Università di Siena
–
Dipartimento di Economia Politica

Three essays on social pressure, nonmonotonic network effects, and asymmetric market demands

Thesis submitted for the degree
Doctor of Philosophy in economics

Candidate: Luca Savorelli *Supervisor*: Prof. Vincenzo Denicolò

February 13, 2011

CONTENTS

Preface i

Summary of the thesis iv

1 Introduction 1

2 Thinness and obesity: food consumption, health, and social pressure 3

2.1 Introduction . . . 4
2.2 'Together Against Slimming Mania': The regulation of role models 8
2.3 The model . . . 11
2.3.1 The individual utility function . . . 11
2.3.2 The determinants of individual body weight . . . 13
2.3.3 The individual problem . . . 13
2.4 Is increasing the ideal body weight desirable? . . . 18
2.4.1 Optimal ideal body weight . . . 18
2.4.2 Should the healthy weight be the ideal body weight? . . 21
2.5 Conclusions . . . 23

References 25

3 Is conformism desirable? Network effects, location choice, and social welfare 30

3.1 Introduction . . . 31
3.2 The model . . . 34
3.2.1 Undercutting incentives and price stage . . . 35
3.2.2 The existence of an equilibrium and the location stage . 37
3.3 Social welfare and conformism desirability . . . 43
3.4 Conclusions . . . 45

References 47

4 **Partial collusion with asymmetric cross-price effects** **49**
4.1 Introduction 50
4.2 A linear Cournot duopoly with asymmetric substitution effects 51
4.3 Partial collusion 55
4.4 Conclusions 59

References **61**

Appendices **64**

A Appendix Chapter 2 **64**
A.1 The relation between satiation and constrained maximization . 64
A.2 Physical effort as an additional choice variable 65
A.3 Ideal body weight when the policy objective neglects the cost of social pressure 67

B Appendix Chapter 3 - proofs **68**
B.1 Lemma 2 68
B.2 Lemma 3 68
B.3 Lemma 5 69
B.4 Proposition 2 69

C Appendix Chapter 4 - proofs **70**
C.1 Proposition 1 70
C.2 Lemma 1 70
C.3 Lemma 2 71
C.4 Proposition 2 72
C.5 Proposition 3 72

Acknowledgements **74**

PREFACE

This thesis merges the two major fields of research I have an interest in: industrial organization and behavioral economics. My laurea thesis at the University of Bologna was dedicated to sequential innovation and patent design. During the first two years as PhD student at the University of Siena, I studied public and private incentives to promote innovation. At the end of the second year, the subjects studied during the doctorate's courses made me profoundly interested in the behavioral aspects of microeconomic theory.

I thus decided to shift the core of my studies from industrial organization to behavioral economics, and in the following two years I focused on inconsistency in dynamic choice. Specifically, my research work focused on models of hyperbolic discounting and, from a broader perspective, on the limits of standard consumer's theory in accounting for real-world behaviors. The majority of the ideas of this thesis originate from those two years of research. The research output consisted in two work-in-progress papers and a working paper, which are not part of this dissertation. The idea underlying those papers, supported by a sociological and psychological literature and casual observation, is that not only people can be present-biased, but also future-biased, weighting in excess future utility with respect to the enjoyment of present life. Modeling this unbalance in discounting utility may describe a class of compulsive behavior, which apparently are not explainable within a rational choice framework. Running under the snow, working in excess of what a rational trade-off between work and life would predict, and being on a diet while being underweight are some examples. This idea is published in my working paper "Know thyself: self awareness and utility misprediction". The other two work-in-progress papers study specific compulsive behaviors, the first focusing on workaholism and the second on eating behavior. From an analytical standpoint, they extend models of geometric discounting (also known as quasi-hyperbolic discounting or present-biased preferences).

The dieting behavior was motivating my paper, titled "The economics of anorexia nervosa: a model of body weight exhaustion". The paper develops a model of hyperbolic discounting accounting for the eating behavior typical of anorexics. While I was writing that paper, I indeed realized that no economic model of rational choice was able to describe in a unique framework the variety of commonly observed eating-weight patterns, from thinness to obesity. The existing models were modeling obesity by using ad hoc assumptions, eventually leading the agent to be overweight whichever his parameters and initial point. Moreover, the research and the accounts of those who were in an anorexic condition share the idea that anorexia is not per se a disease, but the outcome of rational choice driven by several factors. Among them, the social pressure to be thin is considered one of the most relevant, and it does affect also the eating behavior of people in a non-pathological condition. Before proceeding to study the multifaceted aspects of eating in a behavioral framework, I thus thought that there was the need for a rational account in a dynamically consistent framework. This is provided in Chapter 2 of this dissertation:"Thinness and obesity: A model of food consumption, health concerns, and social pressure".

The policy side of the topic was interesting as well. "The economics of anorexia nervosa" was motivated by actual agreements signed by the Fashion industry associations and several governments. Those agreements are now studied as well in Chapter 2. The aim of those agreements is modifying the existing social norms on what is the ideal weight. They raise a number of research questions, e.g., the suitable policy objective (utility or health?), a hot topic in the health economics literature. Given the challenges of the research topic, requiring a brand-new positive and normative analysis, and the interdisciplinary nature of the work, I decided to share the ideas and the work done up to that moment with my coauthor Davide Dragone, whom I met at the University of Bologna when we were research fellows at the department of economics, and with whom I was sharing an interest in dynamic behavior and social norms.

While studying hyperbolic discounting, which is often exploited to model addictions, I came across apparently odd results in empirical works: The estimated elasticities of substitution were highly asymmetric, not only in entity, but also in sign (i.e., good i was substitute of good j while good j was substitute in good i). A deeper search showed me that the asymmetric effect was moreover displayed in a good part of the applied literature in different fields of study, such as industrial organization and marketing. Casual observation (e.g., accessories, or salad and dressing) also suggests that asymmetry is widespread in the real world. Reviewing the theoretical literature of consumer's choice, I realized that there is no consensus on a sound definition of a basic concept such as substitution. Even if it is known that the gross definitions of substitution and complementarity can be asymmetric, the literature has focused on symmetry, from the textbooks to the frontier of research. I thus started investigating which class of utility functions generates this kind of asymmetries in a work still in progress. Nevertheless, for the substitution effect to be asymmetric, it is sufficient for the income effect to be asymmetric for the two goods. I thus exploited this framework to show an immediate application of this feature, adding a piece to the research on the factors facilitating collusion, the fourth and last chapter of this dissertation: "Partial collusion in a duopoly with asymmetric cross-price effects".

Finally, Chapter 3: "Is conformism desirable? Network effects, location choice, and social welfare in a duopoly" is inspired by Chapter 2, and closely interrelated to it. The paper about eating behavior was highly connected to the role of fashion industry, and fashion is a salient phenomenon as long it is a network phenomenon. Chapter 3 has been written entirely in my year (from October 2009 to October 2010) as visiting research student at the London School of Economics and Political Science (LSE), where my research project focused on the interrelation of culture and economics. This project was of inspiration for this paper as well, even though it was related to side research topics with respect to those of this dissertation (but connected to my research fellowship at department of economics at the University of Bologna, about behavioral economics and organizational behavior).

Summing up, I believe that the three main chapters of this dissertation are three original and independent pieces of research, but deeply linked either in terms of the issues they deal with and motivation, or in terms of analytical development. They are not only stand-alone bricks of research, but also they suggest new ways for future research from a theoretical, applied and experimental perspective. Some examples are the evaluation of policies for eating behavior, the study of the endogenous formation of social preferences, the effect of network effects' shapes and asymmetry in cross-price effects in a range of economic models, who could be tested via an experimental design. Not least, I consider completing in my future research the aforementioned works in progress from which this thesis has been originated, proceeding the research of the topics such as suicidal behavior and compulsiveness, which route to what rationality is and to the foundations of economic analysis.

SUMMARY OF THE THESIS

This thesis addresses three main issues: the influence of social pressure on eating behavior and related policies, the desirability of conformism resulting from nonmonotonic network effects, and the role of asymmetric cross-price effects in facilitating collusion. The first chapter illustrates the connections between the subsequent chapters.

The second chapter focuses on policies aimed at favouring good eating practices. The increasing concern of the policy maker about eating behaviour has focused on the spread of obesity and on the evidence of people dieting despite being underweight. As the latter behaviour is often attributed to the social pressure to be thin, some governments have already taken actions to ban ultra-thin ideals and models. Assuming that people are heterogeneous in their healthy weights but exposed to the same ideal body weight, this paper proposes a theoretical framework to assess whether increasing the ideal body weight is socially desirable, both from a welfare and a health point of view. If being overweight is the average condition and the ideal body weight means being thin, increasing the latter may increase welfare by reducing social pressure. By contrast, health may be impaired since people are induced to depart even further from their healthy weight. Given that in the US and in Europe people are on average overweight, we conclude that these policies, even if welfare improving, may exacerbate the obesity epidemic.

The third chapter studies the role of conformism in determining market outcomes and social welfare. It considers a duopoly where the network effect is nonmonotone and the network can be overloaded. The firms choose prices and locations endogenously, and the agent's utility is influenced by the number of people patronizing the same firm she does. We determine the market equilibrium, and we study how the network effect influences social welfare. We compare this setting with the standard horizontal differentiation model with no network effects to understand whether and how conformism is socially desirable. The results show that whether network effects are desirable depends on how people are conformist, and whether overloading is feasible. If overloading is not possible (in either of the firm's network), and the total consumers' mass is sufficiently high, a network effect which is slightly concave increases social welfare. By contrast, if overloading is possible, and the total consumers' mass is sufficiently small, social welfare is increased if the network effect is more concave than in the previous case.

The fourth chapter explores the role of asymmetric cross-price effects in facilitating collusion. Asymmetries in cross-price elasticities have been demonstrated by several empirical studies. We study from a theoretical stance how introducing asymmetry in the substitution effects influences the sustainability of collusion. We characterize the equilibrium of a linear Cournot duopoly with substitute goods, and consider substitution effects which are asymmetric in magnitude. Within this framework, we study partial collusion using Friedman (1971) solution concept. Our main result shows that the interval of quantities supporting collusion in the asymmetric setting is always smaller than the interval in the symmetric benchmark. The asymmetry in the substitution effects thus makes collusion more difficult to sustain. This implies that previous antitrust decisions could be reversed by considering the role of this kind of asymmetry.

1. INTRODUCTION

The chapters of this thesis are linked by the attempt of studying specific research, antitrust and policy issues by introducing in the economics literature aspects of real-world behavior (e.g., social norms and asymmetric preferences).

As aforementioned in the Preface, Chapter 4 was inspired by the same studies from which Chapters 2 and 3 originated. Nevertheless, it is an independent piece of work. From an analytical standpoint, it shares a dynamic perspective with Chapter 2, and a more standard industrial organization analysis with Chapter 3.

Chapter 2 and Chapter 3 are closely linked. From an analytical standpoint, they both exploit quadratic nonmonotonic elements in the utility function. In Chapter 2 quadratic forms are introduced to account for non satiation in eating behavior, costs in terms of health due to departures from one's healthy weight, and costs for an individual to depart from his group ideal weight. In Chapter 3, I consider a nonmonotonic network effect, which as well takes a quadratic form able to account for a range of phenomena, e.g. crowding, or fashion. From the point of view of the motivation, they both attempt to introduce the role of fashion in economic analysis, still an open area for research. The approach of these chapters is line in with Akerlof and Kranton (2000)[1], who consider the fact that individuals belong to social categories, and that each category can be associated with different ideal physical attributes and prescribed behaviors. The existence of these references generates a pressure to conform to the ideal attribute or behavior, and deviations from them are socially sanctioned. Akerlof and Kranton moreover observe that public policies sometimes are directed at manipulating these references, and consequently individual behavior. A notable example is the stigmatization of smokers and

[1] See Chapter 2, references.

the restrictions on smoking advertising aimed at cutting the consumption of tobacco. Moreover, both chapters share an interest not only in positive analysis, but also in its normative side. I also think that the frameworks exploited in Chapter 2 and 3 may be useful to study other analogous phenomena, aside those the two chapters specifically focus on.

The terms "chapter" and "paper" will be used as synonyms throughout the thesis. Acknowledgements specific to each chapter are recognized at the end of the body of the paper, while broader (academic, moral, or both) contributions to the development of this thesis are addressed at its end.

2. THINNESS AND OBESITY: A MODEL OF FOOD CONSUMPTION, HEALTH CONCERNS, AND SOCIAL PRESSURE

Abstract

The increasing concern of the policy maker about eating behaviour has focused on the spread of obesity and on the evidence of people dieting despite being underweight. As the latter behaviour is often attributed to the social pressure to be thin, some governments have already taken actions to ban ultra-thin ideals and models. Assuming that people are heterogeneous in their healthy weights but exposed to the same ideal body weight, this paper proposes a theoretical framework to assess whether increasing the ideal body weight is socially desirable, both from a welfare and a health point of view. If being overweight is the average condition and the ideal body weight means being thin, increasing the latter may increase welfare by reducing social pressure. By contrast, health may be impaired since people are induced to depart even further from their healthy weight. Given that in the US and in Europe people are on average overweight, we conclude that these policies, even if welfare improving, may exacerbate the obesity epidemic.[1]

JEL classification: D91, I18.

Keywords: Body Weight; Diet; Obesity; Social Pressure; Underweight.

[1] This chapter is co-authored with Davide Dragone.

2.1 Introduction

The obesity epidemic is becoming a major problem in Western industrialized societies (Acs and Lyles, 2007). The phenomenon is reaching alarming proportions, in particular in the US, Mexico, and the UK, where 6 out of 10 people are on average overweight or obese (Mazzocchi et al., 2009). Together with the increasing trend of obesity, there is evidence of social pressure to be thin and, especially for women, of a weight bias (Brownell et al., 2005; Owen and Laurel-Seller, 2000). As a consequence, a great number of people are exploiting weight control practices, ranging from weight loss diets to pills (Weiss et al., 2006).[2] Such practices may lead to unhealthy weight control behaviours, which are found to be associated with an increased risk of the onset of eating disorders (Neumark-Sztainer et al., 2006). It has been suggested that reducing the social pressure to be thin would counteract the tendency towards unhealthy dieting. While the debate is ongoing in the US and in Europe, as well as in India and in Australia, some countries have already taken actions in this direction. This paper proposes a theoretical framework to study eating behaviour and the social pressure to be thin, and to assess whether these policy interventions, aimed at affecting food-intake behaviour by modifying the ideals of physical beauty, are desirable.

Such interventions often involve the fashion industry and the media. For instance, the governments of Italy, Germany and Spain have recently signed agreements with the national fashion business associations to set out in detail some aspects of the industry production decisions, communication strategies, and labelling of products. The agreements favour an increase in the production of large size clothes and in the minimum size of models on the catwalks and mannequins in the shops. They also forbid addressing size 46 as a "Plus Size". Along the same lines, India and Israel have banned underweight models from the fashion shows, and the New York City Council has approved a resolution calling upon sponsors of the New York Fashion Week to ban models with a low body mass index (BMI). Private groups and stylists associations have also undertaken specific measures. For example, Unilever has adopted a global communication strategy whereby models and actors are required to be neither excessively slim nor excessively large. The explicit goal of these measures is to reduce unhealthy eating behaviour by affecting the ideal of beauty which is disseminated by the media and the fashion industry.

[2] Weiss et al. (2006) find that 51% of US adults tried to control their weight in the 12 months prior to their enquiry. These figures include people who tried to lose weight and those who tried only not to gain weight.

When reading the documents supporting these decisions, it is clear that both the governments and the fashion industry agree that fashion is a powerful trendsetter. It not only influences what clothes, styles, and colours are trendy, but also defines how a person should appear to be desirable. This includes body shape and body weight. People often recognize that being attractive according to the yardsticks set by the media and the fashion industry is out of reach. It is also true that not everybody judges her or himself according to these supposedly ideal body shapes and weights. There is evidence, however, that plain-looking people tend to find worse jobs and receive lower wages (Hamermesh and Biddle, 1994) and, with reference to body weight, that being overweight leads to multiple forms of prejudice and discrimination (Cawley, 2004; Morris, 2006; Puhl and Heuer, 2009). In other words, there is evidence that those not conforming to the ideal body weight are stigmatized and socially sanctioned.

In this paper we formalize these arguments, considering how a given ideal body weight affects eating behaviour and, consequently, body weight and health. We propose a dynamic model of individual eating behaviour in which a forward-looking agent chooses how much food to consume. Utility depends on food consumption and body weight, the latter being endogenously determined by the difference between calories intake and expenditure. If the agent's body weight is either higher or lower than her *healthy* weight, she may suffer an impairment to her health. She may also suffer a disutility cost if her body weight does not conform to an *ideal* weight that is exogenously determined by society and cannot be influenced by her. Since healthy weight and ideal weight may not coincide, she has to trade off health against the social consequences of her food intake. This results in a variety of eating behaviours, including the possibility of optimally choosing to be on a diet despite being underweight.

Under the assumption that the mentioned policies are effective in increasing the ideal body weight, we provide a normative assessment of their desirability, both in terms of aggregate utility and aggregate health. The distinction is relevant because a welfare-maximizing policy does not coincide with a health-maximizing one. Assuming that people are heterogeneous in their healthy weights, but are exposed to the same ideal body weight, we show that increasing the latter induces people to increase their food intake and their body weight. The desirability of this effect depends on what kind of body weight and eating behaviour are observed prior to the change in the ideal body weight. If people are underweight and stay on a diet, increasing the ideal body weight allows both aggregate welfare and health to be improved. If people are overweight and on a diet, however, increasing the ideal body weight may improve overall utility, but may impair health because it induces people to become even more overweight. A precise assessment of the relevant policy objective and of who will be influenced by the policy is therefore critical for determining the desirability of increasing the ideal body weight. This is particularly relevant for the US and the European countries, for example, where people are on average overweight and where increasing the ideal body weight, even if welfare improving, may exacerbate the obesity epidemic.[3]

This paper makes several contributions to the economic literature on eating behaviour. First, we contribute to the existing body of papers discussing policies aimed at affecting eating behaviour. The current debate is mainly focused on actual and proposed policies against obesity, such as the introduction of fat taxes and thin subsidies (Yaniv et al., 2009), the implementation of educational programmes (Acs and Lyles, 2007; Philipson and Posner, 2008), the regulation of fast foods and food advertisements (Stanton and Acs, 2007), and the enhancement of access to information (Downs et al., 2009; Wansink et al., 2009). To the best of our knowledge, laws and agreements aimed at regulating eating behaviour by affecting the ideal body weight have not been investigated before. These policies have never been given a theoretical foundation and this paper tries to this gap in the literature.

Second, we contribute to the debate on whether one should have health or welfare, of which health is just a component, as the relevant policy function (Lakdawalla et al., 2005). We show that there exist conditions under which increasing the ideal body weight is unambiguously desirable and that there is thus no conflict between improving health and improving welfare. We also show the conditions under which there is a trade-off between health and welfare and the choice of the policy function is thus crucial.

Third, we propose a model that can be used to study eating behaviour leading to being overweight as well as eating behaviour leading to being underweight. The result that being overweight can be the rational outcome of a maximizing agent, which is relevant for the obesity literature, has already been shown (Levy, 2002; Yaniv, 2002; Yaniv et al. 2009; Dragone, 2009). The conclusion that being underweight can also be rational, and the specification of the conditions under which it may occur, are novel results. Our model therefore allows to study both being overweight and being underweight, which tend to be separately considered in the literature, using a single theoretical framework of eating behaviour.

Fourth, in our setup prices and technology are given, and the main drivers

[3] Given the relevance of Europe in the international fashion market, these agreements can also have world-wide consequences for the fashion industry, a global business that in the US has a turnover in excess of \$200 billion. The analysis of the consequences of the agreements for the fashion industry is out of the scope of this paper.

are individual preferences and individual metabolism. This provides a perspective that is complementary to the existing contributions that focus on the changes in market prices, available income and technology, and their consequences on eating behaviour (Yaniv, 2002; Philipson and Posner, 2003, 2008; Cutler et. al., 2003; Chou et al., 2004; Lakdawalla et al., 2005; Yaniv et al., 2009). Interestingly, while our results do not explicitly depend on changes in the economic scenario, the model we propose can be easily extended to take them into account.

