2(t)$ the shadow price of pollution.

The maximum principle asserts that there exists a λ_0 and a continuous and piecewise continuously differentiable functions $\lambda_1(t)$ and $\lambda_2(t)$, such that for all t

$$(\lambda_0, \lambda_1(t), \lambda_2(t)) \neq (0,0,0) \tag{1.21}$$

$$\mathcal{H}(t, k^*(t), x^*(t), c^*(t), \Lambda; \Theta) \geq \mathcal{H}(t, k^*(t), x^*(t), c(t), \Lambda; \Theta) \qquad \forall t \tag{1.22}$$

The necessary first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \qquad \Rightarrow \qquad \lambda_1 = C^{-\sigma} \tag{1.23}$$

$$\frac{\partial \mathcal{H}}{\partial k} = \rho\lambda_1 - \dot{\lambda}_1 \qquad \Rightarrow \qquad \dot{\lambda}_1 = \lambda_1(\rho + \delta - A) - \lambda_2 A \tag{1.24}$$

$$\frac{\partial \mathcal{H}}{\partial x} = \rho\lambda_2 - \dot{\lambda}_2 \qquad \Rightarrow \qquad \dot{\lambda}_2 = \lambda_2 \cdot [\rho + \eta(1 - \frac{2\theta}{\eta}x)] + Bx^{\gamma-1} \tag{1.25}$$

$$\lambda_0 = 1 \qquad \text{or} \qquad \lambda_0 = 0 \tag{1.26}$$

and sufficient conditions for maximisation are the following transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)\cdot k(t) = 0 \tag{1.27}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_2(t)\cdot x(t) = 0 \tag{1.28}$$

$$\lambda_1(t) \geq 0 \tag{1.29}$$

$$\lambda_2(t) \leq 0 \tag{1.30}$$

Since the terminal conditions for capital and pollution as time approaches infinity are left free, it follows that $\lim_{t\to\infty}\lambda_1(t) = 0$ and $\lim_{t\to\infty}\lambda_2(t) = 0$ so necessarily, because of condition 1.26, $\lambda_0 = 1$.

The following system of four differential equations represents the conditions any optimal path has to obey:

$$\dot{k} = (A-\delta)k - \lambda_1^{-\frac{1}{\sigma}} \tag{1.31}$$

$$\dot{x} = Ak - \eta x + \theta x^2 \tag{1.32}$$

$$\dot{\lambda}_1 = \lambda_1(\rho - (A-\delta)) - \lambda_2 A \tag{1.33}$$

$$\dot{\lambda}_2 = \lambda_2(\eta - 2\theta x + \rho) + Bx^{\gamma-1} \tag{1.34}$$

with equations 1.33 and 1.34 representing the Euler equations. In equilibrium, all the variables in the economy grow at a zero rate, so $\dot{k} = \dot{x} = \dot{\psi} = \dot{\lambda} = 0$.

I first start by analyzing the so-called corner solutions, that is to say solutions that assume consumption equal to the minimum subsistence level. Assuming

HP 2.4. $C^*(t) = mc$

and also

HP 2.5. $mc < \frac{\eta^2(A-\delta)}{4\theta A}$ (This condition is necessary to guarantee the level of pollution be real)

it follows that there are two simultaneous steady states represented in the table below:

Equilibrium 1	Equilibrium 2
$k^* = mc/(A-\delta)$ $x^* = \frac{\eta - \sqrt{\eta^2 - \frac{mc\cdot 4\theta A}{A-\delta}}}{2\theta}$ $\lambda_1^* = mc^{-\sigma}$ $\lambda_2^* = -\frac{Bx_1^{*\gamma-1}}{\eta - 2\theta x_1^* + \rho}$	$k^* = mc/(A-\delta)$ $x^* = \frac{\eta + \sqrt{\eta^2 - \frac{mc\cdot 4\theta A}{A-\delta}}}{2\theta}$ $\lambda_1^* = mc^{-\sigma}$ $\lambda_2^* = -\frac{Bx_1^{*\gamma-1}}{\eta - 2\theta x_1^* + \rho}$

Due to the inverse U-shaped function for the pollution decay, this corner solution admits two stationary points, for each value of mc respecting condition 2.5.

Now, I consider interior solutions. From equation 1.32, it is possible to see that considering $\dot{x} = 0$ and rearranging I get

$$k = \frac{x(\eta - \theta x)}{A} \tag{1.35}$$

and, combining equations 1.31, 1.33, 1.34 and considering $\dot{k} = \dot{\psi} = \dot{\lambda} = 0$ I get

$$k = \frac{1}{(A-\delta)} \cdot \left(\frac{AB}{A-\delta-\rho}\right)^{-\frac{1}{\sigma}} \cdot x^{\frac{1-\gamma}{\sigma}} \cdot (\eta - 2\theta x + \rho)^{\frac{1}{\sigma}} \tag{1.36}$$

Any intersection between the two equations 1.35 and 1.36 represents an equilibria[1]. In general, the existence of an equilibria (or multiplicity of equilibria) depends on the choice of the parameters of the model. Equation 1.36 is decreasing in all his domain, whilst equation 1.35 has an inverted-U shape. Graphically, one may have the following cases:

[1]This rearrangement of equations 1.31 - 1.34 is only aimed at expressing the two stationary solutions in the $k - x$ plane and equations 1.35 and 1.36 do not have necessarily an economic interpretation

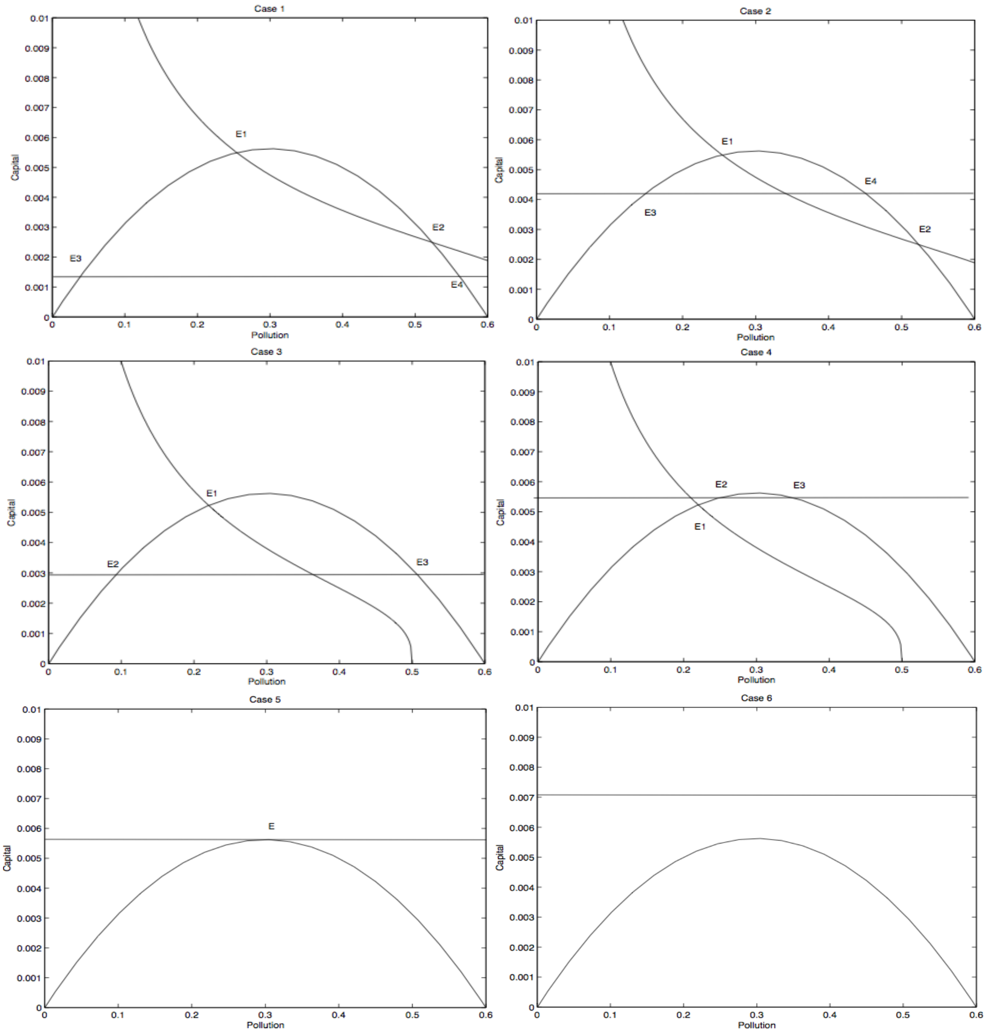


Figure 1.1: *Graphical representation of different cases, ranging from four to zero steady states. These cases are not exhaustive.*

The first two sets of graphs (case 1 and 2) have been obtained using the following set of parameters: $B = 10,000,000$, $A = 0.8$, $\rho = 0.04$, $\theta = 0.05$, $\eta = 0.03$, $\sigma = 3$, $\gamma = 3$, and $\delta = 0.1$. The second two graphs (case 3 and 4) have instead been obtained using $B = 10,000,000$, $A = 0.8$, $\rho = 0.02$, $\theta = 0.05$, $\eta = 0.03$, $\sigma = 3$, $\gamma = 3$, and $\delta = 0.1$. Irrespective of the set of parameters chosen, the last two graphs suggest that if the level of capital at the minimum level of consumption is as high as the maximum level of capital corresponding to the turning point of the decay function for pollution, we can only have one steady state, and if it is higher, no steady states at all. The last two graphs, instead, do not respect H.2.5 because, in graph 5, $mc = \frac{\eta^2(A-\delta)}{4\theta A}$ and in graph 6 $mc > \frac{\eta^2(A-\delta)}{4\theta A}$, with the consequence, respectively, of the existence of only one (or two equal) solutions for pollution, or zero

The first graph represents a case where two interior solutions exist, and those are represented by $E1$ and $E2$. At the same time, this picture shows that there might exist two additional corner solutions, represented by the intersection between the horizontal line (which identifies the minimum level of capital that is necessary to guarantee a consumption equal to the subsistence level and to cover capital's depreciation). Those solutions are, respectively, $E3$ and $E4$.

The second picture depicts instead another case where there are still two interior solutions, but one (represented by $E2$) cannot be considered a feasible equilibria since its level of consumption is lower than the subsistence level. This case therefore leads to only three feasible stable solutions.

Case three represents a different situation where only one interior solution exists, with associated level of consumption higher than mc. Despite the fact that this solution requires a different parameter's set with respect the previous case, the outcome is similar since it generates three feasible steady solutions.

Case four happens when mc is larger than the equilibrium levels of all the interior solutions, but the two corner solutions still respect proposition 2.3. Hence, the number of feasible stable solutions is only two and those are the corner solutions.

Case five occurs when mc is equal to $\frac{\eta^2(A-\delta)}{4\theta A}$. This situation leads to just one stable solution, irrespective of the number of the existing interior solutions. This is because if they existed, they would have necessary a level of equilibrium consumption necessarily lower than the subsistence level. Case six shows, instead, that whatever the number of interior solutions, if $mc > \frac{\eta^2(A-\delta)}{4\theta A}$, no feasible steady state can exist, because of the reason above.

It follows that the next propositions hold:

Proposition 2.1. Necessary and sufficient condition to have one interior stable solution is $\eta < \rho$.

Proposition 2.2. Necessary and sufficient condition to have either two or zero interior stable solutions is $\rho > \eta$, sufficient condition to have two interior solutions is $\rho < \Psi \cdot (\eta/\theta)^{2\sigma+\gamma-1}$ with $\lambda_1 = (1/2)^{\sigma+\gamma-1} \cdot (A-\delta)^{\sigma} \cdot (\frac{AB}{A-\delta-\rho})$

Proposition 2.3.Necessary and sufficient condition to have two (one) corner solution(s) is $mc < (=)\eta^2(A-\delta)/(4A\theta)$.

1. ON THE NON-OPTIMALITY OF IRREVERSIBLE POLLUTION ACCUMULATION FOR AN INFINITELY LIVED PLANNED ECONOMY.

Depending on the choice of the parameters, I can have up to a maximum of four different equilibria. Consider for example case 1 in figure 1, and assume that $\underline{k}$ is lower than the level of capital in equilibrium E2. This implies the existence of four non-trivial stationary solutions. On the other hand, however, if the level of $\underline{k}$ is higher than the level of capital in equilibrium at E2, E2 cannot be considered a valid solution and therefore the number of equilibria are three.

In what follows, I will confine my analysis to the case where multiplicity of steady states occurs because, from an economic point of view, I believe it is the most interesting and the most realistic. It is not unusual indeed to see different countries with characteristics that can be represented by such a configuration of stable solutions. For instance, it is generally agreed that the cleanest cities in the world are located in developed and rich countries, like Canada, Finland, Norway etc. The worst polluted countries are mainly in China and India, that although they are growing at very high rate, they are not certainly rich countries. On the converse, there are natural paradises in very poor countries, like still are in Africa. This to highlight the fact that multiplicity of equilibria is the situation that more represents the actual state of the world, and that is the reason why I decided to focus on it.

In the next section, I will discuss the stability properties of the equilibria, limiting the analysis to a local level. Such a kind of analysis, is then deepened in section 3 by studying the global dynamics of the model. Local analysis is indeed of little relevance when multiplicity of equilibria arises, because it is only able to draw conclusions only on a close neighbourhood of the equilibria, and it is silent about the dynamics in between them.

The first interesting information one can extract from the study of the local dynamics of the equilibria is the occurrence of an eventual poverty trap. From the pictures displayed previously, some equilibria a characterised by low levels of consumption and capital, and some by higher levels of consumption and capital. If more than one equilibria is found to be (saddle) stable, and one provides a lower level of welfare (either because consumption is lower and/or pollution is higher), we may talk about poverty trap, that is to say an equilibria which is socially dominated but from which is difficult to escape.

The second interesting information that will be analysed in the section concerning the global dynamics, is the behaviour of the system in a generic point of the $k - x$ space, which represent the initial conditions, respectively, for capital and pollution. The question I will try to answer is whether the social planner will bring pollution to its irreversibility region or not, starting, as an example, in a neighbourhood of an unstable equilibria or far enough from a stable equilibria.

Local dynamics of the equilibria. The study of the local dynamics of the system around the steady states is usually carried on by linearising the model around them, using a first order taylor expansion. The first order taylor expansion or Jacobian matrix of the sistem 1.31 - 1.34 is therefore:

$$\begin{pmatrix} \dot{\tilde{k}} \\ \dot{\tilde{x}} \\ \dot{\tilde{\lambda}}_1 \\ \dot{\tilde{\lambda}}_2 \end{pmatrix} = \begin{pmatrix} A-\delta & 0 & \frac{1}{\sigma}\lambda_1^{*-\frac{1+\sigma}{\sigma}} & 0 \\ A & -\eta+2\theta x^* & 0 & 0 \\ 0 & 0 & \rho-(A-\delta) & -A \\ 0 & -2\theta\lambda_2^* + B(\gamma-1)x^{*\gamma-2} & 0 & \eta-2\theta x^*+\rho \end{pmatrix} \cdot \begin{pmatrix} \tilde{k} \\ \tilde{x} \\ \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \end{pmatrix}$$

The characteristic polynom of the matrix of coefficient can be written as

$$(\mu^2 - \rho\mu)^2 + (\mu^2 - \rho\mu)z + s \tag{1.37}$$

with

$$z = (A-\delta)(\rho-(A-\delta)) - (\eta-2\theta x^*)(\rho+\eta-2\theta x^*) \tag{1.38}$$

$$\begin{aligned} s = (A-\delta)(A-\delta-\rho)(\eta-2\theta x^*)(\rho+\eta-2\theta x^*) + \\ +\frac{1}{\sigma}\lambda_1^{*-\frac{1+\sigma}{\sigma}}\left\{A^2[-2\theta\lambda_2^* + B(\gamma-1)x^{*\gamma-2}]\right\} \end{aligned} \tag{1.39}$$

Equating the characteristic polynom to zero, and computing the eigenvalues, I get:

$$\mu_{1,2,3,4} = \frac{1}{2}\rho \pm \sqrt{(\rho/2)^2 - \frac{1}{2}z \pm \frac{1}{2}\sqrt{z^2-4s}} \tag{1.40}$$

and the following lemmas hold:

1. If $z < 0$, $0 < s \leq (z/2)^2$ it is a nec. and suff. condition for all μ to be real, 2 positive and two negative.

2. If $s > (z/2)^2$ and $s - (z/2)^2 - \rho^2 \cdot (z/2) > 0$ it is a nec. and suff. condition for all μ to be complex, two with negative real parts and two with positive real parts.

3. If $s < 0$ it is a nec. and suff. condition for one eig. to be negative and either 3 eig. to be positive or one positive and two having positive real parts.

4. If $s > (z/2)^2$ and $s - (z/2)^2 - \rho^2 \cdot (z/2) = 0$ it is a nec. and suff. condition for all μ to be complex and two having zero real part.

It follows from these lemmas that any equilibrium lying on the increasing locus of the marginal rate of decay is saddle-stable ($z < 0$), while the equilibria lying on the decreasing part of the natural rate of decay of pollution are stable if and only if $s > 0$. Since s depends on equilibrium levels of λ_1, λ_2 and x, analytical conditions determining the sign of s cannot be found and therefore we have to rely on numerical simulations.

The next table presents two possible outcomes which are depicted in figure 1 above. The first block is about the results obtained using the parameters of the first two pictures (case 1 and 2) and considering a minimum level of consumption mc very low (in particular, $mc = 0.0005$). It follows from those estimation that the model exhibits two stationary and stable solutions out of four, implying that under the set of parameters used, only the equilibria on the increasing locus of the decay function of pollution are stable. However, the second block shows that, changing just one parameter (in particular, bringing ρ from 0.04 to 0.02, which is the set of parameters used in the third and fourth pictures above) the corner equilibria lying on the decreasing locus of the function of the pollution's decay is stable. This means that lowering the level of impatience of the representative citizen, it is better to stay in the (socially) dominated equilibria than deviating. Of course this result hold in a close neighbourhood of the equilibria provided that capital respects the constraint of being greater than $\underline{k}$. This result can already be an indicator of a non-optimality of irreversible pollution accumulation since if the population has a "low enough" rate of intertemporal preferences, the higher levels of utility they can achieve by increasing capital and consumption are not big enough to compensate the losses due to the growth of pollution.

	E1	E2	E3	E4
Case 1 $mc =$ 0.0005	$k^* = 0.2543$ $x^* = 0.5240$ $\lambda_1^* = 0.1758e+08$ $\lambda_2^* = -0.1450e+08$ $\mu_1 = 0.6995$ $\mu_2 = 0.0556$ $\mu_3 = -0.0156$ $\mu_4 = -0.6595$	$k^* = 0.0025$ $x^* = 0.5240$ $\lambda_1^* = 1.8911e+08$ $\lambda_2^* = -1.5602e+08$ $\mu_1 = 0.6999$ $\mu_2 = 0.0308$ $\mu_3 = 0.0092$ $\mu_3 = -0.6599$	$k^* = 0.0007$ $x^* = 0.0197$ $\lambda_1^* = 4,000,000$ $\lambda_2^* = -12965.5$ $\mu_1 = -0.0280$ $\mu_2 = -0.66$ $\mu_3 = 0.0680$ $\mu_3 = 0.7$	$k^* = 0.0007$ $x^* = 0.5803$ $\lambda_1^* = 4,000,000$ $\lambda_2^* = -1.22e+08$ $\mu_1 = 0.0119$ $\mu_2 = -0.66$ $\mu_3 = 0.0281$ $\mu_3 = 0.7$
Case 3 $mc =$ 0.0005	$k^* = 0.0052$ $x^* = 0.5240$ $\lambda_1^* = 0.1758e+08$ $\lambda_2^* = -0.1450e+08$ $\mu_1 = 0.6996$ $\mu_2 = 0.0386$ $\mu_3 = -0.0186$ $\mu_4 = -0.6796$	$k^* = 0.0007$ $x^* = 0.0197$ $\lambda_1^* = 8e+09$ $\lambda_2^* = -129455.1$ $\mu_1 = -0.028$ $\mu_2 = -0.68$ $\mu_3 = 0.048$ $\mu_3 = 0.7$	$k^* = 0.0007$ $x^* = 0.5803$ $\lambda_1^* = 8e+09$ $\lambda_2^* = -1.17e+08$ $\mu_1 = -0.008$ $\mu_2 = -0.68$ $\mu_3 = 0.028$ $\mu_3 = 0.7$	

Depending on the parameter's set chosen, this model can predict two types of poverty traps, one characterised by low levels of pollution, and one characterised by high levels of pollution. This outcome is in line with the evidence on the environmental quality in different poor countries. It is indeed not rare that very polluted cities in the world are often situated in poor countries. According to the Time, for instance, the most polluted places in the world are in China and India[1]. The evidence suggests moreover that the cleanest cities in the world[2] are located in developed (rich) countries, suggesting that those places are probably near the interior equilibria characterised by relatively low levels of pollution and high levels of percapita income.

The next section instead studies the existence of an optimal path conducing pollution to its irreversibility region, and later the two solutions are compared through simulation, using a dynamic programming algorithm.

1.2.2 Irreversible pollution accumulation

This solution requires that pollution reaches its threshold level. The analysis of this second option available to the planner has a point of non-differentiability (in $x = \bar{x}$) and therefore I use the same approach kept by Tahvonen and Whithagen and I split the problem into two subproblems: the first - or first period problem - where the planner maximises in finite time T the citizen's discounted utility, with the constraint

[1]Linfen (China) is the first most polluted city in the world, where the amount of particulate matters in the air is such that it makes the laundry black before it dries, followed by Sukinda (China) where 60% of the drinking water contains hexavalent chromium at levels more than double international standards and Vapi (India), where levels of mercury in the city's groundwater are reportedly 96 times higher than WHO safety levels, and heavy metals are present in the air and the local produce.

[2]Among the cleanest cities in the world we see Calgary (Canada), Honolulu (Hawaii), Helsinky (Finland), and Ottawa (Canada)

that $x_T = \bar{x}$, and the second period problem, where the maximisation goes from T to infinity, with initial conditions $x_T = \bar{x}$ and k_T equal to the final value of capital in the first period. Of course, in the second period problem the natural decay function for pollution is zero since it has reached the threshold of irreversibility.

The problem can be expressed, then, as follows[1]:

$$\max_{c(t)} W^T = \int_0^T e^{-\rho t}\left[\frac{C(t)^{1-\sigma}-1}{1-\sigma} - \frac{Bx(t)^\gamma}{\gamma}\right]dt \tag{1.41}$$

subject to

$$\dot{k}(t) = (A-\delta)k(t) - C(t) \tag{1.42}$$
$$\dot{x}(t) = Ak(t) - \eta(x(t) + \frac{\theta}{\eta}x(t)^2) \tag{1.43}$$
$$k(0) = k_0 \tag{1.44}$$
$$k(T) \geq \underline{\mathrm{k}} \tag{1.45}$$
$$x(0) = x_0, \qquad x_0 < \bar{x} \tag{1.46}$$
$$x(T) = \bar{x}, \qquad T < \infty \tag{1.47}$$

which represents the so-called "first period problem"[2], immediately followed by the "second period problem" that is

$$\max_{c(t)} W_T = \int_T^\infty e^{-\rho t}\left[\frac{C(t)^{1-\sigma}-1}{1-\sigma} - \frac{Bx(t)^\gamma}{\gamma}\right]dt \tag{1.48}$$

subject to the laws of motion of the two state variables and the initial conditions

$$\dot{k}(t) = Ak(t) - \delta k(t) - C(t) \tag{1.49}$$
$$\dot{x}(t) = Ak(t) \tag{1.50}$$
$$x(T) = \bar{x} \tag{1.51}$$
$$k(T) = k_T \tag{1.52}$$

[1] It is worthwhile here to make some clarifications: let T the number of periods the planner chooses to let pollution reach its own threshold of irreversibility. It might be the case that (i) The planner fixes an arbitrary T and set $x(T) = \bar{x}$ or (ii)The planner chooses the optimal T such that $x(T) = \bar{x}$. Both cases are admissible, however, in the second case further optimality conditions are required and are explained in the text.

[2] Transversality conditions are not required in the first period problem

and that is also subject to the following sign and transversality conditions, which are sufficient conditions for maximisation:

$$\lambda_1(t) \geq 0 \tag{1.53}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)k(t) = 0 \tag{1.54}$$

$$\lambda_2(t) \leq 0 \tag{1.55}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_2(t)(x(t) - \bar{x}) = 0 \tag{1.56}$$

It is possible to prove (proof provided in the appendix) that an optimal path for the first period problem exists, because all the state variables are closed subset of $\mathbb{R}$, and the control $C(t) \in \mathcal{C} \subseteq \mathbb{R}$.

Let us denote the maximised welfare function for the first and second period, respectively, $\hat{W}^T$ and $\hat{W}_T$ for $T < \infty$. The maximised utility function for the whole period is then $W = \hat{W}^T + \hat{W}_T$. If T is considered fixed, nothing has to be added to the problem, otherwise, if the planner wish to chose the optimal T, let's say T^*, the maximum principle requires that in addition to the first order conditions and tranversality conditions, also this condition must be satisfied:

$$\mathcal{H}(k^*(T^*), x^*(T^*), c^*(T^*), \lambda_1(T^*), \lambda_2(T^*), T^*) = 0 \tag{1.57}$$

The existence of an optimal control with free final time is proved in the appendix A, provided we modify the assumptions such that T^* is free to vary in $[T_1, T_2]$ and the theorem is satisfied on the interval $[0, T_2]$. If the planner wishes to maximise the utility by choosing the optimal terminal time T^*, it must be the case that

$$\left.\frac{\partial W}{\partial T}\right|_{T^*} = \left.\frac{\partial \hat{W}^T}{\partial T}\right|_{T^*} + \left.\frac{\partial \hat{W}_T}{\partial T}\right|_{T^*} \tag{1.58}$$

where

$$\begin{aligned} e^{\rho T}\frac{\partial \hat{W}^T}{\partial T} &= \frac{c^T(T)^{1-\sigma} - 1}{1-\sigma} - \frac{B\bar{x}^\gamma}{\gamma} + \lambda_1^T(T)[(A-\delta)k^T(T) - c^T(T)] + \\ &+ \lambda_2^T(T)[Ak^T(T) - \underbrace{\eta(\bar{x} + \frac{\theta}{\eta}\bar{x}^2)}_{=0}] \end{aligned} \tag{1.59}$$

$$\begin{aligned} -e^{\rho T}\frac{\partial \hat{W}_T}{\partial T} &= \frac{c_T(T)^{1-\sigma} - 1}{1-\sigma} - \frac{B\bar{x}^\gamma}{\gamma} + \lambda_{1T}(T)[(A-\delta)k_T(T) - c_T(T)] + \\ &+ \lambda_{2T}(T)[Ak_T(T)] \end{aligned} \tag{1.60}$$

Equation 1.58 is verified when $C_T(T) = C^T(T)$ and $k_T(T) = k^T(T)$, i.e. they are continuous functions at T^*. If instead it is not optimal to reach $\bar{x}$ in finite time, it must be the case that

$$\lim_{T\to\infty} \sup \frac{\partial W}{\partial T} = \lim_{T\to\infty} \sup \frac{\partial \hat{W}^T}{\partial T} + \lim_{T\to\infty} \sup \frac{\partial \hat{W}_T}{\partial T} \geq 0 \tag{1.61}$$

so it is necessary that W does not decrease when T increases without limit.

