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On Poverty Traps, Rational Bubbles, and Wealth Inequality

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Abstract

This paper develops a dynamic general equilibrium (DGE) model with heterogeneous agents to connect three macroeconomic phenomena: persistent poverty traps, sluggish real growth, and rising wealth inequality. The model achieves this by allowing agents, who differ in patience and face a subsistence consumption constraint, to choose portfolios between productive capital and a fixed-supply, unproductive asset susceptible to rational speculative bubbles. The analysis reveals that these bubbles, while rational, induce a positive wealth effect for asset-holders, which, through optimal consumption-smoothing (via agents' Euler equations), reduces the aggregate savings rate, permanently "crowding out" productive capital that crowds out productive investment, leading to lower real wages and output, which in turn exacerbates wealth inequality by pushing constrained agents closer to the poverty trap. A calibration exercise, disciplined by real-world stylized facts, illustrates the model's path-dependence and highlights the particular vulnerability of middle-income economies to such collapses.

Keywords: Poverty Traps, Wealth Inequality, Speculative Bubbles, Endogenous Savings, Portfolio Choice, Heterogeneous Agents, General Equilibrium, Subsistence Consumption.

JEL classification: D31, E21, E44, G12, O11, O40.

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1. Introduction

The modern economic landscape is defined by a striking puzzle: a persistent divergence between the soaring value of wealth, particularly in unproductive assets like housing, and the lackluster growth of real wages and productive investment. Since the 1980s, many advanced economies have seen wealth-to-income ratios climb to levels unseen since the Gilded Age (Piketty, 2014). However, a significant body of research has shown that this trend is not primarily driven by an accumulation of productive capital (machines, equipment, software).[1]

Instead, two stylized facts stand out. First, as Rognlie (2015) demonstrates, the entire post-1970s increase in the aggregate capital share in advanced economies can be attributed to the housing sector, not to productive capital. Second, Knoll et al. (2017a), using a 140-year dataset, confirm this, showing that while the relative price of productive capital has actually *fallen*, housing wealth as a share of national income has tripled since 1950. This suggests a macroeconomic shift *from* productive accumulation *to* unproductive asset valuation. This financialization has coincided with median wage stagnation and concerns about a slowdown in real investment (Gutiérrez and Philippon, 2017).

Existing theoretical frameworks struggle to jointly explain this divergence. Models of development traps (e.g., Galor and Zeira, 1993; Azariadis and Drazen, 1990) excel at explaining persistent poverty but typically abstract from financial markets and asset valuation. Conversely, the literature on rational bubbles (e.g., Tirole, 1985; Martin and Ventura, 2012; Martin and Ventura, 2018; Toda, 2014) provides a rigorous foundation for asset pricing but often in endowment settings where the crucial link between speculation and real production

[1]The contemporary economic landscape presents a stark contrast: while global wealth, particularly in financial and real estate assets (i.e. "unproductive capital") has surged, growth in real output and median wages has remained disappointingly slow in many parts of the world. The coexistence of rapid wealth accumulation, particularly at the extreme upper tail, alongside stagnant real wage growth for the median household presents a significant challenge to standard macroeconomic models. The empirical task of precisely documenting this top-end concentration from imperfect data has itself become a major field of research (Dávila-Fernández and Punzo, 2021a; Madsen and Strulik, 2025; Piketty, 2014; Saez and Zucman, 2016). Recent advances, such as the rigorous Pareto extrapolation framework developed by Émilien Gouin-Bonenfant and Toda (2023), provide increasingly sophisticated tools to confirm the scale of this tail inequality.

is missing. These models can conclude that bubbles *reduce* inequality by transferring wealth to the impatient poor, a result at odds with empirical observation.