Finally, we contribute to the literature on eating behaviour and social pressure. Our approach is in line with Akerlof and Kranton (2000), who consider the fact that individuals belong to social categories and that each category can be associated with different ideal physical attributes and prescribed behaviours. The existence of these references generates a pressure to conform to the ideal attribute or behaviour whereby deviations are socially sanctioned.[4] The ideal attribute can be endogenous to the reference group (e.g., it can be given by the average body weight of peers), in which case obesity may spread in the population, as if it were socially contagious, due to a social multiplier effect (Christakis and Fowler, 2007; Etilé, 2007; Trogdon et al., 2008; Blanchflower et al., 2009). In this paper, we take a different perspective and focus on the case where the influence of a person's body weight on the ideal reference weight is negligible. In other words, we consider a scenario where the ideal weight is exogenously given. This allows to better understand the effects of social pressure on eating behaviour and to study the health and welfare consequences of changing the ideal body weight.

The paper is structured as follows. In the following section, we describe the content of the existing agreements and their underlying rationale. In the third section, we develop a model of individual food consumption and endogenous body weight. In the fourth section, we study the impact of an increase in the ideal body weight on both welfare and health and discuss the implications of existing policies. In the fifth and last section, we present our conclusions and the direction of future research.

[4] As there is no consensus on the definition of what a social norm is, we will not use this term and simply refer to the social cost of not conforming to the ideal body weight. Akerlof and Kranton (2000) introduce the concept of social norm to study prescribed behaviours as well as prescribed attributes. Levy (2002) refers to the existence of social pressure on body weight as a sociocultural norm of appearance, but there also exist alternative definitions of social norms. For example, Bicchieri (2006) defines a social norm in terms of social expectations, while Fehr and Gächter (2000) refer to it as a behavioural regularity.

2.2 'Together Against Slimming Mania': The regulation of role models

Public policies are sometimes directed at manipulating the references of ideal attributes and behaviours and, consequently, individual choices. A notable example is the stigmatization of smokers and the restrictions on smoking advertising aimed at cutting the consumption of tobacco, which have been effective in reducing smoking in the population (Akerlof and Kranton, 2000; Philipson and Posner, 2008). Analogously, in this paper we provide a theoretical evaluation of the welfare and health effects of policies designed to affect the social desirability of body weight with the goal of counteracting the 'slimming mania' that has attracted the attention of the policy maker. In this section, we focus on the regulation of role models with the aim of changing the ideal body weight and the resultant eating behaviour. We describe the agreements that have been approved (or are close to approval) in some countries and discuss the underlying rationale.

On 23 January 2006, the Italian Ministry of Youth and Sports Activities signed an agreement with the fashion industry to prohibit the participation of anorexic models in fashion shows, to increase the production of clothes sized 46-48, and to avoid promoting thin ideals. One year later, on 23 January 2007, the Spanish Ministry of Health and the Spanish fashion industry and stylists associations signed a similar agreement setting out that models and mannequins should approximate the population's biometrics. They prescribed a minimum size for models and mannequins, a minimum BMI for participants of beauty contests and prohibited the designation 'Plus Size' for clothes sized 46. France opted for a tougher measure. To counteract the proliferation of websites promoting anorexia as a lifestyle (the so-called 'Pro-Ana' movement), from 15 April 2008 those who explicitly promote extreme forms of thinness can be punished with a 30,000 euro fine and two years in prison.[5] A few months later, on 11 July 2008, the German Ministry of Health and the German textile and fashion industry also signed a national charter. Significantly

[5] Anorexia nervosa is a mental illness which affects eating behaviour and can originate from multiple causes such as environmental, biological and genetic factors, sexual abuse and traumatic events (the loss of a loved one, going off to college, a new job, loss of job, etc.). The adherence to an ultra-thin ideal is often associated with people suffering from anorexia nervosa; but those who suffer from this disorder frequently have other psychological contributors existing prior to development of the disease. Accordingly, magazines and other media might contribute to the acceleration of the disorder, but it is still a matter of investigation whether they are sufficient to cause it.

titled 'Life Carries Weight - Together Against Slimming Mania', the goal of the charter is 'to initiate a process of reorientation among children, young people and adults regarding prevailing beauty ideals' by promoting and propagating 'a healthy body image' and 'unequivocally reject[ing] the unhealthy ideal of extreme thinness particularly among girls and women'. It also prescribes a minimum BMI for fashion models and prohibits the publication of images or photos of extremely thin models. Finally, it proposes to promote a discussion at the European level aimed at drawing up a European charter to channel individual national efforts. In the wake of these developments, analogous actions have also been taken outside the European Union. In 2006, India's Health Ministry enforced a ban of excessively skinny models, both male and female, from the Lakme Fashion Week in Mumbai. In 2007, the New York City Council approved a resolution calling upon sponsors of the New York Fashion Week - one of the most important international fashion shows - to ban underweight models from walking the runways. Similarly, in June 2010, the Australian Youth Minister announced a new plan to ban hiring models with low BMIs and mandated the stocking of larger size clothes in the stores; the Israel Ministerial Committee of Legislative Affairs supported a bill which prohibits models with a BMI lower than 18.5 from being shown in media advertisements and being hired by modelling agencies.[6]

The above-mentioned agreements are based on three assumptions. First, there exists a thin ideal; second, the fashion industry, the media and advertising have a major role in disseminating the thin ideal;[7] third, such a thin ideal influences people's eating behaviour (specifically, it induces people to constrain food consumption) and is believed to negatively affect the health condition and eating behaviour of part of the population, in particular young women. As this consequence is not being internalized by the fashion stylists, the media and trendsetters in general, a government intervention appears justified.

We now discuss whether the above assumptions are sound. Since the 1960s, a thin, fit body shape for females and a lean, muscular physique for males have represented attractiveness in Western industrialized countries. Owen and Laurel-Seller (2000) report that more than 80% of the models found on the Internet and almost 100% of Playboy centerfolds show underweight or severely

[6]It is worth mentioning that not all governments and fashion industry associations have agreed to undertake this kind of measures. For example, the British Fashion Council has decided not to ban ultra-thin models from the London Fashion Weeks despite the ongoing debate, which started in 2006. This has not prevented some British stylists from independently deciding not to hire ultra-thin models.

[7]For instance, in the German charter it is observed that 'even if the number of super-thin models employed in Germany is far lower than in other European countries, the [fashion] industry still exerts a tremendous influence through the media'.

underweight women.[8] The second assumption, namely that the fashion industry, the media and advertising influence the ideals of beauty and the ideal body weight, is corroborated by a vast psychological and sociological literature (Garner et al., 1980; Fallon, 1990; Wolf, 1991; Kilbourne, 1994; Smolak, 1996; Owen and Laurel-Seller, 2000; Strahan et al., 2006; Swami, 2006; Ahern et al., 2008). The third assumption, which concerns whether the existence of an ideal body weight influences people's eating behaviour, and whether the thin ideal is responsible for people's detrimental eating behaviour, has been a matter of debate. On the one hand, there are some who observe that not everybody judges her or himself according to the ideal weight proposed by the media, especially if it is perceived as unrealistic and unattainable. According to this perspective, the fashion industry and the media should freely choose their marketing strategies and model testimonials, and no external intervention is needed.[9] On the other hand, there are some who underline that, even if a person does not individually care about the prevalent ideal body weight, it can nevertheless be relevant if other people (say, a potential employer, a potential mate, one's peers) discriminate against those who do not conform to the ideal body weight. The existence of a 'weight bias' (where a person encounters overt and covert victimization such as teasing, bullying, social exclusion or avoidance) is a robust finding in the psychological and sociological literature (Brownell et al., 2005; Puhl and Brownell, 2006; Puhl and Heuer, 2009). The weight bias may originate from a variety of interpersonal sources, the most frequent being family members, classmates, and sales clerks. For teens the school is the most commonly mentioned place in which weight stigmatization takes place. Neumark-Sztainer et al. (2002) and Neumark-Sztainer and Eisenberg (2005) find that teens frequently report weight teasing for either being underweight (44% and 36,6% of females and males, respectively) or overweight (45,3% and 50,2%, respectively). Obese adolescents are also less likely to date (Cawley et al., 2006). Adult obese people are less likely to be hired and are paid less than their non-obese coworkers. They are also perceived as having undesirable traits related to job performance and to be less disciplined on the job, and are given inferior professional assignments; they are at a disadvantage in terms of health care insurance and penalized through some companies'

[8] Similar results have been found for Vogue high-fashion models (Morris et al., 1989). Fashion models and celebrities were found to be thinner than 98% of American women (Smolak, 1996). The use of ultra-thin models seems to be based mainly on aesthetic criteria, as most designers clearly state that clothes hang better on thin figures. An explanation why the thin ideal has been promoted by the fashion and advertising industry for the last 50 years is out the scope of this paper.

[9] As an example, see the viewpoint taken by Armstrong and Turner in The Times, 23 September 2006.

benefit programmes for their weight status (Baun II and Ford, 2004; Cawley, 2004; Brownell et al., 2005; Morris, 2006; Conley and Glauber, 2006; Puhl and Heuer, 2009). The extent of these effects depends on variables such as gender, age, and ethnicity. For example, overweight men experience fewer negative effects of body mass on economic outcomes (Conley and Glauber, 2006), and underweight African American women earn less than those with healthy weight (Cawley, 2004). Given the concern for the obesity epidemic, most of the contributions study whether a weight bias exists for overweight and obese individuals. Interestingly, however, there is evidence of a weight bias also for underweight people. For example, Morris (2006) reports that for males mean hourly wages are highest for overweight men and lowest for underweight and morbidly obese men, the difference being around 12%. By contrast, underweight females earn only around 2% less than normal-weight women (the highest paid category), while morbidly obese women earn 12% less. This suggests that the relationship between body weight and labour outcomes such as wage, occupation, and labour force participation may be non-monotonic (Averett and Korenman, 1996; Pagan and Davila, 1997; Cawley, 2004; Baum II and Ford, 2004; Bhattacharya and Bundorf, 2005; Morris, 2006; Atella et al., 2008).

In the following section, we present a model of individual eating behaviour to study the effect of the social pressure to conform to an ideal, exogenous body weight on individual food intake and body weight and to provide a guideline for evaluating the above-mentioned policies.

2.3 The model

2.3.1 The individual utility function

Consider a population made up of distinct groups, indexed by $G \in \{1, 2, ..., M\}$. The individual utility function of agent i belonging to group G depends on the amount of calories she intakes through food consumption $c_i \geq 0$ and on her body weight $w_i > 0$.[10] Specifically, we consider the following utility function

[10] Instead of referring to body weight, one might refer to BMI, which is a measure of body weight normalized by the individual's height and the most used diagnostic tool in the study of weight problems. More precisely, the BMI is defined as an individual's body weight (kg) divided by her squared height (m^2); it depends neither on age nor sex. It is thus clear that, if individual height is constant, it is equivalent to refer to the body weight or the BMI.

$$U_{i,G}(c_i, w_i) = c_i \left(c_i^F - \frac{c_i}{2} \right) - \frac{1}{2} \left(w_i - w_i^H \right)^2 - \frac{\beta}{2} \left(w_i - w^G \right)^2. \tag{2.1}$$

The first term represents utility from food consumption. The parameter $c_i^F > 0$ represents the individual satiation point (where F stands for food) beyond which the marginal pleasure from food is negative.[11] We say agent i is undereating (or on a diet) if she eats less than her satiation level requires, and she is overeating if she eats more. The former case occurs if $c_i < c_i^F$, the latter if $c_i > c_i^F$.

The last two terms represent the effect of body weight on individual utility. We assume that this occurs through two channels: health and social desirability.

The first effect refers to the health consequences of being either overweight or underweight. According to the WHO guidelines (1995; 2000; 2004), this occurs when a person's BMI is higher than 25 or lower than 18.5. When the individual BMI is between 18.5 and 25, a person is considered to have a normal weight.[12] Within this range of values, we assume that there exists for each agent i a BMI that maximizes the agent's health condition; we therefore denote the corresponding body weight as $w_i^H > 0$ (where H stands for healthy).[13] The health consequences of body weight are summarized by the disutility cost the agent suffers if her body weight w_i is different from her healthy weight w_i^H (for a similar assumption, see Philipson and Posner, 2003; Lakdawalla and Philipson, 2005; Etilè, 2007).

The second effect is related to the existence of a socially desirable body weight $w^G \geq 0$. We assume that w^G is exogenously determined and that all agents belonging to group G consider w^G to be the socially desirable body weight.[14] Given that agents are heterogeneous, in general w^G does not coincide with the individual healthy weight. As in Levy (2002) and Etilé (2007), the

[11] Allowing for satiation seems to be a natural assumption as we are focusing on the consumption of food at the individual level and not on the consumption of a generic bundle of goods. In the Appendix, we show that c_i^F is the optimal level of food consumption, resulting from a standard constrained maximization problem. Accordingly, c_i^F depends on the agent's preferences, on her income and on market prices. For notational simplicity, we omit the dependence of c_i^F on these variables.

[12] A person is severely underweight if her BMI is lower than 16.5, moderately underweight if 16.5≤BMI<17, mildly underweight if 17≤BMI<18.5, overweight if 25≤BMI<30, and obese if BMI≥30.

[13] See Dwyer (1996) for a discussion on the definition of healthy weight.

[14] The provocative sentence 'A woman can never be too rich or too thin', attributed to Wallis Simpson, Duchess of Windsor, corresponds to the case where the ideal body weight is zero and the cost of deviating from it is strictly monotone.

specification used in (2.1) implies that having a body weight that is different from w^G is costly for the agent, with the parameter $\beta \geq 0$ measuring the relevance of this cost. We interpret this as the social cost of not conforming to a given body weight due to, for example, discrimination on the job or between peers.[15]

2.3.2 The determinants of individual body weight

Agent i can choose how much food to consume. She knows that her body weight cannot be chosen but is endogenously determined by the balance between calories intake and expenditure. Research on human nutrition shows that the largest source of calories expenditure is due to the functioning of organs and tissues, as measured by the basal metabolic rate (BMR). The BMR is the energy expenditure when a person is at rest (Mifflin et al., 1990). It is not influenced by the level of aerobic activities and largely depends on body weight together with other individual characteristics such as individual metabolism (Broeder et al., 1992; Smith et al., 1997). As in Levy (2002) and Dragone (2009), we assume that body weight changes over time according to the following law of motion:

$$\dot{w}_i(t) = c_i(t) - \delta w_i(t) \tag{2.2}$$

where $\delta > 0$ is a parameter indicating the effect of weight on the burning of calories (e.g. metabolism) and $\delta w_i(t)$ represents the energy expenditure according to the BMR.[16]

2.3.3 The individual problem

The goal of agent i belonging to group G is to choose the amount of food consumption that maximizes her utility. Given an infinite time horizon and a

[15] Similar preferences can be originated by an explicit model of social sanctions on the job market, the marriage market, and in friendship networks. See Cawley et al. (2006), for an application to adolescents and dating activities.

[16] One might also include in the analysis the choice of physical exercise, which includes on-the-job and off-the-job activity. Though this is a relevant source of calories expenditure, it plays a minor role with respect to the BMR. In the Appendix, we show that extending the analysis to include physical exercise does not change the qualitative properties of the solution.

discount rate $\rho > 0$, the agent's intertemporal problem can be formally written as follows:

$$\max_{\{c_i(t)\}} \int_0^{\infty} e^{-\rho t} \left[c_i\left(t\right) \left(c_i^F - \frac{c_i\left(t\right)}{2} \right) - \frac{\left(w_i\left(t\right) - w_i^H\right)^2}{2} - \beta \frac{\left(w_i\left(t\right) - w^G\right)^2}{2} \right] dt \tag{2.3}$$

$$\text{subject to } \dot{w}_i(t) = c_i(t) - \delta w_i(t) \tag{2.4}$$

$$w_i(0) = w_{i0}. \tag{2.5}$$

where $w_{i0} > 0$ is the individual body weight at time zero.

Before solving the dynamic problem, consider what would happen if the agent did not take into account how food consumption affects her body weight, i.e., if she ignored (2.4). In such a case, it would always be optimal to consume up to satiation for all possible body weights, i.e., agent i would choose $c_i(t) = c_i^F$ at all t. The optimal consumption of this agent does not change over time, but her body weight clearly changes, according to (2.4), until the steady state weight $w_i = c_i^F/\delta$ is reached. The value $w_i^F := c_i^F/\delta$ is to be interpreted as the body weight that an agent would reach if she always ate to satiation. It is possible to distinguish between two possible cases. If $w_i^F > w_i^H$, we say that satiation induces being overweight; likewise we say that satiation induces to be underweight if $w_i^H > w_i^F$.[17] The distinction between these two cases will prove useful to distinguish different scenarios and to the interpret and discuss the results of the dynamic problem.

In the following proposition, we characterize the solution of the intertemporal problem of an agent i that takes into account all factors influencing her utility, including the endogenous consequences of her eating behaviour for her body weight.

Let $A = 1 + \beta + \delta(\delta + \rho)$; $B = -\rho + \sqrt{4\left(1+\beta\right) + \left(2\delta + \rho\right)^2}$; $w_i^W = w_i^H - \frac{\delta(\delta+\rho)}{\beta}\left(w_i^F - w_i^H\right)$ and $w_i^C = w_i^F + \frac{1}{\beta}\left(w_i^F - w_i^H\right)$; then the following applies.

Proposition 2.1 *Given the intertemporal problem (2.3)-(2.5):*

1. *there exists a unique steady state level of food consumption and body*

[17] Notice that regardless whether the individual metabolism δ and satiation level c_i^F favour being, say, overweight does not mean that an agent will optimally choose to eat to satiation, nor that she will optimally become overweight in steady state.

weight

$$c_i^* = \delta \frac{w_i^H + \delta(\delta+\rho) w_i^F + \beta w_i^G}{A}, \tag{2.6}$$
$$w_i^* = \frac{w_i^H + \delta(\delta+\rho) w_i^F + \beta w_i^G}{A}; \tag{2.7}$$

2. *the steady state has saddle point stability;*

3. *in steady state, the agent is underweight if $w^G < w_i^W$ and overeating results if $w^G > w_i^C$;*

4. *along the optimal path leading to the steady state, food consumption is given by the following decreasing function of body weight*

$$c_i = \frac{B}{2\delta} c_i^* + \left(\delta - \frac{B}{2}\right) w_i. \tag{2.8}$$

Proof. The current value Hamiltonian function associated with the problem (omitting the time index and the individual index i) is

$$H(\cdot) = c\left(c^F - \frac{c}{2}\right) - \frac{\left(w - w^H\right)^2}{2} - \frac{\beta\left(w - w^G\right)^2}{2} + \lambda(c - \delta w) \tag{2.9}$$

where λ is the relevant costate variable. The necessary and sufficient conditions are:

$$\frac{\partial H}{\partial c} = 0 \Leftrightarrow c^F - c = -\lambda \tag{2.10}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial w} = (\rho+\delta)\lambda + \left(w - w^H\right) + \beta\left(w - w^G\right) \tag{2.11}$$
$$\dot{w} = c - \delta w \tag{2.12}$$

with the relevant transversality condition being $\lim_{t\to\infty} e^{-\rho t}\lambda(t)w(t) = 0$. Differentiating (2.10) with respect to time and substituting (2.11)-(2.10), we can rewrite the above conditions as a linear dynamic system that depends on consumption and weight:

$$\dot{c} = (c - c^F)(\delta+\rho) + (1+\beta)w - w^H - w^G \tag{2.13}$$
$$\dot{w} = c - \delta w. \tag{2.14}$$

The steady state results from $\dot{c} = \dot{w} = 0$. As the trace and the determinant of the 2×2 Jacobian matrix

$$J = \begin{bmatrix} \delta+\rho & 1+\beta \\ 1 & -\delta \end{bmatrix}$$

are, respectively, positive and negative, the steady state has saddle point stability. To check that the optimal trajectory (2.8) leading to the steady state is linear, consider the following ratio between (2.13) and (2.14),

$$\frac{dc}{dw} = \frac{(c - c^F)(\delta + \rho) + (w - w^H) + \beta(w - w^G)}{c - \delta w}, \tag{2.15}$$

and replace (2.8). As the slope of (2.8) is $\delta - \frac{B}{2} < 0$, the optimal path of food consumption crosses the locus $\dot{w} = 0$ from above, guaranteeing that (2.8) is indeed the stable saddle path. To verify the conditions under which being overweight and overeating occurs, take the difference $w_i^* - w_i^H$ and $c_i^* - c_i^F$, respectively, and rearrange. ■

Proposition 1 shows that a forward-looking agent that explicitly considers the dynamics of body weight formation should adapt the level of food consumption to her body weight (see Eq. 2.8). More precisely, it is optimal to constrain food consumption when body weight is high and, conversely, to consume much (eventually beyond satiation) when body weight is low. Following such a consumption path leads to a steady state that positively depends on the individual healthy weight w_i^H, the individual body weight associated with food satiation w_i^F, and the ideal body weight w^G. Interestingly, the qualitative properties of the steady state critically depend on the relative magnitude of these three variables. Consider thus the case where $w_i^H < w_i^F$, i.e., the case where sticking to the satiating food consumption level induces the agent to become overweight. In such a case, $w_i^W < w_i^C$ and three types of steady state may result. If the ideal body weight is low ($w^G < w_i^W$), in steady state agent i is underweight and undereating; if it is high ($w^G > w_i^W$), agent i is overweight and overeating. If instead the ideal body weight is in an intermediate range (if $w^G \in \left(w_i^W, w_i^C\right)$), agent i is overweight and undereating (see Fig. 1).