For what concerns the first period problem, define the current value Hamiltonian associated to the problem 1.41 - 1.47 as

$$\begin{aligned} \mathcal{H}(t, k(t), x(t), c(t), \Lambda; \Theta) &\overset{def}{=} \cdot \left[\frac{C(t)^{1-\sigma} - 1}{1-\sigma} - \frac{Bx(t)^\gamma}{\gamma}\right] + \\ +\lambda_1(t)\Big[(A-\delta)k(t) - c(t)\Big] &+ \lambda_2(t)\Big[Ak(t) - \eta(x(t) + \frac{\theta}{\eta}x(t)^2)\Big] \end{aligned} \tag{1.62}$$

where Λ is the set of shadow prices and Θ represents the set of exogenous parameters of the model where, as before, $\Theta = \{A, B, \sigma, \rho, \delta, \eta, \theta, \gamma, mc\}$.
The necessary first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \qquad \Rightarrow \qquad \lambda_1 = C^{-\sigma} \tag{1.63}$$

$$\frac{\partial \mathcal{H}}{\partial k} = \rho\lambda_1 - \dot{\lambda}_1 \qquad \Rightarrow \qquad \dot{\lambda}_1 = \lambda_1(\rho + \delta - A) - \lambda_2 A \tag{1.64}$$

$$\frac{\partial \mathcal{H}}{\partial x} = \rho\lambda_2 - \dot{\lambda}_2 \qquad \Rightarrow \qquad \dot{\lambda}_2 = \lambda_2 \cdot [\rho + \eta(1 - \frac{2\theta}{\eta}x)] + Bx^{\gamma-1} \tag{1.65}$$

(1.66)

so the following system of four differential equations represents the conditions any optimal path has to obey:

$$\dot{k} = (A-\delta)k - \lambda_1^{-\frac{1}{\sigma}} \tag{1.67}$$

$$\dot{x} = Ak - \eta x + \theta x^2 \tag{1.68}$$

$$\dot{\lambda}_1 = \lambda_1(\rho - (A-\delta)) - \lambda_2 A \tag{1.69}$$

$$\dot{\lambda}_2 = \lambda_2(\eta - 2\theta x + \rho) + Bx^{\gamma-1} \tag{1.70}$$

$$k(0) = k_0 \tag{1.71}$$

$$k(T) \geq \underline{\mathrm{k}} \tag{1.72}$$

$$x(0) = x_0, \qquad x_0 < \bar{x} \tag{1.73}$$

$$x(T) = \bar{x}, \qquad T < \infty \tag{1.74}$$

with equations 1.69 and 1.70 representing the Euler equations.

For the second period problem, define the current value Hamiltonian associated to the problem 1.48 - 1.52 as:

$$\begin{aligned}\mathcal{H}(t, k(t), x(t), c(t), \Lambda; \Theta) &\stackrel{def}{=} \lambda_0 \left[\frac{C(t)^{1-\sigma} - 1}{1-\sigma} - \frac{Bx(t)^{\gamma}}{\gamma} \right] + \\ &+ \lambda_1(t) \Big[Ak(t) - \delta k(t) - C(t) \Big] + \lambda_2(t) \cdot Ak(t) \qquad (1.75)\end{aligned}$$

where Λ is the set of shadow prices and, as before $\Lambda = \{\lambda_0(t), \lambda_1(t), \lambda_2(t)\}$ with λ_1 and λ_2 representing, respectively, the shadow prices of capital and pollution and Θ represents the set of exogenous parameters of the model where, as before, $\Theta = \{A, B, \sigma, \rho, \delta, \eta, \theta, \gamma, mc\}$.

The maximum principle asserts that there exists a λ_0 and a continuous and piecewise continuously differentiable functions $\lambda_1(t)$ and $\lambda_2(t)$, such that for all t

$$(\lambda_0, \lambda_1(t), \lambda_2(t)) \neq (0, 0, 0) \qquad (1.76)$$

$$\mathcal{H}(t, k^*(t), x^*(t), c^*(t), \Lambda; \Theta) \geq \mathcal{H}(t, k^*(t), x^*(t), c(t), \Lambda; \Theta) \qquad \forall t \qquad (1.77)$$

Moreover[1],

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \qquad \Rightarrow \qquad \lambda_1 = C^{-\sigma} \qquad (1.78)$$

$$\frac{\partial \mathcal{H}}{\partial k} = \rho\lambda_1 - \dot{\lambda}_1 \qquad \Rightarrow \qquad \dot{\lambda}_1 = \lambda_1(\rho + \delta - A) - \lambda_2 A \qquad (1.79)$$

$$\frac{\partial \mathcal{H}}{\partial x} = \rho\lambda_2 - \dot{\lambda}_2 \qquad \Rightarrow \qquad \dot{\lambda}_2 = \lambda_2 \cdot \rho + Bx^{\gamma-1} \qquad (1.80)$$

$$\lambda_0 = 1 \qquad \text{or} \qquad \lambda_0 = 0 \qquad (1.81)$$

Since the terminal conditions for capital and pollution as time approaches infinity are left free, it follows that $\lim_{t\to\infty} \lambda_1(t) = 0$ and $\lim_{t\to\infty} \lambda_2(t) = 0$ so $\lambda_0 = 1$. Finally, 1.51 and 1.52 have to be satisfied.

Rearranging equation 1.78 we get an expression for consumption in terms of the shadow price of capital:

$$C = \lambda_1^{-\frac{1}{\sigma}} \qquad (1.82)$$

[1] For notational simplicity, in the following I will use interchangeably the generic variable z instead of $z(t)$ whenever this does not constitute ambiguity.

so the economic system can be represented by the following four differential equations

$$\dot{k} = (A-\delta)k - \lambda_1^{-\frac{1}{\sigma}} \tag{1.83}$$

$$\dot{x} = Ak \tag{1.84}$$

$$\dot{\lambda}_1 = \lambda_1(\rho+\delta-A) - \lambda_2 A \tag{1.85}$$

$$\dot{\lambda}_2 = \lambda_2\rho + Bx^{\gamma-1} \tag{1.86}$$

From equation 1.84 is straightforward to see that in order to keep pollution stable through time ($\dot{x} = 0$) it is necessary to keep capital nil. This means that no production nor consumption can occur in steady state, and since for hypothesis the model guarantees a minimum level of consumption, implying also a strictly positive capital and production, no stationary solution can be found in this second period problem. In the long run, the optimal path will converge to a consumption level equal to the subsistence level mc, with a capital level constant and equal to $\underline{\mathrm{k}}$. This level of capital is such that it produces a level of income which sustains a minimum level of consumption and an investment level which is equal to the depreciation of capital.

The existence of a balanced growth path for capital, pollution and consumption can be reasonably excluded because this would imply a constant and equal rate of growth for all the variables involved, consumption, pollution and capital. Since the marginal utility from consumption is an increasing and concave function of consumption, and the marginal disutility from pollution is an increasing and convex function of pollution (so it grows at a rate that is greater than the rate of growth of the marginal utility from consumption), there will be a point on time $t' \in [T,\infty)$ where an additional unit of pollution will produce a disutility higher than the utility produced by an additional unit of consumption. At this point in time, the optimal path will predict a consumption level equal to mc, production equal to $\underline{y}$ and a minimum level of capital $\underline{k}$ that is necessary to guarantee a level of investments that covers the depreciation, and a subsistence level of consumption. Pollution, from t' onward, will have an instantaneous variation $\dot{x} = A\underline{k}$, while the variation of capital and consumption will be nil.

1.2.3 Paths comparison

The model does not allow to say which of the paths gives higher utility, so direct comparison is necessary. In particular, we are interested to see whether an irreversible solution may provide an higher level of discounted utility than an irreversible one. But this requires first of all the computation of W^∞ and $W = \hat{W}^T + \hat{W}_T$. The analysis presented in paragraph 2.2.1 is only partial, because it is just able to say something about the stability of the two steady states and their associated level of welfare W^∞ if the system is in equilibrium and there are no shocks able to carry on the system far away from them, but is is completely unable to say anything about the behaviour of the system in between of the two fixed points, or in any point of the $x-k$ plane. In order to say something about the behaviour of this economy far away from the equilibria, global analysis is needed. In the next section, I will use an algorithm of dynamic programming to carry on this analysis.

As it was previously anticipated, the problem presented here has the peculiarity of having, for each initial condition and in finite time horizon, two simultaneous optimal paths which respect the first order conditions. In accordance to the possibilities available to the planner, it may be optimal either to increase utility by reducing pollution or, viceversa, by increasing consumption. The first choice takes the system in a path which is converging to the saddle stable equilibria introduced in section 2.2.1 (so in this case a reversible solution is optimal), and the other choice takes the system toward the irreversibility threshold for pollution. Whether it is optimal one or the other, is a question I will try to answer below.

1.3 Global analysis

In this section, I am interested to see whether - in case of multiple equilibria - the system can, starting from a neighbourhood of the unstable equilibria, recover and converge to the socially optimum steady state. Due to the lack of closed form solution of this dynamic model, I need to use computational methods. I use the convenient approach of dynamic programming, which provides the value function and the control variable in feedback form. This allows to find the global dynamics of the state space in the region restricted by arbitrary values of capital and pollution, using a fixed grid size technique.

1.3.1 Discretisation

The first step to do that is to discretize the model identified by equations 1.16 - 1.18

$$\max_{c_t \in \mathcal{C}_t} U_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - B \frac{x_t^{\gamma}}{\gamma} \right] \tag{1.87}$$

subject to

$$\begin{aligned} x_{t+1} &= Ak_t - x_t(\eta - \theta x_t - 1) && (1.88) \\ k_{t+1} &= (A - \delta + 1)k_t - c_t && (1.89) \\ \beta &= (1-\rho) && (1.90) \\ k_0 &= k && (1.91) \\ x_0 &= x && (1.92) \end{aligned}$$

Here $\mathcal{C}_t$ denotes the set of discrete control sequences $C = (C_1, C_2, ...)$ for $C_i \in \mathcal{C}$ The optimal value function V is the unique solution of the discrete Hamilton-Jacobi-Bellman's equation

$$V(k,x) = \max_{c \in \mathcal{C}} \left\{ u_t(k_t, x_t, C_t) + \beta V(k_{t+1}, x_{t+1}) \right\} \tag{1.93}$$

with

$$u_t(k_t, x_t, C_t) = \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - B \frac{x_t^{\gamma}}{\gamma} \right] \tag{1.94}$$

If I define the dynamic programming operator T by

$$T(V)(k,x) = \max_{C \in \mathcal{C}} \left\{ u_t(k_t, x_t, C_t) + \beta V(k_{t+1}, x_{t+1}) \right\} \tag{1.95}$$

then V can be characterised as the unique solution of the fixed point equation

$$V(k,x) = T(V)(k,x) \qquad \text{for all } x, k \in \mathcal{R}^n \tag{1.96}$$

1.3.2 Results

The study of dynamic decision models with multiple equilibria is intricate. Multiple equilibria can arise in models with non-concave pay-off functions, externalities and increasing returns. Recently multiple equilibria have been found also in concave economies (for a survey on models with multiple equilibria, see Deissenberg et al (6)). In terms of dynamics, multiple equilibria are difficult to analyse, since the domain of attraction might not coincide with the stable and unstable equilibria, and multiple optimal paths may exist as well. In the context of my model, multiple (non-trivial) equilibria arise from some parameter constellations. In the following, I will consider only a set of parameters which gives multiple steady states, because I believe this case is the most interesting from a policy point of view. Consider the following parameter set:

$$\begin{aligned} B &= 10,000,000 \\ A &= 0.8 \\ \rho &= 0.04 \\ \theta &= 0.05 \\ \eta &= 0.03 \\ \sigma &= 3 \\ \gamma &= 3 \\ \delta &= 0.1 \\ \beta &= 1-\rho = 0.96 \end{aligned}$$

Those parameters yield the following numerical solution for the two (non-trivial) steady states:

Variable	Equilibrium 1	Equilibrium 2
k^*	0.5494e-2	0.2489e-2
x^*	0.2543	0.5240
λ_1^*	0.1758e+08	1.8911e+08
λ_2^*	-0.1450e+08	-1.5602e+08

The eigenvalues of the first equilibria areμ_1= 0.6995, μ_2= 0.0556, μ_3 =-0.0156 and μ_4=-0.6595 and for the second are μ_1 = 0.6999, μ_2= 0.0308, μ_3 = 0.0092 and μ_4= -0.6599. This information allows us to say only that the first equilibria is saddle stable (however, this conclusion holds only locally and what happens between the two steady states is a black box), and that second is unstable. Nothing can be said about the direction of the instability of this latter equilibria. In other words, from the local analysis nothing can be inferred about whether - starting from initial conditions close to the unstable equilibria - the system will converge to the stable (and pareto dominant) equilibria or not. To this purpose, I studied the global dynamics of this system using a dynamic programming algorithm. Dynamic programming allows to draw the phase diagram of the system in terms of the states variables and it is a convenient tool to study the global dynamics in case of multiplicity of equilibria. The algorithm is described in detail in appendix, and results are depicted in figure 2.

The first thing is to check first of all if the system will converge to the socially dominant equilibria or not, starting in proximity of the unstable one. Numerical simulations show that, for example, assuming a fixed plan horizon of 50 periods, there exist two optimality candidates: one path that brings pollution toward the irreversibility region and the other one that converges to the saddle stable (and socially dominant) equilibria. Basically, an efficiently managed economy may choose to achieve the objective of maximising the utility function by means of two instruments: (i) increasing consumption or (ii) reducing pollution. The first policy implies that the consumption profile of the first periods is left low, capital is allow to increase at a very fast rate, and so also consumption in the subsequent periods. The second policy, viceversa, is described by an high level of consumption in the first period (aimed at reducing the level of capital, responsible for the production of pollution), and a low profile (although increasing) of consumption in subsequent periods. The choice between these two paths cannot be made a priori and the computation of the utility's present value is needed.

Figure 2 and 3 show the phase diagrams in terms of the state variables and the behaviour of the control variable for the two different paths. Numerical simulations show that the utility's present value for the first path (the path diverging towards the irreversibility threshold of pollution, represented in figure 2) is equal to -1.1427e+07, against a present value of -1.4796e+07 for the second path in figure 3, the path converging to the saddle stable equilibria. It is therefore worth increasing capital and consumption up or close to the irreversibility threshold of pollution, if the time horizon is sufficiently low.

Things seem different, however, when the plan horizon is longer. As T grows, hypothetically to infinity, numerical simulations suggest that the optimal path is no longer to bring pollution close or up to the irreversibility region, as it is clearly highlighted in the figures below. Those pictures indeed show a tendency of the system to converge to the stable and socially optimal interior equilibria[1]. This may be explained by the fact

[1] The minimum consumption level is assumed to be set at very low levels such that the solution never hits it

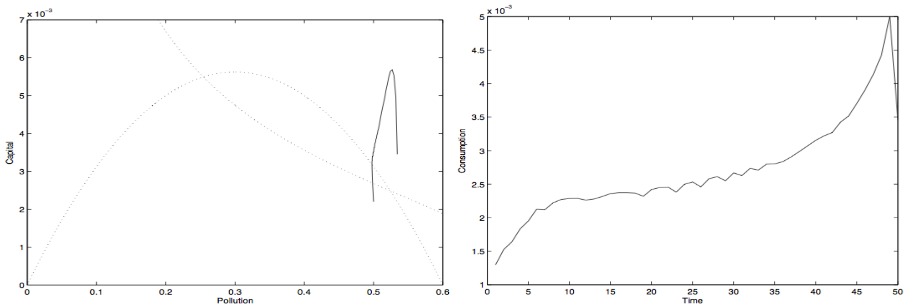


Figure 1.2: Divergent path, T=50, $k_0 = 0.0025$, $x_0 = 0.5$

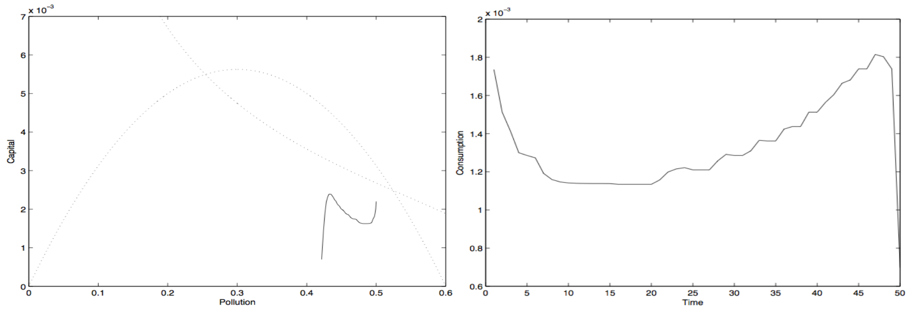


Figure 1.3: Convergent path, T=50, $k_0 = 0.0025$, $x_0 = 0.5$

that the utility of having high levels of consumption for limited amounts of periods followed by minimum levels of consumption and increasing levels of pollution for infinite periods is definitely worse than having moderately high levels of consumption and low levels of pollution forever, especially if the intertemporal rate of preferences is not too high.

As it is possible to see in figure 4, for T=200, the tendency is to keep capital at a level which allows both strictly positive consumption and a reduction of pollution, except for dramatically increase capital, consumption and pollution during the last periods (but this is due to the fact that we are dealing with a routine that in order to be ran has to set a finite time horizon).

So, two identical countries may choose different environmental policies only if they differ in the choice of their planning horizons. Governments who have short term objectives will choose paths which imply a growing stock of pollution through time, whilst governments with longer horizons will choose paths which imply a decrease of pollution and a slower increase in consumption. This convergence to the interior and stable equilibria is found irrespective of the set of initial conditions.

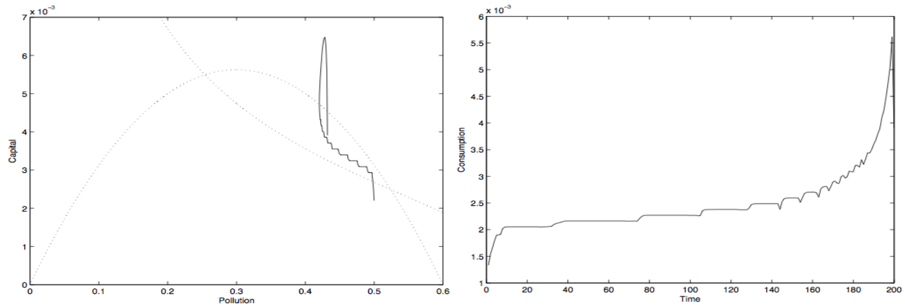


Figure 1.4: Optimal plan, T=200, $\rho = 0.04$, $k_0 = 0.0025$, $x_0 = 0.5$

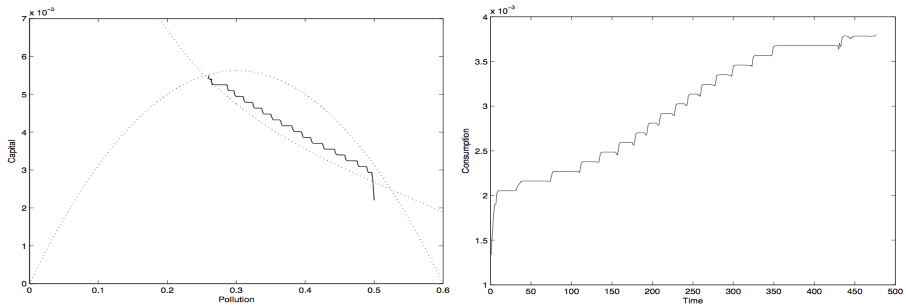


Figure 1.5: Optimal plan, T=∞, ρ=0.04, $k_0 = 0.0025$, $x_0 = 0.5$

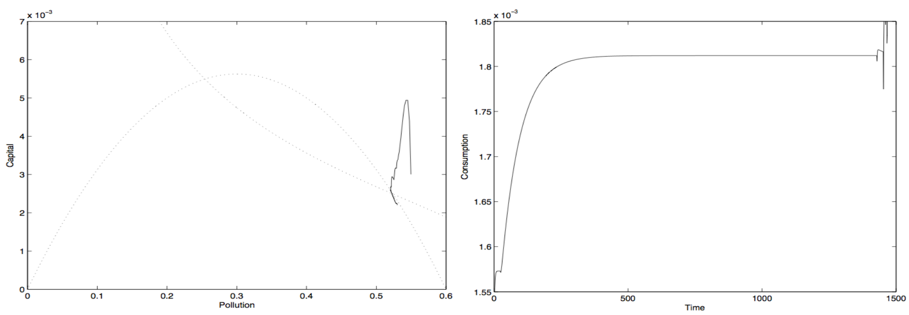


Figure 1.6: Optimal plan, T=1500, ρ=0.04, $k_0 = 0.0022$, $x_0 = 0.5$

Figure 7 displays a path leading pollution to reach its irreversibility threshold. As predicted in the previous section, after pollution has become irreversible, it is optimal for the planner to let capital and consumption to reach their "survival" levels set by $\underline{k}$ and mc, respectively. The picture shows also that pollution grows steadily through time.

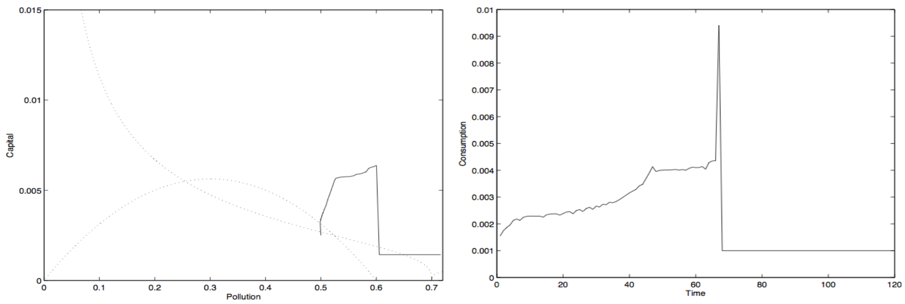


Figure 1.7: Optimal plan, T=∞, $\rho = 0.04$, $k_0 = 0.0025$, $x_0 = 0.5$, $mc = 0.001$

In order to compare the two different environmental policies, it is necessary to compute the present value of all the flows of utility provided in each period by the two paths. The optimal path depicted in figure 5 provides a present value of utility equal to -5.93e+07, whilst the path depicted in figure 7 provides -2.09e+08, with a negative difference of -1.50e+08.

Contrary to Tahvonen and Withagen's result, according to which optimality or not of irreversible pollution accumulation depends on the initial condition for pollution (being optimal when the initial condition for pollution is higher than the level determined by the unstable equilibria), my numerical simulations show a different story. Figure 6 provides a clear example. Initial condition for pollution is 0.53, which is higher than 0.524 characterising the second equilibria. The time span necessary to get into what they call "domain of attraction of the saddle stable equilibria" is however very high, and increases as the initial pollution level increases. Consumption also grows steadily but slowly in proximity of the equilibria: unfortunately the accuracy of the picture is not enough to make it evident. What makes the difference between their model and my model is not the set of initial condition (which is also a poor explanation of the reasons why a country should prefer irreversibility), but the fact that the marginal utility of consumption when consumption is zero, is nil. This is an implication of the fact that they deal with a local pollution problem and not with a global one. Being zero the marginal utility from consumption when there is no consumption at all implies that the population can move elsewhere to satisfy their needs. In my model it is not possible, and there survival (and consumption, although at minimum levels) is always preferred to an additional unit of pollution.

The simulations therefore are clear in highlighting the fact that irreversibility is never optimal, if the planning horizon is infinite, and no matters the initial level of pollution. Different environmental policies can therefore be explained only by different planning horizon of the governments, all other parameters constant.

1.3.3 The effect of ρ on the global dynamics of the system

The representative household's level of impatience, represented by ρ, may play a crucial role in determining the environmental policy chosen by the planner. The more impatient the people are, the more probable is a policy which implies a growing stock of pollution through time. The effects are someway similar to a shortening of the time horizon, and this is confirmed by the simulations. Figure 6 represents the global dynamics of the system assuming a time horizon of 200 periods, and an intertemporal rate of preferences equal to 0.009. The only parameter that distinguishes figure 6 from figure 4 is the level of ρ, but as it is possible to see the dynamics is dramatically different. Convergency to

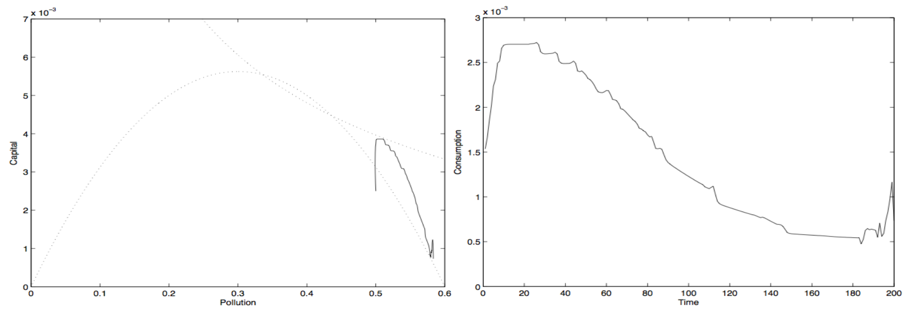


Figure 1.8: Optimal plan, T=200, $\rho = 0.09$, $k_0 = 0.0025$, $x_0 = 0.5$

the saddle stable equilibria is harder to find, since the high discount rate and the finite horizon make worth for the planner choosing a path which implies an high growth rate of consumption for the first periods, which are valued more than future ones, especially because pollution - compared to consumption, grows at much slower rate.

1.4 Conclusion

This paper contributes in the debate about the necessity of a unified global environmental policy kept by all the nations. With a theoretical model of economic growth with pollution accumulation and an endogenous function for the natural decay of pollution, I show with numerical simulation that in an efficiently planned economy, with infinite time horizon plan, irreversible pollution accumulation cannot be an optimal policy. Optimality of such a policy can occur only if the plan of government is short-minded. Since utility depends on both consumption and the level of pollution, in principle the planner can choose to achieve the maximum level of welfare by, alternatively, increasing consumption or reducing pollution. The second strategy pays in the long run, while the first in short run.

It would be important to keep in mind that although each nation decides on its own its environmental policy and it is free to join international agreement, we live in the same planet and if all the countries would be one with the priority to safeguard life, they would not engage in such production of pollution, in each form. So, despite incentives are not enough to give up opportunities to grow in the short run, it would be useful to ask whether such individual policy can be consistent with individual long terms goal. Each country may decide to pollute a lake if there are others from which he can extract utility, but what happens when all the lakes are polluted?

Unfortunately, populist policies and the fact that politicians stay in power for few years and needs to be reelected can - someway - affect the environmental policies undertaken by the countries. Special interests overcome in importance other issues which are in general considered of marginal relevance, like the environment, because a policy undertaken by a single country can only marginally affect it, especially when it deals with global problems.

So, a deep study of the incentives taking a country to engage in global emission's reduction is needed and it can be part of future research.