While the measurement of this phenomenon is now on a firm statistical footing, a robust theoretical mechanism linking financial speculation, real-sector stagnation, and the persistence of poverty remains a key open question. This paper provides such a mechanism by understanding that the potential links between these phenomena – wealth concentration, sluggish real growth, and persistent poverty – require a framework that integrates financial/unproductive assets dynamics with real production and distributional concerns. So, this paper bridges this gap by proposing a model with a novel general equilibrium wage channel. We argue that the rise in unproductive asset values is not a harmless sideshow but a core driver of the puzzle.

The literature shows that the values of financial and capital assets and real estate prices have skyrocketed in many economies (Basco, 2014; Dávila-Fernández and Punzo, 2021b; Piketty, 2014; Saez and Zucman, 2016; Stiglitz, 2015; Policardo and Sanchez Carrera, 2024), growth in productive capacity and median living standards has often lagged (Giombini et al., 2023). Furthermore, the persistence of low-income equilibria and the failure of many economies to converge challenge traditional growth narratives. Existing theoretical frameworks typically address these phenomena in isolation. Models of poverty traps (e.g., Azariadis, 1996; Azariadis and Drazen, 1990; Banerjee and Newman, 1993; Galor and Zeira, 1993; Ghiglino and Sorger, 2002; Grassetti and Carrera, 2025) emphasize threshold effects arising from credit constraints or human capital indivisibilities, but often abstract from financial markets and asset choice. Models of rational asset bubbles (e.g., Hirano and Toda, 2024; Hirano and Toda, 2025a; Hirano and Toda, 2025b; Kamihigashi, 2018; Tirole, 1985; Toda, 2014; Weil, 1989) explore the dynamics of non-fundamental asset pricing but usually operate within representative agent frameworks, limiting their ability to analyze distributional consequences or poverty dynamics. While Kamihigashi (2018), perfectly summarizes the canonical result in general equilibrium theory: in a standard model with infinitely-lived agents and complete markets, rational bubbles are ruled out. An infinitely-lived agent (or a set of agents linked by complete markets) would perform backward induction from infinity, realize the asset has no terminal value, and thus price it at zero today. However, in what follows we develop a model that does not have complete markets, and this is the specific, subtle feature that allows rational bubbles to exist. We consider that the "incompleteness" is the endogenous borrowing constraint on Type L agents, which is micro-founded by their Stone-Geary preferences. This is precisely the class of models (e.g., Martin and Ventura, 2012) where bubbles are sustainable because binding constraints on one set of agents keep the rate of return high enough for the unconstrained agents to willingly hold the bubble.

However, key questions remained unanswered: Why invest in unproductive assets? Why do portfolios differ across agents? How do optimal choices shape aggregate outcomes (growth and distribution) and inequality? Existing literature often tackles these issues in separate silos. Models of poverty traps typically emphasize mechanisms like credit constraints (Galor and Zeira, 1993), human capital thresholds (Banerjee and Newman, 1993; Easterly, 2006), or technological non-convexities, often abstracting from the role of asset markets and portfolio choice. Conversely, the literature on rational asset bubbles (Hirano and Toda, 2024; Tirole, 1985; Toda, 2014; Weil, 1989; Martin and Ventura, 2018) masterfully explains the dynamics of non-fundamental asset pricing but usually employs representative agent frameworks, precluding analysis of inequality or poverty dynamics.