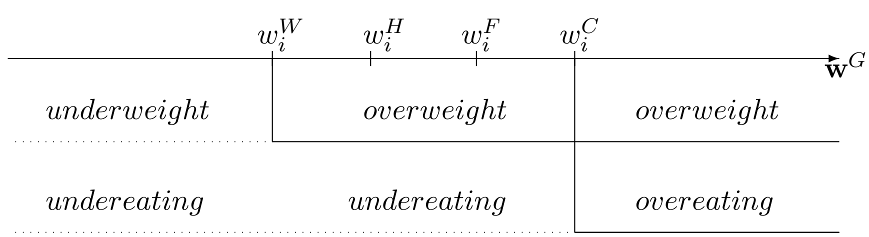

Figure 1: Steady states of an agent that tends to be overweight

In the alternative scenario, where sticking to the satiating food consumption level induces the agent to become underweight, similar results hold: if the ideal body weight is low, the first type of steady state may result; if it is high, the second one may result. The difference with respect to the previous case is that, if the ideal body weight is in the intermediate range, the agent will, in steady state, be overeating and underweight.[18] Overall, this implies that the model allows for four types of steady states to arise, namely

1. overweight and undereating;
2. overweight and overeating;
3. underweight and overeating;
4. underweight and undereating.

The first type of steady state is reached by those that constrain their food consumption in order to avoid becoming more overweight (e.g., overweight people that stay on a diet). The second type of steady state is reached if the ideal weight w^G is high, i.e., $w^G > \max\{w_i^C, w_i^W\}$. Sumo wrestlers and body builders tend to reach this condition as they have specific incentives to have a large body mass. The third type of steady state is typical of people that tend to become underweight (e.g., because they have a very fast metabolism) and must overeat in order to avoid becoming even more underweight. The fourth type of steady state occurs if the ideal weight is low, i.e., $w^G < \min\{w_i^C, w_i^W\}$. People falling into this category are thin and yet constrain their food consumption. Clearly, the last case is what the policy maker has in mind when addressing the slimming mania.

The above steady states can be produced within the same theoretical framework and they are consistent with the empirical observation of people's eating behaviour and the body weight they reach. Given the quadratic specification of the model, this can be easily used to show the implications of the policy actions aimed at changing the ideal body weight at the aggregate level. This is the objective of the next section.

[18] If $w_i^F < w_i^H$, then $w_i^C < w_i^W$, which implies that the formal condition for the underweight/undereating steady state is $w^G < w_i^C$ and that it is $w^G > w_i^W$ for the overweight/overeating steady state. The range of values for w^G to induce an underweight/overeating steady state is (w_i^C, w_i^W). Note that the distance between w_i^W and w_i^C increases as β decreases. If there was no social pressure, i.e. $\beta = 0$, only two steady states would arise, being overweight and undereating (if $w_i^F > w_i^H$) or being underweight and overeating (if $w_i^F < w_i^H$).

2.4 Is increasing the ideal body weight desirable?

All policies mentioned in section 2 point out that the current ideal body weight is below the average body weight and that it would be desirable to increase it. In this section, we show that the desirability of increasing the ideal body weight critically depends on what the new ideal body weight is, on who will be affected by this change, and on what is the objective of the policy maker. The latter point is relevant because there is a lively debate on the proper normative framework for the study of health policies (Hurley, 2000; Brouwer et al., 2008). Philipson and Posner (2008) observe that, from an economic standpoint, the policy function should be aggregate utility and not aggregate health (which is merely a part of the utility function). By contrast, alternative approaches consider eating behaviour as a public health issue (see, e.g., Hurley, 2000; Brouwer et al., 2008), and intervention is recommended to maximize aggregate health (or, equivalently, minimize aggregate health losses). Even though we do not take a stand on this issue, we consider both viewpoints and study whether and how choosing either perspective may lead to different conclusions about the evaluation of the policies. We first determine the ideal body weight that maximizes aggregate utility and the ideal body weight that minimizes aggregate health losses. Then we use these two results to evaluate the German and Spanish proposal contained in the above-mentioned agreements, according to which it is desirable to promote a healthy body weight as the ideal body weight.

2.4.1 Optimal ideal body weight

In section 3, we have considered the optimal behaviour of a generic agent i belonging to group G. We now focus on the whole group, assuming that all its members choose their food intake according to Proposition 1.

For all agents in G the ideal body weight is the same. They are, however, heterogeneous in their individual healthy weights w_i^H and satiation levels c_i^F. Let $f_G(w^H, c^F)$ be the joint density function of the healthy weights w_i^H and of the satiation levels c_i^F of the agents belonging to group G. This allows to write the average utility of group G as follows:

$$\int \int U_{i,G}\left(c_i, w_i\right) f_G\left(w^H, c^F\right) dw^H dc^F. \tag{2.16}$$

Define $\overline{w}^H$ and $\overline{c}^F$ as the average healthy weight and the average satiation level in group G, respectively. By analogy with the individual case, we define $\overline{w}^F := \overline{c}^F/\delta$ as the body weight associated with satiation, saying that the group tends to be overweight if $\overline{w}^F > \overline{w}^H$ and tends to be underweight otherwise.

Let $C = 1/\left\{\beta\ \left(1+\delta^2\right) + \left[1+\delta\ (\delta+\rho)\right]^2\right\}$, then the following Lemma applies:

Lemma 2.1 *Given the joint density function $f_G(w^H, c^F)$ of the healthy weights w_i^H and of the satiation levels c_i^F:*

1. *the ideal body weight that maximizes the average steady state utility of group G is*

$$w^{GU} = \left[1+\beta+\delta\,(\delta+2\,\rho)\right]C\overline{w}^H + \delta^2\left[1+\beta+(\delta+\rho)^2\right]C\overline{w}^F; \quad (2.17)$$

2. *the ideal body weight w^{GU} is between the average healthy weight $\overline{w}^H$ and the body weight associated with satiation $\overline{w}^F$;*

3. *if $w^G = w^{GU}$, on average the group will reach a steady state associated with*

   (a) being overweight and undereating if $\overline{w}^F > \overline{w}^H$, and

   (b) being underweight and overeating if $\overline{w}^F < \overline{w}^H$.

Proof. Consider the following problem

$$\max_{w^G} \int\int U_{i,G}\left(w_i^*, c_i^*\right) f_G(w^H, c^F) dw^H dc^F.$$

Taking the derivative with respect to w^G and simplifying yields

$$\int\int \left\{\delta\left[(1+\beta)^2+(\delta+\rho)^2\right]c_i^F + \left[1+\beta+\delta\,(\delta+2\rho)\right]w_i^H + \right.$$

$$\left. -\left\{\delta^2\rho^2 + \left(\delta^2+1\right)\left[1+\beta+\delta\,(\delta+2\rho)\right]\right\}w^G\right\} f_G(w^H, c^F)dw^H dc^F = 0.$$

Exploiting the additive separability of the foc, it can be simplified as follows:

$$\delta\left[1+\beta+(\delta+\rho)^2\right]\overline{c}^F + \left[1+\beta+\delta\,(\delta+2\rho)\right]\overline{w}^H +$$

$$-\left\{\delta^2\left(\rho^2+1\right) + \left[1+\beta+\delta\,(\delta+2\rho)\right]\right\}w^G = 0.$$

Solving with respect to w^G yields (2.17), the ideal body weight that maximizes the average utility of G. Substituting w^{GU} in the steady state food consumption and body weight of the representative consumer of the group, i.e.,an agent whose satiation level and healthy weight is precisely $\bar{c}^F$ and $\overline{w}^H$, yields the last result in the Lemma. ■

The Lemma above implies that, under the optimal ideal body weight $w^G = w^{GU}$, group G would not reach a steady state associated with being underweight and undereating, nor with being overweight and overeating. In other words, despite being outcomes that solve the intertemporal maximization problem of the agent *given* w^G, they are dominated outcomes that the policy maker would eliminate if he could optimally set the ideal body weight.

We now consider what ideal body weight maximizes the average steady state health of group G, which is equivalent to the solution of the following problem:

$$\min_{w^G} \int \int \left(w_i^* - w_i^H\right)^2 f_G(w^H, c^F) dw^H dc^F.$$

As one would expect, if the policy objective maximizes aggregate health, the ideal body weight must be such that the average steady state weight of the group coincides with its average healthy weight, i.e., $\int \int w_i^* f_G(w^H, c^F) dw^H dc^F = \int \int w_i^H f_G(w^H, c^F) dw^H dc^F := \bar{w}^H$. Whether this steady state is associated with undereating or overeating, however, hinges on the relation between satiation and healthy weight, as stated below.

Lemma 2.2 *Given the joint density function $f_G(w^H, c^F)$ of the healthy weights w_i^H and of the satiation levels c_i^F:*

1. *the ideal body weight that maximizes the average steady state health of group G is*
$$w^{GH} = \bar{w}^H - \frac{\delta\left(\delta + \rho\right)}{\beta} \left(\bar{w}^F - \bar{w}^H\right); \tag{2.18}$$

2. *the body weight w^{GH} is less than the average healthy weight if the group tends towards being overweight, and it is higher if the group tends towards being underweight;*

3. *if $w^G = w^{GH}$, on average the steady state will not be associated with being overweight nor being underweight. The steady state is characterized by*

   (a) *undereating if $\overline{w}^F > \overline{w}^H$, and*

   (b) *overeating if $\overline{w}^F < \overline{w}^H$.*

Proof. See the previous proof, replacing the average utility function with the average loss function. Incidentally, note that w^{GH} is the aggregate analogue of w_i^W of Proposition 1. ■

2.4.2 Should the healthy weight be the ideal body weight?

We now use the model and the results we have derived to evaluate a specific proposal contained in the Spanish and German agreements. It refers to the promotion and propagation of 'a healthy body image [to] unequivocally reject the unhealthy ideal of extreme thinness, particularly among girls and women'. This proposal is aimed at reversing the current trend for 'size zero', and the rising popularity of visibly underweight models and celebrities (Ahern et al., 2008) in order to counteract the slimming mania in female teenagers and women. While it may intuitively seem sound, we show that this proposal does not yield unambiguous results and that trade-offs between utility and health may arise.

In order to assess the desirability of increasing the ideal body weight to the average healthy weight, we rely on three considerations. First, in the countries that are taking measures against unhealthy eating behaviour, there is a wide consensus on the existence of a thin ideal for girls and women. In our model, this amounts to considering an ideal body weight lower than the healthy weight of the target group, $w^G < \bar{w}^H$. Second, some authors (Chou et al., 2004; Lakdawalla et al., 2005) observe that declining food prices and the declining strenuousness of work have a relevant role in the growth of obesity. We interpret this insight by considering a scenario where eating to satiation leads to being overweight, which amounts to assuming $\overline{w}^H < \overline{w}^F$. Third, we interpret the promotion of a healthy body image as an increase in the current ideal body weight to the average healthy weight of a given target group. Given the above considerations, we therefore focus on the case where $w^G < \overline{w}^H < \overline{w}^F$, and we show the welfare and health consequences of increasing the ideal body weight w^G to the average healthy weight $\overline{w}^H$.[19]

Proposition 2.2 *Consider the case where the current ideal body weight is lower than the average healthy weight and satiation leads to being overweight.*

1. *If the group is on a diet and overweight, increasing the ideal body weight to the average healthy weight is welfare improving but health worsening.*

[19] As shown in the Appendix, the proposition holds even if one compares the aggregate health function with a welfare function that neglects the costs of social pressure.

2. *If the group is on a diet and underweight, increasing the ideal body weight to the average healthy weight is welfare improving; it is health improving if the current ideal body weight is very low but health worsening otherwise.*

Proof. We are in the case where $w^{GH} < \overline{w}^{H} < w^{GU} < \overline{w}^{F} < \overline{w}^{C}$ and $w^{G} < \overline{w}^{H}$. As the optimal ideal weight w^{GU} is higher than the average healthy weight $\overline{w}^{H}$, increasing w^{G} (eventually until w^{GU}) is welfare improving irrespective of whether the group is on average underweight or overweight. Let

$$\tilde{w}^{G} = w^{GH} - \frac{\delta\left(\delta+\rho\right)}{\beta}\left(\bar{w}^{F} - \bar{w}^{H}\right) < w^{GH}$$

be the ideal weight that induces the target group to have a health condition identical to the one that would be reached if $w^{G} = \bar{w}^{H}$. If $w^{G} \in [0, \tilde{w}^{G})$, which corresponds to the case where the ideal body weight is very low, the group is on average very underweight and undereating. In such a case, the health condition of the group is improved by increasing the ideal body weight to $\bar{w}^{H}$. By contrast, if $w^{G} \in \left(\tilde{w}^{G}, \bar{w}^{H}\right)$, increasing the ideal weight to $\bar{w}^{H}$ is health worsening. We can distinguish two sub-cases. First, if $w^{G} \in \left(\tilde{w}^{G}, w^{GH}\right)$, the group is mildly underweight and undereating and will become overweight if the ideal weight was changed to $\bar{w}^{H}$. As the new steady state weight would be more distant from $\bar{w}^{H}$ than it was before the intervention, the policy is health worsening. Second, if $w^{G} \in \left(w^{GH}, \bar{w}^{H}\right)$, the group is mildly overweight and undereating, and it would become even more overweight if w^{G} was $\bar{w}^{H}$. In both cases, an increase in the ideal weight is strictly health worsening. ■

The proposition above shows that the desirability of increasing the ideal body weight to the level of a healthy weight may create a trade-off between health and welfare considerations. Under the initial thin ideal, people have an incentive to constrain their food intake and stay on a diet. In a scenario where eating to satiation leads to being overweight, being on a diet helps people to keep their body weight low and reduce the resultant social pressure. If the ideal weight is increased, the social pressure to be thin is reduced, allowing people to optimally increase their food intake. If the ideal weight is increased to match the average healthy weight, on average people will still stay on a diet, but they will not be constraining their food consumption as they did before, and they will reach an overweight condition. This outcome is always welfare improving because of increased utility from food consumption and reduced social pressure;[20] but it is detrimental for health if it induces overweight people to become even more overweight or mildly underweight

[20] It is easy to show that, if $\bar{w}^{H} < \bar{w}^{F}$ average social pressure is minimized for an ideal weight larger than the average healthy weight.

people to become overweight. The only case where it is also health improving concerns the target group being so underweight that an overweight outcome is desirable.

As increasing the ideal body weight to the average healthy weight is unambiguously welfare improving, one might conclude that such a measure is beneficial as the associated benefits outweigh the costs. However, as observed by Philipson and Posner (2008) in their study of health policies, an implicit assumption is often postulated, requiring the government to maximize health. This postulate is based on the fact that targeting health has a direct impact on the probability of people contracting non-communicable diseases in that it can improve the quality of life and decrease the expected costs of public health care. If health is the relevant objective, then the desirability of increasing the ideal body weight critically depends on which target group will be affected. On the one hand, policy interventions may affect only those that are on a very restrictive diet and severely underweight, which is clearly the case that the policy maker has in mind. If such policy interventions are able to affect only this group, increasing the ideal body weight is desirable both from a welfare and a health point of view, and there is no particular need for choosing between the two alternative objective functions we have considered. By contrast, a different assessment holds if increasing the size of models and mannequins does not only affect underweight and undereating teenagers but also adult women that are overweight and on a diet. In this case, welfare would on average increase, but the average health condition might decrease. This trade-off implies that a precise definition of the policy maker's objective function is needed for assessing the desirability of increasing the ideal body weight. This is particularly crucial for the US and the European countries, where people are on average overweight. In such a case, if increasing the ideal weight does not only affect underweight people on a diet but also has an impact on the eating behaviour of other people, these policies, even if welfare improving, may exacerbate the obesity epidemic.

2.5 Conclusions

Inspired by the policies implemented by some European governments to counteract unhealthy eating behaviour, the present paper shows how policy actions aiming to modify the ideal body weight affect individual eating choices and body weight. We propose a model where the utility of a forward-looking agent depends on (i) food consumption, (ii) the health condition and (iii) body weight conforming with an ideal weight. Once the agent is aware of how food consumption affects body weight, and she explicitly takes this information into account when choosing how much to eat, then eating behaviour leading both to be overweight or underweight can be a rational outcome. This result is not due to changes in market prices, income or lifestyle, but is the consequence of individual preferences for food consumption and body weight as well as of the individual metabolism. In particular, we show that the apparent paradox of Western industrialized societies, where many people are overweight despite being on a diet, can be rationalized without resorting to self-control problems.

Allowing our model to include the existence of social pressure to conform to an ideal body weight yields useful insights. On the one hand, we show that it can be optimal to engage in patterns of eating behaviour that are often described as pathological, as in the case of people who constrain food consumption despite being underweight or binge despite being overweight. On the other hand, our model provides a useful tool to evaluate some policy measures that have been recently discussed and implemented. As an example, we have focused on the proposal contained in the German and Spanish agreements and studied the consequences of setting the healthy weight as the ideal body weight. Given the current tendency towards obesity and the existence of a thin ideal, we have shown that such policies are welfare improving but might have adverse effects on health if they induce people to become too overweight.

On a theoretical ground, our model provides a starting point for studying intertemporal eating behaviour and the role of social pressure. Possible extensions include the endogenous definition of the ideal body weight and allowing agents to choose their ideal body weight (or their group) among multiple references. On an empirical ground, the issue whether the mentioned agreements are effective in changing the references on the ideal body weight is still an open question which we plan to address in future research.

Acknowledgements

I thank my supervisor, Vincenzo Denicolò, for useful comments and suggestions. I am also thankful to Oriana Bandiera, Francesca Barigozzi, Maria Bigoni, Marco Casari, Paolo Vanin and the audiences of the Public Economic Theorists 2010 Conference, Istanbul; the Public Economics UK Workshop at the IFS, London; the 2010 Social Economics Conference, Forlì; and the Department of Economics, Bologna. I am grateful to STICERD - LSE, where part of this paper was written while I was a visiting research student. The usual disclaimer applies.