1.5 Appendix

A. Proof of the existence of an optimal path for the first period problem - T fixed

To prove the existence of an optimal path for the first period problem when the final time T is fixed and the pollution at T is equal to its threshold, I use the Filippov - Cesari theorem of existence of an optimal control (Seierstad and Sydsæter (13), p. 132) which requires the convexity of the set

$$N(k,x,\mathcal{C},t) = \left\{ \frac{c^{1-\sigma}-1}{1-\sigma} - \frac{Bx^\gamma}{\gamma} + \omega, (A-\delta)k - c, Ak - \eta(x + \frac{\theta}{\eta}x^2) \right\} \tag{1.97}$$

where $\mathcal{C} \subseteq \mathbb{R}$ represents the set of all admissible controls, and $\omega \leq 0$. The theorem states: Consider the standard optimal control problem 1.41 - 1.47. Assume that:

- There exists an admissible triple $(k(t), x(t), c(t))$.
- $N(k,x,\mathcal{C},t)$ is convex for each (k,x,t).
- $\mathcal{C}$ is closed and bounded.
- There exists two numbers $\bar{k}$ and $\bar{x}$ such that $\|k(t)\| \leq \bar{k}$ and $\|x(t)\| \leq \bar{x}$ for all $t \in [0,T]$ and all admissible pairs $(k(t), x(t), c(t))$.

Then, there exists an optimal pair $(k^*(t), x^*(t), c^*(t))$ (with $c^*(t)$ measurable). In order to prove the convexity of the set in 1.97, let us keep $(k(t), x(t), t)$ fixed, so $k(t) = K$ and $x(t) = X$. Let y_1, y_2, y_3 three arbitrary points in $N(K,X,\mathcal{C},t)$, i.e.

$$y1 = \left\{ \Big(\frac{c_1^{1-\sigma}-1}{1-\sigma} - \frac{BX^\gamma}{\gamma}\Big)e^{-\rho t} + \omega_1, (A-\delta)K - c_1, AK - \eta(X - \frac{\theta}{\eta}X^2) \right\}$$
$$y2 = \left\{ \Big(\frac{c_2^{1-\sigma}-1}{1-\sigma} - \frac{BX^\gamma}{\gamma}\Big)e^{-\rho t} + \omega_2, (A-\delta)K - c_2, AK - \eta(X - \frac{\theta}{\eta}X^2) \right\}$$
$$y3 = \left\{ \Big(\frac{c_3^{1-\sigma}-1}{1-\sigma} - \frac{BX^\gamma}{\gamma}\Big)e^{-\rho t} + \omega_3, (A-\delta)K - c_3, AK - \eta(X - \frac{\theta}{\eta}X^2) \right\}$$

for some $\omega_1, \omega_2, \omega_3 \leq 0$ and $c_1, c_2, c_3 \in \mathcal{C}$. Let λ_1 and λ_2 two positive constants such that $\lambda_1 + \lambda_2 \leq 1$. I need to prove that $y_4 = \lambda_1 y_1 + \lambda_2 y_2 + (1-\lambda_1-\lambda_2)y_3 \in N(K,X,\mathcal{C},t)$. Put $\lambda_1 y_1 + \lambda_2 y_2 + (1-\lambda_1-\lambda_2)y_3 = (z_1, z_2, z_3)$. The first component z_1 is:

$$\begin{aligned} z_1 &= \lambda_1\Big(\frac{c_1^{1-\sigma}-1}{1-\sigma}-\frac{BX^\gamma}{\gamma}\Big)e^{-\rho t}+\lambda_1\omega_1+ \\ &+ \lambda_2\Big(\frac{c_2^{1-\sigma}-1}{1-\sigma}-\frac{BX^\gamma}{\gamma}\Big)e^{-\rho t}+\lambda_2\omega_2+ \\ &+ (1-\lambda_1-\lambda_2)\Big(\frac{c_3^{1-\sigma}-1}{1-\sigma}-\frac{BX^\gamma}{\gamma}\Big)e^{-\rho t}+(1-\lambda_1-\lambda_2)\omega_3 \quad (1.98) \\ &= \Big\{\lambda_1\frac{c_1^{1-\sigma}-1}{1-\sigma}+\lambda_2\frac{c_2^{1-\sigma}-1}{1-\sigma}+(1-\lambda_1-\lambda_2)\frac{c_3^{1-\sigma}-1}{1-\sigma}\Big\}e^{-\rho t}+ \\ &- \frac{BX^\gamma}{\gamma}e^{-\rho t}+\lambda_1\omega_1+\lambda_2\omega_2+(1-\lambda_1-\lambda_2)\omega_3 \quad (1.99) \end{aligned}$$

Since it is known that W^T is concave in c, so $W^{T}" \leq 0$, we have

$$\begin{aligned} &\lambda_1\frac{c_1^{1-\sigma}-1}{1-\sigma}+\lambda_2\frac{c_2^{1-\sigma}-1}{1-\sigma}+(1-\lambda_1-\lambda_2)\frac{c_3^{1-\sigma}-1}{1-\sigma} \\ \leq\ & \frac{[\lambda_1c_1+\lambda_2c_2+(1-\lambda_1-\lambda_2)c_3]^{1-\sigma}-1}{1-\sigma} \\ =\ & \frac{c_4^{1-\sigma}-1}{1-\sigma} \end{aligned}$$

with $c_4=\lambda_1c_1+\lambda_2c_2+(1-\lambda_1-\lambda_2)c_3$. Then, $c_4 \in \mathcal{C}$. Using this result, from the last inequality we see that

$$z_1 \leq \left(\frac{c_4^{1-\sigma}-1}{1-\sigma}-\frac{BX^\gamma}{\gamma}\right)e^{-\rho t}+\lambda_1\omega_1+\lambda_2\omega_2+(1-\lambda_1-\lambda_2)\omega_3 \quad (1.100)$$

Define $\omega_4 = z_1 - \left(\frac{c_4^{1-\sigma}-1}{1-\sigma}-\frac{BX^\gamma}{\gamma}\right)e^{-\rho t}$. Then, from 3.11,

$$\omega_4 \leq \lambda_1\omega_1+\lambda_2\omega_2+(1-\lambda_1-\lambda_2)\omega_3 \leq 0 \qquad \text{since } \omega_1,\omega_2,\omega_3 \leq 0$$

The second and third components, z_2 and z_3 are found similarly to the first:

$$\begin{aligned} z_2 &= \lambda_1[(A-\delta)K-c_1]+\lambda_2[(A-\delta)K-c_2]+(1-\lambda_1-\lambda_2)[(A-\delta)K-c_3] \\ &= (A-\delta)K-(\lambda_1c_1+\lambda_2c_2+(1-\lambda_1-\lambda_2)c_3) \\ &= (A-\delta)K-c_4 \end{aligned}$$

$$\begin{aligned} z_3 &= \lambda_1[AK-\eta(X-\frac{\theta}{\eta}X^2)]+\lambda_2[AK-\eta(X-\frac{\theta}{\eta}X^2)]+ \\ &\quad +(1-\lambda_1-\lambda_2)[AK-\eta(X-\frac{\theta}{\eta}X^2)] \\ &= AK-\eta(X-\frac{\theta}{\eta}X^2) \end{aligned}$$

Piecing all this together, we see that we have found a $c_4 \in \mathcal{C}$ and a $\omega_4 \leq 0$ such that $\lambda_1 y_1+\lambda_2 y_2+(1-\lambda_1-\lambda_2)y_3 = \{(\frac{c_4^{1-\sigma}-1}{1-\sigma}-\frac{BX^\gamma}{\gamma})e^{-\rho t}+\omega_4, (A-\delta)K-c_4, AK-\eta(X-\frac{\theta}{\eta}X^2)\}$. Hence, $\lambda_1 y_1 + \lambda_2 y_2 + (1-\lambda_1-\lambda_2)y_3 \in N(K, X, \mathcal{C}, t)$ and thus $N(K, X, \mathcal{C}, t)$ is convex.

B. The dynamic programming algorithm

The algorithm approximates the solution on a grid Γ covering a compact subset Ω of the state space. I pick a reasonable set Ω and consider only trajectories which remain in Ω in all future times. I assume that for any point $(k, x) \in \Omega$ there exists at least one control value c such that $(k_{t+1}, x_{t+1}) \in \Omega$ holds. Denoting the nodes of the grid Γ by (k^i, x^j), $i = 1, ..., n$ and $j = 1, ..., m$, the approximation V^Γ satisfy

$$V^\Gamma(k^i, x^j) = T(V^\Gamma)(k^i, x^j) \tag{1.101}$$

for all nodes (k^i, x^j) of the grid, where the value of V^Γ for points (k, x) which are not grid points (these are needed for the evaluation of T) is determined by bilinear interpolation. Basically, the standard computational algorithm that is used here can be summarised as follows (cite larson):

1. The first step is to set up a grid for the state variables. Each level of capital k and each level of pollution x are quantised, respectively, to N_k and N_x equidistant levels, from 0 to, respectively, $\bar{k}$ and $\bar{x}$. In total, then, the grid points for the state variables are $N_k \cdot N_x$. The control variable c is quantised to N_c equidistant levels, from 0 to $\bar{c}$.

2. For each point in the grid $(k(i), x(j))$, $i = 1, ..., N_k$ and $j = 1, ..., N_x$, each control $c(h)$, $h = 1, ..., N_c$ is applied, and the next state is computed according to the formulas given by equations 1.88 and 1.89. Let us call the next-state value of k and x, respectively, $k1$ and $x1$. Notice that $k1$ and $x1$ are tri-dimensional matrices whose generic element is represented by $k1(i, j, h)$ and $x1(i, j, h)$ and whose dimensions are $N_k \cdot N_x \cdot N_c$. Furthermore, the elements of $k1$ and $x1$ are, in general, not grid points. I then check whether each element of $k1 \in [0, \bar{k}]$ and $x1 \in [0, \bar{x}]$. If they do not belong to those intervals, their values are replaced with "missing".

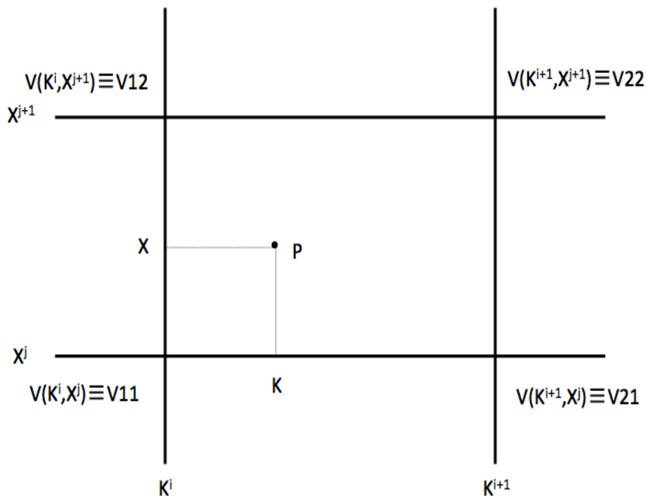
3. Define the number of periods T, and set up an index $l = 1$. Evaluate k_0 and x_0.

4. The procedure is backward. At the final time T, citizens consume what is left in terms of capital, so $c_T = k_T$ irrespective to the value of x. So, for each point in the grid $(k(i), x(j))$, $i = 1, ..., N_k$ and $j = 1, ..., N_x$ I compute the value function at time T which is nothing but

$$V^{\Gamma}(k(i), x(j), T) = \frac{k(i)^{(1-\sigma)} - 1}{1-\sigma} - B\frac{x(j)^{\gamma}}{\gamma}$$
$$i = 1, ..., N_k, j = 1, ..., N_x \tag{1.102}$$

   I then store in memory $V^{\Gamma}(k(i), x(j), T)$ and $c(i, j, T) = k(i)$ constant across the j and the T-dimensions.

5. At time $T - l$, for each $i = 1, ..., N_k$, $j = 1, ..., N_x$ and $h = 1, ..., N_c$ I compute the next-period value function $V1(k1(i, j, h), x1(i, j, h))$ interpolating the existing values of $V^{\Gamma}(k(i), x(j), T - l + 1)$ stored in memory. Of course, if either $x1(i, j, h)$ or $k1(i, j, h)$ (or both) are "missing values", also $V1(k1(i, j, h), x1(i, j, h))$ will be "missing". I need to interpolate those values because in general $k1(i, j, h)$ and $x1(i, j, h)$ are not grid points, and I know the value of $V^{\Gamma}(k(i), x(j), T - l + 1)$ only for grid points. Notice that $V1(k1(i, j, h), x1(i, j, h))$ is a tri-dimensional matrix whose dimensions are $N_k \cdot N_x \cdot N_c$. The procedure is the following: the fact that in general $k1(i, j, h)$ and $x1(i, j, h)$, $i = 1, ..., N_k$, $j = 1, ..., N_x$ and $h = 1, ..., N_c$ do not lie on the grid means that I am in the situation in which I have to compute the value function knowing its approximation on four equidistant points around it. Graphically,



I want to approximate a function on the point $P = (k, x)$ that represents my next period values of the state variable once the control is applied, knowing the value function in the points $V11$, $V12$, $V21$ and $V22$. The function in P is computed according to the following formula:

$$\begin{aligned} V(P) = & \frac{V11}{(k^{i+1}-k^i)(x^{j+1}-x^j)} \cdot (k^{i+1}-k)(x^{j+1}-x) \\ & \frac{V21}{(k^{i+1}-k^i)(x^{j+1}-x^j)} \cdot (k-k^i)(x^{j+1}-x) \\ & \frac{V12}{(k^{i+1}-k^i)(x^{j+1}-x^j)} \cdot (k^{i+1}-k)(x-x^j) \\ & \frac{V22}{(k^{i+1}-k^i)(x^{j+1}-x^j)} \cdot (k-k^i)(x-x^i) \end{aligned} \tag{1.103}$$

6. For each $i = 1, ..., N_k$, $j = 1, ..., N_x$ and $h = 1, ..., N_c$ I compute

$$\begin{aligned} V'(k(i), x(j), c(h), T-l) = & \frac{c(h)^{1-\sigma}-1}{1-\sigma} - B\frac{x(j)^{\gamma}}{\gamma} + \\ & \beta V1(k1(i,j,h), x1(i,j,h)) \end{aligned} \tag{1.104}$$

   After that, for each $i = 1, ..., N_k$, $j = 1, ..., N_x$ I chose the maximum variable over the h-dimension (control), by direct comparison. Those values are stored $c(i, j, T-l)$ and $V^{\Gamma}(k(i), x(j), T-l)$

7. The value of l takes $l + 1$.

8. I check whether l is equal to T. If it is not, I go back to point 5. If $l = T$, then

9. Define three vectors $k^*(t)$, $x^*(t)$ and $c^*(t)$, $t = 1, ..., T$ which represent the optimal trajectories of capital, pollution and consumption starting from the initial conditions x_0 and k_0. Set $k^*(1) = k_0$ and $x^*(1) = x_0$.

10. Set time $t = 1$.

11. Find the ith and jth elements in the vectors of quantized k and x, which are closer to the values $k^*(t)$ and $x^*(t)$. If $(k(i), x(j)) = (k^*(t), x^*(t))$ then the state is a grid point, and $c^*(t)$ is read directly as $c(i, j, t)$. If $(k(i), x(j)) \neq (k^*(t), x^*(t))$, $c^*(t)$ is computer through bilinear interpolation using values of $c(i, j, t)$ at the closest grid points for k and x.

12. Check whether $t = T + 1$. If this equality is satisfied, go to point 15. Else, go to the next point.

13. Compute k^*_{t+1} and x^*_{t+1} according to the following equations:

$$k^*(t+1) = (A + \delta - 1)k^*(t) - c^*(t) \quad (1.105)$$

$$x^*(t+1) = Ak^*(t) - x^*(t)(\eta - \theta x^*(t) - 1) \quad (1.106)$$

14. Time t takes value $t + 1$. Go to point 11.

15. The value function is computed as follows: set time $t = T$ and an index $l = 1$. At final time T, the value function is computed according to the following formula:

$$V^*(T) = \frac{c^*(T)^{1-\sigma} - 1}{1 - \sigma} - B\frac{x^*(T)\gamma}{\gamma} \quad (1.107)$$

16. Check wheter $t = 0$. If so, end the program, otherwise go to the next point.

17. Time t takes values $T - l$.

18. The value function at time t is now

$$V^*(t) = \frac{c^*(t)^{1-\sigma} - 1}{1 - \sigma} - B\frac{x^*(t)^\gamma}{\gamma} + \beta V^*(t+1) \quad (1.108)$$

19. The index l takes value $l + 1$. Go to point 16.

This computational procedure is very appealing for a number of reasons. First, because thorny questions about existence and uniqueness are avoided; as long as there is at least one feasible control sequence, then the direct-search procedure guarantees that the absolute maximum utility is achieved. Furthermore, extremely general types of systems equations and constraints can be handled. Constraints actually reduce the computational burden by decreasing the admissible sets of states and controls. Finally, the optimal control is obtained as a true feedback solution in which the optimal control for any admissible state and stage is determined. However, to the best of my knowledge there is not any algorithm able to identify whether multiple solutions exist, and this one makes no exception. Identify them may be difficult, because it requires repeated simulation of the same routine, and a bit of luck. If indeed multiple solutions exist, since in general they do not provide the same values of discounted flows of utility, the path which provides the highest value is generally chosen by the routine. So, most of the time, one does not even realise that multiple paths satisfying the first order conditions exist. They can only be found choosing appropriate grids, and this is a very difficult task because it requires first of all the knowledge about the existence of multiple solutions, and a good guess about the direction of the two paths. Finally, luck is always welcome.

Codes

In this section, I report the matlab codes that I used to draw the pictures and to compute the present value of the flow of utilities, in order to compare the two paths. The first program is the following, named

PROGRAM fp_main:

code:

```
clear

fp_parameters
fp_step1
fp_nextstates


fp_interpolation
save I1 I
save C1 C
```

PROGRAM fp_parameters:

code:

```
% parameters of the problem
B=10000000;
A=0.8;
rho=0.04;
theta=0.05;
eta=0.03;
sigma=3;
gamma=3;
delta=0.1;
beta=1-rho;

% definition of the grid for capital, pollution and consumption


kgrid=linspace(0.00,0.006,100); %This commnand creates a vector of 30
equidistant points from 0 to 0.1. The sintax is linspace(begin, end, number
of points)
xgrid=linspace(0.2,0.6,120);
cgrid=linspace(0.00,0.005,60);

% Number of periods
T=50;
```

PROGRAM fp_step1:

code:

```
% At time T (final), no control is applied because I assume that people
% consume what is left, so c(T)=(A-delta+1)k(T). So, for every state in
% kgrid and every x in xgrid, I compute and store C(i,j,t) (consumption↙
associated to the
% i-th state at time T) as well as I(i,j,t) (value function for each state↙
at
% time T).

% Initialisation of the matrix of solution for consumption:
C=NaN(length(kgrid),length(xgrid),T);

t=T;
for i=1:length(kgrid)
    for j=1:length(xgrid)
        I(i,j,t)=(((A-delta+1)*kgrid(i))^(1-sigma)-1)/(1-sigma)-B*xgrid(j)↙
^gamma/gamma;
        C(i,j,t)=(A-delta+1)*kgrid(i);
    end
end
```

PROGRAM fp_nextstates:

code:

```
for i=1:length(kgrid)
    for h=1:length(cgrid)
        K1(i,h)=(A-delta+1)*kgrid(i)-cgrid(h);
        if K1(i,h)<0 || K1(i,h)>max(kgrid)
            K1(i,h)=NaN;
        end
    end
end
for i=1:length(kgrid)
    for j=1:length(xgrid)
        X1(i,j)=A*kgrid(i)-xgrid(j)*(eta-theta*xgrid(j)-1);
        if X1(i,j)<0
                X1(i,j)=0;
        elseif X1(i,j)>max(xgrid)
                 X1(i,j)=NaN;
        end
    end
end
```

PROGRAM fp_interpolation:

code:

```
for z=1:T-1
t=T-z;
for i=1:length(kgrid)
    for j=1:length(xgrid)
        for h=1:length(cgrid)

if isnan(K1(i,h))==0 && isnan(X1(i,j))==0

tmpk=abs(kgrid-K1(i,h));
[k k]=min(tmpk);
ck=kgrid(k);
tmpx=abs(xgrid-X1(i,j));
[x x]=min(tmpx);
cx=xgrid(x);

else
        I1(i,j,h,t)=NaN;
        V(i,j,h,t)=NaN;
end
```

```
    if ck==K1(i,h) && cx==X1(i,j)  % This means that X1 and K1 are grid↙
points
        I1(i,j,h,t)=I(k,x,t+1);
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +↙
beta*I1(i,j,h,t);

    elseif ck==K1(i,h) && X1(i,j)<xgrid(x) && x>1% This means that k1 is a↙
grid point but not x1. I have to interpolate between xgrid(x-1) and xgrid(x)
        I1(i,j,h,t)=I(k,x-1,t+1)+(X1(i,j)-xgrid(x-1))*(I(k,x,t+1)-I(k,x-1,↙
t+1))/(xgrid(x)-xgrid(x-1));
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +↙
beta*I1(i,j,h,t);

    elseif ck==K1(i,h) && X1(i,j)>xgrid(x) && x<length(xgrid)% This means↙
that k1 is a grid point but not x1. I have to interpolate between xgrid(x)↙
and xgrid(x+1)
        I1(i,j,h,t)=I(k,x,t+1)+(X1(i,j)-xgrid(x))*(I(k,x+1,t+1)-I(k,x,t+1))/↙
(xgrid(x+1)-xgrid(x));
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +↙
beta*I1(i,j,h,t);

    elseif cx==X1(i,j) && K1(i,h)<kgrid(k) && k>1% This means that X1 is a↙
grid point but not K1. I have to interpolate between kgrid(k-1) and kgrid(k)
        I1(i,j,h,t)=I(k-1,x,t+1)+(K1(i,h)-kgrid(k-1))*(I(k,x,t+1)-I(k-1,x,↙
t+1))/(kgrid(k)-kgrid(k-1));
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +↙
beta*I1(i,j,h,t);

    elseif cx==X1(i,j) && K1(i,h)>kgrid(k) && k<length(kgrid) % This means↙
that X1 is a grid point but not K1. I have to interpolate between kgrid(k-1)↙
and kgrid(k)
        I1(i,j,h,t)=I(k,x,t+1)+(K1(i,h)-kgrid(k))*(I(k+1,x,t+1)-I(k,x,t+1))/↙
(kgrid(k+1)-kgrid(k));
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +↙
```

```
beta*I1(i,j,h,t);

    elseif K1(i,h)<kgrid(k) && X1(i,j)<xgrid(x) && k>1 && x>1
        % I need to interpolate between kgrid(k-1), kgrid(k), xgrid(x-1)
        % and xgrid(x)
        D=(xgrid(x)-xgrid(x-1))*(kgrid(k)-kgrid(k-1));
        I1(i,j,h,t)=I(k-1,x-1,t+1)/D*(xgrid(x)-X1(i,j))*(kgrid(k)-K1(i,
h))...
            +I(k-1,x,t+1)/D*(X1(i,j)-xgrid(x-1))*(kgrid(k)-K1(i,h))...
            +I(k,x-1,t+1)/D*(xgrid(x)-X1(i,j))*(K1(i,h)-kgrid(k-1))...
            +I(k,x,t+1)/D*(X1(i,j)-xgrid(x-1))*(K1(i,h)-kgrid(k-1));
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +
beta*I1(i,j,h,t);

    elseif K1(i,h)>kgrid(k) && X1(i,j)<xgrid(x) && x>1 && k<length(kgrid)
         % I need to interpolate between kgrid(k), kgrid(k+1), xgrid(x-1)
         % and xgrid(x)
         D=(xgrid(x)-xgrid(x-1))*(kgrid(k+1)-kgrid(k));
         I1(i,j,h,t)=I(k,x-1,t+1)/D*(xgrid(x)-X1(i,j))*(kgrid(k+1)-K1(i,
h))...
             +I(k,x,t+1)/D*(X1(i,j)-xgrid(x-1))*(kgrid(k+1)-K1(i,h))...
             +I(k+1,x-1,t+1)/D*(xgrid(x)-X1(i,j))*(K1(i,h)-kgrid(k))...
             +I(k+1,x,t+1)/D*(X1(i,j)-xgrid(x-1))*(K1(i,h)-kgrid(k));
         V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma
+ beta*I1(i,j,h,t);

    elseif K1(i,h)<kgrid(k) && X1(i,j)>xgrid(x) && k>1 && x<length(xgrid)
        % I need to interpolate between kgrid(k-1), kgrid(k), xgrid(x) and
        % xgrid(x+1)
        D=(xgrid(x+1)-xgrid(x))*(kgrid(k)-kgrid(k-1));
        I1(i,j,h,t)=I(k-1,x,t+1)/D*(xgrid(x+1)-X1(i,j))*(kgrid(k)-K1(i,
h))...
            +I(k-1,x+1,t+1)/D*(X1(i,j)-xgrid(x))*(kgrid(k)-K1(i,h))...
            +I(k,x,t+1)/D*(xgrid(x+1)-X1(i,j))*(K1(i,h)-kgrid(k-1))...
            +I(k,x+1,t+1)/D*(X1(i,j)-xgrid(x))*(K1(i,h)-kgrid(k-1));
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +
beta*I1(i,j,h,t);

    elseif K1(i,h)>kgrid(k) && X1(i,j)>xgrid(x) && k<length(kgrid) &&
x<length(xgrid)
        % I need to interpolate between kgrid(k), kgrid(k+1), xgrid(x), and
        % xgrid(x+1)
        D=(xgrid(x+1)-xgrid(x))*(kgrid(k+1)-kgrid(k));
        I1(i,j,h,t)=I(k,x,t+1)/D*(xgrid(x+1)-X1(i,j))*(kgrid(k+1)-K1(i,
h))...
            +I(k,x+1,t+1)/D*(X1(i,j)-xgrid(x))*(kgrid(k+1)-K1(i,h))...
            +I(k+1,x,t+1)/D*(xgrid(x+1)-X1(i,j))*(K1(i,h)-kgrid(k))...
            +I(k+1,x+1,t+1)/D*(X1(i,j)-xgrid(x))*(K1(i,h)-kgrid(k));
        V(i,j,h,t)=(cgrid(h)^(1-sigma)-1)/(1-sigma)-B*xgrid(j)^gamma/gamma +
beta*I1(i,j,h,t);

    end

end
        end
end
```

```
for i=1:length(kgrid)
    for j=1:length(xgrid)
        [c h]=max(V(i,j,:,t));
        C(i,j,t)=cgrid(h);
        I(i,j,t)=c;
    end
end
end
```

PROGRAM fp_submain:

code:

1. ON THE NON-OPTIMALITY OF IRREVERSIBLE POLLUTION ACCUMULATION FOR AN INFINITELY LIVED PLANNED ECONOMY.

```
load C1.mat
load I1.mat
fp_parameters
fp_initialconditions
fp_retrievekx

fp_vstar

fp_plot
```

PROGRAM fp_initialconditions:

code:

```
%Declaration of the initial conditions. n represents the number of couples
%of initial conditions.

n=1;
k0=linspace(0.0025,0.0025,n);
x0=linspace(0.5,0.5,n);

IC=[k0;x0];
```