This paper integrates insights from three main strands of macroeconomic literature. First, it builds upon the literature on poverty traps and non-linear dynamics. Seminal contributions by Nelson (1956), Leibenstein (1957), and later formalized by Galor and Zeira (1993), Banerjee and Newman (1993), and Azariadis and Drazen (1990), established the possibility of multiple equilibria in economic development. These models typically rely on technological non-convexities, credit market imperfections, or externalities in human capital accumulation. Our model contributes to this literature by generating multiple equilibria through a preference-based mechanism (subsistence consumption), which interacts directly with general equilibrium factor prices. This approach is closer in spirit to models with minimum consumption requirements, such as Matsuyama (2002). Second, this paper connects to the extensive literature on asset pricing, portfolio choice and rational bubbles (Forbes et al., 2016; Hirano and Toda, 2024; Ikeda and Shibata, 1995). Following Samuelson (1958), Tirole (1985) (and the more recent, sophisticated work by Hirano and Toda, 2025a; Hirano and Toda, 2025b; Kamihigashi, 2018; Toda, 2014) provided the canonical OLG framework where bubbles on intrinsically useless assets can exist if the economy is dynamically inefficient. Other approaches explores bubbles in production economies (e.g., Santos and Woodford, 1997) and their potential welfare implications (Martin and Ventura, 2018). Our contribution is to embed a rational bubble within a heterogeneous-agent DGE framework, allowing us to explicitly analyze its impact on productive investment (crowding out) and wealth distribution, phenomena often overlooked in representative-agent bubble models. Third, our work relates to the growing literature on wealth and income inequality. The empirical findings of Piketty (2014), Saez and Zucman (2016), Policardo and Sanchez Carrera (2024), and others have highlighted the role of capital returns and asset price appreciation in driving top wealth shares (Émilien Gouin-Bonenfant and Toda, 2023).[2]

Our paper surpasses the literature by providing a mechanism where asset bubbles, interacting with heterogeneous savings behavior driven by patience and subsistence constraints, directly generate divergence in wealth accumulation and suppress wage growth, consistent with the observed "r (return on capital) $> g$ (economic growth rate)" phenomenon during periods of high asset inflation (Jakurti, 2025). We specifically address the theoretical and empirical critique (Garleanu et al., 2008; Hori and Im, 2023; Bahloul Zekkari, 2024; Martin and Ventura, 2012) regarding the portfolio composition of the wealthy by showing that even with endogenous portfolio choice,[3] bubbles still crowd out aggregate productive investment

[2]Theoretical models exploring these dynamics often focus on heterogeneous returns to wealth (e.g., Benhabib et al., 2011) or bequest motives (De Nardi, 2004).

[3]See: https://www.ecb.europa.eu/press/economic-bulletin/focus/2024/html/ecb.ebbox202405_07~33327d5fab.en.html

via a general equilibrium wealth effect, impacting wages regardless of who holds the bubbly asset.[4]

In summary, this paper's contribution is to push the literature frontier in a new direction. It moves the debate beyond "do bubbles exist?" to "what are the real consequences of bubbles in a production economy with pre-existing vulnerabilities?" By introducing the General Equilibrium (GE) wage channel and the subsistence-based poverty trap, this paper provides a robust theoretical foundation for the intuitive, real-world argument that speculative manias in "unproductive" assets (like real estate) are not a harmless sideshow. Instead, they actively undermine the productive base of the economy, suppress wages, and exacerbate the divide between capital-holders and wage-earners. Our model suggests that financial speculation is not a harmless side-show; it can have first-order, negative effects on the real economy by diverting savings, suppressing real wages, and exacerbating inequality. This provides a theoretical basis for considering policies aimed at mitigating unproductive speculation, suggesting they may enhance both efficiency and equity. In this sense, our proposed model offers several key ideas.[5] First, it endogenously generates multiple stable steady states: an efficient high-income equilibrium (K_H^*) and a stable poverty trap (K_L^*). The bifurcation arises directly from the interaction of the subsistence constraint with the general equilibrium wage level, without resorting to exogenous rules. Second, it demonstrates that rational

[4]Specifically, this paper relates to models studying the interaction between finance and growth (e.g., Greenwood and Jovanovic, 1990; Acemoglu and Zilibotti, 1997). While much of this literature focuses on the beneficial role of financial development, our model highlights a potentially detrimental channel where financial markets, through speculative bubbles in unproductive assets, can actively impede real capital accumulation and exacerbate inequality.