References

Acs, Z. J., Lyles, A., eds. 2007. *Obesity, Business and Public Policy*. Elgar.

Ahern, A. L., Bennett, K. M., Hetherington, M. M. 2008. Internalization of the ultra-thin ideal: positive implicit associations with underweight fashion models are associated with drive for thinness in young women. *Eating Disorders,* **16,** 294-307.

Akerlof, G., Kranton, R. 2000. Economics and identity. *Quarterly Journal of Economics* **115**, 715-753.

Armstrong, L. 2006. Thin argument does a fat lot of good. *The Times*, Sept. 23. Available at http://www.timesonline.co.uk/tol/news/uk/article1074455.ece

Atella, V., Pace, N., Vuri, D., 2008. Are employers discriminating with respect to weight? European evidence using quantile regression. *Economics & Human Biology*, **6**, 305–329.

Averett, S., Korenman, S. 1996. The economic reality of the beauty myth. *Journal of Human Resources,* **31**, 304-330.

Baum II, C. L., Ford, W. F. 2004. The wage effects of obesity: a longitudinal study. *Health Economics,* **13**, 885-899

Bhattacharya, J., Bundorf, M. K. 2005. The incidence of the healthcare costs of obesity. *NBER, Working Paper* 11303.

Bicchieri, C. 2006. *The Grammar of Society: The Emergence and Dynamics of Social Norms*. Cambridge University Press.

Blanchflower, D. G., Oswald, A. J., Van Landeghem, B. 2009. Imitative obesity and relative utility. *Journal of the European Economic Association,* **23**, 528-538.

Broeder, C. E., Burrhusm, K. A., Svanevik, L. S., Wilmore, J. H. 1992. The effects of aerobic fitness on resting metabolic rate. *American Journal of Clinical Nutrition,* **55**, 795-801.

Brouwer, W. B. F., Culyer, A. J., van Exel, N., Rutten, F. F. H. 2008. Welfarism vs. extra-welfarism. *Journal of Health Economics,* **27**, 325-338.

Brownell, K. D., Puhl, R. M., Schwartz, M. B., Rudd, L., eds. 2005. *Weight Bias: Nature, Consequences, and Remedies.* The Guilford Press.

Cawley, J. 2004. The impact of obesity on wages. *Journal of Human Resources,* **39**, 451-474.

Cawley, J., Joyner, K., Sobal, J. 2006. Size matters: The influence of adolescents' weight and height on dating and sex. *Rationality and Society,* **18**, 67-94.

Chou, S., Grossman, M., Saffer, H. 2004. An economic analysis of adult obesity: results from the Behavioral Risk Factor Surveillance System. *Journal of Health Economics,* **23**, 565-587.

Christakis, N., Fowler, J. 2007. The spread of obesity in a large social network over 32 years. *The New England Journal of Medicine,* **357**, 370-379.

Conley, D., Glauber, R. 2006. Gender, body mass, and socioeconomic Status: New evidence from the PSID, in M. Grossman, B. Lindgren (eds.). *The Economics of Obesity* (Advances in Health Economics and Health Services Research), vol. 17, Emerald Group Publishing Limited, 253-275.

Cutler, D. M., Glaeser, E. L., Shapiro, J. M. 2003. Why have Americans become more obese? *Journal of Economic Perspectives,* **17**, 93-118.

Downs, J. S., Loewenstein, G., Wisdom, J. 2009. Strategies for promoting healthier food choices. *American Economic Review,* **99,** 159-164.

Dragone, D. 2009. A rational eating model of binges, diets and obesity. *Journal of Health Economics,* **28**, 799-804.

Dwyer, J. 1996. Policy and healthy weight. *Preventive Medicine,* **25**, 30-33.

Etilé, F. 2007. Social norms, ideal body weight and food attitudes. *Health Economics* 1**6**, 945-966.

Fallon, A. 1990. Culture in the mirror: sociocultural determinants of body image, in T. Cash, T., Pruzusky (eds.). *Body Images: Development Deviance and Change*. The Guilford Press, 80-109.

Fehr, E., Gächter, S. 2000. Fairness and retaliation: The economics of reciprocity. *Journal of Economic Perspectives,* **14,** 159-181.

Garner, D. M., Garfinkel, P. E., Schwartz, D., Thompson, M. 1980. Cultural expectations of thinness in women. *Psychological Reports,* **47**, 483-491.

Hamermesh, D. S., Biddle, J. E. 1994. Beauty and the labor market. *American Economic Review,* **84,** 1174-1194.

Hurley, J. 2000. *An overview of the normative economics of the health sector*, in A. J. Culyer, J. P. Newhouse (eds.). Handbook of Health Economics, vol. 1, 55-118.

Kilbourne, J. 1994. Still killing us softly: Advertising and the obsession with thinness, in P. Fallon, T. Katzman, S. C. Wooley (eds.), *Feminist Perspectives on Eating Disorders*. The Guilford Press, 395-418.

Lakdawalla, D., Philipson, T. J., Bhattacharya, J. 2005. Welfare-enhancing technological change and the growth of obesity. *American Economic Review,* **95,** 253-257.

Levy, A. 2002. Rational eating: can it lead to overweightness or underweightness? *Journal of Health Economics*, **21**, 887-899.

Mazzocchi, M., Traill, W. B., Shogren J. F. 2009. *Fat Economics: Nutrition, Health, and Economic Policy*. Oxford University Press.

Mifflin, M. D., St Jeor, S. T., Hill L. A., Scott B. J., Daugherty S. A., Koh Y. O. 1990. A new predictive equation for resting energy expenditure in healthy individuals. *American Journal of Clinical Nutrition*, **51,** 241-247.

Morris, S. 2006. Body mass index and occupational attainment. *Journal of Health Economics*, **25**, 347-364.

Neumark-Sztainer, D., Falkner, N., Story, M., Perry, C., Hannan, P., and Mulert, S. 2002. Weight-teasing among adolescents: Correlations with weight status and disordered eating behaviors. *International Journal of Obesity,* **1,** 123-131.

Neumark-Sztainer, D., Eisenberg, M. 2005. Weight bias in a teen's world. in K. D. Brownell, R. M. Puhl, M. B. Schwartz, L. Rudd (eds.), *Weight Bias: Nature, Consequences and Remedies*. The Guilford Press, 69-79.

Neumark-Sztainer, D., Wall, M., Eisenberg, M., Story, M., Hannan, P. J. 2006. Overweight status and weight control behaviors in adolescents: longitudinal and secular trends from 1999 to 2004. *Preventive Medicine,* **43**, 52-59.

Owen, O. R, Laurel-Seller, E. 2000. Weight and shape ideals: Thin is dangerously in. *Journal of Applied Social Psychology,* **30**, 979-990.

Pagán, J., Dávila, A. 1997. Obesity, occupational attainment, and earnings. *Social Science Quarterly,* **78**, 756-770.

Philipson, T. J., Posner, R. A. 2003. The long-run growth in obesity as a function of technological change. *Perspectives in Biology and Medicine,* **46**, 87-107.

Philipson, T. J., Posner, R. A. 2008. Is the obesity epidemic a public health problem? A review of Zoltan J. Acs and Alan Lyles's obesity, business and public policy. *Journal of Economic Literature,* **46**, 974-982.

Puhl, R. M., Heuer, C. A. 2009. The stigma of obesity: a review and update. *Obesity,* **17**, 941-964.

Puhl, R., Brownell, K.D. 2006. Confronting and coping with weight stigma: An investigation of overweight and obese individuals. *Obesity,* **14,** 1802-1815.

Smith, D. A., Dollman, J., Withers, R. T., Brinkman, M., Keeves, J. P., Clark, D. G. 1997. Relationship between maximum aerobic power and resting metabolic rate in young adult women. *Journal of Applied Physiology,* **82,** 156-163.

Smolak, L. 1996. Methodological implications of a developmental psychopathology approach to the study of Eating Problems, in L. Smolak and M. P. Levine (eds). *The Developmental Psychopathology of Eating Disorders: Implications for Research, Prevention, and Treatment*, Lawrence Erlbaum Associates, 31-55.

Stanton, K. R.,Acs, Z. J. 2005. The infrastructure of obesity and the obesity epidemic: implications for public policy. *Applied Health Economics and Health Policy,* **4,** 139-146.

Strahan, E. J., Wilson, A. E., Cressman, K. E., Buote, V. M. 2006. Comparing to perfection: How cultural norms for appearance affect social comparisons and self-image. *Body Image,* **3,** 211-227.

Swami, V. 2006. The influence of body weight and shape in determining female and male physical attractiveness, in M. V. Kindes (ed). *Body Image: New Research.* Nova Biomedical Books, 35-61.

Trogdon, J. G., Nonnemaker, J., Pais, J. 2008. Peer effects in adolescent overweight. *Journal of Health Economics,* **27,** 1388-1399.

Turner, J. 2006. Size 0: not the thin end of the wedge, *The Times*, Sept. 23. Available at timesonline.co.uk/tol/comment/columnists/janice_turner/article647974.ece

Wansink, B., Just, D. R., Payne, C. R. 2009. Mindless eating and healthy heuristics for the irrational. *American Economic Review,* **99,** 165-169.

WHO expert consultation 2004. Appropriate body-mass index for Asian populations and its implications for policy and intervention strategies. *The Lancet,* **363**(9403), 157-163.

WHO 1999. Physical status: the use and interpretation of anthropometry. Report of a WHO Expert Committee. *WHO Technical Report Series,* **854**. Geneva: World Health Organization.

WHO 2000. Obesity: preventing and managing the global epidemic. Report of a WHO Consultation. *WHO Technical Report Series,* **894**. Geneva: World Health Organization.

Wolf, N. 1991. *The beauty myth: How images of beauty are used against women*. William Morrow.

Yaniv, G. 2002. Non-adherence to a low-fat diet: an economic perspective. *Journal of Economic Behavior and Organization,* **48**, 93-104.

Yaniv, G., Rosin, O., Tobol, Y. 2009. Junk-food, home cooking, physical activity and obesity: the effect of the fat tax and the thin subsidy. *Journal of Public Economics,* **93**, 823-830.

3. IS CONFORMISM DESIRABLE? NETWORK EFFECTS, LOCATION CHOICE, AND SOCIAL WELFARE IN A DUOPOLY

Abstract

In this paper we study a duopoly where the network effect is nonmonotone and the network can be overloaded. The firms choose prices and locations endogenously, and the agent's utility is influenced by the number of people patronizing the same firm she does. We determine the market equilibrium, and we study how the network effect influences social welfare. We compare this setting with the standard horizontal differentiation model with no network effects to understand whether and how conformism is socially desirable. The results show that whether network effects are desirable depends on *how* people are conformist, and whether overloading is feasible. If overloading is not possible (in either of the firm's network), and the total consumers' mass is sufficiently high, a network effect which is slightly concave increases social welfare. By contrast, if overloading is feasible, and the total consumers' mass is sufficiently small, social welfare is increased if the network effect is more concave than in the previous case.

JEL classification: C72, D43, L13.

Keywords: network effects, horizontal differentiation, duopoly, overloading, nonmonotonicity.

3.1 Introduction

Yesterday evening I met some friends for a pub crawl in London. We first went to George IV, the pub of the university, but it was crowded, with many people standing outside. We thus decided to go to the Ship Tavern, which is the closest. Once entered the pub, we realized that it was almost empty. Finally, even though it was a little farer away from the university, we moved to the Shakespeare's Head, on Kingsway, where we found a fair amount of customers, and sat and drunk our drinks.

This little narrative entails the main ingredients of this paper: the non-monotonicity of network effects, the distance of goods, and the positioning of retailers. As the story suggests, network effects are not always increasing in the number of people participating in the network. Casual observation suggests that people prefer a fair amount of the others sharing a place, or exhibiting the consumption of a good. This fact can be noticed also in the market for fashion products: you may want neither to be the only weird person wearing a kind of clothes nor that everyone dresses like you.[1]

Excessive crowding could moreover generate disutility as well as a standard overloaded network, or traffic jam. The marketing literature has shown extensive evidence about how retail crowding affects consumers' behavior (e.g. Eroglu and Harrell, 1986; Eroglu et al. 2005). Individuals vary in their tolerance to crowding and excessive crowding can decrease hedonic utility or generate disutility. Nevertheless, crowding tolerance does not only depend on individual factors, but also has cultural roots which make the average level of crowding tolerance differing across cultures.[2] For example, Kaya and Weber (2003) study a sample of American and Turkish students, showing that the Turkish students have a higher perception of crowding with respect to the Americans. Pons and Laroche (2007) study a sample of Canadian and Mexican students and find that the perceived level of crowding in the same situation is on average higher among the Mexican students. In an analogous study, Pons et al. (2006) find that Lebanese students' average perception of crowding is

[1] For an explanation of this phenomenon in terms of signalling see Pesendorfer (1995), considering fashion cycles.

[2] Tabellini (2008) observes that in the economic literature the notion of culture has been defined in different ways as: a) a selection mechanism among multiple equilibria or in repeated interactions; b) the set of beliefs manipulated by earlier generations regarding the consequences of the individual's actions; c) primitives values and individual preferences. Guiso et al. (2006) suggest that, whichever the definition, the identification of the cultural influence should exploit those aspects not changing along the individual's life and typically inherited from generation to generation.

higher than Canadian students'. These differences can be explained by national cultural dimensions, such as the degree of individualism, i.e. the extent to which people are expected to look after only themselves or the closest relatives. On the basis of the work by Hofstede (2001)[3], America and Canada score high in individualism, while Mexico, Turkey, and Lebanon score low. Mooij and Hofstede (2002) hypothesize that, more in general, converging of technology and income will lead to heterogeneity in consumer behavior based on cultural differences, with relevant implications for social welfare.

For these reasons, the aim of this paper is to understand whether and how the shape of network effects influences social welfare. While casual observation is confirmed by the evidence shown above, the majority of research on network effects and externalities has concentrated on monotonicity. We thus focus on network effects which are nonmonotone in the number of people consuming a good at the same location (in a dimension of the product). We introduce moreover the possibility for overloading.

We consider a duopoly where the firms can choose prices and locations, and where the utility of a consumer is influenced by the number of people patronizing the same firm. We determine the market equilibrium and the incentives for each firm to undercut the rival both at the price stage and at the location stage. We then proceed to study how the network effect influences social welfare, and the role played by its concavity, nonmonotonicity, and the possibility of overloading. Finally, we compare this setting with the standard horizontal differentiation model to understand when conformism is socially desirable.

We find that the firms have no incentives to undercut at the price stage, while at the location stage there are incentives for displacing the location to capture the rival's market. The introduction of nonmonotonicity and overloading thus imposes further conditions on the existence of a subgame perfect equilibrium in pure strategies. The equilibrium can exist either at the increasing or at the decreasing part of the network effect, or both. It can also exist when the network is overloaded. We moreover show that the overloading of the network raises prices and thus has anti-competitive effects. We observe that

[3] The most widely used measurements of culture across the social sciences are the Hofstede's dimensions. The Hofstede's book is one the most quoted in the Social Science Citation Index, but, surprisingly, it is little known among the economists. Hofstede (2001) proposes four dimensions of national culture: individualism (IDV), power distance (PDI), uncertainty aversion (UAI), and masculinity (MAS). They are based on 117000 questionnaires surveyed in the period 1967 – 1973 at the IBM Corporation, with 88000 employees responding, across 72 countries and 20 languages. They are stable across years, as numerous studies have subsequently validated them, and they exhibit a high degree of correlation with competing frameworks.

the endogenous determination of the locations allows the firms to differentiate only horizontally. This suggests that the choice of differentiating vertically by firms should entail some rigidity in the location choice of the firms.

Comparing the social welfare in the case with network effects to the case in which they are absent, we find that the firms' profits are increasing in the network effects only if the network can be overloaded. The consumers' surplus is decreasing in the concavity of the network effect, while by contrast profits are increasing. The results show that whether network effects are socially desirable depends on *how* people are conformist, and whether overloading is feasible. If overloading is not possible (in either of the firms' network), and the total consumers' mass is sufficiently high, a network effect which is slightly concave increases social welfare. By contrast, if overloading is feasible, and the total consumers' mass is sufficiently small, social welfare is increased if the network effect is more concave than in the previous case.

We extend the existing literature along the following lines: in this paper the network effect can be nonmonotonic and negative; it does not depend directly on the total size of the network, but only on the size of consumers patronizing the same store; finally, the network can be overloaded. Pesendorfer (1995) proposes a model of fashion cycles, where the consumers' utility displays nonmonotonic features similar to those adopted in this paper. He gives the conditions under which the consumers could be better off by banning fashion. Our approach is different from Pesendorfer's since we do not rely on signalling arguments, and we do not focus explicitly on fashion goods, even though our model could be applied to the fashion market. Nevertheless we share Pesendorfer's interest in the welfare analysis of exclusivist-conformist effects. Yang and Barrett (2002) study a continuous time optimization model considering a monopoly characterized by nonconcave and nonmonotonic network externalities. Their model differs from ours since strategic considerations are absent, and they do not study neither the case of overloaded networks, nor social welfare. Lambertini and Orsini (2005) study the existence of the equilibrium in a duopoly with Bertrand competition and endogenous choice of the locations. In their framework the network externality can be only monotone increasing, and the welfare analysis is not explicitly performed. We extend their contribution allowing it for nonmonotonicity and overloading. Our paper also relates to Grilo et al. (2001), who studies as well a duopoly with Bertrand competition, but with exogenously given locations of the firms. Even if they allow the network effect to be nonmonotone, in fact they focus on the role played by the *total* size of the network in the market equilibrium and mostly on the case of monotonic externalities, without considering the possibility for overloading. Our paper generalizes the functional form of the network externality adopted in their paper, and focuses on the role played by the number of people patronizing the same store.[4] Moreover, in our paper the firms locate endogenously, and we perform welfare analysis studying when the network effects are beneficial. Finally, Grilo and Friedman (2005) consider a circular city model where consumers care about the others' identity. They study the optimal number of firms entering the market, and compare the case in which the network effect depends on consumers' identity with the case in which consumers are anonymous. Here we focus on anonymous consumers who care only about the number of people patronizing their store, we consider a standard differentiation model with quadratic transportation costs, and our welfare analysis targets the role of network effects on social welfare.

The paper is organized as follows. In the following section we present the duopoly sequential game and study the existence of an equilibrium. In section 3 we perform the social welfare analysis and we discuss the desirability of conformism, and in the last section we provide conclusions and directions for future research.

3.2 The model

Consider two firms, A and B, whose locations are x_A and x_B, where x_i, $i = A, B$ belongs to the compact subset $[\underline{x}, \overline{x}] \subset \mathbb{R}$. They sell at price p_i, have no production costs, and their locations are determined endogenously. The consumers are uniformly distributed over the interval $[0, 1]$ and n is their total mass. A consumer's indirect utility function is given by

$$U_i = K - p_i - t(x - x_i)^2 + E(n_i),$$

where K is the gross utility from consumption; $t(x - x_i)^2$ is the total transportation cost, where $x \in [0, 1]$ is the location of the consumer and $t > 0$ is the unit transportation cost; n_i is the number of consumers patronizing store i, such that $\sum_i n_i = n$. The last term represents the network effect function. Analogously to Grilo et al. (2001), we define it as follows:

$$E(n_i) = \alpha n_i - \beta n_i^2, \tag{3.1}$$

[4] Of course, the total mass of consumers is going to play as well an important role in the determinants of the equilibrium and of social welfare.

and throughout the paper we will assume that $\alpha, \beta \geq 0$.[5] Notice that the network effect depends only on the number of consumers buying from the firm i, and not on the total mass of consumers. The network effect function is depicted in Figure 1. When the consumers' mass n is lower than $\alpha/2\beta$, it is clear that the individuals can face only an increasing network effect. By contrast, if $n > \alpha/2\beta$ the function can be nonmonotone. We will exploit this fact to explore the role of monotonicity in this environment. The "OverLoading" threshold is n_{OL} [6], over which the excessive crowding of the market generates disutility.[7] The model reduces to a standard positional (or status) goods model setting $\alpha = 0$, to a standard linear network effect model setting $\beta = 0$. The comparison with the case in which the network effect is absent can be easily obtained by setting $\alpha = \beta = 0$.