PROGRAM fp_retrievekx:

code:

```
% I generate a matrix of dimension 3,T. In the first row k*, in the second
% row x*, in the third row c*

sol=NaN(3,T,n);


z=1;
while z<=n
sol(1,1,z)=IC(1,z);
sol(2,1,z)=IC(2,z);
z=z+1;
end


 t=1;
 for z=1:n
 if sol(1,t,z)<=min(kgrid) || sol(1,t,z)>=max(kgrid) || sol(2,t,z)<=min
(xgrid) || sol(2,t,z)>=max(xgrid)
     display('Initial conditions out of the grid')
     display('Change the grid, or the initial conditions')
 end
 end
 z=1;
 while z<=n
     t=1;
    while t<=T-1
    if sol(1,t,z)>min(kgrid) && sol(1,t,z)<max(kgrid) && sol(2,t,z)>min
(xgrid) && sol(2,t,z)<max(xgrid)
```

```
    tmpk=abs(kgrid-sol(1,t,z));
    [k k]=min(tmpk);
    ck=kgrid(k);
    tmpx=abs(xgrid-sol(2,t,z));
    [x x]=min(tmpx);
    cx=xgrid(x);

    else
            sol(1,t,z)=NaN;
            sol(2,t,z)=NaN;
            sol(3,t,z)=NaN;
    end

    if ck==sol(1,t,z) && cx==sol(2,t,z)
          % this means that x and k are grid points so c* is read directly
          sol(3,t,z)=C(k,x,t);
          sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          if sol(1,t+1,z)<=mc/(A-delta)
              sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
              sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          end
          sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);

    elseif ck==sol(1,t,z) && sol(2,t,z)<xgrid(x)&& x>1
          % I need to interpolate between xgrid(x-1) and xgrid(x) - linear
```

```
          % interpolation since k is a grid point.
          sol(3,t,z)=C(k,x-1,t)+(sol(2,t,z)-xgrid(x-1))*(C(k,x,t)-C(k,x-1,↙
t))/(xgrid(x)-xgrid(x-1));
          sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          if sol(1,t+1,z)<=mc/(A-delta)
              sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
              sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          end
          sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);

    elseif ck==sol(1,t,z) && sol(2,t,z)>xgrid(x) && x<length(xgrid)
          % I need to interpolate between xgrid(x) and xgrid(x+1) - linear
          % interpolation since k is a grid point
          sol(3,t,z)=C(k,x,t)+(sol(2,t,z)-xgrid(x))*(C(k,x+1,t)-C(k,x,t))/↙
(xgrid(x+1)-xgrid(x));
          sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          if sol(1,t+1,z)<=mc/(A-delta)
              sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
              sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          end
          sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);

    elseif sol(1,t,z)<kgrid(k) && cx==sol(2,t,z) && k>1
          % I need to interpolate between kgrid(k-1) and kgrid(k) - linear
          % interpolation since x is a grid point
          sol(3,t,z)=C(k-1,x,t)+(sol(1,t,z)-kgrid(k-1))*(C(k,x,t)-C(k-1,x,↙
t))/(kgrid(k)-kgrid(k-1));
          sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          if sol(1,t+1,z)<=mc/(A-delta)
              sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
              sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          end
          sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);
```

```
    elseif sol(1,t,z)>kgrid(k) && cx==sol(2,t,z) && k<length(kgrid)
           % I need to interpolate between kgrid(k) and kgrid(k+1) - linear
           % interpolation since x is a grid point
           sol(3,t,z)=C(k,x,t)+(sol(1,t,z)-kgrid(k))*(C(k+1,x,t)-C(k,x,t))/
(kgrid(k+1)-kgrid(k));
           sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
           if sol(1,t+1,z)<=mc/(A-delta)
               sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
               sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
           end
           sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);

    elseif sol(1,t,z)<kgrid(k) && sol(2,t,z)<xgrid(x) && k>1 && x>1
           % Bilinear interpolation between kgrid(k-1), kgrid(k), xgrid(x-1)
           % and xgrid(x)
           D=(kgrid(k)-kgrid(k-1))*(xgrid(x)-xgrid(x-1));
           sol(3,t,z)=C(k-1,x-1,t)*(kgrid(k)-sol(1,t,z))*(xgrid(x)-sol(2,t,
z))/D...
               +C(k,x-1,t)*(sol(1,t,z)-kgrid(k-1))*(xgrid(x)-sol(2,t,z))/D...
               +C(k-1,x,t)*(kgrid(k)-sol(1,t,z))*(sol(2,t,z)-xgrid(x-1))/D...
               +C(k,x,t)*(sol(1,t,z)-kgrid(k-1))*(sol(2,t,z)-xgrid(x-1))/D;
           sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
```

```
           if sol(1,t+1,z)<=mc/(A-delta)
               sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
               sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
           end
           sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);


    elseif sol(1,t,z)<kgrid(k) && sol(2,t,z)>xgrid(x) && k>1 && x<length
(xgrid)
           % Bilinear interpolation between kgrid(k-1), kgrid(k), xgrid(x),
           % xgrid(x+1)
           D=(kgrid(k)-kgrid(k-1))*(xgrid(x+1)-xgrid(x));
           sol(3,t,z)=C(k-1,x,t)*(kgrid(k)-sol(1,t,z))*(xgrid(x+1)-sol(2,t,
z))/D...
               +C(k,x,t)*(sol(1,t,z)-kgrid(k-1))*(xgrid(x+1)-sol(2,t,z))/D...
               +C(k-1,x+1,t)*(kgrid(k)-sol(1,t,z))*(sol(2,t,z)-xgrid(x))/D...
               +C(k,x+1,t)*(sol(1,t,z)-kgrid(k-1))*(sol(2,t,z)-xgrid(x))/D;
           sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
           if sol(1,t+1,z)<=mc/(A-delta)
               sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
               sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
           end
           sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);

    elseif sol(1,t,z)>kgrid(k) && sol(2,t,z)<xgrid(x) && x>1 && k<length
(kgrid)
           % Bilinear interpolation between kstar(k), kstar(k+1), xstar(x-1)
           % and xstar(x)
           D=(kgrid(k+1)-kgrid(k))*(xgrid(x)-xgrid(x-1));
           sol(3,t,z)=C(k,x-1,t)*(kgrid(k+1)-sol(1,t,z))*(xgrid(x)-sol(2,t,
z))/D...
               +C(k+1,x-1,t)*(sol(1,t,z)-kgrid(k))*(xgrid(x)-sol(2,t,z))/D...
               +C(k,x,t)*(kgrid(k+1)-sol(1,t,z))*(sol(2,t,z)-xgrid(x-1))/D...
               +C(k+1,x,t)*(sol(1,t,z)-kgrid(k))*(sol(2,t,z)-xgrid(x-1))/D;
           sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
           if sol(1,t+1,z)<=mc/(A-delta)
               sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
               sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
           end
           sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);
```

```
    elseif sol(1,t,z)>kgrid(k) && sol(2,t,z)>xgrid(x) && k<length(kgrid) &&↙
x<length(xgrid)
          D=(kgrid(k+1)-kgrid(k))*(xgrid(x+1)-xgrid(x))
          sol(3,t,z)=C(k,x,t)*(kgrid(k+1)-sol(1,t,z))*(xgrid(x+1)-sol(2,t,↙
z))/D...
              +C(k+1,x,t)*(sol(1,t,z)-kgrid(k))*(xgrid(x+1)-sol(2,t,z))/D...
              +C(k,x+1,t)*(kgrid(k+1)-sol(1,t,z))*(sol(2,t,z)-xgrid(x))/D...
              +C(k+1,x+1,t)*(sol(1,t,z)-kgrid(k))*(sol(2,t,z)-xgrid(x))/D;
          sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          if sol(1,t+1,z)<=mc/(A-delta)
              sol(3,t,z)=(A-delta+1)*sol(1,t,z)-(mc/(A-delta));
              sol(1,t+1,z)=(A-delta+1)*sol(1,t,z)-sol(3,t,z);
          end
          sol(2,t+1,z)=A*sol(1,t,z)-sol(2,t,z)*(eta-theta*sol(2,t,z)-1);



   end
  t=t+1;
 end
 sol(3,T,z)=sol(1,T,z);
 z=z+1;
 end
```

PROGRAM fp_vstar:

code:

```
% This program computes - backward in time - the value functions for the
% optimal path found in the program 1_retreivekx4 given the initial
% conditions IC.
z=1;
while z<=n
t=T;

Vstar(T,z)=(sol(1,T,z)^(1-sigma)-1)/(1-sigma)-(B*sol(2,T,z)^gamma)\gamma;
for i=1:T-1
    Vstar(t-i,z)=(sol(3,t-i,z)^(1-sigma)-1)/(1-sigma)-(B*sol(2,t-i,z)^gamma)↙
/gamma + beta*Vstar(t-i+1,z);

end
z=z+1;
end
```

PROGRAM fp_plot:

code:

```
%plotting tools

1_parameters5
domain=0:0.001:0.6;
C=1/(A-delta)*(A*B/(A-delta-rho))^(-1/sigma);

for j=1:length(domain)
F1(j)=domain(j)*(eta-theta*domain(j))/A;
F2(j)=C*domain(j)^((1-gamma)/sigma)*(eta-2*theta*domain(j)+rho)^(1/sigma);
end

plot(domain,F1, ':',domain,F2,':')
axis([0 0.6 0 0.007])
hold on

% Creation of the variables for plotting (cancel the last three periods
% because of not beautiful graph

solplot=sol;

for z=1:n
plot(solplot(2,:,z),solplot(1,:,z))
end
xlabel('Pollution')
ylabel('Capital')
hold on
```

Bibliography

[1] Bellman, Robert, 1957. *Dynamic Programming.* Princeton N.J., Princeton University Press.

[2] Bellman R., and Dreyfus S., 1962. *Applied Dynamic Programming.* Princeton N.J., Princeton University Press.

[3] Caputo M., 2005. *Foundations of Dynamic Economic Analysis. Optimal Control Theory and Applications.* Cambridge University Press.

[4] Dasgupta S., 1982. *Control of Resources.* Basil Blackwell, Oxford. 3

[5] Dasgupta S., Laplante B., Wang H. and Wheeler D. 2002. *"Confronting the Environmental Kuznets Curve"*, Journal of Economic Perspectives, no.1, Vol. 16, pp. 147-168.

[6] Deissenberg, Ch., Feichtinger, G., Semmler, W., Wirl, F., 2003. *Multiple equilibria, history dependence, and global dynamics in intertemporal optimization models.* In: Barnett, W., Deissenberg, Ch., Feichtinger, G. (Eds.), *Economic Complexity: Nonlinear Dynamics, Multi-agents Economies, and Learning*, ISETE vol. 14. Elsevier, Amsterdam, pp. 91-122. 22

[7] Forster, B., 1975. *Optimal Pollution Control with a Nonconstant Exponential Rate of Decay.* Journal of Environmental Economics and Management 2, pp. 1-6. 3

[8] Holling C.S., 1973, *"Resilience and Stability of Ecological Systems"*, Review of Ecology and Systematics, vol. 4, pp. 1-23. 3

[9] Keeler E., Spence M. and Zeckhauser R. 1972. *"The Optimal Control of Pollution"*, Journal of Economic Theory, no. 4, pp. 19-34. 2

[10] Larson, Robert, 1967. *A Survey of Dynamic Programming Computational Procedures.* IEEE Transaction on Automatic Control, pp. 767-774.

[11] Nordhaus William, 2007. *A Review of the Stern Review on the Economics of Climate Change.* Journal of Economic Literature 45, pp. 686-702.

[12] Ryder H.E., and Heal G.M., *"Optimal Growth with Intertemporally Dependent Preferences"*, The Review of Economic Studies, Vol. 40, No. 1, pp. 1-31.

[13] Seierstad A. and K. Sydsæter, 1987. *Optimal Control Theory with Economic Applications*, North Holland. 30

[14] Stern, Nicholas, 2007. *The Economics of Climate Change. The Stern Review.* Cambridge and New York, Cambridge University Press. 3

[15] Stokey, Nancy. 1998. *"Are There Limits to Growht?"*, International Economic Review, No.1, Vol. 39, pp. 1-31. 2, 6

[16] Tahvonen O., and Withagen C., 1996. *"Optimality of Irreversible Pollution Accumulation"*, Journal of Economic Dynamics and Control, no. 20, pp. 1775-1795. 1, 2

2

Is Democracy Good for the Environment? Quasi-Experimental Evidence from Regime Transitions.

Abstract

This paper tests the hypothesis that democratisation is conducive to less environmental depletion due to human activity. Using Interrupted Time Series (ITS) design for a panel of 47 transition countries and two indexes of pollution, CO2 emissions and PM10 concentrations, I find that democracies and dictatorships have two different targets of environmental quality, with those of democracies higher than those of dictatorships. Income inequality may as well alter this targets, but with opposite effects in the two different regimes.

JEL Classification: C23, D31, H23, Q58, Q51

Keywords: Democracy; Environment; Cointegration; Interrupted Time Series; Segmented Regression.

2.1 Introduction

The relation between various measures of pollution and democracy is highly debated. Two different views about the effect of democracy on the environment have been put forward: a dominant thinking of the 1970s was that democracy and its associated liberties to pollute, consume and procreate would generate ecological catastrophes, and the inability of governments to control the "tragedy of the commons" is an evident example of such a failure of democracies in the management of environmental issues (Desai, (8), Hardin, (16)). Democracy is also known to be a means for redistributing income and power to the poor; Rodrik (44) for example, presents evidence that democracy is associated with an higher share of wages in GDP and thus lower inequality and Milanovic (39) finds that democracies with greater inequality of factor income redistribute more to the poor, so to those whose consumption has an higher marginal impact on the environment.

At present, however, there is a growing empirical research on the possible affinities between democracy and ecology. There are several reasons for such a positive relationship[1], basically: i. democracies respect individual rights and so environmentalists are free to market their ideas and transform them into environmental legislation; ii. the necessity of democratic government to be elected (or reelected) makes them more responsive to their citizens; iii. open political systems are more likely to learn from scientists and other concerned citizens than are autocracies; vi. democratic states tend to cooperate with each other within international environmental agencies, and finally, v. because democracies all have free market economies, business in the market can be subject both to environmental incentives and sanctions (Dasgupta and Maler (6), Schultz and Crockett (30) and Payne (28)). Furthermore, democracies respect human life more than autocracies and therefore they are more responsive to life-threatening environmental degradation (Gleditsh and Sverdrup (13))

In the empirical literature, Bhattarai and Hamming (3), for example, use a measure of institutional quality (measured by an index of political rights and civil liberties) to account for the role of different policy regimes in the causes of deforestation in Latin America, Africa and Asia. Torras and Boyce (34) use a similar technique for a panel data survey of a variety of air and water pollution indicators. Gallagher and Thacker

[1] for a systematisation of the argument, see Payne (1995)

(12) introduce a concept of "stock of democracy" to study its implications through time. They all find positive evidence that civil liberties and political rights are associated with more pro-environmental behaviour.

In this paper, I make use of Interrupted Time Series (ITS) design to study the effect of democratisation on two indicators of air quality, CO2 emissions and the level of PM10, in a panel of 47 transition countries during the period 1950-2002 (for CO2) and 1990-2002 for the other measure. Panel data estimation methods allow control for unobservable country-specific effects that result in a missing-variable bias in cross-sectional studies. I included in my sample only transition countries because I believe it is a natural and transparent way to study the effect of democratisation. The inclusion of countries which have not shifted regime may bias the estimate due to the fact that other events (and not democracy) may be responsible for such a change (like, for instance, technological progress). This effect may be more easily disentangled in transition countries due to the possibility of running segmented regressions, as it will become clearer later.

The method of ITS is a powerful quasi-experimental approach for evaluating the effects of interventions. Segmented regression analysis of ITS data allows us to assess, in statistical terms, how much an intervention changed an outcome of interest, immediately and over time, instantly or with delay, and whether factors other than the intervention could explain the change. If a positive association between democracy and various measure of environmental quality is found in the literature, without comparing whether different regimes have the opposite effect, nothing can be said, in principle, about the goodness of that regime in promoting good policies for environmental protection. One example will be illuminating. Suppose we want to study the effect through time of democracy on the level of CO2 emissions, and suppose we find a negative and significant effect, so democratic regimes tend to decrease emissions. But what if also dictatorships reduced them? If that was the case, it would not be possible to state that democracies are good for the environment, because this would imply that non-democracies are not, which is not necessarily true. If both democracies and non-democracies have a positive effect on the environmental quality, it is certainly another reason that is driving this result.

So, contrary to the other methods commonly used in the literature, this approach is suitable to measure the effect of democratisation through time, taking into account of other time-variant effects, and moreover, it is capable of identifying whether other factors (possibly unknown) are driving the results. Some important observations coming from the empirical evidence are worth considering: due to the fact that income and democratic institutions are positively correlated (Barro (4), Przeworski and Limongi (43)), and also that income and cleaner/more efficient technologies are positively correlated (Hausman (21)), cross-country empirical works do not establish causation between regime and various measures of pollution.

Regressions in panel data are usually done by assuming fixed effect models, which can solve the issue of accounting for time invariant heterogeneity, and often the distinction between democracies and others are done by using dummies. This approach has the pitfall of not disentangling the effects of democracy from other effects in play during periods of democracy. It is well known that other factors may influence the performance of a country. In the ideal situation, one could observe the same country simultaneously under two different types of regime, and this would guarantee that all the other variables are held constant. But since this is not feasible (what one can observe is the same country at different points in time), a simple regression, even if it is able to control for time invariant unobservables, could be biased due to the time variant unobservable variables. Several effects may cause this bias: *History*, such as events which occurred during the period considered providing an alternative explanation of a given phenomenon, *maturation*, or processes through which the country produces changes as a function of time per se (like for example the maturation of technology towards less energy intensive production processes) or even *instability*, referred at the fact that all time series are unstable even when no treatments are applied are all typical unobservable effects that may bias these estimates.

This paper is organised as follows: section 2.2 motivate the paper, section 2.3 describes the dataset used and some summary statistics, section 2.4 illustrates the estimation techniques and results, and section 2.5 concludes.

2.2 Motivations

Does democracy really benefit environmental quality? Using a sample of 47 transition countries, I observe that during the period 1990-2002 the average concentration of PM10 recorded during spells of dictatorship is about 1.36 times bigger than the concentration recorded during democratic periods, despite the average level of GDP is 2.25 times bigger during democracy than during dictatorship spells[1].

The average intensity of CO2 emissions produced per unit of GDP is 1.17 times larger in periods of dictatorship than in periods of democracy[2]. Several countries show clearly a decrease in the intensity of emission per unit of GDP in proximity to the regime shift:

Figure 2.1

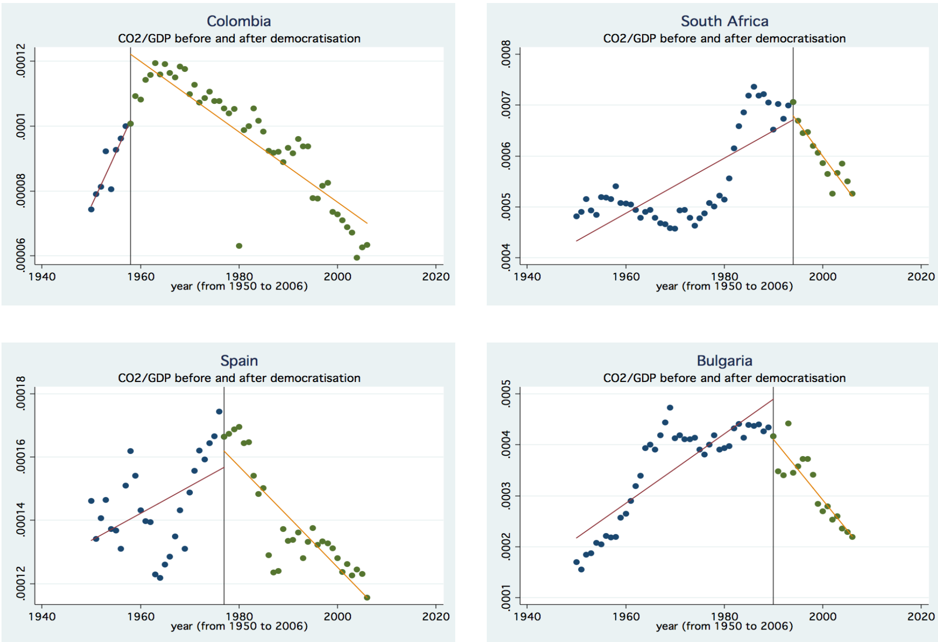


Figure 2.1 shows, for Colombia, South Africa, Spain and Bulgaria, the actual (dotted) and fitted (line) levels of intensity of emissions (expressed in tons of carbon per unit of GDP). The vertical line in each subfigure represents the date of the regime shift. All the four countries have experienced a transition to democracy after long periods of dictatorship. After the regime shift we observe a reverse pattern for emissions; while before democratisation the tendency is to increase the intensity of CO2 emissions in production, later we observe a decline, which is persistent through time.

[1]The average concentration of PM10 during democracy is 69.67906 and during dictatorship is 89.14407. GDP during democratic periods is, on average, 117,648.9 against 52,189.88 during dictatorship.

[2]This average is computed over all the 53 years and over all the countries, conditioned to periods of democracy or dictatorship. The data for periods of democracy show an intensity of CO2 emissions (in Kg of carbon) per unit of income of 0.1269 against 0.1485 during periods of dictatorship.

Variations in percapita emissions are consistently reduced during democratic periods as the following simple fixed-effects regression that includes all the sample of 47 countries and 53 years shows (standard errors in parenthesis)[1]:

$$\Delta\text{CO2}_{it} = \underset{(1.89233)}{8.229862} + \underset{(0.0079892)}{0.1170303}\Delta\text{GDP}_{it} + \underset{(3.356346)}{-17.3097}\text{Dem}_{it} \tag{2.1}$$

with $\Delta CO2_{it}$ denoting predicted variations in the level of percapita CO2 emissions expressed in Kg of carbon occurred between $t-1$ and t for country i, ΔGDP_{it} variations in the level of percapita income occurred between $t-1$ and t for country i, and Dem_{it} is a dummy variable coded 1 during periods of democracy, and 0 otherwise. All the estimated coefficients are significant at 1% level.

As previously anticipated, variations in CO2 emissions are consistently reduced during periods characterised by democratic institutions. The coefficient related to the variable Dem denotes exactly how the variation in the level of emissions decreases as a consequence of democratisation. In other words, it represents the average "kink" in the two fitted lines before and after the regime change, for the whole sample of countries.

2.3 Data and Descriptive Statistics

I consider percapita CO2 emissions and PM10 concentrations as dependent variables. Data on CO2 are from Marland, G., T.A. Boden, and R. J. Andres (22) and are available for the period 1950-2002. They refer to emissions from burning fossil fuels and manufacturing cement, expressed in Kg of carbon. Their estimates are based on energy data from the United Nations and cement data from the US Geological Survey (for an analytical description of the estimation procedures see (35). Data for PM10's concentration are available for the period 1990-2002 and the source is World Bank - World Development indicators' Database. Among the independent variables, I consider percapita GDP, from Maddison (21) and two trends, one for democracy (D) and another for dictatorship (A), extrapolated from Przeworski's dataset (for a list of variables and sources, refer to table A - Data and Sources in the Appendix). D (A) represents the number of consecutive years since the last regime change to democracy (dictatorship). It is coded 0 the year of transition to democracy (dictatorship), and 1, 2, 3 after one, two, three consecutive years of democracy (dictatorship) and so on.

[1] All the results are also significant at 1% level

During periods of dictatorship (democracy) it is coded zero. The last independent variable used is *Inequality*, from the EHII dataset of the University of Texas Inequality Project (UTIP). This measure is an estimate of the Theil's index of household's income inequality derived from econometric relationships between UTP-UNIDO (dataset that calculates the industrial pay inequality measures based on the UNIDO Industrial statistics), other conditioning variables, and the World Bank's Deninger and Squire Dataset[1].

Table B in the appendix lists all the countries included in the sample, and their respective regime changes to and from democracy. They are mainly developing countries, with the exception of the countries distinguished by an asterisk which are considered by IMF (World Economic Outlook updated to april 2009) developed or advanced. The following table 1 shows the summary statistics of the primary explanatory variables:

2.4 Estimation techniques and results.

2.4.1 Diagnostic tests and model selection

The first step to decide the best way to estimate a relationship of interest, is to choose the appropriate econometric tool. In order to do so, it is necessary to verify the characteristics of the data generating processes, by testing for eventual unit root. I test the occurrence of unit root on the series percapita CO2 emissions, percapita GDP, PM10 concentrations and inequality using four tests for panel data: Levin, Lin and Chu (LLC), Im, Pesaran and Shin (IPS), ADF and PP-Fisher tests. All these tests suggest that the data generating processes of CO2 emissions and GDP have unit root, while PM10 concentration and inequality appear stationary[2].

For what concerns the dependent variable PM10, given its stationarity, it cannot be cointegrated with any other variable and therefore a standard Least Squares Dummy Variable (LSDV) model for testing the effects of democracy on its level is appropriate, as it should produce stationary (possibly normal) residuals.

For what concerns instead the model for CO2 emissions, I cannot say a priori if the same LSDV model is appropriate, and a test for cointegration is mandatory so to

[1]For a detailed presentation of the techniques used to construct such an index, visit the University of Texas' website at http://utip.gov.utexas.edu/default.html

[2]For a detailed description of these tests, the tests that will follow and results, refer to the appendix.

Table 2.1: Summary statistics of the primary explanatory variables

Variable	Description	Obs	mean	Std.Dev.	min	max
$CO2$	Kg of carbon per-capita	2255	563.663	834.6266	0	4470
$PM10$	Concentrations of particulate matters, micrograms per cubic meter	611	72.10575	48.19759	11.9218	274.45
GDP	Percapita GDP, 1990$	2452	2778.26	2677.37	289.15	16572.83
Dem	Dummy for Democracy (Przeworski)	2234	.3531782	.4780645	0	1
D	Trend for Dem	2234	3.176813	6.653377	0	44
A	Trend for (1-Dem)	2234	10.27261	12.0507	0	50
$Ineq$	Household's income inequality (UTIP)	1129	42.00001	6.666231	19.81	62.32
$D \cdot INEQ$	Product of inequality and the trend for democracy	1124	160.2534	323.7205	0	1857.66
$A \cdot INEQ$	Product of inequality and the trend for autocracy	1124	491.798	544.5511	0	2232.621
$Year$	Year (from 1950 to 2002)	2491	1976	15.30013	1950	2002
N		2491				

exclude possible problems related to spurious regression[1]. If indeed the two series were cointegrated, standard OLS techniques would produce superconsistent estimates of the parameters, but if they were not, the model would suffer of the spurious regression problem with invalid inference of the parameters of interest.

In the Engle-Granger approach, cointegration is tested by verifying that the residual series generated by the regression of one I(1) variable over another I(1) variable is stationary[2]. To verify whether percapita CO2 emissions and percapita GDP are cointegrated, I use the approach suggested by Kao (27). The global ADF statistic for Kao residual cointegration test with the null of no cointegration shows a t-stat of 3.499820 with a p-value of 0.0002, so this test strongly suggests that those two series are cointegrated.