[5]Specifically, this paper introduces two key ingredients into an otherwise standard neoclassical growth model with heterogeneous agents: i) Agent Heterogeneity and Subsistence: Following the poverty trap literature, we include two types of infinitely-lived dynasties: a "patient" unconstrained group (Type H, high β_H) and an "impatient" constrained group (Type L, low β_L). Crucially, Type L agents face a subsistence consumption level ($\bar{c}$), modeled via a Stone-Geary utility function. This creates an endogenous non-linearity: when wages fall too low, these agents prioritize survival and their optimal savings rate drops to zero. ii) Productive vs. Unproductive Assets (Endogenous Portfolio Choice): Agents can allocate their endogenously determined savings between productive capital (K_p), which generates output and factor incomes, and an unproductive asset (K_u), such as land or gold, which has a fixed supply and offers returns only through price appreciation. The portfolio choice is endogenous, driven by a standard no-arbitrage condition based on expected returns.

speculative bubbles in the unproductive asset are sustainable equilibrium paths. Crucially, these bubbles are not neutral. They create a powerful wealth effect that alters optimizing agents' intertemporal consumption decisions. Via their Euler equations, agents respond to increased perceived wealth by consuming more and saving less in aggregate. Third, this endogenous reduction in aggregate savings permanently "crowds out" productive capital. The economy converges to a bubble-sustained equilibrium (K^*_{bub}) with a lower real capital stock ($K^*_{bub} < K^*_H$), consequently leading to lower real output and lower real wages. This directly links financial speculation to real economic stagnation through a general equilibrium channel operating via optimal consumption-savings responses. Fourth, the model provides a clear mechanism for rising inequality driven by bubbles. Patient (Type H) agents, who endogenously hold more wealth and thus a larger share of the unproductive asset, disproportionately benefit from bubble-driven capital gains. Impatient (Type L) agents, relying more on wages, suffer directly from the wage suppression caused by the crowding out of productive capital. This fall in wages pushes them closer to their subsistence constraint, potentially triggering a collapse into the poverty trap and widening the gap between the two groups. This addresses critiques by showing how inequality can arise even with optimal portfolio choices for all, driven by preference heterogeneity and general equilibrium effects.[6]

To numerically illustrate our results, we provide a calibration exercise, distinguishing between parameters representative of high-, middle-, and low-income countries based on empirical evidence. Simulations illustrate the path dependence inherent in the model and demonstrate the differential impact of speculative bubbles. While high-income economies appear relatively resilient, middle-income economies are shown to be particularly vulnerable to bubble-induced collapses into the poverty trap. Our analysis suggests that the composition of asset accumulation has first-order effects on real economic performance and distribution.

[6]It avoids the empirically questionable assumption that wealthy agents exogenously favor unproductive assets; here, portfolio choices are optimal for all, but outcomes differ due to preference heterogeneity. See: https://www.hhs.se/sv/houseoffinance/research/featured-topics/2025/why-the-rich-get-richer-new-research-shows-its-about-risk-and-patience/, https://gualtiazza.github.io/papers/AKRS.pdf

Policies aimed at managing speculation in unproductive assets may therefore be viewed not just as tools for financial stability or redistribution, but potentially as crucial components of a strategy for sustainable growth and poverty reduction, especially in developing economies.

The remainder of the paper is structured as follows. Section 2 details the model environment. Subsection 2.1 defines the agents' intertemporal portfolio allocation and characterizes the competitive equilibrium—subsection 2.2 comments on the agent's behavior driven by the permanent income hypothesis. Section 3 analyzes the steady states and poverty traps, while subsection 3.1 studies the bubble effect and wealth inequality. Section 4 presents a calibration and simulation analysis to illustre the results. Section 5 concludes.

2. The Model Setup

We consider an infinite-horizon discrete-time economy populated by a representative firm and two types of infinitely-lived dynastic households.

Firms and Production. A representative competitive firm produces a single final good Y_t using productive capital $K_{p,t}$ and labor L_t. The production function is Cobb-Douglas with constant returns to scale:

$$Y_t = K_{p,t}^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

with total labor supply normalized to $L_t = L = 1$. Firms rent capital and hire labor in competitive factor markets to maximize static profits:

$$\Pi_t = Y_t - r_{p,t} K_{p,t} - w_t L_t. \tag{2}$$

The first-order conditions yield the standard endogenous factor prices:

$$\text{Rental rate on productive capital:} \quad r_{p,t} = \alpha K_{p,t}^{\alpha-1}, \tag{3}$$

$$\text{Wage rate:} \quad w_t = (1-\alpha) K_{p,t}^{\alpha}. \tag{4}$$

The economy features two types of assets for storing wealth.