The market is modelled as a two stage game, where in the first stage the firms choose their locations over the interval $[\underline{x}, \overline{x}] \subset \mathbb{R}$, and in the second stage choose prices. The equilibrium is derived by backward induction as a subgame perfect Nash equilibrium in pure strategies.[8] We proceed to derive the solution of the second stage of the game.

3.2.1 Undercutting incentives and price stage

We denote with $\widehat{x}$ the position of the consumer indifferent between buying from A or B. Her position can be derived by setting $U_A(\widehat{x}) = U_B(\widehat{x})$, and by

[5] We thus assume that the function is always concave. Considering the possibility for convexity could be a useful extension of this paper. For a case in which the externality function is convex, see Yang and Barret (2002).

[6] It can be alternatively thought as congestion, or a vanity effect. Grilo et al. (2005) give two definitions in the same paper for vanity. The first is given by considering only the externality $E(n_i)$, and it says that vanity is displayed in consumer behavior when $\alpha < 0$, because consumers are always worse off when the size is increasing. This is true, but incomplete, since as noticed in the text, $E(n_i)$ can be negative also with $\alpha > 0$ and $n_i > \alpha/\beta$. In the same paper Grilo et al. (2005) state as well that consumer's preferences display vanity or conformity depending on whether the value of the externality is lower or greater than the transportation costs. Yang and Barret (2002) tribute the nonmonotonic shape of the network externality to the sum of a functionality effect and an exclusivity effect.

[7] The possibility for overloading was allowed also in Grilo et al. (2005), Young and Barret (2002), but the two papers nevertheless do not consider overloading (Grilo et al. limit the analysis to $n_i < \alpha/2\beta$, with brief considerations in the conclusions for the other case, and Young and Barret leave it for future extensions).

[8] We limit ourselves to pure strategies and concave profit functions. When the pure-strategy equilibrium does not exist, a mixed-strategy equilibrium always exists in a finite strategic-form game. (see e.g. Nash, 1950; Osborne and Pitchik, 1987).

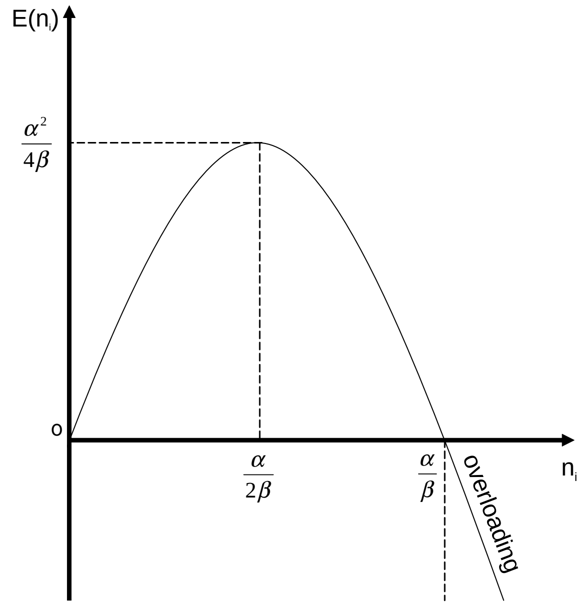

Figure 3-1 The network effect function

satisfying the condition $n_A = \widehat{x}n$ and $n_B = (1-\widehat{x})n$. Then the indifferent consumer is given by:

$$\widehat{x} = \frac{p_A - p_B + t(x_B^2 - x_A^2) - \alpha n + \beta n^2}{2[t(x_B - x_A) - \alpha n + \beta n^2]}. \tag{3.2}$$

Focusing on the case of an interior equilibrium, the two firms maximize the profit $\Pi_i = p_i n_i$ choosing the price level. The solution for the prices at this stage is given by

$$p_A^* = \frac{t}{3}(x_B - x_A)(2 + x_B + x_A) - \alpha n + \beta n^2 \tag{3.3}$$

$$p_B^* = \frac{t}{3}(x_B - x_A)(4 - x_B - x_A) - \alpha n + \beta n^2. \tag{3.4}$$

Notice that whenever $n > n_{OL,}$ that is the total size of the market allows at least one of the firms network to be overloaded, firms increase prices with respect to the standard case where $\alpha = \beta = 0$. This leads to the following Proposition.

Proposition 3.1 *If the total network size allows at least one of the firms' network to be overloaded, the network effect is anti-competitive.*

Any of the two firms can have incentives, provided that the other is playing p_i, to capture the whole market by undercutting the price. In the following Lemma we check that in fact an equilibrium in prices does exist.

Lemma 3.1 *Undercutting at the price stage is never profitable.*

Proof. The two firms are symmetric, we will thus provide the proof only for firm A's incentives to undercut. Firm A considers (3.4) as given, and to undercut sells at a price such that

$$\widehat{x}(p_A) = \frac{6n(\alpha - \beta n) + 3p_A + 2t(x_A - x_B)(2 + x_A + x_B)}{6[n(\alpha - \beta n) + t(x_\alpha - x_\beta)]} = 1.$$

Solved for p_A and inserting into firm A's profits gives

$$\Pi_A^c = -\frac{2}{3}t(x_A - x_B)(x_A + x_B - 1)$$

i.e. firm A's undercutting profits, which are positive if and only if either $x_A > x_B \wedge x_B < 1/2 \wedge x_A + x_B < 1$, or $x_B > 1/2 \wedge x_A < x_B \wedge x_A + x_B > 1$. Let Π_A^* be the equilibrium profits, then undercutting is profitable if and only if $\Pi_A^* - \Pi_A^c < 0$, that is

$$\frac{n[t(x_A - x_B)(x_A + x_B - 4) - 3(\alpha n - \beta n^2)]^2}{18[t(x_B - x_A) - (\alpha n - \beta n^2)]} < 0,$$

which is never verified for any of the relevant values of the locations and of the parameters. The proof of the theorem for firm B's can be analogously obtained by inverting the indexes. ■

The equilibrium prices at the price stage are thus p_A^* and p_B^*. What follows takes into account the conditions for positive prices. In the next subsection we proceed to derive the condition for the existence of the equilibrium and the endogenous location choices of the firms.

3.2.2 The existence of an equilibrium and the location stage

We first study the conditions for a subgame-perfect equilibrium in pure strategies to exist. We then derive the candidates for the optimal locations, confining ourselves to the case of concave profit functions, and study whether there are any undercutting incentives to change the locations. Finally, we study the characteristics of the equilibrium.

Inserting p_i^* into the profit functions, $\Pi_i(x_A, x_B)$ depends only on the firms' locations. Considering the symmetry of the two firms, the second order conditions (SOCs henceforth) are given by

$$\frac{\partial^2 \Pi_i}{\partial^2 x_i^2} < 0. \tag{3.5}$$

The condition for (3.5) to be verified are stated in the following Lemma.

Lemma 3.2 $\frac{\partial^2 \Pi_i}{\partial^2 x_i^2} < 0$ *if and only if the conditions of either of the following cases are satisfied:*

1. $0 < \alpha n \leq \frac{\alpha^2}{2\beta} \wedge \alpha n - \beta n^2 > \frac{3}{2}t$;
2. $\frac{\alpha^2}{2\beta} < \alpha n < \frac{\alpha^2}{\beta} \wedge (\alpha n - \beta n^2 < \frac{9}{8}t \vee \alpha n - \beta n^2 > \frac{3}{2}t)$;
3. $\alpha n \geq \frac{\alpha^2}{\beta}$.

The proof is in the Appendix.

Let's now consider the solution of the location stage. Deriving the first order conditions, solving for x_A and x_B gives five critical points. The only candidate equilibrium such that the SOCs are verified is at the locations $x_A = -\frac{1}{4}$ and $x_B = \frac{5}{4}$, which are consistent with the results in the previous literature on product differentiation.[9]

The nonmonotonic network effects introduce conditions on the positivity of profits, as stated in the following lemma.

Lemma 3.3 *Consider the two locations* $x_A = -\frac{1}{4}$ *and* $x_B = \frac{5}{4}$*. The firms' profits*

$$\Pi_i = \frac{3}{4}nt - \frac{n}{2}(\alpha n - \beta n^2), \tag{3.6}$$

are positive if and only if either:

1. $0 < \alpha n \leq \frac{3}{2}t$;
2. $\alpha n > \frac{3}{2}t \wedge \alpha n - \beta n^2 < \frac{3}{2}t$;

for any $\alpha, \beta, n, t > 0$.

[9] The equilibrium locations of the firms are outside the extremes of the consumers' positions, and are the same of those in the game without network effects. What is interesting is that the existence of an externality does not influence the strategic choice of the firms, see also Lambertini and Orsini (2005).

The proof is in the Appendix. Note that the first term of (3.6) represents the profits of the firms absent the network effect. The second part thus represents the component of the profits that is owed to the presence of the effects. If the term in brackets is positive, the network effect affects negatively the profits of the firms. In other words, profits decrease if the network effect is sufficiently low so as not to allow for overloading even if all consumers would be served by the same firm.

At $(-\frac{1}{4}, \frac{5}{4})$ the equilibrium prices are $p_i = \frac{3}{2}t - E(n)$. As long as $E(n) < \frac{3}{2}t$, the prices are thus above the marginal costs (here set equal to zero). Therefore, at the location stage each firm may displace its location so as to undercut the rival and capture the whole market. The following Lemma states the conditions under which this may happen.

Lemma 3.4 *At the two locations $x_A = -\frac{1}{4}$ and $x_B = \frac{5}{4}$, firm A monopolizes the market by displacing its location if and only if*

$$\alpha n \geq \frac{3}{2}t \wedge \frac{5}{6}t \leq \alpha n - \beta n^2 < \frac{3}{2}t. \tag{3.7}$$

Proof. The setting is symmetric as in the undercutting price case. We thus consider the possibility for firm A to displace its location, given $x_B = \frac{5}{4}$. Thus we solve for which location $\widehat{x}(x_A, \frac{5}{4}) = 1$, which gives

$$x_A^{'} = \frac{1}{2} - \frac{[48t\ (\alpha n - \beta n^2) + 9t^2]^{\frac{1}{2}}}{4t};\ x_A^{''} = \frac{1}{2} + \frac{[48t\ (\alpha n - \beta n^2) + 9t^2]^{\frac{1}{2}}}{4t} \tag{3.8}$$

This means that the locations of firm A such that all the consumers buy from A are symmetrical with respect to the centre of the location interval. We thus study the incentive to undercut B when $x_A = x_A^{'}$. In this case, the profits from displacement are given by:

$$\Pi_A^{dis} = \frac{n[48t\ (\alpha n - \beta n^2) + 9t^2]^{\frac{1}{2}}}{2} - 2n(\alpha n - \beta n^2) - \frac{3}{2}nt. \tag{3.9}$$

Firm A's displacement profits are positive if and only if $0 < n < \frac{\alpha}{\beta}$ and $\alpha n - \beta n^2 < \frac{3t}{2}$, i.e. if the network effect evaluated at the total size is not overloading, and the level of the externality is sufficiently low. The profits for firm A at the $(x_A = -\frac{1}{4}, x_B = \frac{5}{4})$ equilibrium are given by

$$\Pi_A = \frac{n(\alpha n - \beta n^2)}{2} - \frac{3}{4}nt \tag{3.10}$$

and they are positive, according to Lemma 3, if and only if $\alpha n > \frac{3}{2}t \wedge \alpha n - \beta n^2 < \frac{3}{2}t$; or always when $0 < \alpha n < \frac{3}{2}t$. Let's call $\triangle\Pi = \Pi_A^{dis} - \Pi_A$.

Firm A has an incentive to displace its location and capture the rival's share whenever $\triangle\Pi > 0$. Taking into account the conditions $\Pi_A^{dis} > 0$ and $\Pi_A > 0$, this happens if and only if either of the following holds:

1. $\frac{5}{6}t < \alpha n < \frac{3}{2}t \wedge (\alpha n - \beta n^2) > \frac{5}{6}t$;
2. $\alpha n \geq \frac{3}{2}t \wedge \frac{5}{6}t \leq (\alpha n - \beta n^2) < \frac{3}{2}t$.

The first can be ruled out by checking the SOCs. ■

Considering jointly Lemmata 1 - 4, in the following proposition we state the main result of this section.

Proposition 3.2 *The two locations $x_A = -\frac{1}{4}$ and $x_B = \frac{5}{4}$ are the unique subgame perfect equilibrium of the game in pure strategies if and only if either of the following holds:*

1. $\alpha n - \beta n^2 > \frac{3}{2}t \wedge \frac{\alpha-(\alpha-6\beta t)^2}{2\beta} < n < \frac{3t}{2\alpha}$;
2. $\alpha n - \beta n^2 < \frac{5}{6}t \wedge n > \frac{3\alpha+\sqrt{3}(3\alpha-10\beta t)^{\frac{1}{2}}}{4\beta}$;

and no subgame perfect equilibrium in pure strategies exists otherwise.

Proof. The proof is readily obtained by merging the conditions of Lemmas 2 - 4. Let's first consider the monotonic increasing part of $E(n)$, i.e. when $n < \frac{\alpha}{2\beta}$. By Lemma 1, the SOCs hold only if $E(n) > \frac{3}{2}t$, which implies $\frac{\alpha-(\alpha-6\beta t)^2}{2\beta} < n$; by Lemma 2 profits are positive for both firms if $0 < \alpha n \leq \frac{3}{2}t$, i.e. $0 < n \leq \frac{3t}{2\alpha}$. Thus, joining the two inequalities we get $\frac{\alpha-(\alpha-6\beta t)^2}{2\beta} < n \leq \frac{3t}{2\alpha}$, which leads to the first part of the proposition. Notice that by Lemma 4 in this range of n no undercutting incentives are at work, and of course if $t > \frac{\alpha^2}{3\beta}$ no subgame perfect equilibrium in pure strategies exists.

Consider now the monotonic decreasing part of $E(n)$. By Lemma 2, if $n > \frac{\alpha}{\beta}$ the SOCs are always met in the relevant range of the parameters. By Lemma 3 the positive profits' condition hold, since $E(n) < 0$. Finally, by Lemma 4 there are no incentives for displacement in this region. Let's now consider the case in which $\frac{\alpha}{2\beta} < n < \frac{\alpha}{\beta}$. In this region, SOCs require $E(n) < \frac{3}{2}t$ and profits' positivity is obtained if $E(n) < \frac{9}{8}t$. The latter is a more stringent condition. Nevertheless, by Lemma 4 in this region incentives for displacement are at work if $\frac{5}{6}t \leq E(n) < \frac{3}{2}t$, thus a subgame perfect equilibrium exists only for $E(n) < \frac{5}{6}t$, which corresponds to $n > \frac{3\alpha+\sqrt{3}(3\alpha-10\beta t)^{\frac{1}{2}}}{4\beta}$.This provides the second part of the Proposition. ■

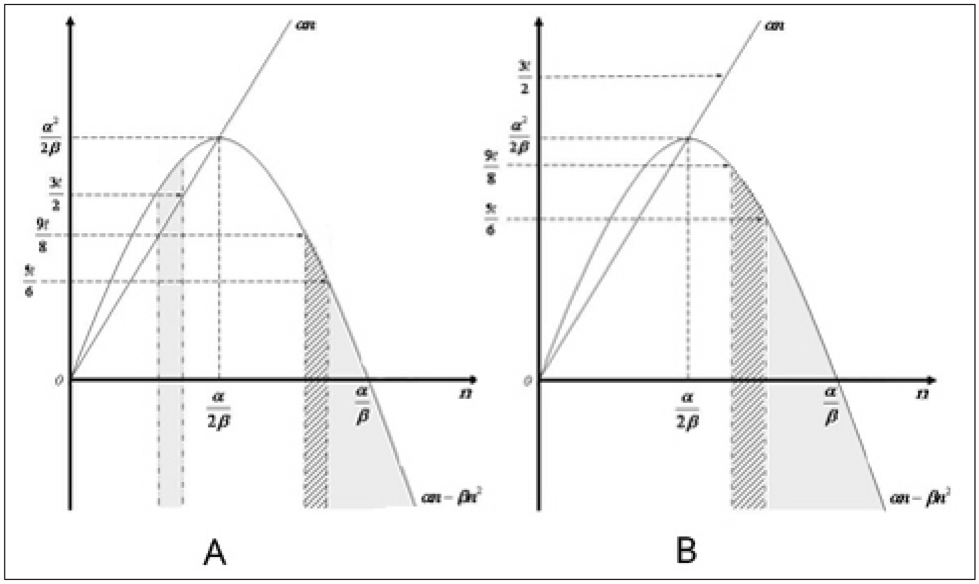

Figure 3-2 Overloading is feasible.

The intuition for the proposition is represented in Figures 2-3, which show four possible cases emerging. Remember that the figure represents the conditions on the *total size* of the network, but the parameters are determined by the network effect on consumers' preferences for the number of people *patronizing the same store*.

In Figure 2 the total network size can be greater than α/β, i.e. there is the possibility for overloading. Consider A: the parabola represents the network effect evaluated at n, while the straight line is the linear positive component of it, which obviously cuts it in its maximum. The shadowed areas represent the locus where an equilibrium in pure strategies exists. On the right part of the parabola, the dashed area represents the part of the existence locus which is eroded by incentives to displace.

In this case an equilibrium exists both on the increasing side and on the decreasing size of the network effect. Consider now figure B: when the transportation cost is sufficiently high (that is when 3/2 of it are greater than the maximum value of the network effect), the equilibrium does not exist any more on the left side of the parabola. On the contrary, on the right side both the undercutting and the equilibrium spaces are increased. In both case A and case B notice that an equilibrium may exist as well in the overloading area[10].

[10] As we will show in Lemma 5, the firms have unilateral incentives to deal with a market which could be overloaded.

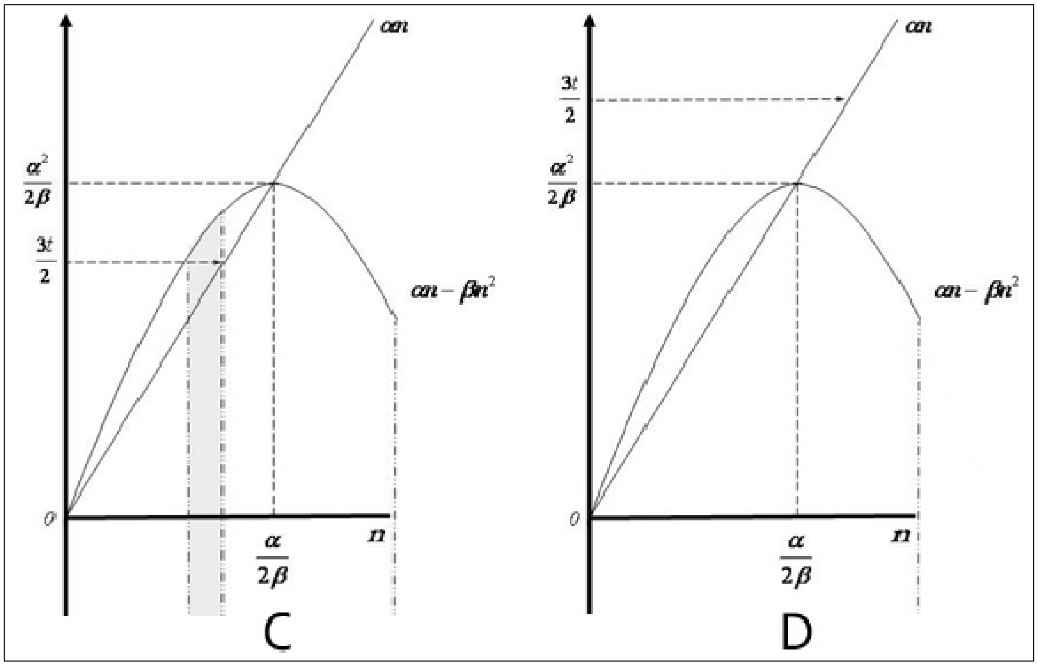

Figure 3-3 Overloading is not feasible.