Maddala and Wu (32) combined tests from individual cross-sections (trace and maximum eigenvalue tests) reject the hypothesis of absence of cointegration at 1% level, and, as expected, accept the hypothesis that there exists one cointegrating relation[3]. So, substantial evidence points out that a cointegrating relation between emissions and income exists, so the use of a procedure that takes into account this fact is justified.

The choice of estimating the model using a single equation or a Vector Error Correction Model (VECM) is made by testing for weak exogeneity of the variables on the right-hand side of the equation describing the long run relationship of CO2 with GDP. Estimating with a single equation when the variables are not weakly exogenous is potentially inefficient and it does not lead to the smallest variance against alternatives, because information is lost. Using the approach suggested by Urbain (48), I test weak exogeneity by testing whether the error correction term embedded in the short run ECM[4] is significant in the equation determining ΔGDP_{it}. In particular, weak exogeneity requires that ΔGDP_{it} does not depend on disequilibrium changes represented by

[1]Spurious regressions occur whenever one regresses a nonstationary variable over another nonstationary variable which are not cointegrated. In general, the estimated coefficients appear to be significant but they lack any real and plausible correlation, since what is correlated is just a common trend and nothing else. Under a spurious regression, the tendency of both series to be growing is picked up by the regression model, even though each series is growing for very different reasons and at a rates which are uncorrelated, and produce nonstationary residuals.

[2]Since in my dataset there are only two nonstationary variables, it follows that at most one cointegrating relation may exist.

[3]In performing these tests, it's been assumed that there is no deterministic trend.

[4]$\hat{\epsilon}_{it-1}$ is computed as a residual of the long run relationship explaining percapita CO2 emissions.

$\hat{\epsilon}_{it-1}$. Results of this test, deeply detailed in the appendix, show that GDP is weakly exogenous so the estimation of the model for CO2 emissions using a simple equation approach is suitable for my purpose.

2.4.2 Interrupted Time Series

Interrupted time series (ITS) are a powerful quasi-experimental approach for evaluating the effects of interventions. Segmented regression analysis of ITS data allows to assess, in statistical terms, how much an intervention changed an outcome of interest, immediately and over time, instantly or with delay, and whether factors other than the intervention could explain the change. In this paper, I make use of this ITS design to estimate the effects of democratisation on the level of emissions produced, in a panel of 47 countries for the period 1950-2002 for CO2 and 1990-2002 for PM10. For each country in my sample, the levels of emissions and concentration of particulate matters, real percapita incomes and the regime are measured at regularly spaced intervals over time. To determine the effect of a regime change on the level of the two indexes of pollution, I generate two series (namely, D and A representing the number of consecutive years since the last regime change to, respectively, democracy and dictatorship) which are divided into two or more segments at changing points, when the regime type varies. So, if a country, at a given date, has shifted from dictatorship to democracy, and there is only one shift in the period taken into consideration, then the time series is divided into two segments. If the number of shifts is n, the number of segment will be, in general, $n+1$. In detail, D and A are trend variables which are evaluated progressively according to the age of the democracy or autocracy. The advantage of using those two variables is that they allow assessment of the effect of the regime through time as well whether other effects may explain the change. If indeed the coefficients relative to those trends have the same sign, this methods allows to conclude that other factors (like general technological progress) may have played a major role in the determination of the environmental policy, and not the regime.

2.4.3 Econometric specifications and results

Due to the nature of data generating process of my variables of interest, I have to specify two different models for estimating the effect of the regime on CO2 and on the level of PM10 concentrations. Previous tests for cointegration between CO2 and

GDP suggest that those two variables have a long run relationship; they move closely together over time and their difference is stable. Test for weak exogeneity shows that GDP is weakly exogenous with respect to CO2 so cointegration approaches which use a single equation are appropriate and no loss of information (and efficiency) occur. For estimating the effect of the regime on the level of CO2 emission, then, I will use the procedure proposed by Engle and Granger (12). The Engle-Granger Error Correction Model (ECM) is appropriate in this setting and has the advantage of incorporating both long run and short run effects; if the model is in disequilibrium the ECM will provide information on the speed of adjustment to equilibrium. Moreover, all the terms in the model are stationary, and so standard regression techniques are valid, and finally, the fact that CO2 and GDP are cointegrated of order CI(1,1) implies that an ECM exists (and, conversely, an ECM generates cointegrated series) and so ECM is immune from the spurious regression problem[1].

For what concerns the estimation of the effects of the two different regimes on the level of PM10 concentrations, I use a LSDV model to take into account of country specific fixed effects, and since tests for unit root for the PM10 series show that it is stationary, standard OLS procedures can be applied. In both econometric specifications I introduce two different trends, or, more precisely, two interrupted series of trends, D and A, which represent, respectively, the number of consecutive years since the last democratisation or autocratisation. During periods of dictatorship, D is coded zero, and during periods of democracy, A is coded zero. In this setup, D and A represent the effect through time on emissions or on concentration of pollution of the type of regime. The other independent variables included in the model are inequality and the cross effects of inequality with the two trends for regime. The reason for their inclusion is that depending on who is the decisive political actor in the two different regimes, inequality may have different effects. So, on one hand, when the population as a whole is involved in the decision process (through the electoral mechanism), different levels of income inequality change the median voter's wealth and so the perceived value of income (or consumption) relative to the environmental quality, and therefore inequality is expected to worsen the environmental quality. On the other hand, during

[1]This can be referred to Granger's representation theorem for dynamic modelling, in Engle and Granger (12)

autocratic periods, if increased inequality is in favor of the dictator[1], it could increase environmental quality, since the rich dictator values the environment more, compared to consumption.

In order to estimate the relation between different regimes and CO2 emissions, consider the following ARDL(1,1)

$$\begin{aligned} CO2_{it} &= \alpha_{0i} + \gamma_1 CO2_{it-1} + \sum_{j=1}^{n} \delta_{0j} X_{jit} + \sum_{j=1}^{n} \delta_{1j} X_{jit-1} + \\ &+ \sum_{z=3}^{4} \delta_{2z} X_{4it} X_{zit} + \sum_{z=3}^{4} \delta_{3z} X_{4it-1} X_{zit-1} + \epsilon_{it} \end{aligned} \tag{2.2}$$

with $j = 1, ..., 4$ and

$$\begin{aligned} X_1 &= GDP \\ X_2 &= D \\ X_3 &= A \\ X_4 &= INEQ \end{aligned}$$

with some manipulations (shown in appendix) we end up with the usual ECM:

$$\begin{aligned} \Delta CO2_{it} &= \sum_{j=1}^{n} \delta_{0j} \Delta X_{jit} + \sum_{z=2}^{3} \delta_{2z} W_z - (1-\gamma_1)\Big[CO2_{it-1} - \frac{\alpha_{0i}}{1-\gamma_1} - \\ &- \sum_{j=1}^{n} \frac{\delta_{0j}+\delta_{1j}}{1-\gamma_1} X_{jit-1} - \sum_{z=2}^{3} \frac{\delta_{2z}+\delta_{3z}}{1-\gamma_1} X_{4it-1} X_{zit-1}\Big] + \eta_{it} \end{aligned} \tag{2.3}$$

$$W_z = \Big(X_{4it}\Delta X_{zit} + \Delta X_{4it} X_{zit} - \Delta X_{4it}\Delta X_{zit}\Big) \tag{2.4}$$

[1] It is indeed well known that many countries in Africa and the Caribbean suffer and have suffered from "kleptocratic" regimes (Acemoglu, Robinson and Verdier, (1)) run by individulas who use their power to transfer a large fraction of society's resources to themselves. Two of the most "kleptocratic successes" are those of Mubutu Sese Seko in the Democratic Republic of Congo (Zaire) and Rafael Trujillo in the Dominican Republic. To have an idea of the numbers, in the 70's, 15-20% of Congo's operating budget went directly to Mubutu, and in 1977 Mubutu's family took $71 million from the National Bank for personal use and in the 80's his personal fortune was estimated in $5 billion (Leslie, (20), p. 72). In the Dominican Republic, Trujillo became in power after he elected himself in a fraudulent election and at the end of his regime, the fortune of Trujillo's family amounted to about 100% of GDP at current prices and the family "controlled almost 80% of the country's industrial production" (Moya Pons, (25), p. 398). Other examples of "keptocratic regimes" include Haiti under the Duvaliers, Nicaragua under the Somozas, Uganda under Idi Amin, Liberia under Charles Taylor and the Philippines under Ferdinand Marcos, but the list can go on, if we include less extreme cases.

where the expression in square brackets represents the long run relationship between CO2 and GDP. If the model is in equilibrium, this expression is equal to zero, and if it is not, the coefficient $-(1-\gamma_1)$ represents the speed of adjustment toward it. Therefore, I first estimate the long run relationship between CO2 and the other covariates as follows:

$$\begin{aligned} CO2_{it} &= \beta_{0i} + \beta_1 GDP_{it} + \beta_2 D_{it} + \beta_3 A_{it} + \beta_4 INEQ_{it} + \\ &+ \beta_5 D_{it} \cdot INEQ_{it} + \beta_6 A_{it} \cdot INEQ_{it} + \epsilon_{it} \end{aligned} \tag{2.5}$$

with

$$\begin{aligned} \beta_j &= \frac{\delta_{0j} + \delta_{1j}}{1-\gamma_1} \qquad j = 1, ..., 4 \\ \beta_h &= \frac{\delta_{2h} + \delta_{3h}}{1-\gamma_1} \qquad h = 5, 6 \end{aligned}$$

using a standar LSDV to account for specific, country-level time invariant effects (whose coefficient is represented by β_{0i}) and using cluster-robust standard errors. The results of this estimation are shown in table 2. The second step is to estimate the residuals from equation 2.5 and proceed to the estimation of the unrestricted error correction model

$$\begin{aligned} \Delta CO2_{it} &= \delta_{01} \Delta GDP_{it} + \delta_{02} \Delta D^*_{it} + \delta_{03} \Delta A^*_{it} + \delta_{04} INEQ + \\ &+ \delta_{22} W_{Dit} + \delta_{23} W_{Ait} - (1-\gamma_1)\hat{\epsilon}_{it-1} + \eta_{it} \end{aligned} \tag{2.6}$$

where W_{Dit} and W_{Ait} are defined as:

$$\begin{aligned} W_{Dit} &= D_{it} \cdot \Delta INEQ_{it} + \Delta D^*_{it} \cdot INEQ_{it} - \Delta D^*_{it} \cdot \Delta INEQ_{it} \\ W_{Ait} &= A_{it} \cdot \Delta INEQ_{it} + \Delta A^*_{it} \cdot INEQ_{it} - \Delta A^*_{it} \cdot \Delta INEQ_{it} \end{aligned}$$

and $\hat{\epsilon}_{it-1}$ is

$$\begin{aligned} \hat{\epsilon}_{it-1} &= CO2_{it-1} - \beta_{0i} - \beta_1 GDP_{it-1} - \beta_2 D_{it-1} - \beta_3 A_{it-1} + \\ &- \beta_4 INEQ_{it-1} - \beta_5 D_{it-1} \cdot INEQ_{it-1} - \beta_6 A_{it-1} \cdot INEQ_{it-1} \end{aligned}$$

Results of the estimation of equation 2.6 are shown in table 3. Table F in the appendix shows the coefficients relative to the fixed effects estimated in equation 2.5.

As previously written, the series PM10 is stationary and therefore there cannot exist any cointegrating relation. It follows that a simple LSDV model is appropriate to estimate the effects of the regime, since the residuals are expected to be stationary (results of the tests of unit root on the residuals are shown in the appendix). In order to conform to the EKC literature, I estimate the following equation including among the regressors also the square and cubic powers of GDP[1]:

$$\begin{aligned} PM10_{it} &= \beta_{0i} + \beta_1 GDP_{it} + \beta_2 GDP_{it}^2 + \beta_3 GDP_{it}^3 + \beta_4 D_{it} + \beta_5 A_{it} + \\ &+ \beta_6 INEQ_{it} + \beta_7 D_{it} \cdot INEQ_{it} + \beta_8 A_{it} \cdot INEQ_{it} + \epsilon_{it} \end{aligned} \quad (2.7)$$

and the results are shown in table 2.

As it is possible to see, the trends for democracy and dictatorship have, respectively, a negative effect on the level of emissions/pollution, in both models and all the coefficients (except that relative to inequality for the model of CO2 emissions) are statistically significant at the standard 5% level. For model 1 (PM10), the elasticity of the concentration of PM10 with respect to one year increase in democracy ($\partial PM10/\partial D$), computed at the mean of inequality is equal to -2.15, and the same elasticity computed for one year increase in dictatorship ($\partial PM10/\partial A$) is 0.1192. For the second model, relative to CO2 emissions, $\partial CO2/\partial D$ = -1.267 and $\partial CO2/\partial A$=4.102. It is possible to say therefore that in both models (even in the second all the conclusions apply only to the long run period) democracy is good for the environment, and moreover the argument is reinforced by the fact that non-democracies are not. Table 3 shows the results from the estimation of the unrestricted ECM for CO2 emissions:

The main results are reported in table 3. Variations in the level of $CO2$ are, of course, positively related to variations in global production, and the coefficient attached to $\hat{\epsilon}$ represents the speed of adjustment towards the equilibrium. It is worth noting that the coefficient related to ΔD_{it}^* is not significant at the standard 5% level (in fact, it has a p-value of 0.574), while the coefficient related to the variation in the trend for dictatorship is positive and significatively related to increases in the variation of emissions. This effect may be the cause of the slower reactions of democracies with respect to autocracies. Democracies are indeed constrained by consultations, elections, every decisions has often to pass the evaluations of another independent organism, and this procedure takes time. On the contrary, in dictatorship the decisions are taken by one individual only and they have not to be scrutinised by other independent powers.

[1]It is important to notice that the sign and the significancy of the other covariates remains unchanged if those two terms are excluded from the model.

Table 2.2: Results of the estimation of the model for PM10 and the long run equilibrium relationship for CO2

	(1) PM10	(2) Percapita CO2
Percapita GDP	0.00375**	0.163***
	(3.32)	(29.39)
D	-4.254***	-33.41*
	(-4.59)	(-2.28)
A	1.091*	36.96***
	(2.18)	(9.23)
$INEQ$	-0.310*	-0.576
	(-2.01)	(-0.22)
$D \cdot INEQ$	0.0500**	0.764*
	(2.73)	(2.37)
$A \cdot INEQ$	-0.0231*	-0.781***
	(-2.12)	(-8.63)
r2	0.345	0.703
N	316	1110

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The reasons for this non-significancy for the variations of the democratic level on the variation of emissions is probably to to this difference in the time a policy takes to be implemented into the two different regimes.

Table 2.3: Unrestricted Error Correction Model for CO2 emissions

	Δ Percapita CO2
Δ Percapita GDP_{it}	0.0938***
	(4.31)
$\hat{\epsilon}_{it-1}$	-0.128***
	(-4.77)
ΔD^{*}_{it}	-11.50
	(-0.56)
ΔA^{*}_{it}	34.13*
	(2.52)
$\Delta INEQ_{it}$	-2.629
	(-1.50)
W_{Dit}	0.579
	(1.91)
W_{Ait}	0.00380
	(0.07)
N	1033

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2.5 Conclusion

Despite the different views about the effect of democracy on the environmental management, in this paper I show that democracy and environmental quality are positively correlated. To show that, I use the powerful approach of ITS design in cointegration analysis to show that democratic countries and autocratic ones have two different targets of environmental quality, with those for democracy higher than those for autocracies. Previous works on democracy and environmental quality were indeed unable to assert that democracy is really good for the environment because they did not show that non-democracies are not. Segmented regression analysis of ITS allows not only to see the effect of democracy through time, but also if this effect differs from the effect of autocracy. The weakness of the previous works in this field was therefore that not comparing the results with those for dictatorships, the positive effect of democracy on the environment might not be due to democracy per se, but from other effects, like maturation, or technological progress, common to both regimes. In this panel of 47 transition countries, this approach shows that democratisation is consistently associated to a reduction of CO2 emissions and PM10 concentrations, but this process may be quite slow because - at least in the ECM relative to CO2 emissions - it is detectable only in the long run. Due to the fact that democratic institutions tend to be slower than autocratic ones in taking decisions and acting, in the short run we do not observe a negative effect of democracy on the level of emissions, while the positive effect of dictatorship is quite consistent. Inequality has two different effects depending on the incumbent regime: in any case it counterbalances the global effect of the regime. In democracy, increased inequality means that the decisive citizen is poorer and so less willing to pay for environmental protection, while inequality during periods of autocracy, under the assumption that this inequality favor the dictator at the expense of the rest of the citizens, it may retain the negative effect of the regime since it increases the dictator's income and so it increases his demand for environmental quality. The overall effect, however, is that dictatorships tend to be associated to a worse environment than democracies.

2.6 Appendix

Diagnostic tests

Panel Unit root test

The first step to decide the best way to estimate a relationship of interest, is to choose the appropriate econometric tool. In order to do so, it is necessary to verify the characteristics of the data generating processes: for each nondeterministic series (CO2 emissions, percapita GDP, PM10 concentrations and Inequality), I test whether they have stationary mean and variance. Available tests for unit root on panel data are based on the Dickey Fuller test (or its augmented version), so I test the following:

$$y_{it} = \rho_i y_{it-1} + X'_{it}\delta_i + \epsilon_{it} \tag{2.8}$$

with $i = 1, ..., N$ and $t = 1, ..., T_i$. ρ_i represents the autoregressive coefficient for country i, X_{it} is matrix representing the exogenous variables in the model, including any fixed effects and individual trends and ϵ_{it} are the errors which are assumed i.i.d.. If $|\rho_i| = 1$, then y_i contains a unit root. For practical purpose, the tests for unit root are performed using the following basic ADF specification:

$$\Delta y_{it} = \alpha_i y_{it-1} + \sum_{j=1}^{p_i} \beta_{ij} \Delta y_{it-j} + X'_{it}\delta_i + \epsilon_{it} \tag{2.9}$$

with $\alpha_i = \rho_i - 1$.

Levin, Lin and Chu test assumes that the unit root process is common to all the cross sections so it assumes $\alpha_i = \alpha$ for every i. The test is performed by testing the null hypothesis $H_0 : \alpha = 0$ against the alternative $H_1 : \alpha < 0$ for all the cross section units. Their procedure derives estimates of α from proxies for Δy_{it} and y_{it} that are standardised and free of autocorrelation and deterministic components. For a given set of lag orders p_i, their procedure begins by estimating two additional sets of equations,

$$\Delta y_{it} = \sum_{j=1}^{p_i} \beta_{ij} \Delta y_{it-j} + X'_{it}\delta \tag{2.10}$$

$$y_{it} = \sum_{j=1}^{p_i} \beta_{ij} \Delta y_{it-j} + X'_{it}\delta \tag{2.11}$$

and denoting $(\hat{\beta}, \hat{\delta})$ the estimated coefficients of equation 2.10 and $(\dot{\beta}, \dot{\delta})$ those of equation 2.11. They define, then, $\Delta\bar{y}_{it}$ and $\bar{y}_{it-1}$ by taking, respectively, Δy_{it} and y_{it-1} and removing the autocorrelations and deterministic components using their respective auxiliary estimates

$$\Delta\bar{y}_{it} = \Delta y_{it} - \sum_{j=1}^{p_i} \hat{\beta}_{ij} \Delta y_{it-j} - X'_{it}\hat{\delta} \tag{2.12}$$

$$\bar{y}_{it-1} = y_{it-1} - \sum_{j=1}^{p_i} \dot{\beta}_{ij} \Delta y_{it-j} - X'_{it}\dot{\delta} \tag{2.13}$$

and standardise both $\Delta\bar{y}_{it}$ and $\bar{y}_{it-1}$ by dividing by the regression standard error

$$\Delta\tilde{y}_{it} = \frac{\Delta\bar{y}_{it}}{s_i}$$

$$\tilde{y}_{it-1} = \frac{\bar{y}_{it-1}}{s_i}$$

where s_i are the estimated standard errors from estimating each ADF in equation 2.9. The estimate of α is then obtained from the pooled proxy equation

$$\Delta\tilde{y}_{it} = \alpha\tilde{y}_{it-1} + \eta_{it} \tag{2.14}$$

which, under the null, a modified t-statistics for the resulting $\hat{\alpha}$ is asymptotically normally distributed

$$t_\alpha^* = \frac{t_\alpha - (N\tilde{T})S_N\hat{\sigma}^{-2}se(\hat{\alpha})\mu_{m\tilde{T}^*}}{\sigma_{m\tilde{T}^*}} \rightarrow N(0,1)$$

where t_α is the standard statistic for $\hat{\alpha} = 0$, $\hat{\sigma}^2$ is the estimated variance of the error term η, $se(\hat{\alpha})$ is the standard error of $\hat{\alpha}$, and

$$\tilde{T} = T - (\sum_i p_i/N) - 1,$$

S_N is the mean of the ratios of the long run standard deviation for each individual, and it is estimated using kernel-based techniques, and $\mu_{m\tilde{T}^*}$ and $\sigma_{m\tilde{T}^*}$ are adjustment terms for the mean and standard deviation (for more details, refer to the original article of Levin, Lin and Chu (29)). In order to perform this test, I include individual constant terms (fixed effects), and an individual trend, so my X_{it} matrix is a $2NT$x$2N$ matrix, where the first NTxN block is a matrix of dummy variables, each representing one single country, and the other block going from row $NT+1$ to $2NT$ and from column $N+1$ to $2N$ is a matrix of trends, one for each single countries. All the other terms in the matrix are equal to zero.

If the test for common unit root fails, it might be convenient to check whether individual unit root exists. Tests available for that are Im, Peasaran and Shin, Fisher AD and PP tests. Im, Pesaran and Shin test begin by specifying a separate ADF regression for each cross-section according to equation 2.9 with the null

$$H_0 : \alpha_i = 0 \text{ for every } i = 1, 2, ..., N$$

and the alternative hypothesis is

$$H_1 : \begin{cases} \alpha_i = 0 & \text{for } i = 1, 2, ..., N_1 \\ \alpha_i < 0 & \text{for } i = N_1 + 1, N_1 + 2, ..., N \end{cases}$$

That can be interpreted as a nonzero fraction of the individual processes is stationary. After having estimated the separate ADF regressions, the average of the t-statistics for α_i from the individual ADF regressions, $t_{iT_i}(p_i)$,

$$\bar{t}_{NT} = \left(\sum_{i=1}^{N} t_{iT_i}(p_i) \right) / N$$

is adjusted to arrive to the desired statistics. Critical values are provided in the Im, Pesaran and Shin's paper (25) for different number of cross sections and time periods when $p_i = 0$ for all i, but in the general case when the lag order in equation 2.9 may be nonzero for some cross-sections, they show that a properly standardised $\bar{t}_{NT}$ has an asymptotic standard normal distribution

$$W_{\bar{t}_{NT}} - \frac{\sqrt{N}\left(\bar{t}_{NT} - N^{-1}\sum_{i=1}^{N} E(\bar{t}_{iT}(p_i))\right)}{\sqrt{N^{-1}\sum_{i=1}^{N} Var(\bar{t}_{iT}(p_i))}} \rightarrow N(0,1)$$

and the expression for the expected mean and variance of the ADF regression t-statistics, $E(\bar{t}_{iT}(p_i))$ and $Var(\bar{t}_{iT}(p_i))$ are provided by Im, Pesaran and Shin for various values of T and p and different test equation assumptions. I will use, in this paper, one lag and, as deterministic component, an individual constant, without introducing any trend term. Finally, there are other two tests for checking individual unit root: Fisher ADF and Fisher PP tests, which are based upon the idea by Maddala, Wu and Choi, and combine the p-values from individual unit root tests. They work as follows: define π_i the p-value from individual unit root test for cross-section i, then under the null of unit root for all the N cross sections, we have the asymptotic result that

$$-2\sum_{i=1}^{N} \log(\pi_i) \rightarrow \chi^2_{2N}$$

and also, Choi demonstrates that

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(\pi_i) \rightarrow N(0,1)$$

where Φ^{-1} is the inverse of a standard normal cumulative distribution function. For both the Fisher tests, I specify as exogenous variables an individual constant (fixed effect) and an individual time trend.

Table C in this appendix shows the results of the tests for all the nondeterministic series. These tests accept the hypothesis of unit root only for two out of four series, percapita GDP and percapita CO2 emissions. Since the level of concentration of particulate matters (PM10) is stationary, it cannot be cointegrated with any other variable, while in principle CO2 and GDP could be. For what concerns the dependent variable PM10, then, a standard Least Squares Dummy Variables (LSDV) model for testing the effect of democracy on its level it is appropriate, as it should produce stationary (possibly normal) residuals. For what concerns the model for CO2 emissions, we cannot say a priori whether the same LSDV model is appropriate, and a test for cointegration is mandatory. If indeed the two series were cointegrated, that model would produce superconsistent estimates of the parameters, but if they were not, the model would suffer of the problem of the spurious regression with invalid inference of the parameters of interest. In this case, the tendency of both series to be growing leads to correlation which is picked up by the regression model, even though each series is growing for very different reasons and at a rates which are uncorrelated. Thus, in absence of cointegration (which will be tested in the next subsection) correlation between nonstationary series does not imply the kind of causal relationship that might be inferred from stationary series, so standard estimation techniques like OLS cannot be used.

Panel cointegration test

If two or more variables are nonstationary or I(1), if they are not cointegrated, the residual series obtained by regressing one I(1) variable over another I(1) variable is expected to be nonstationary, or I(1). This would lead to spurious regression since the estimated coefficients do not reflect a real relationship between those two variables, but simply correlated time trends. This however would be different if those two variables were cointegrated, and the coefficients would benefit from the “superconsistency” property. Since in my dataset there are only two nonstationary variables, namely, CO2 and GDP, it follows that at most one cointegrating relation may exist. In the Engle-Granger approach, cointegration is tested by verifying that the residual series generated by the regression of one I(1) variable over another I(1) variable is stationary. To verify whether percapita CO2 emissions and percapita GDP are cointegrated, I use the approach suggested by Kao (27). Kao uses a two-step procedure to test for cointegration: in the first step, he regress the dependent variable (which is I(1)) over the independent (also I(1)) specifying cross-section specific intercepts and homogeneous coefficient: he basically regresses

$$CO2_{it} = X_{it}'\alpha_i + \beta GDP_{it} + e_{it} \tag{2.15}$$

where X_{it} is a matrix of dummy variables representing each single country and assuming $CO2_{it} = CO2_{it-1} + u_{it}$ and $GDP_{it} = GDP_{it-1} + \epsilon_{it}$ for $t = 1, ..., T$ and $i = 1, ..., N$. Then Kao runs the pooled auxiliary regression

$$e_{it} = \rho e_{it-1} + \sum_{j=1}^{p} \psi_j \Delta e_{it-j} + v_{it} \tag{2.16}$$

Assuming $p = 1$[1], this augmented Dikey Fuller test for panel data reject at 1% level the hypothesis that $\rho = 1$. The global ADF t-statistic for Kao residual cointegration test with the null hypothesis of no cointegration shows a t-stat of 3.499820 with a p-value of 0.0002, so this test strongly suggests that those two series are cointegrated (detailed results of this test are in this appendix - table D).