1. Productive Capital (K_p): This is the physical capital used in production. It accumulates through investment and depreciates at a constant rate $\delta_p \in (0, 1)$. The aggregate stock $K_{p,t}$ is the sum of holdings across all households. The (gross) return is:

$$R_{p,t+1} = \underbrace{r_{p,t+1}}_{\text{Rental}} + \underbrace{(1 - \delta_p)}_{\text{Undepreciated portion}}$$

2. Unproductive Asset (K_u): This represents assets like land, gold, or other intrinsically useless stores of value. Its aggregate physical supply is fixed at $\bar{K}_u$. We assume it does not depreciate ($\delta_u = 0$). Its price in terms of the consumption good is $p_{u,t}$, which is endogenously determined by market clearing. The (gross) return is from price appreciation:

$$R_{u,t+1} = \frac{p_{u,t+1}}{p_{u,t}}$$

Households. The economy is populated by two types of infinitely-lived households (dynasties),[7] indexed by $i \in \{H, L\}$. Both types supply one unit of labor inelastically. They differ in their population mass (n_H, n_L with $n_H + n_L = 1$) and their preferences. Let $k_{p,i,t}$ and $k_{u,i,t}$ denote the holdings of productive and unproductive assets by an agent of type i at the beginning of period t. Let $u_i(c)$ be the period utility function for type i. The intertemporal consumption-savings choice (Euler equation) relates marginal utility across time:

$$u_i'(c_{i,t}) \geq \beta_i E_t[u_i'(c_{i,t+1}) R_{p,t+1}] \quad (= \text{ if } k_{p,i,t+1} > 0) \tag{5}$$

Derived from comparing consumption today vs. saving in K_p and consuming tomorrow.

$$u_i'(c_{i,t}) = \beta_i E_t[u_i'(c_{i,t+1}) \cdot R_{p,t+1}]$$

[7] Weil (1989) studies the infinitely-lived agents with altruism.

Substituting factor prices and $R_{p,t+1}$:

$$u_i'(c_{i,t}) = \beta_i E_t[u_i'(c_{i,t+1}) \cdot (\alpha K_{p,t+1}^{\alpha-1} + (1-\delta_p))]$$

Type H: "Unconstrained" Agents. These agents represent dynasties with high patience (high β_H) or potentially higher initial wealth. They solve a standard intertemporal optimization problem with log-utility:

$$\max_{\{c_{H,t},k_{p,H,t+1},k_{u,H,t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_H^t \log(c_{H,t}) \tag{6}$$

subject to the period budget constraint:

$$c_{H,t} + k_{p,H,t+1} + p_{u,t}k_{u,H,t+1} = w_t + [r_{p,t} + (1-\delta_p)]k_{p,H,t} + p_{u,t}k_{u,H,t} \tag{7}$$

and standard non-negativity constraints on consumption and asset holdings ($c_{H,t} \geq 0,\ k_{p,H,t+1} \geq 0,\ k_{u,H,t+1} \geq 0$). Notice that, for all feasible paths they satisfy $c_{H,t}$ well above any poverty threshold and the optimal consumption choice is interior. Consequently the Kuhn–Tucker conditions never bind for consumption and the usual first-order (Euler) condition holds. Formally, they solve

$$\max_{\{c_{H,t},k_{p,H,t+1},k_{u,H,t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_H^t \log(c_{H,t}) \tag{8}$$

subject to (7) and non-negativity of assets. Because solutions are interior, the first-order condition for consumption is:

$$\frac{1}{c_{H,t}} = \beta_H\, E_t\left[\frac{1+r_{p,t+1}}{c_{H,t+1}}\right],$$

with the usual transversality condition. Economically, this assumption captures that H agents are sufficiently wealthy (or patient) that they never cut consumption to subsistence levels and thus behave according to smooth intertemporal optimization.