Let's now consider in Figure 3 the case in which the total network size is such that overloading is impossible. In this case, the equilibrium does not exist anymore on the right side of the parabola. It is thus striking that the existence of the equilibrium in that area depends on the possibility for overloading. Figure 3C shows that an equilibrium can exist on the increasing side of the network effect if the transportation rate is sufficiently low (as in figure 2A). When this is not verified, figure 3D shows the case in which no equilibrium exists.

A further comment on the above proposition concerns the *kind* of differentiation of the products. As observed in Grilo et al. (2001), the differentiation in the model can be interpreted as *horizontal* if $0 < x_A + x_B < 2$, and *vertical* if either $x_A + x_B > 2$ or $x_A + x_B < 0$. The following Corollary thus stems directly from Proposition 1.

Corollary 1 *If the firms choose endogenously their locations, at the equilibrium only horizontal differentiation can result.*

This result thus suggests that the choice of vertical differentiation should be driven by some rigidities in the location choice, so that horizontal differentiation is either non-available or does not entail a market equilibrium. We now proceed in the following section to perform the analysis of social welfare.

3.3 Social welfare and conformism desirability

In this section we study how the presence of the network effect influences social welfare, and the role played by concavity, monotonicity, and overloading. The consumer's surplus evaluated at the equilibrium values obtained in Proposition 1 is given by

$$\begin{aligned} CS &= \int_0^{\widehat{x}} K - p_A - t(x-x_A)^2 + \alpha n_A - \beta n_A^2 \, dx + \\ &\quad \int_{\widehat{x}}^{1} K - p_B - t(x-x_B)^2 + \alpha n_B - \beta n_B^2 \, dx \\ &= K - \frac{13}{48}t + \frac{1}{4}(2\alpha n - \beta n^2). \end{aligned} \tag{3.11}$$

The last term is the component of the consumer's surplus deriving from the network effect. The following Lemma states when the presence of the externality positively affects CS and industry profits.

Lemma 3.5 *The network effect increases*

1. *consumers' surplus if and only if* $0 < n < \frac{\alpha}{\beta}$;
2. *firms' profits if and only if* $n > \frac{\alpha}{\beta}$.

See the Appendix for the proof. An interpretation of this result is that a positive network effect in the utility function (which depends only on the number of people consuming the good at the same firm) in equilibrium translates into a negative effect of opposite sign in the firms' profits, depending on the *total mass*. The consumers' preferences nevertheless determine the shape of the network effect evaluated at n. The firms thus have an inherent preference for markets in which the parameter α is low and concavity β is high. Looking at (3.11) and (3.6), notice that the consumers' surplus is decreasing, and the profits are increasing in the concavity of the effect.

Comparing the parts of the above Lemma, it is thus clear that, for some range of the parameters, the network effect increases industry profits while decreasing consumer's surplus. This happens when the size of the market allows it for overloading in the network patronizing either of the firms. Summing up, the net effect on social welfare is ambiguous, and there is the suspicion that the degree of concavity is crucial at determining social welfare. Social welfare is given by the sum of the consumer surplus and industry profits, that is:

$$SW = CS + \sum \Pi_i = K + (\frac{3}{2}n - \frac{13}{48})t + \frac{\alpha n}{2}(1 - 2n) + \frac{\beta n}{4}(4n - 1). \quad (3.12)$$

Notice that SW is increasing in β if and only if $n > \frac{1}{4}$. We now want to compare (3.12) with the situation in which the effect is absent, that is if $\alpha = \beta = 0$. Social welfare with no externality is then given by:

$$SW^{ne} = K + (\frac{3}{2}n - \frac{13}{48})t. \quad (3.13)$$

The comparison of SW and SW^{ne} leads to the following Proposition, which states under what conditions the presence of the network effect increases social welfare.

Proposition 3.3 *Let* $\alpha, \beta > 0$.

The network effect increases social welfare if and only if either of the followings holds:

1. $\frac{1}{2} < n < \frac{\alpha}{\beta} \wedge \beta < \frac{\alpha(2-4n)}{n-4n^2}$;
2. $\frac{\alpha}{\beta} < n < \frac{1}{4} \wedge \beta > \frac{\alpha(2-4n)}{n-4n^2}$.

The proof is in the Appendix. The first part of the above proposition states the condition when the total mass of consumers does not allow for overloading. In this case, the network effect is desirable only if the mass of the population is sufficiently high and the concavity of the network effect is sufficiently small. The second part of the proposition considers the possibility for overloading. With respect to the other case, the externality is socially desirable only if the consumers' mass is low and the concavity sufficiently high. Notice, referring to Lemma 5, that the industry profits are increased by the network effect only in case 2.

Is thus conformism desirable? The above results show that the answer to this question depends on *how* people are conformist. If the consumers' mass does not allow for overloading (in either of the two firms), and is sufficiently high, then a network effect not too much concave increases social welfare. By contrast, if the consumers' mass allows for overloading, and is sufficiently small, a network effect more concave than in the previous case is needed to increase social welfare.

3.4 Conclusions

In this paper we studied how the existence of nonmonotonic network effects influences the welfare and the equilibrium properties of a Bertrand duopoly where the choice of locations is endogenous. We have shown the conditions for a firm to capture the rival's share at the location stage, and when an equilibrium in pure strategies exists. We then asked whether and how the presence of networks effects influences social welfare, and the answer is that it depends on the shape of the networks effect. Indeed, consumer's surplus is increasing while industry profits are decreasing in its level of concavity, thus creating ambiguity in the overall effect on social welfare. Considering social welfare, the determinants of the results are three: the consumer mass, the possibility of overloading, and the concavity of the network effect. We found that social welfare is increased by a network effect with small concavity when the consumers' mass is high and overloading is not feasible. By contrast, when the consumers' mass is low and overloading is feasible, social welfare is increased by a network effect that is highly concave.

The present work suggests that *how* people are conformist is an important part of the study of network effects. Nevertheless, it limits its analysis to a linear Bertrand duopoly, and to a specific network effect shape. Future research could explore a more general framework in which the network effect has a general shape, and in which the transmission of cultural values of collectivist societies versus individualism is challenged. By contrast, studying in detail markets with specific characteristics, such as the physical location of retailers or of restaurants and pubs, as well as the fashion designers choices in the space characteristics of the clothes, could provide useful insights on the determinants of social welfare, and eventually on regulation and public policy. Finally, it is interesting noticing that Mooij and Hofstede (2002) studying 14 different countries, without suggesting a causal effect, find that the number of café per million of inhabitants is negatively correlated with the degree of individualism. Given the existence of evidence about differences in the tolerance of crowding across cultures, an empirical study could attempt to find how different attitudes towards socialization marginally impacts the density and the locations of shops and retailers.

Acknowledgements

I thank my supervisor, Vincenzo Denicolò, and Raimondello Orsini for useful suggestions. I am also grateful to STICERD - The London School of Economics and Political Science, where part of this chapter has been written while I was a visiting research student. The usual disclaimer applies.

References

Eroglu, S., & Harrell, G.D. 1986. Retail crowding: theoretical and strategic implications. *Journal of retailing*, **62**, 346–363.

Eroglu, S.A., Machleit, K., & Barr, T.F. 2005. Perceived retail crowding and shopping satisfaction: the role of shopping values. *Journal of business research*, **58**, 1146–1153.

Friedman, J.W., & Grilo, I. 2005. A market with a social consumption externality. *The japanese economic review*, **56**, 251–272.

Grilo, I., Shy, O., & Thisse, J.F. 2001. Price competition when consumer behavior is characterized by conformity or vanity. *Journal of public economics*, **80**, 385–408.

Guiso, L., Sapienza, P., & Zingales, L. 2006. Does culture affect economic outcomes? *The journal of economic perspectives*, **20**, 23–48.

Hofstede, G. 2001. *Culture's consequences: Comparing values, behaviors, institutions, and organizations across nations.* Sage Pubns.

Kaya, N., & Weber, M.J. 2003. Cross-cultural differences in the perception of crowding and privacy regulation: American and Turkish students. *Journal of environmental psychology*, **23**, 301–309.

Lambertini, L., & Orsini, R. 2005. The existence of equilibrium in a differentiated duopoly with network externalities. *The japanese economic review*, **56**, 55–66.

Nash, J.F. 1950. Equilibrium points in n-person games. *Proceedings of the national academy of sciences of the united states of america*, 48–49.

Osborne, M.J., & Pitchik, C. 1987. Equilibrium in Hotelling's model of spatial competition. *Econometrica: Journal of the econometric society*, 911–922.

Pesendorfer, W. 1995. Design innovation and fashion cycles. *The american economic review*, **85**, 771–792.

Pons, F., & Laroche, M. 2007. Cross-cultural differences in crowd assessment. *Journal of business research*, **60**, 269–276.

Pons, F., Laroche, M., & Mourali, M. 2006. Consumer reactions to crowded retail settings: Cross-cultural differences between North America and the Middle East. *Psychology and marketing*, **23**, 555–572.

Tabellini, G. 2008. Institutions and culture. *Journal of the european economic association*, **6**, 255–294.

Yang, Y.N., & Barrett, C.B. 2002. Nonconcave, nonmonotonic network externalities. *Taipei economic enquiry*, **38**, 1–22.

4. PARTIAL COLLUSION WITH ASYMMETRIC CROSS-PRICE EFFECTS

Abstract

Asymmetries in cross-price elasticities have been demonstrated by several empirical studies. In this paper we study from a theoretical stance how introducing asymmetry in the substitution effects influences the sustainability of collusion. We characterize the equilibrium of a linear Cournot duopoly with substitute goods, and consider substitution effects which are asymmetric in magnitude. Within this framework, we study partial collusion using Friedman (1971) solution concept. Our main result shows that the interval of quantities supporting collusion in the asymmetric setting is always smaller than the interval in the symmetric benchmark. The asymmetry in the substitution effects thus makes collusion more difficult to sustain. This implies that previous Antitrust decisions could be reversed by considering the role of this kind of asymmetry.

JEL classification: L14, D62.

Keywords: asymmetry, substitutes, Cournot duopoly, collusion, folk theorem.

4.1 Introduction

In this paper we study the sustainability of collusion in a Cournot duopoly where market demands are asymmetric in the magnitude of substitution effects. The evidence of asymmetric cross-price elasticities is shown in several empirical studies (e.g. Berry et al., 1995; Serthuraman et al., 1999; Kim and Cotterill, 2008; Rojas and Peterson, 2008). For example, in Berry et al. (1995) the cross-price semi-elasticity between Nissan Sentra and Ford Escort is 1.375, while between Ford Escort and Nissan Sentra is 8.024; in the US market for processed cheese Kim and Cotterill (2008) find that the cross-price elasticity between Weight Watchers and Kraft is 0.25, while between Kraft and Weight Watchers is 0.04; the cross-price elasticity between Bud and Old Style is 0.003, while the cross-price elasticity between Old Style and Bud is 0.242 (Rojas and Peterson, 2008); finally, in a meta-analysis of 1060 cross-price effects, 19 grocery product categories and 280 brands, Serthuraman et al. (1999) provide an empirical generalization of this asymmetric price effect. The evidence supports the idea that in general cross-price elasticities are not symmetric, and this fact is also compatible with a theoretical perspective, since aggregate market demands need not satisfy any symmetry condition (see e.g. Diewert, 1980; Bonfrer et al. 2006), and also at the individual level the Slutsky matrix need not be symmetric (because the income effect need not be symmetric).

In this paper we consider a linear duopoly model with Cournot competition and substitute products. We extend Singh and Vives (1984) by allowing for asymmetry in the magnitude of the substitution effects, deriving the equilibrium quantities, prices and profits. We compare them to those in a symmetric equivalent duopoly setting, namely a duopoly in which the substitution parameters are symmetric and equal to the average of the parameters in the asymmetric case. Then, using the folk theorem solution concept and the penal code according to Friedman (1971), we analyze partial collusion. We derive the range of collusive quantities that makes the collusion sustainable for both firms, and compare it to the symmetric equivalent case.

This paper adds the two following main contributions to the existing literature. First, even though an extensive literature on collusion considers other kinds of asymmetries in market demands and in the characteristics of firms (for an overview see e.g. Feuerstein, 2005), to the best of our knowledge, the effect of asymmetric cross-price effects on the stability of collusion has not yet been studied from a theoretical stance. Our main result shows that, given the discount rate of each firms, the interval of quantities which supports the collusion is always smaller than in the symmetric equivalent case. Intuitively, if firms are asymmetric the intervals of the quantities making the collusion stable are no longer coincident, and the effect of asymmetry is to shift each firm's interval in opposite directions. This happens since a high collusion quantity raises the relative value of the deviation strategy for the weak firm, whose production decision is relatively more influenced by the other. Similarly, a low level of the collusion quantity makes the deviating behavior more convenient for the strong firm. Only the intermediate levels of collusive quantity are therefore supported by both firms. In this stylized setting, we thus conclude that the asymmetry in the substitution effect makes collusion more difficult to sustain. This implies that previous antitrust decisions could be reversed by considering the role of this kind of asymmetry.

The second result is related to the characterization of the equilibrium in the asymmetric Cournot duopoly. We find that the firm whose price is relatively more influenced by the other firm's production decisions sets a lower quantity, and sells at a lower price with respect to the rival, getting lower profits. An intuition for this result is that, since the two goods are strategic substitutes (but asymmetrically) the production decisions are driven by the firm which is relatively less influenced by the rival. The symmetric equivalent firm equilibrium prices, profits and quantities lie between those of the strong and the weak firm.

The paper is structured as follows. In the next section we characterize the equilibrium condition for the asymmetric Cournot duopoly and compare it to the symmetric equivalent case. In the third section we derive the solution of the collusion supergame, and we state the results on the implicit collusion stability. In the fourth section we draw the final remarks and directions for future theoretical and empirical research. The proofs are relegated to the appendix.

4.2 A linear Cournot duopoly with asymmetric substitution effects

In this section we consider a Cournot duopoly which extends Singh and Vives (1984) allowing for asymmetry in the magnitude of the substitution effects. This extension does not need any particular assumption, since market demands need not satisfy any symmetry conditions such as those required by the Slutsky equation (see e.g. Diewert, 1980; Bonfrer et al., 2006). Moreover, if the income effect in the Slutsky equation is not symmetric, the individual demands are not symmetric in the cross-price effect as well. Mastroleo and Savorelli (2010) show which class of individual utility functions underlies the asymmetry in cross substitution effects.

Let us consider a duopoly where firm i and firm j face the following inverse demand function

$$p_i = a - q_i - b_i q_j \ , \{i, j\} = \{1, 2\} \tag{4.1}$$

where $b_i \in [0, 1]$, $a > 0$ and $q_{i,j} \geq 0$.We normalize the own-price effect to one, but this is not going to affect the results qualitatively. If $b_i \neq b_j$, we say that the demands are *asymmetric*, and if $b_i = b_j$ we say that they are *symmetric*. In a Cournot competition setting, each firm chooses the production quantity to maximize its profits as follows:

$$\max_{q_i} \pi_i = (p_i - c)q_i \tag{4.2}$$

where c is a constant marginal cost. The solution of the model leads to the equilibrium quantities and profits

$$q_i^C = \frac{(b_i - 2)(a - c)}{b_i bj - 4} \tag{4.3}$$

$$\pi_i^C = \frac{(b_i - 2)^2 (a - c)^2}{(b_i bj - 4)^2} \tag{4.4}$$

$$\Pi^C = \pi_i + \pi_j = \frac{[(b_i - 2)^2 + (b_j - 2)^2](a - c)^2}{(b_i bj - 4)^2}. \tag{4.5}$$

We assume henceforth that $a > c$ to guarantee that the Cournot equilibrium quantities are positive. To have an intuition about the strategic interaction driving the results, let us consider the following best response functions:

$$q_i^C = \frac{(a - c - b_i q_j^C)}{2} \tag{4.6}$$

$$q_j^C = \frac{(a - c - b_j q_i^C)}{2}. \tag{4.7}$$

By simple inspection it is immediately apparent that the two goods are strategic substitutes. Then, taking the cross derivative of each best response function leads to the following Remark.

Remark 2 *Consider* $\frac{dq_i^C}{dq_j^C} = -\frac{b_i}{2}$ *and* $\frac{dq_j^C}{dq_i^C} = -\frac{b_j}{2}$ *, then:*

1. *if* $b_i > b_j$ *,*$\frac{dq_i^C}{dq_j^C} < \frac{dq_j^C}{dq_i^C}$*;*

2. *if* $b_i < b_j$ *,*$\frac{dq_i^C}{dq_j^C} > \frac{dq_j^C}{dq_i^C}$*.*

The above Remark states that, if e.g. $b_i > b_j$, an expansion in firm j production has an impact on i's quantity choice greater than the impact that an equivalent expansion by i has on the quantity choice of j. For this reason, we call i the *weak substitute* and j the *strong substitute* (and vice versa if $b_i < b_j$). Consistently, the strong substitute produces a higher quantity and gets higher profits with respect to the weak. Henceforth, the equilibrium results associated with the asymmetric demands will be denoted by the superscript ASY.

To allow for a comparison with the asymmetric setting, we exploit as benchmark a special case of the symmetric setting, such that $b_i = b_j = \frac{b_i+b_j}{2}$. The substitution parameters are chosen to be the mean of the parameters in the asymmetric case. We call the benchmark setting a *symmetric equivalent,* and we denote the associated equilibrium values by the superscript SE. In the symmetric equivalent case the solution of problem 4.2 is

$$q_i^{SE} = q_j^{SE} = \frac{2(a-c)}{4+b_i+bj} \quad (4.8)$$

$$\pi_i^{SE} = \frac{4(a-c)^2}{(4+b_i+b_j)^2}. \quad (4.9)$$

To provide an intuitive comparison, in Figure 1 (next page) we represent the asymmetric Cournot equilibrium and its symmetric equivalent in the space of quantities when $b_j > b_i$.

The best response functions are represented by the thick lines in the asymmetric case, and by the thin straight lines in the symmetric equivalent. The SE equilibrium is North West of the asymmetric Cournot equilibrium, thus the weak substitute j produces a lower quantity with respect to the SE case, and the strong substitute i a higher quantity. By considering the two parabolas, representing the contour lines of the profit functions at the Cournot equilibrium, it is straightforward to notice the asymmetry in the region of the possible equilibria which are Pareto superior to the Cournot-Nash.

The following Proposition characterizes the solution to problem 4.2 with respect to the equilibrium prices and the quantity levels, and compares them to the symmetric equivalent case.