Maddala and Wu (32) combined test from individual cross-sections[2] is specified as follows: consider the following VAR representation for each cross-section unit:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \epsilon_t \tag{2.17}$$

where Y is a vector of I(1) variables (in my case, CO2 and GDP). Subtracting Y_{t-1} on the left and right hand side of equation 2.17 and adding and subtracting $A_2 Y_{t-1}$ from the right hand side, we get

$$\Delta Y_t = \Pi Y_{t-1} + \Gamma \Delta Y_{t-1} + \epsilon_t \tag{2.18}$$

with $\Pi = (A_1 + A_2 - I)$ and $\Gamma = -A_2$. Granger's representation theorem asserts that if the coefficient matrix Π has a reduced rank $r < k$, then there exists kxr matrices α and β each with rank r such that $\Pi = \alpha \cdot \beta'$ and $\beta' Y_t$ is I(0). r is the number of cointegrating relations and each column of β is the cointegrating vector, and the elements of α are known as the adjustment parameters in the VEC model. Johansen's

[1] Similar results are obtained for longer lags specifications, up to four.

[2] Maddala and Wu use the results obtained by Fisher and Johansen (14)

method is to estimate the Π matrix from an unrestricted VAR and to test whether we can reject the restrictions implied by the reduced rank of Π. In performing this test, I assume that the level data Y_t have no deterministic trends and the cointegrating equations have only interceipts, so

$$H(r) : \Pi Y_{t-1} = \alpha(\beta' Y_{t-1} + \rho_0) \tag{2.19}$$

To determine the number of cointegrating relations r conditional on assumption 2.19, we proceed sequentially from $r = 0$ to $r = k - 1$ until we fail to reject. The trace statistic for the null hypothesis of r cointegrating relations is computed as

$$LR_{tr}(r|k) = -T \sum_{i=r+1}^{k} log(1 - \lambda_i) \tag{2.20}$$

where λ_i is the largest eigenvalue of the Π matrix.

The maximum eigenvalue statistic tests the null hypothesis of r cointegrating relations against the alternative of $r + 1$ relations. The test statistics is computed

$$\begin{aligned} LR_{max}(r|r+1) &= -T \cdot log(1 - \lambda_{r+1}) \\ &= LR_{tr}(r|k) - LR_{tr}(r+1|k) \end{aligned} \tag{2.21}$$

If the test statistics are continuous, the significance levels for each cross-section unit, denoted by π_i for $i = 1, 2, ..., N$, are independent uniform (0,1) variables, and $-2 \log \pi_i$ has a χ^2 distribution with two degrees of freedom. The approach proposed by Maddala and Wu to test cointegration in panel is to combine tests from individual cross sections to obtain a test statistic for the full panel, using the additive property of the χ^2 variables: if π is the p-value from an individual cointegration test for the cross-section i, under the null hypothesis for the panel we have

$$-2 \sum_{i=1}^{N} log(\pi_i) \rightarrow \chi^2_{2N} \tag{2.22}$$

Both trace and maximum eigenvalue tests reject the hypothesis of absence of cointegration at 1% level, and accept the hypothesis that there exists one cointegrating relation[1]. So, substantial evidence points out that a cointegrating relation between emissions and income exists, so the use of a procedure that takes into account this fact is justified.

[1]In performing this test, it's been assumed that there is no deterministic trend

Weak exogeneity test

In the model for the estimation of the of the effect of the regime on the level of CO2 emissions, I have only two non-stationary series, while the other variables are either deterministic or I(0). It follows that only one cointegrating relation can exist, so in principle it is possible to estimate this relationship using a single equation. However, estimating with a single equation is not free of drawbacks, as it is potentially inefficient, and so it does not lead to the smallest variance against alternative approaches. In general, information is lost unless the right-hand side variables in the cointegration vector are weakly exogenous. Weak exogeneity of these variables is indeed a prerequisite to assert that no useful information is lost when we condition on these variables without specifying their generating process. In practical terms, it must be the case that GDP is weakly exogenous with respect to the level of CO2 emissions. As pointed out by Urbain (48), testing for weak exogeneity requires testing whether the error-correction term embedded in the short-run ECM ($\hat{\epsilon}_{it-1}$ computed as a residual of the long run relationship equation) is significant in the equation determining ΔGDP_{it}. In particular, weak exogeneity requires that ΔGDP_{it} does not depend on the disequilibrium changes represented by $\hat{\epsilon}_{it-1}$.

Consider the following long run relationship for the dependent variable percapita $CO2$ emissions:

$$\begin{aligned} CO2_{it} &= \beta_{0i} + \beta_1 GDP_{it} + \beta_2 D_{it} + \beta_3 A_{it} + \beta_4 INEQ_{it} + \\ &+ \beta_6 D_{it} \cdot INEQ_{it} + \beta_7 A_{it} \cdot INEQ_{it} + \epsilon_{it} \end{aligned} \tag{2.23}$$

where the betas are combinations of parameters deriving from the ARDL model (for more details, please refer to the appendix). Testing for weak exogeneity requires, in order:

1. Estimating the coefficients of equation 2.23

2. Computing the estimated residual series as

$$\begin{aligned} \hat{\epsilon}_{it} &= CO2_{it} - \hat{\beta}_{0i} - \hat{\beta}_1 GDP_{it} - \hat{\beta}_2 D_{it} - \hat{\beta}_3 A_{it} - \hat{\beta}_4 INEQ_{it} + \\ &- \hat{\beta}_6 D_{it} \cdot INEQ_{it} - \hat{\beta}_7 A_{it} \cdot INEQ_{it} \end{aligned}$$

3. Estimating the following equation[1]:

$$\begin{aligned}\Delta GDP_{it} &= \gamma_1 \Delta CO2_{it} + \gamma_2 \Delta D^*_{it} + \gamma_3 \Delta A^*_{it} + \gamma_4 \Delta INEQ_{it} \\ &+ \gamma_5 W_{Dit} + \gamma_6 W_{Ait} - (1-\alpha)\hat{\eta}_{it-1} + \delta \hat{\epsilon}_{it-1} \qquad (2.24)\end{aligned}$$

   where the short-run interaction effects between the two trends and inequality are

$$\begin{aligned}W_{Dit} &= D_{it} \cdot \Delta INEQ_{it} + \Delta D^*_{it} \cdot INEQ_{it} - \Delta D^*_{it} \cdot \Delta INEQ_{it} \\ W_{Ait} &= A_{it} \cdot \Delta INEQ_{it} + \Delta A^*_{it} \cdot INEQ_{it} - \Delta A^*_{it} \cdot \Delta INEQ_{it}\end{aligned}$$

   and $\hat{\eta}_{it}$ is the error correction term for the equation defining the relationship between GDP and $CO2$, and it is estimated from the long run relationship between GDP and $CO2$, given by

$$\begin{aligned}GDP_{it} &= \theta_{0i} + \theta_1 CO2 + \theta_2 D_{it} + \theta_3 A_{it} + \theta_4 INEQ_{it} + \\ &+ \theta_6 D_{it} \cdot INEQ_{it} + \theta_7 A_{it} \cdot INEQ_{it} + \eta_{it} \qquad (2.25)\end{aligned}$$

   and $\hat{\epsilon}_{it-1}$ is already defined at point 2.

4. Checking and testing the significancy of the coefficient attached to $\hat{\epsilon}_{it-1}$, δ, in the equation defined at point 3. If δ is significant in equation 2.24, then we cannot say that GDP is weakly exogenous and then a multivariate model for estimation is necessary. If instead δ is found to be non-significant, a standard error correction model with one single equation is enough to estimate efficiently the relations of interest, so no loss of information occurs.

Results from the estimation of equation 2.24 for $CO2$ show that t-test for δ accepts the null of $\delta = 0$ at the standard level of 5%. This coefficient amounts to -0.0382668, with a standard error of 0.1013909, a t-statistic of -0.38 and a p-value of 0.706, so GDP is shown to be weakly exogenous.

It is possible to conclude, then, that the estimation of the model for CO2 emissions using a single equation approach is appropriate, and there is no loss of information (and efficiency).

[1] ΔD^* and ΔA^* are equal, respectively, to $\Delta D \cdot Dem$ and $\Delta A \cdot (1 - Dem)$

List of tables

Appendix Table A - Data and Sources		
Variable	Description	Source
Democracy Variable, Dem	Data for the period 1951-2002, it is coded 1 when the regime can be classified as democratic, 0 otherwise. It is equal to 1-REG , where REG is the the index in Przeworski's database "REG02" that is coded 1 if a country is under dictatorship, 0 otherwise.	Adam Przeworski, http://politics.as.nyu.edu/object/przeworskilinks.html
CO2	Percapita emissions of CO2 estimates expressed in metric tons of carbon	Marland, G., T.A. Boden, and R. J. Andres, 2008. *Global, Regional, and National CO2 Emissions*. In "Trends: A Compendium of Data on Global Change". Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., http://cdiac.ornl.gov/trends/emis/overview.html
Year	Year the observation is referred to. 1951 - 2006	-
GDP	Percapita GDP in in 1990 GK$. GK refers to the method used to estimate this data (Geary-Khamis method). For a description of the methodology, see reference (47)	Angus Maddison, http://www.ggdc.net/maddison/
Democracy Trend, D	This variable indicates how many subsequent years a country has been democratic. It is coded 0 the year of the switch to democracy, and 1, 2, 3 etc... after one, two or three periods since democratisation, if further switches have not took place. It is coded 0 if Democracy variable is equal to 0	Data elaborated from Przeworski's datasets, http://politics.as.nyu.edu/object/przeworskilinks.html
Dictatorship Trend, A	This variable indicates how many subsequent years a country has been autocratic. It is coded 0 the year of the switch to dictatorship, and 1, 2, 3 etc... after one, two or three periods since autocratisation, if further switches have not took place. It is coded 0 if Democracy variable is equal to 1.	Data elaborated from Przeworski's datasets, http://politics.as.nyu.edu/object/przeworskilinks.html
Inequality	Theil index of household's income inequality (EHII dataset), annual observations	University of Texas Income Inequality Project (UTIP) http://utip.gov.utexas.edu/data.html
PM10	Annual average concentration level of particulate matters expressed in micrograms per cubic meter	World Bank, World Development indicators (WDI) http://databank.worldbank.org/ddp/home.do?Step=1&id=4
Forest	Percentage of country's surface covered by forest, annual observations	World Bank, World Development indicators (WDI) http://databank.worldbank.org/ddp/home.do?Step=1&id=4

Appendix Table C - Panel Unit root test summary				
Exogenous variables: individual effect and individual trend **Lags included: 1**				
Series : CO2				
Method	Stat.	Prob**	Cross Section	Obs
Null: Common unit root process				
Levin, Lin & Chu	1.58923	0.9440	46	2164
Null: Individual unit root process				
Im, Pesaran and Shin W-Stat	3.77347	0.9999	46	2164
ADF-Fisher Chi-Square	55.8579	0.9989	46	2164
PP-Fisher Chi-Square	106.053	0.1501	46	2210
Series : PM10				
Method	Stat.	Prob**	Cross Section	Obs
Null: Common unit root process				
Levin, Lin & Chu	-19.4936	0.0000	47	517
Null: Individual unit root process				
Im, Pesaran and Shin W-Stat	-3.13628	0.0009	47	517
ADF-Fisher Chi-Square	131.894	0.0061	47	517
PP-Fisher Chi-Square	188.949	0.0000	47	564
Series : GDP				
Method	Stat.	Prob**	Cross Section	Obs
Null: Common unit root process				
Levin, Lin & Chu	-0.20043	0.4206	47	2357
Null: Individual unit root process				
Im, Pesaran and Shin W-Stat	3.46602	0.9997	47	2357
ADF-Fisher Chi-Square	66.3949	0.9862	47	2357
PP-Fisher Chi-Square	58.8221	0.9983	47	2404
Series : Inequality				
Method	Stat.	Prob**	Cross Section	Obs
Null: Common unit root process				
Levin, Lin & Chu	-5.08281	0.0000	37	973
Null: Individual unit root process				
Im, Pesaran and Shin W-Stat	-3.28362	0.0005	37	973
ADF-Fisher Chi-Square	139.410	0.0000	37	973
PP-Fisher Chi-Square	231.032	0.0000	37	1026
***Probabilities for Fisher tests are computed using an asymptotic Chi-Square distribution. All other tests assume asymptotic normality*				

Table B - Regime changes (Year)

	Przeworski (1950-2002)		
Country	No.	Switch to dem.	Switch to dict.
Albania	1	1992	-
Bangladesh	1	1991	-
Bolivia	3	1979, 1982	1980
Brazil	2	1979	1964
Bulgaria	1	1990	-
Burundi	2	1993	1996
Central Afr. Rep.	1	1993	-
Chile	2	1990	1973
Colombia	1	1958	-
Congo (Brazzaville)	2	1992	1997
Cote d'Ivoire	1	2000	-
Czechoslovakia*	1	1990	-
Ecuador	3	1979	1963, 2000
El Salvador	1	1984	-
Ghana	5	1969, 1979, 1993	1972, 1981
Greece*	2	1974	1967
Guinea-Bissau	1	2000	-
Haiti	1	1994	-
Hungary	1	1990	-
Indonesia	1	1999	-
Kenya	1	1998	-
Korea Rep.*	3	1960, 1988	1961
Laos	1	-	1959
Lesotho	1	1993	-
Madagascar	1	1993	-
Malawi	1	1994	-
Mali	1	1992	-
Mexico	1	2000	-
Moldova	1	1996	-
Nepal	2	1991	2002
Nicaragua	1	1984	-
Niger	3	1993, 2000	1996
Nigeria	4	1979, 1999	1966, 1983
Pakistan	5	1972, 1988	1956, 1977, 1999
Panama	3	1852, 1989	1968
Peru	7	1956, 1963, 1980, 2001	1962, 1968, 1990
Philippines	2	1986	1965
Poland	1	1989	-
Portugal*	1	1976	-
Romania	1	1990	-
Senegal	1	2000	-
Sierra Leone	4	1996, 1998	1967, 1997
South Africa	1	1994	-
Spain*	1	1977	-
Sri Lanka	2	1989	1977
Venezuela	1	1959	-
Zambia	1	1991	-

Appendix Table D - Kao residual cointegration test between CO2 and GDP				
Included observations 2491				
Null Hypothesis: No cointegration				
Trend assumption: No deterministic trend				
	t-stat		Prob	
ADF	3.499820		0.0002	
Residual variance			32630.21	
HAC variance			54563.01	
Augmented Dickey Fuller test equation				
Dependent variable $\Delta\hat{\epsilon}_{it}$				
Method: Least squares				
Included observations 2164				
Variable	Coeff.	Std.Err.	t-stat	Prob
$\hat{\epsilon}_{it-1}$	-0.010256	0.005722	-1.792398	0.0732
$\Delta\hat{\epsilon}_{it-1}$	0.026135	0.021942	1.191083	0.2338

Appendix Table E - Johansen Fisher Panel cointegration test for CO2 and GDP				
Included observations 2491				
Trend assumption: No deterministic trend (restricted constant)				
Lags interval (in first difference): 1 1				
Unrestricted cointegration rank test (trace and maximum eigenvalue)				
Hypothesized no. of CE(s)	Fisher stat* from trace test	Prob	Fisher stat* from max eigen. test	Prob
None	185.7	0.0000	180.7	0.0000
At most 1	83.10	0.7353	83.10	0.7353
** Prob are computed using asymptotic Chi-square distribution*				

Appendix Table G - Panel Unit root test summary				
Exogenous variables: none				
Lags included: 1				
Series : Residuals from the estimation of the long run relationship of CO2 and the other covariates (ref. table 2)				
Method	Stat.	Prob**	Cross Section	Obs
Null: Common unit root process				
Levin, Lin & Chu	-7.72560	0.0000	40	971
Null: Individual unit root process				
ADF-Fisher Chi-Square	211.779	0.0000	40	971
PP-Fisher Chi-Square	303.568	0.0000	46	1030
Series : Residuals from the estimation of the relationship of PM10 and the other covariates (ref. table 2)				
Method	Stat.	Prob**	Cross Section	Obs
Null: Common unit root process				
Levin, Lin & Chu	-7.96817	0.0000	25	206
Null: Individual unit root process				
ADF-Fisher Chi-Square	123.941	0.0000	25	206
PP-Fisher Chi-Square	157.196	0.0000	25	206
***Probabilities for Fisher tests are computed using an asymptotic Chi-Square distribution. All other tests assume asymptotic normality*				

	PM10	Percapita CO2		PM10	Percapita CO2
Albania	30.81*	-178.3	Madagascar	0	-170.3
	(2.30)	(-1.48)		(.)	(-1.46)
Bangadesh	209.1***	-82.42	Malawi	70.41***	-32.93
	(22.13)	(-0.75)		(8.19)	(-0.27)
Bolivia	108.9***	-144.0	Mali	0	0
	(8.72)	(-1.23)		(.)	(.)
Brazil	15.11	-437.5***	Mexico	-4.078	-104.5
	(0.82)	(-3.61)		(-0.19)	(-0.88)
Bulgaria	49.82**	959.8***	Moldova	34.30*	146.3
	(2.63)	(8.39)		(2.59)	(1.14)
Burundi	54.62***	-60.08	Nepal	58.47***	-85.51
	(5.91)	(-0.48)		(6.79)	(-0.71)
Central African Rep.	62.16***	-99.27	Nicaragua	0	-344.1**
	(7.21)	(-0.82)		(.)	(-3.02)
Chile	25.58	-281.6*	Niger	0	0
	(1.19)	(-2.32)		(.)	(.)
Colombia	58.60***	-240.6*	Nigeria	135.9***	7.297
	(3.54)	(-1.98)		(10.71)	(0.06)
Congo	0	-140.5	Pakistan	204.7***	-77.72
	(.)	(-1.09)		(17.93)	(-0.69)
Cote d'Ivoire	62.82***	-134.1	Panama	20.78	-305.0*
	(4.13)	(-1.12)		(1.06)	(-2.54)
Czechoslovakia	-17.26	2207.3***	Peru	53.31**	-264.8*
	(-0.82)	(18.50)		(3.20)	(-2.21)
Ecuador	18.64	-207.6	Philippines	48.12***	-134.7
	(1.13)	(-1.80)		(3.84)	(-1.14)
El Salvador	41.90**	-273.7*	Poland	6.935	1696.4***
	(3.21)	(-2.33)		(0.36)	(15.24)
Ghana	33.13**	-101.9	Portugal	19.07	-473.3***
	(3.19)	(-0.80)		(0.98)	(-4.09)
Greece	41.67*	95.11	Romania	3.696	798.1***
	(2.11)	(0.79)		(0.24)	(5.96)
Guinea-Bissau	0	0	Senegal	84.17***	-128.9
	(.)	(.)		(8.09)	(-1.10)
Haiti	0	-164.6	Sierra Leone	89.54***	20.29
	(.)	(-1.38)		(9.31)	(0.14)
Hungary	-15.56	622.0***	South Africa	-9.145	1628.8***
	(-0.78)	(5.92)		(-0.53)	(12.39)
Indonesia	83.89***	-206.3	Spain	13.50	-185.4
	(5.02)	(-1.68)		(0.67)	(-1.55)
Kenya	47.71***	-63.74	Sri Lanka	70.38***	-312.3**
	(5.00)	(-0.52)		(4.78)	(-2.70)
Korea Rep.	3.441	158.9	Venezuela	25.70	74.90
	(0.16)	(1.42)		(1.40)	(0.59)
Laos	0	0	Zambia	95.66***	83.29
	(.)	(.)		(11.42)	(0.70)
Lesotho	70.66***	0			
	(6.51)	(.)			

t statistics in parentheses

$* \, p < 0.05$, $** \, p < 0.01$, $*** \, p < 0.001$

The econometric model for the estimation of the unrestricted ECM for CO2 emissions

Consider the following ARDL(1,1)

$$\begin{aligned} CO2_{it} &= \alpha_{0i} + \gamma_1 CO2_{it-1} + \sum_{j=1}^{n} \delta_{0j} X_{jit} + \sum_{j=1}^{n} \delta_{1j} X_{jit-1} + \\ &+ \sum_{z=3}^{4} \delta_{2z} X_{4it} X_{zit} + \sum_{z=3}^{4} \delta_{3z} X_{4it-1} X_{zit-1} + \epsilon_{it} \end{aligned} \tag{2.26}$$

with $j = 1, ..., 4$ and

$$\begin{aligned} X_1 &= GDP \\ X_2 &= \sqrt{D} \\ X_3 &= \sqrt{A} \\ X_4 &= Ineq \end{aligned} \tag{2.27}$$

with some manipulations

$$\begin{aligned} CO2_{it} - CO2_{it-1} + CO2_{it-1} &= \alpha_{0i} + \gamma_1 CO2_{it-1} + \sum_{j=1}^{n} \delta_{0j} X_{jit} + \sum_{j=1}^{n} \delta_{1j} X_{jit-1} + \\ &+ \sum_{j=1}^{n} \delta_{0j} X_{jit-1} - \sum_{j=1}^{n} \delta_{0j} X_{jit-1} + \sum_{z=2}^{3} \delta_{2z} X_{4it} X_{zit} + \\ &+ \sum_{z=2}^{3} \delta_{3z} X_{4it-1} X_{zit-1} + \epsilon_{it} \end{aligned}$$

$$\begin{aligned} \Delta CO2_{it} &= \alpha_{0i} - (1 - \gamma_1) CO2_{it-1} + \sum_{j=1}^{n} \delta_{0j} \Delta X_{jit} + \sum_{j=1}^{n} (\delta_{0j} + \delta_{1j}) X_{jit-1} + \\ &+ \sum_{z=2}^{3} \delta_{2z} X_{4it} X_{zit} + \sum_{z=2}^{3} \delta_{3z} X_{4it-1} X_{zit-1} + \epsilon_{it} \\ &= \alpha_{0i} - (1 - \gamma_1) CO2_{it-1} + \sum_{j=1}^{n} \delta_{0j} \Delta X_{jit} + \sum_{j=1}^{n} (\delta_{0j} + \delta_{1j}) X_{jit-1} - \\ &- \sum_{z=2}^{3} \delta_{2z} \Delta X_{4it} \Delta X_{zit} + \sum_{z=3}^{4} (\delta_{2z} + \delta_{3z}) X_{4it-1} X_{zit-1} + \\ &+ \sum_{z=2}^{3} \delta_{2z} X_{4it} \Delta X_{zit} + \sum_{z=2}^{3} \delta_{2z} \Delta X_{4it} X_{zit} \end{aligned}$$

and we end up with the usual ECM:

$$\begin{aligned}
\Delta CO2_{it} &= \sum_{j=1}^{n} \delta_{0j} \Delta X_{jit} + \sum_{z=2}^{3} \delta_{2z} W_z - (1-\gamma_1)\Big[CO2_{it-1} - \frac{\alpha_{0i}}{1-\gamma_1} - \\
&- \sum_{j=1}^{n} \frac{\delta_{0j}+\delta_{1j}}{1-\gamma_1} X_{jit-1} - \sum_{z=2}^{3} \frac{\delta_{2z}+\delta_{3z}}{1-\gamma_1} X_{4it-1} X_{zit-1}\Big] \\
W_z &= \Big(X_{4it}\Delta X_{zit} + \Delta X_{4it} X_{zit} - \Delta X_{4it} \Delta X_{zit}\Big)
\end{aligned}$$

Bibliography

[1] Acemoglu D., Robinson J.A. and Verdier T., 2004. *Kleptocracy and Divide-and-Rule: A Model of Personal Rule*. Journal of the European Economic Association, Vol. 2, No. 2/3, Papers and Proceedings of the Eighteenth Annual Congress of the European Economic Association, pp. 162-192. 58

[2] Barro, Robert, 1996. *Getting it Right*. Cambridge MA: MIT Press.

[3] Barro, Robert 1996b. *Democracy and Growth*. Journal of Economic Growth 1, pp. 1-27.

[4] Barro, Robert J., 1999. *Determinants of Democracy* Journal of Political Economy 107, pp. 158-183. 50

[5] Belk, R.W., 1988. *Possession and the Extended Self*, Journal of Consumer Research 5, pp. 39-47.

[3] Bhattarai M., and Hammig, M., 2001. *Institutions and the Environmental Kuznets Curve for Deforestation: a Cross Country Analysis for Latin America, Africa and Asia*. World Development 29, pp. 995-1010. 48

[7] Blanchflower, David G., and Andrew J. Oswald, 1998. *What Makes a Young Entrepreneur?* Journal of Labor Economics 16, pp. 26-60.

[8] Campbell, Donald T., 1969. *Reforms as experiments*. American Psychologist 24, pp. 409-429.

[9] Congleton, Roger D., 1992. *Political Institutions and Pollution Control*. The Review of Economics and Statistics 74, pp. 412-421.

[6] Dasgupta, P. and Mäler, K. G., 1995. *Poverty, Institutions, and the Environmental Resource-Base*, in Behrman, J., Srinivaan, T.N. (eds.), Handbook of Development Economics, vol. 3A. Elsevier Science, Amsterdam. (Chapter 39). 48, 86

[8] Desai, Uday, 1998. *Environment, Economic Growth, and Government*, Ecological Policy and Politics in Developing Countries. Albany, NY. State University Press of New York Press, 1-45. 48, 86

[12] Engle, R.F. and C.W.J. Granger, 1987. *Cointegration and Error Correction: Representation, Estimation and Testing.* Econometrica 55, pp. 251-276. 57

[13] Farzin, Y.H. and Bond, C.A., 2006, *Democracy and Environmental Quality*, Journal of Development Economics 81, pp. 213-235.

[14] Fisher R.A., 1932. *Statistical Methods for Research Workers, 4th edition, Edimburgh.* Oliver & Boyd. 68

[15] Frankel J.A. and Rose A.K., 2005. *Is Trade Good for the Environment? Sorting Out the Causality*, The Review of Economics and Statistics 50, pp. 85-91.

[12] Gallagher K. P., and Thacker S. C., 2008. *Democracy, Income and Environmental Quality*, Political Economy Research Institute (PERI) Working Paper No. 164, University of Massachusetts Amhrest. 49

[17] Gaspart F., and Platteau, J.P., 2007. *Heterogeneity and Collective Action for Effort Regulation: Lessons from the Senegalese Small-Scale Fisheries*, in *Inequality, Cooperation and Environmental Sustainability*, edited by Baland J.M., Bardhan P., and S. Bowles, Princeton University Press.

[13] Gleditsch, Nils P., and Bjorn Sverdrup, 2003. *Democracy and the Environment.* In *Human Security and the Environment: International Comparisons*, edited by Edward Paper and Michael Redclift. London: Elgar. 48, 86

[15] Grossman, G.M., and Krueger, A.B., 1995, *Economic Growth and the Environment*, Quarterly Journal of Economics 110, pp. 353-377. 102

[16] Hardin, Garret, 1968. *The Tragedy of the Commons.* Science 162, pp. 1243-1248. 48, 86

[21] Hausman, Jerry, 1979. *Individual Discount Rates and the Purchase and Utilization of Energy-using Durables*, Bell Journal of Economics 10, pp. 33-54. 50

[22] Heerink N., Mulatu A., and Bulte E., 2001. *Income Inequality and the Environment: Aggregation Bias in Environmental Kuznets Curves.* Ecological Economics 38, pp. 359-367.