Type L: "Constrained" Agents. These agents represent dynasties with low patience ($\beta_L < \beta_H$) or lower initial wealth. Their crucial feature is a subsistence consumption requirement $\bar{c} > 0$. Their preferences are represented by a Stone-Geary utility function:

$$\max_{\{c_{L,t}, k_{p,L,t+1}, k_{u,L,t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_L^t \log(c_{L,t} - \bar{c}) \tag{9}$$

subject to the period budget constraint:

$$c_{L,t} + k_{p,L,t+1} + p_{u,t} k_{u,L,t+1} = w_t + [r_{p,t} + (1 - \delta_p)] k_{p,L,t} + p_{u,t} k_{u,L,t} \tag{10}$$

the subsistence constraint $c_{L,t} > \bar{c}$, and non-negativity constraints ($k_{p,L,t+1} \geq 0, k_{u,L,t+1} \geq 0$). The marginal utility of consumption is:

$$u'(c_{L,t}) = \frac{1}{c_{L,t} - \bar{c}},$$

which diverges to infinity as $c_{L,t} \to \bar{c}$.[8] This ensures that the subsistence constraint binds whenever the household's wealth or income is insufficient to maintain $c_{L,t} - \bar{c} > 0$. When the constraint is *non-binding*, the first-order condition for consumption takes the standard Euler form:

$$\frac{1}{c_{L,t} - \bar{c}} = \beta_L \, E_t \left[\frac{1 + r_{p,t+1}}{c_{L,t+1} - \bar{c}} \right].$$

However, when the constraint *binds* (i.e., $c_{L,t} = \bar{c}$), the Euler equation no longer holds with equality, and the household is cornered at its subsistence level. In this case, asset

[8] The marginal utility

$$u'(c_{L,t}) = \frac{1}{c_{L,t} - \bar{c}}$$

diverges as $c_{L,t} \to \bar{c}$, ensuring that type L agents never consume below the subsistence level. Economically, this introduces the possibility of corner solutions and can generate poverty traps if wealth or returns are insufficient. Their demand for assets is therefore more rigid, and they are more sensitive to fluctuations in wages or capital returns.

accumulation halts, and the intertemporal condition becomes an inequality:

$$\frac{1}{c_{L,t} - \bar{c}} > \beta_L \, E_t\left[\frac{1 + r_{p,t+1}}{c_{L,t+1} - \bar{c}}\right], \quad \text{with} \quad c_{L,t} = \bar{c}, \; k_{p,L,t+1} = 0.$$

The inequality reflects potential borrowing constraints or the non-negativity constraint on savings. If the subsistence constraint binds ($c_{L,t} \approx \bar{c}$), the LHS approaches infinity, and the agent saves zero. That is, the Type L equation only holds if $c_{L,t} > \bar{c}$. If the constraint binds, $u'_L \to \infty$ and optimal savings fall to zero. Economically, this formulation captures the presence of *poverty traps*: once income or wealth falls below a critical threshold, the household cannot save or invest productively and remains trapped at subsistence. Conversely, if productivity or transfers are high enough to raise consumption above $\bar{c}$, the household re-enters the interior regime where standard intertemporal optimization applies.