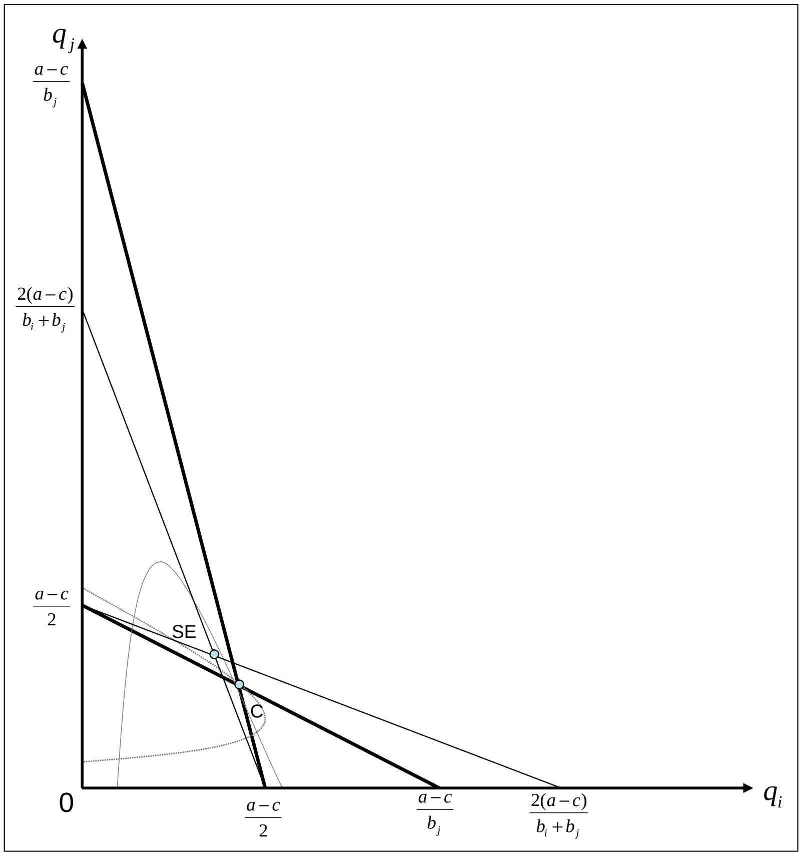

Figure 4-1 The symmetric equivalent and Cournot equilibria.

Proposition 4.1 *Let* $q_i^{SE} = q_j^{SE} = q^{SE}$ *and* $p_i^{SE} = p_j^{SE} = p^{SE}$. *Then the following holds:*

1. $b_i^{ASY} > b_j^{ASY} \Longleftrightarrow q_i^{ASY} < q^{SE} < q_j^{ASY}$;

2. $b_i^{ASY} > b_j^{ASY} \Longleftrightarrow p_i^{ASY} < p^{SE} < p_j^{ASY}$;

3. $b_i^{ASY} > b_j^{ASY} \Longleftrightarrow \pi_i^{ASY} < \pi^{SE} < \pi_j^{ASY}$.

The above Proposition states that in the symmetric equivalent case quantities, prices, and profits lie between the weak substitute's and the strong substitute's. The strategic weakness thus leads to lower quantities, prices and profits with respect to both the symmetric equivalent and the strong substitute.

4.3 Partial collusion

In this section we will investigate how asymmetry in substitutability affects the stability of collusion. As in the standard symmetric case, the implicit level of collusion should not necessarily be the monopoly quantity. The reason is that there are infinite quantities higher than the monopoly one that still provide profits greater than in the Cournot game. Moreover, as we will show in Remark 3, when the discount rate is sufficiently high the monopoly quantity does not make the collusion stable. Given a common discount factor δ, we study the minimum individual quantity produced by a firm that allows collusion to be stable, and to what extent asymmetry influences the stability of collusion.

We will proceed through the following steps. First, we will set up the Cournot supergame and study the collusion and deviation strategies, finding the interval of quantities for each firm that allows collusion to be sustainable. Second, we will state in Proposition 2 which is the interval of collusive quantities that makes collusion sustainable for both. Finally, in Proposition 3 we will state our main result, showing whether asymmetry in the substitution effects makes collusion easier with respect to the symmetric equivalent benchmark.

Over an infinite horizon, the two firms play grim trigger strategies in a Cournot supergame $\Gamma(\infty)$. We use the folk theorem solution concept according to Friedman (1971). In the first stage of the game, $t = 0$, the firms follow a collusive strategy σ^* and maximize the joint profits. In general, profits division is not equal. In the remaining time horizon, if both the firms played the collusive strategy in the previous period, the firms continue to play the collusive strategy σ^*. Otherwise, if at least one firm deviates from the collusive strategy playing σ^D, the firms play the Cournot-Nash strategy σ^C.

We first consider the collusive strategy σ^*. Since there are infinite potential collusive outcomes, we use joint-profit maximization as the selection criterion for the collusive focal point (as in e.g. d'Aspremont et al., 1983; and in asymmetric environments, e.g. Rothschild, 1999). Accordingly, we solve the following problem:

$$\max_{q_i, q_j} \pi_i + \pi_j \tag{4.10}$$

which leads to the equilibrium quantities and profits:

$$q_i^* = q_j^* = \frac{a-c}{2+b_i+bj} \tag{4.11}$$

$$\pi_i^* = \frac{(b_j+1)(a-c)^2}{(2+b_i+b_j)^2}. \tag{4.12}$$

We can then state the following remark, which characterizes the joint-profit maximization equilibrium.

Remark 3 *When firm i and firm j maximize the joint profit, they produce the same quantity, and the weak substitute obtains a level of profit lower than the strong.*

The intuition for this Remark is that, since the two firms are technologically symmetric, when maximizing the joint profit they take into account the strategic externality deriving from the asymmetry in the strategic substitutability, and internalize it by playing like symmetric firms. Thus, it seems reasonable for this equilibrium to be a focal point also for lower levels of joint profits.

When the firms play the σ^* strategy, each firm gets the profits $\pi_i^* = q_i^*(a - c - q_i^* - b_i q_j^*)$. Taking into account the characterization in Remark 2, the firms set $q_i^* = q_j^*$. We can therefore rewrite the above expression as

$$\pi_i^* = q_j^*(a - c - (1+b_i)q_j^*).$$

Notice again that with asymmetric demands the collusive quantities are still symmetric, but the profits are not.

We now derive the equilibrium values for firm i playing σ^D, denoting the solutions with the superscript D. In the period of deviation, firm i solves the following maximization problem:

$$\max_{\{q_i^D \mid q_j = q_j^*\}} \pi_i = (p_i - c)q_i = q_i^D(a - c - q_i^D - b_i q_j^*)$$

and obtains the following per-period quantities and profits:

$$q_i^D = \frac{(a-c-b_i q_j^*)}{2} \tag{4.13}$$

$$\pi_i^D = \frac{(a-c-b_i q_j^*)^2}{4}. \tag{4.14}$$

Thus, the flow of profits for the deviating firm i is

$$\Phi_i^D = \pi_i^D(q_j^*) + \frac{\delta}{1-\delta}\pi_i^{ASY} \tag{4.15}$$

where $\delta \in [0,1]$ is the discount rate common to i and j.

In what follows we will study the problem of collusion stability. We will first derive the interval of quantities of the rival firm that makes the collusion sustainable, and then the interval of quantities on which both the firms agree to collude.

For firm i, the collusive strategy is sustainable only if $\Phi_i^D \leq \pi_i^*(q_j^*)/(1-\delta)$.Then, the following lemma states for which values $q_{i,j}^*$ the collusion is stable for each single firms.

Lemma 4.1 *The collusion is stable for firm i if and only if* $\underline{q}_j^{Col} < q_j^* < \overline{q}_j^{Col}$*,and for firm j if and only if* $\underline{q}_i^{Col} < q_i^* < \overline{q}_i^{Col}$*, where:*

$$\begin{aligned}
\overline{q}_i^{Col} &= (a-c)\frac{(b_ib_j-4)[b_i(\delta-1)-2]+A_i}{(b_ib_j-4)[\delta b_i^2-(2+b_i)^2]} \\
\underline{q}_i^{Col} &= (a-c)\frac{(b_ib_j-4)[b_i(\delta-1)-2]-A_i}{(b_ib_j-4)[\delta b_i^2-(2+b_i)^2]}
\end{aligned}$$

$$\begin{aligned}
\overline{q}_j^{Col} &= (a-c)\frac{(b_ib_j-4)[b_j(\delta-1)-2]+A_j}{(b_ib_j-4)[\delta b_j^2-(2+b_j)^2]} \\
\underline{q}_j^{Col} &= (a-c)\frac{(b_ib_j-4)[b_j(\delta-1)-2]-A_j}{(b_ib_j-4)[\delta b_j^2-(2+b_j)^2]}
\end{aligned}$$

and $A_i = [\delta b_i\ (\delta b_i(b_i-2)^2 + b_i(b_i^2-b_j^2) + 8(b_i-b_j))]^{\frac{1}{2}}$, $A_j = [\delta b_j\ (\delta b_j(b_j-2)^2 + b_j(b_j^2-b_i^2) + 8(b_j-b_i))]^{\frac{1}{2}}$.

Henceforth we will consider only real-valued boundaries quantities.[1] We call $Q_i = \{\underline{q}_i^{Col} < q_i^* < \overline{q}_i^{Col}\}$ and $Q_j = \{\underline{q}_j^{Col} < q_j^* < \overline{q}_j^{Col}\}$ the sets of sustainable collusion quantities for firm i and j. As it is evident by inspection, the two interval do not coincide as long $b_i \neq b_j$. Some collusive quantities are thus sustainable for firm i, but not for firm j. Since it could also be the case that the two intervals do not overlap, in the following Lemma we state when there is room for collusion among firms if the two firms are asymmetric.

[1] When $b_i > b_j$, A_i is always positive and A_j is positive when $\frac{(8b_i-(8+b_i^2)b_j+b_j^3)}{((b_j-2)^2b_j)} < d$ and, in addition, to be real valued, if $b_j < \frac{13-\sqrt{41}}{8} \simeq 0,825$, it must be $b_i < \frac{2(2-[(b_j-2)^2(b_j+1)]^{\frac{1}{2}})}{b_j}$. When $b_i < b_j$, A_j is always positive and analogous conditions can be obtained by inverting indexes.

Lemma 4.2 *If $b_i \neq b_j$, then $Q_i \neq Q_j$ and $Q_i \cap Q_j$ always exists.*

The above Lemma means that there is always an interval of quantities on which the two firms can agree to collude, and which is sustainable for both. Let us call $Q^{ASY} = Q_i \cap Q_j$ the set of collusive quantities which are sustainable for both firm i and firm j. By contrast, when $b_i = b_j$, $\underline{q}_i^{Col} = \underline{q}_j^{Col} = \underline{q}^{Col}$ and $\overline{q}_i^{Col} = \overline{q}_j^{Col} = \overline{q}^{Col}$, and the two sets coincide. In this case, we call the set of collusive quantities Q^{SYM}. The following proposition characterizes the set of collusive quantities that makes the collusion stable for each case.

Proposition 4.2 *When:*

1. $b_i < b_j \Leftrightarrow Q^{ASY} = \{\underline{q}_j^{Col} < q_{i,j}^* < \overline{q}_i^{Col}\}$;
2. $b_i = b_j \Leftrightarrow Q^{SYM} = \{\underline{q}^{Col} < q^* < \overline{q}^{Col}\}$;
3. $b_i > b_j \Leftrightarrow Q^{ASY} = \{\underline{q}_i^{Col} < q_{i,j}^* < \overline{q}_j^{Col}\}$.

First, notice that setting $b_1 = b_2 = 1$, the boundary values in the above Lemma reduce to the well known symmetric case

$$\frac{(9-5\delta)(a-c)}{3(9-\delta)} < q_{i,j}^* < \frac{a-c}{3}.$$

Second, setting $\delta = 0$, the boundaries of the stable collusion quantities are $[\frac{a-c}{2+b_j}, \frac{a-c}{2+b_i}]$ in case 1, and the reverse in case 2. The lower boundary is thus greater than the collusive monopoly quantity, $\frac{a-c}{2+b_j+b_i}$. This leads to the following remark.

Remark 4 *There always exist a value $0 \leq \delta \leq 1$ such that the collusive monopoly equilibrium is not stable.*

The above remark points out that, as in the symmetric case, the monopoly quantity is not always a feasible quantity, and when this is the case the analysis of partial collusion thus becomes more relevant.

We proceed to answer the key research question of this paper, i.e. whether asymmetry in the substitution effects makes collusion more difficult to sustain. Without qualitatively affecting our results, we normalize the parameter space by setting $b_j = 1 - b_i$.We define the index of asymmetry as $\gamma = |b_j - b_i| = |1-2b_i|$.When the two values are symmetric, $b_i = b_j = \frac{1}{2}$ and $\gamma = 0$, while $\gamma = 1$ when one parameter is 0 and the other 1, the maximum level of asymmetry. We then use the average value of the parameter as benchmark, and we state the main result of this paper in the following proposition.

Proposition 4.3 *The interval of collusive quantities in the symmetric equivalent case is always larger than the one in the asymmetric case.*

The intuition behind this proposition can be understood by highlighting the strategic interaction between the two firms. Each firm determines what is the level of production of *the other* firm that makes the collusion stable for *itself*. When firms collude, it is optimal for both to produce the same quantity. Then, it can immediately be seen from (4.13) that, if the weak substitute wants to deviate from the agreement, he is going to produce a lower deviation quantity with respect to the deviation quantity of the strong substitute. The Cournot quantity in the following period is lower as well. The reason is that a high collusion quantity raises the relative value of the deviation strategy for the weak substitute. Analogously, a low level of the collusion quantity makes the deviating behavior more convenient for the strong substitute. The asymmetry in the strategic substitution effect translates into asymmetric partial collusion strategies, which still overlap, but are no longer coincident. Each firm's collusive interval is shifted in opposite directions, and only the intermediate levels of collusive quantity are supported by both firms. We can therefore conclude that the asymmetry in the substitution effect makes collusion more difficult to sustain with respect to the symmetric benchmark case.

4.4 Conclusions

In this paper we generalized Singh and Vives (1984) to account for asymmetry in the substitution effects, and to study its implications for implicit collusion. The first result we found characterizes Cournot equilibrium: The symmetric benchmark firm equilibrium prices, profits and quantities are lower than those of the strong, and higher than those of the weak. The second result states that the asymmetry in the substitution effects makes partial collusion more difficult to support with respect to its symmetric benchmark.

Future extensions of this model could explore the robustness of the results under different settings and kinds of competition (e.g. Bertrand competition, semi-collusion, R&D). Our feeling is that, in frameworks other than collusion, introducing asymmetries in the cross-price effect could lead to new theoretical insights particularly useful to empirical analysis.

Finally, this paper suggests that the asymmetry in the substitution effects is a relevant issue when evaluating the possibility of implicit collusion among firms. If the empirical estimations of market demands do not take into account this kind of asymmetry, it is likely that the extent to which the firms can collude is overestimated. We thus think that there is room for an empirical re-assessment of previous estimations, and that in this light perhaps some anti-trust decisions could be reversed. Moreover, the theory proposed in this paper could be usefully tested by using experimental economics methodology.

Acknowledgements

I am grateful to my supervisor, Vincenzo Denicolò, for useful suggestions. I am also thankful to Fabio Pinna, Philipp Schimdt-Dengler, Pasquale Schiraldi, and the seminar's audience at the London School of Economics for their useful comments and suggestions. I am also grateful to STICERD - London School of Economics and Political Science, where part of this work has been written in my year as visiting research student.

References

Berry, S., Levinsohn, J., & Pakes, A. 1995. Automobile prices in market equilibrium. *Econometrica: Journal of the econometric society*, **63**, 841–890.

Bonfrer, A., Berndt, E.R., & Silk, A.J. 2006. Anomalies in Estimates of Cross-Price Elasticities for Marketing Mix Models: Theory and Empirical Test. *Nber working paper*.

D'Aspremont, C., Jacquemin, A., Gabszewicz, J.J., & Weymark, J.A. 1983. On the stability of collusive price leadership. *The canadian journal of economics / revue canadienne d'economique*, **16**, 17–25.

Diewert, W.E. 1980. Symmetry conditions for market demand functions. *The review of economic studies*, **47**, 595–601.

Feuerstein, S. 2005. Collusion in industrial economics - A survey. *Journal of industry, competition and trade*, **5**, 163–198.

Friedman, J.W. 1971. A non-cooperative equilibrium for supergames. *The review of economic studies*, **38**, 1–12.

Kim, D., & Cotterill, R.W. 2008. Cost pass-through in differentiated product markets: the case of us processed cheese. *Journal of industrial economics*, **56**, 32–48.

Mastroleo, M., & Savorelli, L. 2009. Asymmetric substitutability and complementarity in individual demand. *Mimeo*.

Rojas, C., & Peterson, E.B. 2008. Demand for differentiated products: Price and advertising evidence from the US beer market. *International journal of industrial organization*, **26**, 288–307.

Rothschild, R. 1999. Cartel stability when costs are heterogeneous. *International journal of industrial organization*, **17**, 717–734.

Sethuraman, R., Srinivasan, V., & Kim, D. 1999. Asymmetric and neighborhood cross-price effects: some empirical generalizations. *Marketing science*, **18**, 23–41.

Singh, N., & Vives, X. 1984. Price and quantity competition in a differentiated duopoly. *The rand journal of economics*, **15**, 546–554.

Appendices

A. APPENDIX CHAPTER 2

A.1 The relation between satiation and constrained maximization

We now show that the satiation level c^F can be interpreted as the solution of a standard maximization problem under budget constraint, which implies that this level can be seen as an indicator of market prices and available income.

For simplicity, consider the choice between two goods: healthy food c and junk food x. Let the market prices be p_c and p_x, respectively, and M be the available income. Consider a well-behaved utility function $V(c, x, w)$ and assume that the marginal utility from both goods is strictly positive, i.e., no satiation is assumed. The agent will choose c and x to spend all her income. Substituting $x = \frac{M}{p_x} - \frac{p_c}{p_x}c$ in the utility function, one obtains

$$V\left(c, \frac{M}{p_x} - \frac{p_c}{p_x}c, w\right) := U(c, w).$$

Taking the first order derivative with respect to time c and equating to zero yields

$$V_c\left(c, \frac{M}{p_x} - \frac{p_c}{p_x}c, w\right) - \frac{p_c}{p_x}V_x\left(c, \frac{M}{p_x} - \frac{p_c}{p_x}c, w\right) = 0 \tag{A.1}$$

or, equivalently,

$$U_c(c, w) = 0 \tag{A.2}$$

By construction, the solution of (A.2) is $c = c^F$. This implies that also (A.1), the constrained maximization problem, is solved by $c = c^F$ and that the optimal level of consumption of healthy food c^F depends on income and market prices. If healthy food is a normal good, c^F increases if the available income increases or if the price of food decreases. This implies that the argument, suggested, among others, by Lakdawalla et al. (2005), according to which obesity is spreading because the available income is increasing and the price of caloric food is decreasing, can be accommodated by considering an increasing c^F. This implies that also w^F is increasing which, as shown in Proposition 1, facilitates the occurrence of overweight outcomes (unless the ideal body weight is very low).

Notice that the condition $V_c - \frac{p_c}{p_x} V_x = 0$ can be rearranged to obtain the standard optimality condition, where the marginal rate of substitution equals the price ratio, $\frac{V_c}{V_x} = \frac{p_c}{p_x}$, at $c = c^F$. If instead c is such that $V_c - \frac{p_c}{p_x} V_x = U_c >$ $(<)0$, then $\frac{V_c}{V_x} > (<) \frac{p_c}{p_x}$, i.e., the amount of healthy food c is lower (larger) than the optimal level c^F. In other words, for a given income and for given market prices, an agent eating beyond (below) satiation can be interpreted as eating more (less) healthy food than required by the optimality condition, where the marginal rate of substitution equals the price ratio.