[23] Holtz-Eakin Douglas, David Joulfaian and Harvey S. Rosen, 1994. *Sticking it Out: Entrepreneurial Survival and Liquidity Constraints.* Journal of Political Economy 102, pp. 53-75.

[24] Hwang Hsiang-Chih, 2006. *Water Quality and the End of Communism: does a Regime Change Lead to a Cleaner Environment?.* Brandeis University Working Paper.

[25] Im, K.S., M.H. Pesaran and Y. Shin, 2003. *Testing for Unit Roots in Heterogeneous Panels.* Journal of Econometrics 115, pp. 53-74. 66

[26] International Monetary Fund, 2009. World Economic Outlook Report.

[27] Kao Chihwa, 1999. *Spurious Regression and Residual-Based Tests for Cointegration in Panel Data*, Journal of Econometrics 90, pp. 1-44. 55, 68

[20] Leslie, Winsome J., 1987. *The World Bank and Structural Transformationin Developing Countries:The Case of Zaire.* Boulder, CO: Lynne Rienner Publishers. 58, 86

[29] Levin A., C.F. Lin and C. Chu, 2002. *Unit Root Test in Panel Data: Asymptotic and Finite-Sample Properties.* Journal of Econometrics 108, pp. 1-24. 65

[30] Levine, James, 1997. *Badass Sells*, Forbes 159, pp. 142-147.

[31] Li Q. and Reuveny R., 2006. *Democracy and Environmental Degradation.* International Studies Quarterly 50, pp. 935-956.

[32] Maddala G.S. and Shaowen Wu, 1999. *A Comparative Study of Unit root test with Panel Data and a New Simple Test*, Oxford Bulletin of Economics and Statistics 61, 631-652. 55, 68

[21] Maddison Angus, 2009. *Statistics on World Population, GDP and Per Capita GDP, 1-2006 AD*, updated March 2009, available at http://www.ggdc.net/maddison/ 52, 96

[22] Marland, G., T.A. Boden, and R. J. Andres, 2008. *Global, Regional, and National CO2 Emissions*. In "Trends: A Compendium of Data on Global Change". Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn. 52, 96

[35] Marland G., and R.M. Rotty, 1984. *Carbon Dioxide Emissions from Fossil Fuels: a Procedure for Estimation and Results for 1950-82*. Tellus 36(B), pp. 232-261. 52

[36] Mason R., 1981. *Conspicuous Consumption: A Study of Exceptional Consumer Behaviour*, St. Martin's Press, New York, NY.

[37] Midlarsky Manus, 1995, *Environmental Influences on Democracy: Aridity, Warfare and a Reversal of the Causal Arrow*, The Journal of Conflict Resolution 39, pp. 224-262.

[38] Midlarsky Manus, 1998, *Democracy and the Environment: An Empirical Assessment*, Journal of Peace Research 35, pp. 341-361.

[39] Milanovic Branko, 2000. *The Median Voter Hypothesis, Income Inequality and Income Redistribution: an Empirical Test with the required data*, European Journal of Political Economy, September 2000. 48

[25] Moya Pons, Frank, 1995. *The Dominican Republic: A National History*. Princeton, NJ: MarkusWeiner. 58, 86

[26] Olson, Mancur. 1965. *The Logic of Collective Action*. Harvard University Press.

[28] Payne Rodger A., 1995, *Freedom and the Environment*, Journal of Democracy 6, pp. 41-55.

[43] Przeworski A. and Limongi F., 1993. *Political Regimes and Economic Growth*, The Journal of Economic Perspectives 7, pp. 51-69. 48, 86

[44] Rodrik Dani, 1999. *Democracies Pay Higher Wages*, Quarterly Journal of Economics 114, pp. 707-738. 50

[30] Schultz, C. B., and T. R. Crockett, 1990 *Economic Development, Democratization, and Environmental Protection in Eastern Europe*. Boston College Environmental Affairs Law Review 18, pp.5384. 48

[34] Torras M., and Boyce J. K., 1998. *Income, Inequality and Pollution: a Reassessment of the Environmental Kuznets Curve*. Ecological Economics 25, pp. 147-160. 48, 86

[47] United Nations, 1992. *Handbook of the International Comparison Program.* 48

[48] Urbain, J.P., 1992. *On Weak Exogeneity in Error Correction Models*. Oxford Bulletin of Economics and Statistics 54, pp. 187-207. 72

[49] Veblen T., 1934. *The Theory of the Leisure Class*, Random House, Inc., New York, NY. 55, 70

[50] Waller M., and Millard, F., 1992. *Environmental Politics in Eastern Europe*. Environmental Politics 1, pp. 159-185.

[51] Wittfogel Karl, 1957. *Oriental Despotism: a Comparative Studies of Total Power*, New Haven CT: Yale University Press.

3

Democratisation, Environmental and Income Inequality.

Abstract

Empirical economists commonly agree upon the fact that the demand for environmental goods is increasing with income, so as long as democratisation shifts the decisive power from a rich individual (autocrat or dictator) to a poorer one (decisive voter), such a regime change should be associated to worse environmental conditions. Assuming that citizens' wealth does not depend on inherited endowments, but only on each individual's share on total production, I will show with a theoretical model that contrary to the expectations, democratisation may have mixed effects on the level of environmental quality, depending on the size of the price and income effects on the demand for environmental quality associated to a decrease in the decisive political actor's wealth. If indeed a poorer individual desires less environmental quality than a richer individual (income effect), he also desires less of the goods responsible for degradation (price effect) so the overall result on the environment is ambiguous. Assuming instead that society is composed by two classes of individuals, one supplying an embodied factor of production and one supplying capital, and assuming moreover that the decisive voter belongs to the first class of individuals while the autocrat does not, democratisation is shown to be beneficial for the environment, the better the effect on the environment, the bigger the difference in wealth between the two decisive political actors.

JEL Classification: D02, D31, O13, Q56

Keywords: Democracy; Environment; Environmental Inequality; Income Inequality.

3.1 Introduction

Theoretical and empirical literature about the effects of democracy on the level of environmental quality is still inconclusive and does not provide clear and unilateral answers to the effect of different regimes on the environment. In the seventies, the dominant thinking was that democracy and its associated liberties to consume and procreate would have generated ecological catastrophes (Desai, (8), Hardin, (16)); recently, on the contrary, scholars have found positive effect of democracy on the environmental quality, the main reason being the necessity for democratic governments to be elected or re-elected, which makes them more prone to respect human life, and then more responsive to life threatening environmental degradation (Dasgupta and Maler (6), Schultz and Crockett (30), Payne (28), Gleditsh and Sverdrup (13)).

This paper is aimed at contributing on the debate about whether democracy may be good or not for the environment. The questions I want to answer are: what is the role of a political regime in the determination of a country's environmental policy? Are democracies more environmentalist than autocracies? In order to answer these questions, it is important first of all to understand the difference between those two regimes. In a (direct) democracy, decisions are taken by majority voting, while in a dictatorship they are taken by one or a group of few individuals. In practice, therefore, the decisive political actor is different, and what makes them differ is often their wealth.

It is well known that wherever in the world, at any time in history, kings, queens and dictators have been the richest and powerful people. From north to south, in developed and underdeveloped countries, in communist and non-communist countries, the only characteristic that this variegated group of people has in common is a lofty position and vast fortune. Forbes, in an article appeared few years ago[1], estimated the Saudi Arabia King Abdullah Bin Abdulaziz's wealth in around 45% of the country's $340 billion GDP. Brunei's Sultan Haji Hassanal Bdkiah has an estimated fortune of $20 billion, and benefits from petroleum and natural gas fields. The President of the United Arab Emirates, Sheikh Khalifa bin Zayed Al Nahyan own the 90% of the 2.5 million barrels a day exported from the UAE and he alone represents two third of the GDP. Mutubu Sese Seko in the Democratic Republic of Congo (Zaire) during the 70's benefitted of around 15-20% of Congo's operating budget and in the 80's his fortune was estimated in $5 billion (Leslie (20), p. 72). Rafael Trujillo in the Dominican Republic had a fortune which amounted to 100% of the national GDP at current prices and his family controlled almost 80% fo the country's industrial production (Moya Pons (25), p. 398). These are only few examples, but the list of very rich rulers is not over and probably it would be a manuscript on its own. From this perspective, a regime change from dictatorship to democracy shifts the decision power from the dictator to the median voter, implying an impoverishment of the decisive political actor and therefore a different willingness to pay for environmental protection.

[1]The article is available online at http://www.forbes.com/2006/05/04/rich-kings-dictators_cz_lk_0504royals.html

If the demand for environmental quality was an increasing function of income, dictatorship would be a panacea for the environment. Numerous empirical studies suggest that willingness to pay for environmental improvement is an increasing function of wealth (Hökby and Söderqvist (17), Kriström and Riera (18), Miles, Pereyra and Rossi (24)). Richer people tend to express higher willingness to pay for environmental improvement than poorer people, that is, environmental quality is typically found to be a normal good. Because of this, an institutional shift from democracy to dictatorship should result in an improvement of a country's environmental quality. As pointed out by Boyce, (4), however, this beneficial effect may not necessarily bring to the expected result, because other factors come into play to mitigate or even nullify it.

First, because an increase in the decisive political actor's income raises his "price" of the environmental protection. Increasing income means increasing the share the individual gets out of production, if production is kept constant. Since production requires some environmental depletion, it is easy to realise that environmental protection is more costly for the rich because a decrease in global GDP results in a greater decrease in his own income than on the poor's, who gets a lower share. Moreover, even though richer individuals may desire more environmental quality (income effect), they also desire more of the goods and services responsible for environmental degradation (price effect), so the expected environmental improvement will occur only if the negative "price effect" does not outweigh the positive "income effect".

Second, because richer people have more resources and capabilities to substitute environmental quality with private consumption, giving raise to the so-called environmental inequality. If indeed the willingness to pay (WTP) for environmental protection is an increasing function of income, different is the elasticity of such willingness to pay. Martini and Tiezzi (23) indeed find, in a panel of Italian households from 1999 to 2006, that despite the willingness to pay for environmental protection is higher for richer people, the elasticity of WTP is less than one, and therefore decreases as income increases, suggesting a lower capability for poorer people to substitute environment with private consumption.

Substantial evidence shows indeed that the less wealthy and powerful members of societies may be the ones exposed to much heavier environmental degradation than the more well off, being able to avoid it, simply relocating to cleaner living areas or using their political power to drive out polluting industries from their neighborhoods. Gray and Shadbegian (14), for instance, obtain some evidence from US data on the paper industry for the period 1985-1997 that polluting emissions are significantly lower in areas with more children, older people and fewer poor people than in areas with young poor people without children, and Bina Agarwal (2), for instance, documented how the degradation of forest resources in rural India had particularly severe effects on poor women, via impacts of their time, income and nutrition. Pastor, Sadd and Hipp (27), for instance, provide substantial evidence that minorities residential areas have an higher likelihood to host various environmental hazards, and Sheila Foster (11) reports that poor african american neighbourhoods in Chester (Pennsylvania) often experience a clustering of waste facilities in their areas.

In this paper, I provide a theoretical model through which I analyse the effects of democratisation on the environment, meaning by democratisation a shift of the decisive power from a rich individual (dictator) to a poorer one (decisive voter). I will assume that citizens differ only by their wealth, meaning by wealth only income (or share they get out of production) and excluding any inherited endowment, to keep the model as simple as possible.

In the first instance, I will analyse the result of democratisation on the level of environmental quality assuming that there is no environmental inequality, so all the citizens experience the same level of pollution.

I will later remove this assumption by introducing a class model of experienced pollution, and assuming that the society is composed by two types of individuals - employers and employees. Employers supply capital in the production process, while employee supply their physical labor and therefore cannot avoid their own exposure to pollution. Employers, on the other hands, supplying capital, can relocate far away (at a cost) from their polluting factories, being not necessary their physical activity in the production process.

As it is often unexpected, income inequality per se between the two decisive political actors may not be responsible of different environmental policies undertaken by the two different regimes, but environmental inequality is the crucial variable affecting them. Income inequality, therefore, has an indirect role as long as it induces environmental inequality. In countries where the majority of the population is represented by employees whose services require physical presence in the firm, democratisation is expected to be beneficial for the environment, and this is in line with the recent empirical evidence.

The next section introduces the model. Section 3.3 presents some empirical evidence and discusses some cases of regime transition, having regards to the effect on environmental policy of democratisation. Section 3.4 concludes.

3.2 The model

This section presents the model, with the aim of analysing the effect of a regime change (in particular, democratisation) on the optimal level of environmental quality. In the first instance, I will assume that the decisive political actor in autocracy is richer than in democracy, so democratisation results in a shift of the decisive power from a rich individual (the dictator) to a poor one (the median voter). Poorer people have a tendency to value consumption more than environmental quality, but richer people consume more of the good which are responsible for pollution. The first part of the model will deal with this issue, assuming that the environment is a pure public good whose exposure cannot avoided by anyone at any cost.

The second part instead will introduce a model of class differences in experienced environmental quality to take into account of the effect of environmental inequality. Assuming that the decisive political actors in democracy and autocracy belong to different classes, and in particular, assuming that the dictator is a capitalist employer who supply only capital in the production process and therefore is able to relocate his home far away from the polluting sources, while the median voter is a employee who supply physical labor and therefore is exposed to emission, I will show that democracy is beneficial to the environment, the more beneficial the higher the difference in wealth between the dictator and the median voter.

3.2.1 Democratisation and income inequality

Throughout this section, I will assume that:

A. 1 *The decisive political actor under an autocratic regime receives a larger share of GDP than under a democratic one.*

Denote then by g the level of production, or GDP, and by e the level of environmental quality, with both g and e assured to be positive. Environment is an essential factor of production, so there cannot be positive production without some environmental depletion. Equation 3.1 represents the relation between production and environmental quality, i.e., the transformation locus between environment and income:

$$g = f - \frac{1}{2}e^2 \tag{3.1}$$

with f a positive constant. The maximum achievable level of production is bounded above due to technological constraints and this boundary is represented by f. A level of production equal to f can be achieved only by completely depleting the environment. Conversely, the highest environmental quality requires absence of production.

Suppose than that the society is composed by N individuals, with all having identical preference function. Each generic individual i gets a share of total production s^i, with $\sum_{i=1}^{N} s^i = 1$ (notice that, in general, in general $s^i \neq s^j$ for $i \neq j$) as a "personal wage" available for consumption, and therefore, the amount each individual can consume in each period is represented by

$$y^i = s^i g \tag{3.2}$$

and therefore, the "personal" marginal rate of transformation between income and environmental quality is represented by

$$\frac{\partial y^i}{\partial e} = -es^i. \tag{3.3}$$

Each individual's utility is a nonseparable function of income and environmental quality and it is denoted by

$$u^i = u(y^i, e) \qquad i = 1, ..., N \tag{3.4}$$

with u^i representing the level of utility for individual i, y^i denotes income (or consumption) of the same individual, and e the level of environmental quality. Environmental quality is a public good which is experienced uniformly across the population, hence the absence of the superscript (this assumption, however, will be removed later). The utility function u^i is assumed increasing and concave with respect to both arguments, and have positive cross derivates. In other words, I have $u_e > 0$, $u_y > 0$, $u_{ee} < 0$, $u_{yy} < 0$, $u_{ey} > 0$ and $u_{ye} > 0$, where u_x is the first derivative of the utility function with respect to the generic variable x and u_{xz} is the first derivative of

u_x with respect to another variable z. These assumptions imply that the utility generated by increasing income or environmental quality is positive, but it increases at a decreasing rate, and that the pleasure derived by income (cleaner environment) is higher, the higher is the environmental quality (income).

Under assumption A.1., if the level of experienced environmental quality does not differ between different people, the effect of democratisation (autocratisation) can be read as the effect of an impoverishment (enrichment) of the decisive political actor. Denote with v the decisive political actor under democracy, and with a the decisive individual under autocracy. Assumption A.1. has the only implication that $s^v < s^a$, so the problem the decisive citizen faces is

$$\begin{aligned} \max_e u^i &= u(y^i, e) \qquad i = v, a && (3.5) \\ \text{s.t.} \quad y^i &= s^i \cdot g && (3.6) \\ g &= f - \frac{1}{2}e^2 && (3.7) \end{aligned}$$

so they decide their optimal levels of environmental quality so as to maximise their utility subject to their wealth and the available technology.
The first order condition for utility maximisation requires

$$\begin{aligned} -s^i e \cdot u_{y^i}(y^i, e) &= -u_e(y^i, e) \qquad \text{or} && (3.8) \\ -es^i &= -\frac{u_e(y^i, e)}{u_{y^i}(y^i, e)} && (3.9) \end{aligned}$$

where u_{y^i} represents the derivative of the utility function for individual i with respect to the level of income y^i, and u_e represents the derivative of the utility for individual i with respect to the environmental quality. Equation 3.9 simply says that as a necessary condition for maximising utility is the equality between the "personal" marginal rate of transformation between income and environment and the marginal rate of substitution between income and environment. The level of environmental quality e chosen by individual i, therefore, will be

$$e^* = \frac{1}{s^i} \cdot \frac{u_e(y^i, e)}{u_{y^i}(y^i, e)}. \qquad (3.10)$$

A first question that comes into mind is whether the effect of a marginal increase in the share of income of this citizen increases the level of environmental quality or not. Taking the first derivative of e^* with respect to s^i, we get that

$$\begin{aligned} \frac{\partial e^*}{\partial s^i} &= \frac{1}{s^i}\Bigg[-\frac{1}{s^i}\left(\frac{u_e(y^i, e)}{u_{y^i}(y^i, e)}\right) + \\ &+ g\left(\frac{u_{ey^i}(y^i, e)u_{y^i}(y^i, e) - u_e(y^i, e)u_{y^iy^i}(y^i, e)}{(u_{y^i}(y^i, e))^2}\right)\Bigg] && (3.11) \end{aligned}$$

which is positive (so the effect of an increase in s^i is good for the environment) when

$$\frac{1}{s^i g} < -\frac{u_{y^i y^i}(y^i, e)}{u_{y^i}(y^i, e)} + \frac{u_{ey^i}(y^i, e)}{u_e(y^i, e)}. \tag{3.12}$$

Defining

$$\frac{u_e(y^i, e)}{u_{y^i}(y^i, e)} = MRS^i_{e,y} \tag{3.13}$$

$$-es^i = MRT^i_{e,y} \tag{3.14}$$

it is easy to see that equation 3.12 can be rewritten as[1]

$$\frac{\partial MRT^i_{e,y}/\partial y}{MRT^i_{e,y}/y} < \frac{\partial MRS^i_{e,y}/\partial y}{MRS^i_{e,y}/y} \tag{3.15}$$

so, a positive variation in the share of income is associated to a positive variation in the level of environmental quality only if the "price effect" due to an higher share of income s^i (which makes also the marginal rate of transformation between environment and income steeper) is lower than the "income effect" due to the relative variation in the marginal rate of substitution between the two goods. In other words, becoming richer is beneficial for the environment only if the elasticity of $MRT^i_{e,y}$ is lower than the elasticity of $MRS^i_{e,y}$, both with respect to income.

Denoting by e^v and e^a the optimal level of environmental quality chosen, respectively, by the decisive citizen in democracy and by the dictator, we have that

$$e^v = \frac{u_{e^v}(y^v, e^v)}{s^v \cdot u_{y^v}(y^v, e^v)} \tag{3.16}$$

$$e^a = \frac{u_{e^a}(y^a, e^a)}{s^a \cdot u_{y^a}(y^a, e^a)} \tag{3.17}$$

The optimal level of environmental quality selected in democracy will be lower than the level selected in dictatorship ($e^v < e^a$) if

$$\frac{MRS^v_{e,y}}{s^v g} < \frac{MRS^a_{e,y}}{s^a g} \tag{3.18}$$

or, rearranging,

$$\frac{MRS^v_{e,y}}{s^v} < \frac{MRS^a_{e,y} - MRS^v_{e,y}}{s^a - s^v} \tag{3.19}$$

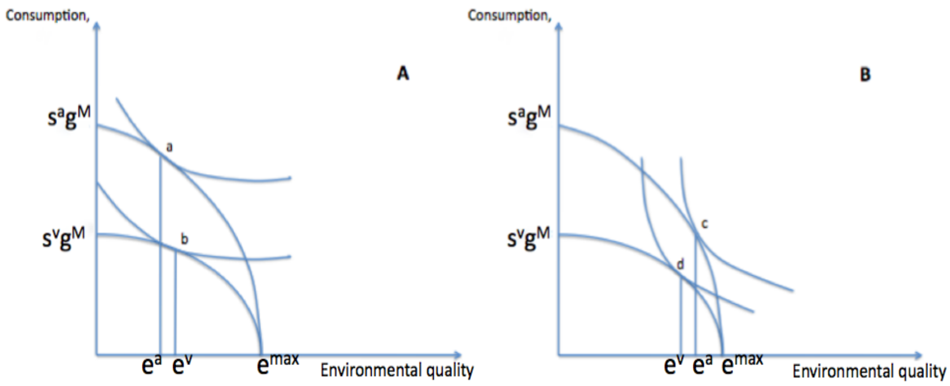
Equation 3.18 highlights the ambiguous effect (depicted in figure 1) of a regime shift. Since the level of environmental quality experienced by both the dictator and the decisive citizen in democracy is actually the same, what makes the difference in determining which regime is more pro-environment depends basically on the difference between "relative" marginal willingness to pay for environmental protection. An example may clarify the interpretation of equation 3.18. Suppose the decisive political actor in democracy is willing to pay 2 unit of income to increase

[1]For the computation of the derivative with respect to y of the marginal rate of transformation, we make use of equation 3.6 according to which $s^i = y^i/g$. Substituting s^i into 3.14 and taking the derivative with respect to y^i, we get $-e/g$, which, once divided by $-es^i$ gives $1/s^i g$.

by one unit environmental quality. Suppose instead that the dictator is willing to pay 5, because he's richer so he values environment more. However, the difference in wealth between those two decisive citizens is huge. Suppose that the dictator gets 10% of the global production, so $s^a = 0.1$. Assume that the median voter gets only 1%, so $s^v = 0.01$. Assume that the country's GDP (g) is, for simplicity, 1000. It follows that, in percentage, the decisive political actor in democracy is willing to give up 0.2% of his income to protect the environment while the same decisive citizen under dictatorship is willing to pay 5%. In this case, of course, democracy is bad for the environment. If inequality between those two citizens instead was not too high, and the marginal rate of substitution for the median voter was 6 against 8 for the dictator, $s^v = 0.05$, $s^a = 0.1$, and keeping constant g at 1000, we have that proportionally to income, the median voter is willing to give up 12% of to increase environmental quality against 8% for the dictator. This case, of course, implies that democracy is good for the environment.

Figure 1 provides a graphical explanation of the argument above. Suppose that the decisive citizen's income increase from $s^v g$ to $s^a g$. Whether this shift is good for the environment or not, depends on the variation of the ratio between the two marginal rates of substitution between y and e for the dictator and the decisive citizen in democracy. Of course, the dictator has a $MRS_{e,y}$ higher than the median voter's $MRS_{e,y}$, but if this increase in the willingness to pay for environmental quality do not compensate the increase in the share of income the dictator gets, dictatorship is bad for the environment (figure 1A). Figure 1B depicts the opposite case, so when democracy is bad for the environment, because the marginal utility from consumption of the decisive voter in democracy is much higher than the dictator's one, so a shift from democracy to dictatorship is good for the environment. Equation 3.18 mans exactly what is depicted in figure 1.

Figure 1.



From another point of view, equation 3.19 shows that we can see democratisation as a bad regime for the environment if the median voter's relative marginal willingness to pay for environmental quality does not compensate what would be the difference of the marginal willingness to pay under the two different regimes, weighted by the difference in income between the two decisive citizens.

3.2.2 A model of class differences in experienced environmental quality

In order to introduce environmental inequality, i.e., the fact that environment is experienced in different ways by different people, I will make the assumption that the population is divided into two classes, capitalists and workers. Capitalists supply capital in the production process and receive the rents their capital generates. Workers can only sell their labour in the firms owned by the capitalists and therefore cannot avoid their exposure to the pollution produced by the firms they are employed in, either because during the job they are exposed to emissions, or because they cannot relocate too far away from the firms since they have to show up at work every morning. Of course, this is not the case for the capitalist, who can choose the best location to live because he does not need to physically sell his own labour. Under the assumption that the decisive political actor in autocracy is a capitalist and in democracy is a worker, the optimal level of environmental quality chosen by those two decisive citizens will be different than in the previous case. The worker is assumed to have an experienced environmental quality which reflects the real status of the environment, therefore his maximisation problem remains unchanged. The capitalist, on the contrary, may decide to increase his own perception[1] of environmental quality, for example, by relocating in a cleaner area, but this can be done at a cost. The price may be related, for instance, to the transportation costs the capitalist must pay in order to reach the city where his firms are located, so the price may be a function of the capitalist's home distance to the industrial district.

Denote by e_x^a the capitalist's (or dictator) level of experience environmental quality. e_x^a is no more equal to the actual, or "real" level, since it can be increased by relocating far away from the pollution source. In detail, the equation for e_x^a is given by

$$e_x^a = e + hc \tag{3.20}$$

where e is the real level of environmental quality, c is the cost of relocating in a cleaner area and h is a positive constant denoting the level of "productivity" of the cost of living far from the pollution sources. In this model, we will assume that

$$e_x^a \geq e \tag{3.21}$$

so the level of experienced environmental quality cannot be lower than the real level. This assumption guarantees that c is an effective cost, so it cannot be negative (the employer basically cannot accept bribes in order to live close to the polluting factory).

Equation 3.20 can be rephrased as follow: if one defines c the monetary expense the employer pays to experience a better environment, c can be read as

$$c = p(e_x^a - e) \tag{3.22}$$

[1] "Perceived" environmental quality may differ to "experienced" environmental quality in the sense that the perception of the environment may be subjective and mistaken. In this paper however, I consider the words "perceived" and "experienced" as a synonymous because I exclude the possibility that a person may perceive pollution differently from other people in the same class or even wrongly.

that is to say, the price for an experienced environmental improvement (p) multiplied by the difference in the experienced environmental quality chosen by the employer and the real environmental quality ($e_x^a - e$). If this is the case, it is possible to notice, by equation 3.20, that, taking e from the right hand side of the equation to the left, the price for an experienced environmental improvement p is defined as $1/h$.

$$p \equiv \frac{1}{h} \tag{3.23}$$

Since the capitalist - or employer - pays a cost $c > 0$, his level of income available for consumption will be reduced of that amount, so

$$y^a = s^a g - c \tag{3.24}$$

and technology remains identified by equation 3.1.