Summary Table: Type H vs. Type L

	Type H	Type L
Patience	High (β_H)	Low (β_L)
Utility	$\log(c)$	$\log(c - \bar{c})$ (Stone–Geary)
Subsistence constraint	None	$c > \bar{c}$
Consumption near lower bound	Finite marginal utility	$u'(c) \to \infty$
Behavior	Standard intertemporal savings	Constrained, possible corner solutions

2.1. Portfolio Choice and Equilibrium

Both types of agents choose an **Intertemporal Portfolio Allocation (No-Arbitrage)**. That is, if an agent holds positive amounts of both assets ($k_{p,i,t+1} > 0$ and $k_{u,i,t+1} > 0$), they must expect the same return from both:

$$E_t[R_{p,t+1}] = E_t[R_{u,t+1}] \tag{11}$$

Substituting the gross returns:

$$E_t[r_{p,t+1} + (1 - \delta_p)] = E_t\left[\frac{p_{u,t+1}}{p_{u,t}}\right] \tag{12}$$

This condition must hold for any agent holding both assets. This condition determines the price path $p_{u,t}$. If expected returns differ, agents will optimally shift their entire portfolio to the higher-return asset (unless constrained by borrowing limits or non-negativity on holdings).

Transversality Condition. To rule out Ponzi schemes or unbounded accumulation, optimal plans must satisfy:

$$\lim_{t\to\infty} \beta_i^t \, \mathbb{E}_0\big[u_i'(c_{i,t})\, k_{i,t+1}^p R_{p,t+1}\big] = 0,$$

ensuring that the discounted marginal value of remaining wealth vanishes in the long run.

Notice that the model has two risk-free assets (K_p and K_u) that must, by no-arbitrage, yield the same return ($R_p = R_u$). Hence, with identical returns, the portfolio composition (θ_1) of any unconstrained agent (Type H) is indeterminate. Therefore, we may claim that the patient, unconstrained Type H agents are the ones who accumulate all the wealth and are therefore the ones who absorb both assets. This clarifies why they are the agents who experience the wealth effect that drives the crowding-out mechanism. More precisely, a key feature of our model, and any model with multiple risk-free assets, is the no-arbitrage condition, which states that in equilibrium, all held assets must yield the same expected return. In our framework:

$$E_t[R_{p,t+1}] = E_t[R_{u,t+1}] \tag{13}$$

This condition is necessary to ensure that agents are willing to hold both productive capital (K_p) and the unproductive asset (K_u). However, this equality also makes the two assets perfect substitutes from the perspective of any individual, unconstrained agent. This creates an indeterminacy in the individual portfolio composition (θ_i) for the patient, unconstrained (Type H) agents. While the model can determine the *aggregate* asset demands that must equal the aggregate supplies (K_p and $\bar{K}_u$), it cannot pin down *which* Type H agent holds which asset. We resolve this standard indeterminacy by making a common assumption of a symmetric equilibrium.

Remark 1 (Symmetric Portfolios)**.** *All agents within the unconstrained (Type H) group are identical. We assume they all follow an identical, symmetric strategy. Therefore, every Type H agent holds the same portfolio composition, which mirrors the aggregate portfolio of their group.*

This assumption implies that if the Type H group in aggregate holds all assets in the economy (which is the case in the steady states we analyze, as Type L agents are constrained and hold no wealth), then each individual Type H agent's portfolio will consist of:

- The share of productive capital:

$$\theta_H(t) = \frac{K_{p,t}}{K_{p,t} + p_{u,t}\bar{K}_u} \tag{14}$$

- The share of the unproductive asset:

$$1 - \theta_H(t) = \frac{p_{u,t}\bar{K}_u}{K_{p,t} + p_{u,t}\bar{K}_u} \tag{15}$$

Implications for the Crowding-Out Mechanism. This clarification is not merely technical; it is central to the paper's main mechanism. By assuming all Type H agents hold an identical (and thus diversified) portfolio, a rise in the bubble price ($p_{u,t} \uparrow$) has a **uniform, positive wealth effect on all Type H agents simultaneously. The total wealth of each Type H agent, $A_H(t)$, unambiguously increases. Following their optimal consumption rule (derived from the Permanent Income Hypothesis for log-utility), all Type H agents will increase their consumption ($c_H \uparrow$). This synchronized increase in consumption leads directly to a fall in the aggregate savings rate, which in turn "crowds out" investment in new productive capital ($K_{p,t+1} \downarrow$) and suppresses the general equilibrium wage. This assumption thus provides a clear and direct link from the financial bubble to the consumption-smoothing behavior of the unconstrained class, which ultimately harms the wage-dependent constrained class.