A.2 Physical effort as an additional choice variable

Consider the case where the agent can choose both the amount of food consumption c_i and the amount of physical effort $e_i \geq 0$. We now show that, in this setup, the optimal solutions and steady state share the same properties illustrated in section 3. Let the individual utility function $U_{i,G}(c_i, w_i, e_i)$ be additively separable and strictly concave with respect to all its arguments. To account for the fact that exerting physical effort burns calories, we consider the following law of motion for body weight

$$\dot{w}_i(t) = c_i(t) - \delta w_i(t) - \varepsilon e_i(t) \tag{A.3}$$

with $\varepsilon > 0$. The current-value Hamiltonian is (omitting the arguments and the time index to simplify the notation)

$$H(\cdot) = U(c, w, e) + m(c - \delta w - \varepsilon e), \tag{A.4}$$

with m being the relevant costate. The necessary conditions for an internal solution are

$$H_c(\cdot) = U_c(\cdot) + m = 0 \tag{A.5}$$

$$H_e(\cdot) = U_e(\cdot) - m\varepsilon = 0 \tag{A.6}$$

$$\dot{m} = (\rho + \delta) m - U_w \tag{A.7}$$

$$\dot{w} = c - \delta w - \varepsilon e \tag{A.8}$$

The additional insight with respect to the basic model concerns the optimal choice of physical effort e. From (A.5) and (A.6) one obtains the following optimality condition:

$$\frac{U_c(\cdot)}{U_e(\cdot)} = -\frac{1}{\varepsilon}. \tag{A.9}$$

The above equation means that, irrespective of the shadow value of weight m, the optimal choice of consumption and effort depends on the marginal rate of substitution between food consumption and physical exercise. As ε is positive, both along the transition path and in steady state it is optimal to choose c and e so that the marginal utility both of food consumption and physical exercise have different signs. As an example, consider the case where the utility from physical effort is $\alpha e_i - \gamma e_i^2/2$, with $\alpha \geq 0$ and $\gamma > 0$. The first term represents the benefit of physical exercise and the second term its disutility cost. Then the necessary conditions are

$$\begin{aligned} c - c^F &= m && \text{(A.10)} \\ \alpha - \gamma e &= \varepsilon m && \text{(A.11)} \\ \dot{m} &= (\rho + \delta)\, m + (\beta + 1)\, w - w^H - \beta w^G && \text{(A.12)} \\ \dot{w} &= c - \delta w - \varepsilon e && \text{(A.13)} \end{aligned}$$

which implies the following linear relation between optimal consumption and effort

$$e = \frac{\alpha}{\gamma} + \frac{\varepsilon}{\gamma}\left(c - c^F\right) \tag{A.14}$$

The above equation has two implications. First, food consumption and physical effort are complements, implying that, with respect to the benchmark model, the agent can eat more, everything else equal, if she exerts more effort. Second, we can solve the problem by focusing on body weight and consumption only, as information on the optimal effort can be simply derived by (A.14). Differentiating with respect to time (A.10), substituting the value of e from (A.11) and the value of m from (A.10), the dynamics of food consumption is equivalent to (2.13), which implies the same asymptotic stability of the benchmark model. Let $F = (1 + \beta)\, \varepsilon^2 + \gamma\, [1 + \beta + \delta\, (\delta + \rho)]$, then the steady state is

$$\begin{aligned} c^{**} &= \frac{1}{F}\left\{\alpha\varepsilon\, (1 + \beta) + \left[(1 + \beta)\, \varepsilon^2 + \gamma\delta\, (\delta + \rho)\right] c^F + \delta\gamma\left(\beta w^G + w^H\right)\right\} \\ e^{**} &= \frac{1}{F}\left\{\alpha\, [1 + \beta + \delta\, (\delta + \rho)] + \varepsilon\left[(1 + \beta)\, c^F - \delta\left(\beta w^G + w^H\right)\right]\right\} \\ w^{**} &= \frac{1}{F}\left[\left(\gamma c^F - \alpha\varepsilon\right)(\delta + \rho) + \left(\varepsilon^2 + \gamma\right)\left(\beta w^G + w^H\right)\right]. \end{aligned}$$

It can be shown that, as in the benchmark model, both an overweight and underweight steady state can emerge.

A.3 Ideal body weight when the policy objective neglects the cost of social pressure

One may wonder whether the desirability of increasing the ideal body weight depends on the fact that the policy maker explicitly takes into account the cost of social pressure. We now show that considering the case where only the utility of consumption and health are considered does not change the welfare and health assessment contained in Proposition 2. The problem is the following:

$$\max_{w^G} \int\int \left[c_i^* \left(c_i^F - \frac{c_i^*}{2} \right) - \frac{1}{2} \left(w_i^* - w_i^H \right)^2 \right] f_G(w^H, c^F) dw^H dc^F. \tag{A.15}$$

Taking the derivative with respect to w^G and simplifying yields

$$\int\int \left\{ (\beta\delta - \rho) c_i^F + (\beta + \delta\rho) w_i^H - \beta \left(1 + \delta^2\right) w^G \right\} f_G(w^H, c^F) dw^H dc^F = 0$$

Exploiting the additive separability of the foc, it can be simplified as follows:

$$(\beta\delta - \rho) \bar{c}^F + (\beta + \delta\rho) \bar{w}^H - \beta \left(1 + \delta^2\right) w^G = 0.$$

Replacing $\bar{c}^F = \delta \bar{w}^F$ yields the following ideal weight:

$$\hat{w}^G = \frac{(\beta\delta - \rho) \delta \bar{w}^F + (\beta + \delta\rho) \bar{w}^H}{\beta \left(1 + \delta^2\right)}$$

If $w^H < w^F$, the ideal weight $\hat{w}^G$ maximizing (A.15) is on the right of $\bar{w}^H$, i.e., $\hat{w}^G \in \left[\bar{w}^H, w^{GU}\right]$. If instead $w^H > w^F$, then $\hat{w}^G$ is on the left of $\bar{w}^H$, more precisely $\hat{w}^G \in \left[w^{GU}, \bar{w}^H\right]$. This implies that the third statement of Lemma 1 and the results contained in Proposition 2 still hold even if the relevant policy objective only includes utility from consumption and health.

B. APPENDIX CHAPTER 3 - PROOFS

B.1 Lemma 2

The second derivative $\frac{\partial^2 \Pi_A}{\partial^2 x_A^2} = \frac{\partial^2 \Pi_B}{\partial^2 x_B^2}$ with respect to locations is given by

$$\begin{aligned}\frac{\partial^2 \Pi_i}{\partial^2 x_i^2} &= \frac{1}{9[n(\alpha-\beta n)+t(x_A-x_B)]^3} nt\{-6(\alpha^3 n^3 - \beta^3 n^6) - \\ &\quad -t^2(x_A-x_B)^2[4\beta n^2(x_A+2x_B-6)+t(x_A-x_B)(x_A+3x_B-8)] + \\ &\quad +2n^2 t(\alpha^2+\beta^2 n^2-\beta)[x_A(x_A-10)-3x_B(x_B-4)-\frac{1}{2}] + \\ &\quad +18\alpha^2\beta n^4 + 2\alpha n[2t^2(x_A-x_B)^2(x_A+2x_B-6)-9\beta^2 n^4]\}.\end{aligned}$$

We then study when $\frac{\partial^2 \Pi_i}{\partial^2 x_i^2} < 0$ in the relevant parameter space, that is $\alpha, \beta, t, n > 0$ and $x_A, x_B \in \mathbb{R}$. Given the complexity of the SOCs, we split the parameter space of n in three sub-intervals, which correspond to the network effect function evaluated at n to be: a) monotonic increasing ($0 < n \leq \alpha/2\beta$), b) monotonic decreasing up to overloading ($\alpha/2\beta < n \leq \alpha/\beta$), and c) monotonic decreasing and overloaded ($n > \alpha/\beta$). Thus, solving the inequalities, $\frac{\partial^2 \Pi_i}{\partial^2 x_i^2} < 0$ if and only if either:

a) $0 < n \leq \frac{\alpha}{2\beta} \wedge \alpha n - \beta n^2 > \frac{3}{2}t$;

b) $\frac{\alpha}{2\beta} < n < \frac{\alpha}{\beta} \wedge (\alpha n - \beta n^2 < \frac{9}{8}t \vee \alpha n - \beta n^2 > \frac{3}{2}t)$;

c) $n \geq \frac{\alpha}{\beta}$.

B.2 Lemma 3

By Lemma 1, each firm's profits are given by $\Pi_i = -\frac{2}{3}t(x_A - x_B)(x_A + x_B - 1)$. Substituting the equilibrium values $x_A = -\frac{1}{4}$ and $x_B = \frac{5}{4}$ and rearranging, one gets $\Pi_i = \frac{3}{4}nt - \frac{n}{2}(\alpha n - \beta n^2)$. The second part is the network effect evaluated at the consumer mass. In this paper we assume that $\alpha, \beta, n, t > 0$. It is clear that, whenever the overall network effect evaluated at the consumers' mass is positive ($\alpha n - \beta n^2 > 0$), profits are reduced from its presence. Then, profits are positive whenever $\alpha n - \beta n^2 < \frac{3t}{2}$. Since this implies $\alpha n < \frac{3t}{2} + \beta n^2$, this is clearly verified whenever $0 < \alpha n \leq \frac{3}{2}t$; if $\alpha n > \frac{3t}{2}$, then the it must be that $\alpha n - \beta n^2 < \frac{3}{2}t$.

B.3 Lemma 5

Part one of the Lemma can be easily checked by solving $2\alpha n - \beta n^2 > 0$. To prove part 2, recall now from (3.6) that firms' profit are given by $\Pi_i = \frac{3}{4}nt - \frac{n}{2}(\alpha n - \beta n^2)$. Then the result is easily obtained by solving $\alpha n - \beta n^2 < 0$.

B.4 Proposition 2

To check when the network effects are beneficial we consider the difference $\Delta SW = SWN - SW = \frac{\alpha n}{2}(1-2n) + \frac{\beta n}{4}(4n-1)$. Notice that the transportation rate cancels out. Thus, whether $\Delta SW \lesseqgtr 0$ does not depend on the cost of transportation. First, $\Delta SW > 0$ if and only if

$$0 < \frac{\alpha n}{2}(1-2n) + \frac{\beta n}{4}(4n-1). \tag{B.1}$$

Notice that for any $\alpha, \beta > 0$, this condition is never verified if and only if and only if $\frac{1}{4} < n < \frac{1}{2}$. Established this point, then consider two cases, when overloading is feasible and when it is not feasible. In the first case, $0 < n < \frac{\alpha}{\beta}$. It needs be $\frac{\beta n}{4}(4n-1) < \frac{\alpha n}{2}(1-2n)$, that is $\beta < \frac{\alpha(2-4n)}{n-4n^2}$. Since $\beta > 0$, to be verified it needs that $\frac{\alpha(2-4n)}{n-4n^2} > 0$. This is verified either if $n > \frac{1}{2}$ or if $n < \frac{1}{4}$, but the second solution can be discarded since it would imply that (B.1) is not verified. Analogously, in the second case, $n > \frac{\alpha}{\beta}$ (B.1) is verified if and only if $\beta > \frac{\alpha(2-4n)}{n-4n^2}$, which is verified if and only if either $n < \frac{1}{4}$ or $n > \frac{1}{4}$, but the latter can be discarded since (B.1) would be then negative.

C. APPENDIX CHAPTER 4 - PROOFS

C.1 Proposition 1

Part 1. By simple inspection, as long as $b_i^{ASY} > b_j^{ASY}$ we know that $q_i^{ASY} < q_j^{ASY}$. Then it is sufficient to solve the inequalities $q_i^{ASY} < q^{SE}$ and $q^{SE} < q_j^{ASY}$ for values of the parameters of the problem, that is $b_{i,j} \in [0,1]$ and $a > c > 0$. Part 2. From the solutions of the symmetric and asymmetric problem it is simple to get $p_i^{ASY} = \frac{a(b_i-2)+[b_i(b_j-1)]c-2}{(b_ib_j-4)}, p^{SE} = \frac{2a+(2+b_i+b_j)c}{4+b_i+b_j}, p_j^{ASY} = \frac{a(b_j-2)+[b_j(b_i-1)]c-2}{(b_ib_j-4)}$ and analogously to Part 1 solving the corresponding inequalities. Part 3. The solution to the inequality is analogous and straightforward by considering 4.8 and 4.4.■

C.2 Lemma 1

Let us consider the inequalities $\Phi_i^D \leq \pi_i^*(q_j^*)/(1-\delta)$, that is:

$$\pi_i^D(q_j^*) + \frac{\delta}{1-\delta}\pi_i^{ASY} \leq \frac{\pi_i^*(q_j^*)}{1-\delta}$$

$$\frac{(a-c-b_iq_j^*)^2}{4} + \frac{\delta}{1-\delta}\frac{(b_i-2)^2(a-c)^2}{(b_ibj-4)^2} \leq \frac{q_j^*(a-2q_j^*-c)}{1-\delta}$$

The solution of the associated equation gives the two values

$$\begin{aligned}\overline{q}_i^{Col} &= (a-c)\frac{(b_ib_j-4)[b_i(\delta-1)-2]+A_i}{(b_ib_j-4)[\delta b_i^2-(2+b_i)^2]}\\ \underline{q}_i^{Col} &= (a-c)\frac{(b_ib_j-4)[b_i(\delta-1)-2]-A_i}{(b_ib_j-4)[\delta b_i^2-(2+b_i)^2]}\end{aligned}$$

where
$A_i = [\delta b_i\ (\delta b_i(b_i-2)^2 + b_i(b_i^2-b_j^2) + 8(b_i-b_j))]^{\frac{1}{2}}$.
Inverting the indexes we obtain analogous values for firm j, that is

$$\overline{q}_j^{Col} = (a-c)\frac{(b_ib_j-4)[b_j(\delta-1)-2]+A_j}{(b_ib_j-4)[\delta b_j^2-(2+b_j)^2]}$$
$$\underline{q}_j^{Col} = (a-c)\frac{(b_ib_j-4)[b_j(\delta-1)-2]-A_j}{(b_ib_j-4)[\delta b_j^2-(2+b_j)^2]}$$

where
$A_j = [\delta b_j\ (\delta b_j(b_j-2)^2 + b_j(b_j^2-b_i^2)+8(b_j-b_i))]^{\frac{1}{2}}$.

We want the interval of collusive quantities for both firm i and j to be real valued. When $b_i > b_j$, A_i's radicand is always positive and A_j's is positive when $\frac{(8b_i-(8+b_i^2)b_j+b_j^3)}{((b_j-2)^2b_j)} < d$ and, in addition, if $b_j < \frac{13-\sqrt{41}}{8} \simeq 0,825$, it must be $b_i < \frac{2(2-[(b_j-2)^2(b_j+1)]^{\frac{1}{2}})}{b_j}$. When $b_i < b_j$, A_j is always positive and analogous conditions can be obtained by inverting indexes.■

C.3 Lemma 2

We will first show that, when $b_i > b_j \iff \underline{q}_j^{Col} < \underline{q}_i^{Col} < \overline{q}_j^{Col} < \overline{q}_i^{Col}$; the second part of the proof showing $b_i < b_j \iff \underline{q}_i^{Col} < \underline{q}_j^{Col} < \overline{q}_i^{Col} < \overline{q}_j^{Col}$ is analogous, and can be obtained by inverting the indexes.

Let us consider $b_i > b_j$. We first prove that $\underline{q}_j^{Col} < \overline{q}_j^{Col}$. This can be done solving the inequality

$$(a-c)\frac{(b_ib_j-4)[b_j(\delta-1)-2]-A_j}{(b_ib_j-4)[\delta b_j^2-(2+b_j)^2]} < (a-c)\frac{(b_ib_j-4)[b_j(\delta-1)-2]+A_j}{(b_ib_j-4)[\delta b_j^2-(2+b_j)^2]}$$

considering the relevant range of the parameters a, b_i, b_j, c, δ, and the conditions stated in Lemma 1 for the quantities to be real-valued, this inequality always holds. Then, we prove that $\underline{q}_i^{Col} < \overline{q}_i^{Col}$. The following inequality

$$(a-c)\frac{(b_ib_j-4)[b_i(\delta-1)-2]-A_i}{(b_ib_j-4)[\delta b_i^2-(2+b_i)^2]} < (a-c)\frac{(b_ib_j-4)[b_i(\delta-1)-2]+A_i}{(b_ib_j-4)[\delta b_i^2-(2+b_i)^2]}$$

always holds, considering as above the relevant range of the parameters and the condition for real-values quantities. Finally, to ensure that the two intervals always overlap, we prove that $\underline{q}_j^{Col} < \overline{q}_i^{Col}$, that is

$$(a-c)\frac{(b_ib_j-4)[b_j(\delta-1)-2]+A_j}{(b_ib_j-4)[\delta b_j^2-(2+b_j)^2]} < (a-c)\frac{(b_ib_j-4)[b_i(\delta-1)-2]-A_i}{(b_ib_j-4)[\delta b_i^2-(2+b_i)^2]}$$

always holds. The remaining inequalities can be derived by transitivity.■

C.4 Proposition 2

Proposition 2 follows directly by Lemma 1 and 2. The two asymmetric firms can agree on the collusive quantity belonging to the interval where the two individual partial collusion strategies overlap.■

C.5 Proposition 3

Let us normalize the parameter space by setting $b_j = 1 - b_i$.Then, consider $b_i < b_j$. By Proposition 2, $\underline{q}_i^{Col} < q_{i,j}^* < \overline{q}_j^{Col}$ and, since assuming positive real-valued outputs, we call $l^{ASY} = \overline{q}_j^{Col} - \underline{q}_i^{Col}$ the length of the set of collusive quantities in the asymmetric case, which equals:

$(a-c)\frac{-(4+b_i(b_i-1))^2[2(\delta-15)+b_i(-5+2\delta b_i^2(\delta-1)+\delta(12+\delta)+b_i(5-3\delta(2+\delta))]+8[B_1+B_2]}{[4+b_i(b_i-1)]^2[(b_i-3)^2+\delta(b_i-1)^2][\delta b_i^2-(2+b_i)^2+\delta b_i^2]}$

where:

$B_1 = [\delta b_i(b_i(17-2b_i+\delta(b_i-2)^2-8))]^{\frac{1}{2}}$,

and

$B_2 = [\delta(7+\delta+b_i(b_i(11+2b_i+(b_i-2)\delta)-20)]^{\frac{1}{2}}$.

In the standard equivalent case, when $b_i = b_j = \frac{1}{2}$, the interval $l^{SE} = \overline{q} - \underline{q}$ is given by

$$l^{SE} = (a-c)\frac{\delta 16}{5(\delta-25)}.$$

Then, considering the inequality $l^{ASY} > l^{SE}$, $(a-c)$ cancel out and the resulting inequality is verified for all the values of the parameters of the problem (b_i, b_j, δ) and it is independent also from the calibration of a and c.

The case in which $b_i < b_j$ can be proved analogously by considering the length $l^{ASY} = \overline{q}_i^{Col} - \underline{q}_j^{Col}$ and comparing it to l^{SE}. The results and the proof are analogous, and can be obtained simply by inverting indexes.■

Acknowledgements

My first and special thanks go to my supervisor, Vincenzo Denicolò. I have been knowing Vincenzo for more than ten years. He has been soundly helpful in all the many moments of need, from my undergraduate studies to the first doctorate years, as well as in recent times. Besides his fundamental help in the scientific work of this dissertation, for all these years Vincenzo has morally supported me well beyond his duties as a supervisor: I am hugely in debt with him both on the human and the scientific side. I can not say now whether I will ever be able to reciprocate his efforts towards me in the future, but I hope this thesis to be at least a little, first step in that direction. In the meanwhile, I hope he knows that my appreciation is well deeper than what these few lines are able to express.

I am sincerely thankful to Giulio Ecchia, my advisor for the research fellowship at the Department of Economics at the University of Bologna. I am particularly thankful to Giulio for addressing and supporting me in my visiting experience at the London School of Economics. That visiting period has been decisive for the development of this thesis, and made me more enthusiastic than ever towards life. I thank Giulio also for his moral support, and the pleasurable days spent working together in the last years.

I thank my coauthor Davide Dragone for the lively collaboration, truly enjoying the work, the discussions, and the time we spent together: It really renewed my interest towards research in a period of disenchantment. I am proud of the first chapter of this thesis, whose energy is real expression of the motivation of two collegues working on something they really believe in.

While I was thinking, writing, presenting this dissertation other people warmly supported me both when succeeding and having troubles: my girlfriend Giulia, my brother Marco, my parents, Andrea and Paola, my grandparents, Gino and Anna, and my two best friends, Matteo and Filippo.

All those abovementioned trained my eyes to discovery, helped me when the tempest was pushing me out of track, and blew into my sails when the direction was good.

The usual disclaimer applies: all the errors remaining in this thesis, and in life, are mine.