The new problem the decisive political actor in dictatorship faces is then

$$\max_{e,c} u^a = u(y^a, e_x^a) \tag{3.25}$$

subject to equation 3.20 and 3.24. The first order conditions for utility maximisation are:

$$\frac{\partial u^a}{\partial e} = 0 \quad \Rightarrow \quad e^{a*} = \frac{u_{e_x^a}(y^a, e_x^a)}{s^a u_{y^a}(y^a, e_x^a)} \tag{3.26}$$

$$\frac{\partial u^a}{\partial c} = 0 \quad \Rightarrow \quad \frac{1}{h} = \frac{u_{e_x^a}(y^a, e_x^a)}{u_{y^a}(y^a, e_x^a)} \tag{3.27}$$

Equation 3.27 represents the equilibrium level of expenditure, which must equate the productivity of the "insurance" against pollution and the amount of the environmental quality the dictator is willing to give up to become one unit richer. Combining equations 3.26 and 3.27 I get

$$e^{a*} = \frac{1}{s^a h} \tag{3.28}$$

so the optimal level of environmental quality chosen by the dictator depends negatively only on the share he can get out of production and the productivity of the expenditure in increasing the perception of environmental quality.

The reason for this result is simple: a richer citizen enjoys environmental quality more than a poor one, but his opportunity cost to pay for environmental protection is higher than for a poor citizen. If he can increase his own perception of environmental quality of an amount that is greater than the amount of environment he is willing to give up to become one unit richer, then it is worthwhile to pay.

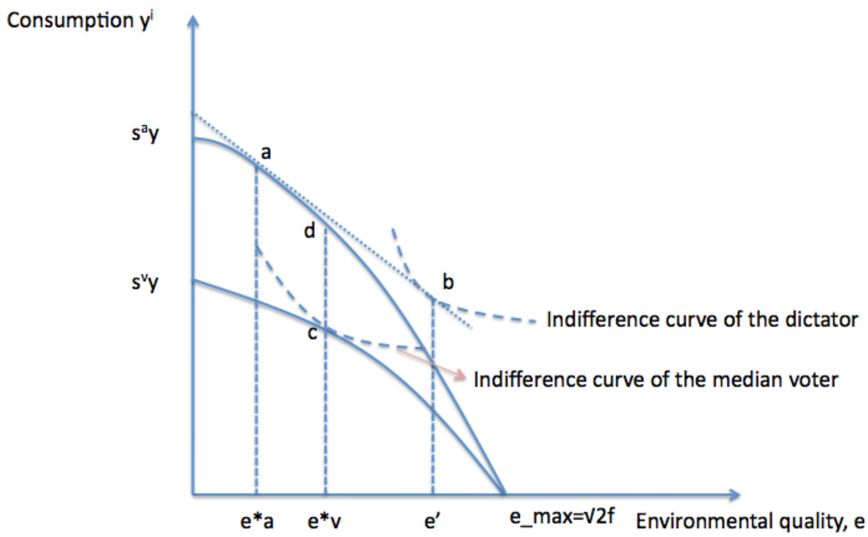
In general, taking the equilibrium relations of the two decisive political actors, represented by equation 3.28 and equation 3.16, I get that the optimal level of environmental quality will be lower in autocracy if

$$h > \frac{s^v}{s^a} \cdot \frac{u_{y^v}(y^v, e^v)}{u_{e^v}(y^v, e^v)} \tag{3.29}$$

that is to say, if the level of productivity of the cost devoted to prevent exposure to pollution is greater than the amount of environmental quality the median voter is willing to give up to become a unit richer, times the ratio of the income share of production of the two decisive actors. So, income inequality between the dictator and the median voter is not a good predictor of the effect of a regime change on the environmental policy undertaken by a country, it is environmental inequality that matters, as long as the expenditure for substituting environment with private consumption is productive (i.e. increase the perception of environmental quality) enough. This result is exacerbated when income inequality between the two political actors is big, making autocracy more prone to choose bad environmental policies even for lower levels of h.

Figure 2 represents two possible equilibria that a country may end-up in, depending on the type of regime. Assume that a dictator gets a larger share of output than a potential median voter in a democracy; if the dictator can pay to avoid his own exposure to pollution and the median voter cannot, the dictator will choose a lower optimal level of environmental quality. Denote by $s^v y - e_{max}$ and $s^a y - e_{max}$ the private transformation locus between income of the potential median voter and the dictator, respectively. At point c the median voter solves his own maximisation problem, equating his $MRS_{e,y}$ (that is to say, the amount of money he is willing to spend to improve the environment) with the amount of money he would get by producing more (this is expressed in terms of e, and is equal to $-se$). Denote this equilibrium level of environmental quality by e^{*v}. The dictator optimises his utility function with respect to two controls, the level of perceived environmental quality and the cost of insurance against exposure to pollution. The equilibrium condition is represented by equation 3.28.

Figure 2



The question is: if the dictator can protect himself against pollution by paying an insurance, will his optimal level of e be lower than the optimal level of the median voter? The answer is yes, provided that at point d the amount of income the dictator has to give up to increase environmental quality by one unit ($-\partial y/\partial e$), is greater than his willingness to pay (in terms of income) to increase quality by one unit ($MRS_{e,y}$).

If this is so, the dictator has the interest to reduce his optimal level of environmental quality such that he can buy the insurance and increase is perceived level of environmental quality. In figure 2, e^{*a} represents the "real" level of environmental quality chosen by the dictator, while e' is the "experienced" level of environmental quality. If this is the case, then, democracy is definitely better for the environment.

3.3 Motivations - Empirical evidence and some case studies

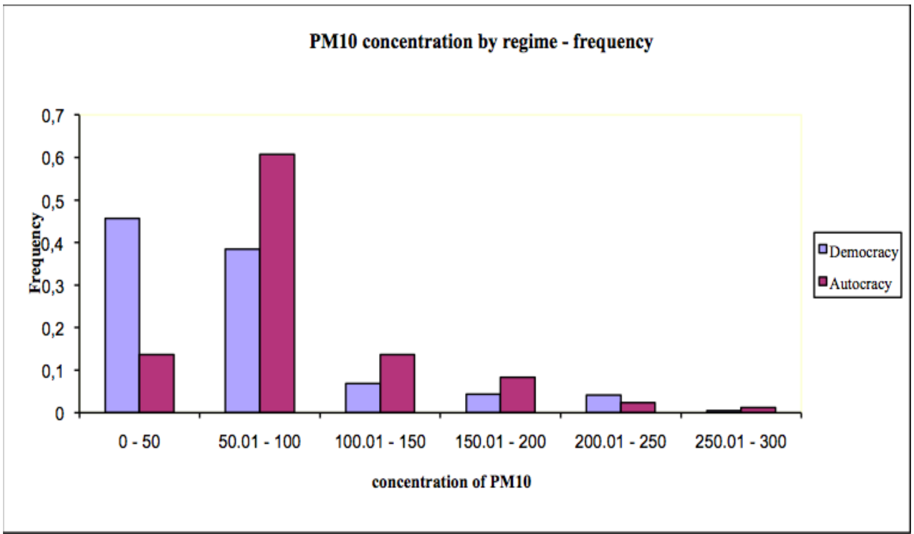
As the previous section shows, in the presence of environmental inequality between the two decisive political actors in dictatorship and democracy, democracy is likely to be beneficial for the environment, the better the effect on the environment, the bigger the income inequality between those two citizens.

Looking at some simple raw statistics[1], using a sample of 47 transition countries, I observe that during the period 1990-2002 the average concentration of PM10 recorded during spells of dictatorship is about 1.36 times bigger than during democratic periods, despite the average level of percapita GDP is 2.36 times bigger during democracy than during dictatorship spells[2].

[1]Data for CO2 emissions are from Marland, G., T.A. Boden, and R. J. Andres (22), for PM10 concentrations are from World Bank - World Development indicators' Database, data for percapita GDP are from Maddison (21), and data for democracy are from Przeworski's dataset, available at http://politics.as.nyu.edu/object/ przeworskilinks.html

[2]This average is computed over all 13 years and all over the countries, conditioned to periods of democracy or dictatorship. The average concentration of PM10 during democracy is 69.67906 and during dictatorship is 89.14407. Percapita GDP during democratic periods is, on average, 4,281.742 against 1,813.18 during dictatorship.

Figure 3.1



Across the period 1950-2002, the average intensity of CO2 emissions produced per unit of GDP is 1.17 times larger in periods of dictatorship than in periods of democracy[1]. Several countries show clearly a decrease in the intensity of emission per unit of GDP in proximity to the regime shift:
a simple fixed-effect regression of variations of percapita CO2 emissions over a constant, variations of percapita income and a dummy for democracy represented in equation 3.30, shows that percapita emissions are consistently reduced during democratic periods[2]:

$$\Delta \mathrm{CO2}_{it} = \underset{(1.89233)}{8.229862} + \underset{(0.0079892)}{0.1170303} \Delta \mathrm{GDP}_{it} + \underset{(3.356346)}{-17.3097} \mathrm{Dem}_{it} \tag{3.30}$$

with $\Delta CO2_{it}$ denoting variations in the level of percapita CO2 emissions expressed in Kg of carbon occurred between $t-1$ and t for country i, ΔGDP_{it} variations in the level of percapita income occurred between $t-1$ and t for country i, and Dem_{it} is a dummy variable coded 1 during periods of democracy, and 0 otherwise.

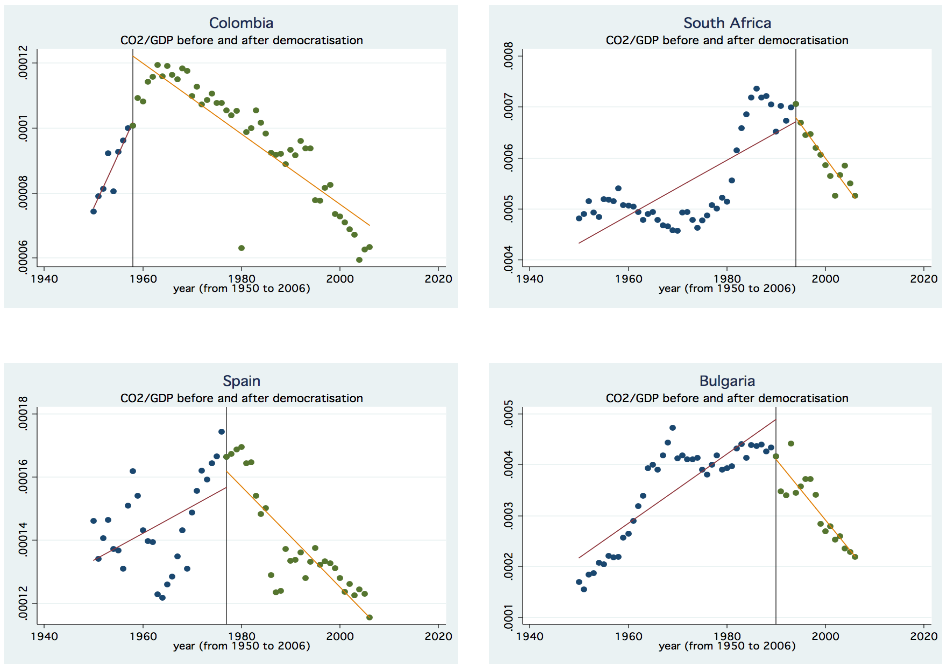
The estimated coefficients of equation 3.30 indicate that emissions are positively correlated to income, and that one dollar increase of GDP requires 0.117 Kg more carbon. However, for each year a country has been democratic, we assist to an average decrease in percapita carbon utilisation of 17.3 Kg. Graphically, the estimated coefficient related to the dummy for democracy represents the average "kink" in the two fitted lines in figure 3.2, before and after the regime shift, for the whole sample of countries.

[1]This average is computed over all the 53 years and over all the countries, conditioned to periods of democracy or dictatorship. The data for periods of democracy show an intensity of CO2 emissions (in Kg of carbon) per unit of income of 0.1269 against 0.1485 during periods of dictatorship.

[2]All the results are significant at 1% level, standard errors in parenthesis

Figure 2.1 shows, for Colombia, South Africa, Spain and Bulgaria, the actual (dotted) and fitted (line) levels of intensity of emissions (expressed in tons of carbon per unit of GDP). The vertical line in each subfigure represents the date of the regime shift. All the four countries have experienced a transition to democracy after long periods of dictatorship. After the regime shift we observe a reverse pattern for emissions; while before democratisation the tendency is to increase the intensity of CO_2 emissions in production, later we observe a decline, which is persistent through time.

Figure 3.2



In early democracies, environmental policies may be often out of the public agenda. The needs of protecting human rights are instead one of the main reason for democratisation. The protection of human rights often manifests as the need of a job for everyone, social reforms aimed at reducing poverty and pressures for meeting the basic needs for the whole population. In this last category falls the right of everyone to have an healthy life for the present and future generations, and therefore environmental management becomes of crucial importance. How a country chooses to achieve the goal of a better environment however varies between different nations, and sometimes it is a direct consequence of other policies undertaken and may not be explicitly regulated. In Colombia, for example, the sharp reduction in the intensity of emission is associated with the land reform which was aimed both at creating jobs and reducing poverty. Under the President Carlos Lleras Restrepo (1966-1970) the Colombian Instutute of Agrarian Reform (INCORA) promoted the redistribution of usable land to the peasants and unemployed workers in the country, issuing more than 60.000 titles in 1968 and 1969 alone. As a result of that, after few years the economy was more diversified than before, labor productivity was higher, and inequality lower. He also implemented an aggressive and broad program of social and economic reforms, creating, among others, a national saving fund, an institute for the family wellbeing, the institute to protect non-renewable resources, an agency to promote export etc. His successor Misael Pastrana Borrero (1970-1974) carried on the development increasing economic growth through encouraging housing construction and giving financial incentives to commercial agriculture. As a result jobs increased as well as wages, and he also promoted the first national environmental legislation in Latin America.

In South Africa the picture is similar. The election of the new government in 1994 was meant to put in place new policies to promote development, directed mainly at alleviating poverty, creating jobs and meeting the basic needs for the majority of South Africans. In this last context, it was necessary to define a clear policy objectives in the area of environmental quality and the use of natural resources. The Constitution provided (and provides) a powerful safeguard in shaping future economic and social development in an environmentally sustainable way: it lays down among the fundamental rights of every citizen that "every person shall have the right to an environment that is not detrimental to his or her health and wellbeing" and that "every person shall have the right to access to all information held by the state or any of its organs at any level of government insofar as such information is required for the protection or exercise of any of his or her rights".

In Spain the decrease in the intensity of emission coincides with an increase in the investments in environmental areas. During the sixties and the early seventies Spain have had an uncontrolled process of industrialisation (Font and Morata (10)). The institutionalisation of the environmental policy started in the seventies, with the creation in 1971 of the Interministerial Commission for the Environment, but investments in the environmental areas were not significant until 1978 (De Esteban and Lopéz Lopéz (7)), when the new Constitution introduced an article that ascribes as duty to ensure rational use of natural resources, to protect and improve the quality of life and to defend or restore the environment.

Lastly, Bulgaria, contrary to other countries, had very bad environmental conditions and democratisation was the result of (mainly) environmentalist protests. Bulgaria's dramatic environmental conditions were inherited by the inefficient and obsolete technologies used during the communist era. Any regulation for environmental protection was missing and it was estimated that two thirds of Bulgarian suffered of health problems due to pollution, in 1988 the top three causes of death in Bulgaria were cardiovascular illnesses, cancer, and respiratory illnesses[1]. The first demonstration against the Communist party in Bulgaria took place in Rousse at the Romanian border in the north of the country. On february 1988 pram-pushing mothers marched through the main street of Rousse protesting that governments was doing nothing on behalf of the "international proletariat" about the chemical plant in Girgiu on the other side of the border in Romania, which for decades had been belching chlorine pollution in the air in Rousse. A committee to save Rousse was founded and its activities were directed not against the regime of Nicolae Ceausescu in Romania but against Bulgaria and Todor Zhivkov. Here, environmental problems were the driving force toward democratisation. For what concerns this work, however, the issue is not whether democracy causes an environmental improvement of whether environmental issues cause democracy. What is important here is that environmental goals can be obtained only through democratic institutions. What comes first is not crucial, it is only that environmental policies were undertaken after the regime transition to democracy, necessary to achieve those objectives.

This paper contributes with a theoretical model to address the question of whether democrati-

[1]U.S. Library of Congress, http://countrystudies.us/bulgaria/29.htm

sation is good for the environment or not, taking into consideration the issues of price effects for environmental protection and substitutability of the environment with private consumption. This model, moreover, provides a possible explanation of the recent empirical evidence, which sees democracy associated to a more pro-environmental policies.

3.4 Conclusion

This theoretical model sheds some lights on the possible mechanisms underlying the observed empirical evidence of an environmental improvement followed by democratisation. In the absence of environmental inequality, a shift of the decision power from the dictator to the median voter may lead to an environmental improvement only if the "relative" (with respect to income) marginal willingness to pay for an additional unit of environmental quality is greater for the median voter than the dictator. This results is simply due to the fact that the dictator, appropriating of a larger share of global production, faces the tradeoff between taking possession of an additional fraction of income or giving up an amount of income and having a better environment. As long as the fraction of income the dictator can take possession increases, the opportunity cost to pay for environmental protection increases as well. On the contrary, since the median voter is poorer than the dictator, his demand for environmental quality will be lower, but also its opportunity cost to pay for additional environmental protection is lower due to the fact that his own share of income is low. The global effect of a regime change from dictatorship to democracy is than ambiguous, and depends primarily on the interaction of the differences of wealth between the two decisive political actors and their marginal willingness to pay for environmental protection. If instead one assumes that the dictator can enjoy a better environment because of his ability to pay in order to avoid exposure to pollution, the basic result of this model is that environmental quality chosen by the dictator does not depend on utility anymore, but only on his own share of global production he can take possession of, and the productivity of the expenditure done for reducing the exposure to pollution. Basically, what I called "insurance for exposure to pollution" reduces the marginal utility from environmental quality and therefore reduces the marginal willingness to pay for environmental protection. As long as this insurance's productivity is higher than the amount of environmental quality the dictator is willing to give up to become one unit richer, then it is worth for the dictator paying. The total amount the dictator will spend in such an insurance depends on its productivity, stopping when the amount of environmental amenities he can renounce to become marginally richer is exactly equal to the level of productivity of the insurance. The direct effect of this insurance on the level of environmental quality is detrimental: the dictator will find it worthwhile to increase production because doing so he can maximise his own consumption (or wealth) and pay for reducing his own exposure to pollution generated by the productive activity. Global production will be greater (and correspondently, the global level of environmental amenities will be lower), the greater the productivity of the insurance policy, because the dictator's perceived environmental quality will be higher.

Concluding, income inequality per se is not a valid indicator to predict whether a change of the decision power from a dictator to the median voter will result in a more environmental protection or not. Wealth inequality instead plays an important role in conjunction with environmental inequalities to explain the recent evidence of beneficial effects of democracy on the environmental quality. The explanation for this effect is simple and relies on the assumption that in authoritarian regimes the person who holds the decision power is likely to be rich and so he can afford - contrary to the decisive citizen in a democracy which is supposed to be poorer - this insurance against exposure to pollution. If this is the case, as the empirical evidence on the distribution of the environmental burden suggests, we can easily explain why democratisations are associated to reductions in human induced emissions.

Some scholar may wonder, however, if this effect may be induced by the environmental Kuznet Curve, assuming that democratisation occurs at the turning point. Looking at the statistics of equation 3.30, and to the row statistics related to concentration of PM10, it can be reasonably excluded that this sharp reduction in the intensity of CO2 emission or concentration of PM10 was the result of generic development, like technological progress or changes in the composition of GDP or any reason that has been adducted to justify the Environmental Kuznet Curve (EKC). Even assuming that an EKC exists for such a kind of pollution indicators, the average percapita income during periods of democracy is far below the Selden and Song's (32) minimum estimate of the "turning point" for CO2 emissions, which was supposed to be around $6,241.00, and for PM10 this turning point has been estimated by Grossman and Krueger (15) in less than $5,000.00 (These estimates are computed using real percapita GDP at constant prices, with reference to 1985 US$. My data for GDP are expressed in 1990 US$, but having them expressed in 1985 US$ the average percapita income during democratic spells between 1990 and 2002 is 4,162,79, and 1,914.93 during dictatorship. Increasing the time of observation from 1950 to 2002, I get that percapita GDP in 1985 US$ during democracy is, on average, 4,286.08, and during dictatorship, 2,405.68, in any case far below the estimated turning points). Moreover, as Stern (33) pointed out in one of his recent surveys, any estimation of the EKC's turning point tend to be larger, the greater is the number of developing and underdeveloped countries included in the sample, which makes the EKC of dubious existence. The fact that intensity of emissions have declined simultaneously to an increase in income, therefore, is not an indicator that the turning point of the EKC has reached, since there are good reasons to believe to stay in the increasing part of the EKC (so, to the left of the turning point).

The evidence is therefore supporting the basics of the model. Democratisation, in a society where the majority of the population is not self-employed and supply physical labour to the production process, is likely to be beneficial for the environment, the bigger the effect, the larger the difference in the share of income the two decisive citizens can take possession of.

Bibliography

[1] Acemoglu D., Robinson J.A. and Verdier T., 2004. *Kleptocracy and Divide-and-Rule: A Model of Personal Rule*. Journal of the European Economic Association, Vol. 2, No. 2/3, Papers and Proceedings of the Eighteenth Annual Congress of the European Economic Association, pp. 162-192. 58

[2] Agrawal, B., 1992. *The Gender and Environment Debate: Lessons from India*, Feminist Studies 18, pp. 119-158. 87

[3] Bhattarai M., and Hammig, M., 2001. *Institutions and the Environmental Kuznets Curve for Deforestation: a Cross Country Analysis for Latin America, Africa and Asia*. World Development 29, pp. 995-1010. 48

[4] Boyce J., 2007. *Inequality and Environmental Protection*, in *Inequality, Cooperation and Environmental Sustainability*, Princeton University Press. 87

[5] Costa D., 1997. *Less of a luxury: the rise of recreation since 1888*. National Bureau of Economics Research, Working Paper no. 6054.

[6] Dasgupta, P. and Mäler, K. G., 1995. *Poverty, Institutions, and the Environmental Resource-Base*, in Behrman, J., Srinivaan, T.N. (eds.), Handbook of Development Economics, vol. 3A. Elsevier Science, Amsterdam. (Chapter 39). 48, 86

[7] De Esteban Alonso and A. Lopéz Lopéz, 1993. *Environmental Policy* in: "Spain and EC Evaluated", London. 100

[8] Desai, Uday, 1998. *Environment, Economic Growth, and Government*, Ecological Policy and Politics in Developing Countries. Albany, NY. State University Press of New York Press, 1-45. 48, 86

[9] Dinan T., and Lim D., 2000. *Who Gains and Who Pays Under Carbon Allowance Trading?* Washington, D.C.: Congressional Budget Office.

[10] Font N. and F. Morata, 1998. *Spain: Environmental Policy and Public Administration. A Marriage of Convenience Officiated by the EU?* in: Hanf K. and A.I. Jansen, "Governance and the Environment in Western Europe; Politics, Policy and Administration", Longman. 100

[11] Foster, Sheila, 1998. *Justice from the Ground-Up: Distributive Inequalities, Grassroots Resistance, and the Transformative Politics of the Environmental Justice Movement.* California Law Review 86, pp. 775-841. 87

[12] Gallagher K. P., and Thacker S. C., 2008. *Democracy, Income and Environmental Quality*, Political Economy Research Institute (PERI) Working Paper No. 164, University of Massachusetts Amhrest. 49

[13] Gleditsch, Nils P., and Bjorn Sverdrup, 2003. *Democracy and the Environment.* In *Human Security and the Environment: International Comparisons*, edited by Edward Paper and Michael Redclift. London: Elgar. 48, 86

[14] Gray and Shadbegian, 2004. *Optimal Pollution Abatement - Whose Benefits Matter, and How Much?*, Journal of Environmental Economics and Management 47, pp. 518-534. 87

[15] Grossman G.M. and A.B. Krueger, 1995. *Economic Growth and the Environment*, The Quarterly Journal of Economics 110, pp. 353-377. 102

[16] Hardin, Garret, 1968. *The Tragedy of the Commons.* Science 162, pp. 1243-1248. 48, 86

[17] Hökby S., Söderqvist T., 2001. *Elasticities of demand and willingness to pay for environmental services in Sweden.* Beijer discussion paper series No. 137, Beijer International Institute of Ecological Economics, The Royal Swedish Academy of Sciences, Stockholm. 87

[18] Kriström B., Riera P., 1996. *Is The income elasticities of environmental improvements less than one?.* Environmental and Resource Economics 7, pp. 45-55. 87

[19] Larsen R.E., 2001. *Revealing demand for nature experiences using purchase data on equipment and lodging.* Discussion paper No. 305, Statistic Norway, Research Department.

[20] Leslie, Winsome J., 1987. *The World Bank and Structural Transformationin Developing Countries:The Case of Zaire.* Boulder, CO: Lynne Rienner Publishers. 58, 86

[21] Maddison Angus, 2009. *Statistics on World Population, GDP and Per Capita GDP, 1-2006 AD*, updated March 2009, available at http://www.ggdc.net/maddison/ 52, 96

[22] Marland, G., T.A. Boden, and R. J. Andres, 2008. *Global, Regional, and National CO2 Emissions.* In "Trends: A Compendium of Data on Global Change". Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn. 52, 96

[23] Martini C., and Tiezzi S., 2010. *Is the Environment a Luxury? An Empirical Investigation using Revealed Preferences and Household Production.* Quaderni del Dipartimento di Economia Politica, n. 599. 87

[24] Miles D., Pereyra A., and Rossi M., 2002. *The Consistent estimation of income elasticity of environmental amenities in Uruguay.* Stud. Econ. no. 17. 87

[25] Moya Pons, Frank, 1995. *The Dominican Republic: A National History.* Princeton, NJ: MarkusWeiner. 58, 86

[26] Olson M., 1965. *The Logic of Collective Action, Public Goods and the Theory of Groups*, Harvard University Press.

[27] Pastor M. Jr, Jim Sadd and John Hipp, 2001. *Which Came First? Toxic Facilities, Minority Move-in, and Environmental Justice.* Journal of Urban Affairs 23, pp. 1-21. 87

[28] Payne R.A., 1995. *Freedom and the Environment*, Journal of Democracy 6, pp. 41-55. 48, 86

[29] Pereyra A., and Rossi M., 1998. *Are environmental goods a luxury?* Working paper no. 13, University of Uruguay.

[30] Schultz, C. B., and T. R. Crockett, 1990 *Economic Development, Democratization, and Environmental Protection in Eastern Europe.* Boston College Environmental Affairs Law Review 18, pp.5384. 48, 86

[31] Scruggs L., 1998. *Political and Economic Inequality and the Environment*, Ecological Economics 26, pp. 259-275.

[32] Selden T.M., and D. Song, 1994. *Environmental Quality and Development: Is there a Kuznets Curve for Air Pollution Emissions?* Journal of Environmental Economics and Management 27, pp. 147-162. 102

[33] Stern, David, 2004. *The Rise and Fall of the Environmental Kuznets Curve.* World Development 32, pp. 1419-1439. 102

[34] Torras M., and Boyce J. K., 1998. *Income, Inequality and Pollution: a Reassessment of the Environmental Kuznets Curve.* Ecological Economics 25, pp. 147-160. 48

[35] Veblen T., 1899/1934. *The Theory of the Leisure Class.* New York, Modern Library.