In equilibrium, with both assets held in positive amounts by at least one agent type, this condition determines the path of the unproductive asset price $p_{u,t}$. Then, let us define the market equilibrium of this economy.

Definition 1 (Recursive Competitive Equilibrium)**.** *A recursive competitive equilibrium, i.e. given initial asset holdings* $\{k^p_{i,0}, k^u_{i,0}\}_{i\in\{H,L\}}$*, a sequence of prices* $\{w_t, r^p_t, p^u_t\}_{t=0}^{\infty}$ *and allocations* $\{c_{i,t}, k^p_{i,t+1}, k^u_{i,t+1}\}_{t=0}^{\infty}$ *for* $i \in \{H, L\}$*, is characterized by value functions* $V_i(k_{p,i}, k_{u,i}, K_p)$*, policy functions for consumption* $c_i(\cdot)$*, next-period productive capital* $k'_{p,i}(\cdot)$*, next-period unproductive assets* $k'_{u,i}(\cdot)$ *for each agent type* $i \in \{H, L\}$*, pricing functions for wages* $w(K_p)$ *and productive returns* $r_p(K_p)$*, a price function for the unproductive asset* $p_u(K_p, \mathbf{k}_u)$*, and an aggregate law of motion for productive capital* $K'_p = G(K_p, \mathbf{k}_u)$ *(where* $\mathbf{k}_u$ *represents the distribution of unproductive assets across types) such that:*

1. ***Household Optimization:*** *The policy functions solve the households' Bellman equations, given the pricing functions and the aggregate law of motion. Agents' policies satisfy their Euler and No-Arbitrage conditions given prices.*

2. ***Firm Optimization:*** *The pricing functions* $w(K_p)$ *and* $r_p(K_p)$ *satisfy equations (3) and (4), i.e.* $w_t = (1-\alpha)K^{\alpha}_{p,t}$ *and* $r_{p,t} = \alpha K^{\alpha-1}_{p,t}$.

3. ***Market Clearing:*** *For all aggregate states* $(K_p, \mathbf{k}_u)$*:*

$$\text{Labor:} \quad n_H \cdot 1 + n_L \cdot 1 = 1 \tag{16}$$

$$\text{Productive Capital:} \quad K_p = n_H k_{p,H} + n_L k_{p,L} \tag{17}$$

$$\text{Unproductive Asset:} \quad \bar{K}_u = n_H k_{u,H} + n_L k_{u,L} \tag{18}$$

$$\text{Goods Market:} \quad K^{\alpha}_p = n_H c_H + n_L c_L + [K'_p - (1-\delta_p)K_p] \tag{19}$$

4. ***Rational Expectations:*** *Agents' expectations about future prices and returns are consistent with the equilibrium law of motion* $G(K_p, \mathbf{k}_u)$ *and the resulting price paths.*

Together, these optimality and equilibrium conditions define the dynamic evolution of consumption, portfolio composition, and prices across both dynasties. Differences in patience ($\beta_H > \beta_L$) and subsistence needs ($\bar{c}$) drive persistent wealth inequality and differential exposure to productive versus unproductive assets.

2.2. Analysis of Agents' Behavior: The Permanent Income Hypothesis

Let's analyze some important details about the behavior of Type H and Type L agents driven by the permanent income hypothesis (PIH).[9]

Analysis of Type H Agent Behavior. The Type H agent solves the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t \log(c_{H,t}) \tag{20}$$

Subject to their budget constraint:

$$A_{t+1} = R_{t+1}(A_t + w_t - c_t) \tag{21}$$

Where:

- A_t is their financial wealth at time t (which is $A_t = R_t k_{p,t} + p_{u,t} k_{u,t}$).
- w_t is their labor income at time t.
- R_{t+1} is the gross return on their assets from t to $t+1$. In this model, the no-arbitrage condition ensures $R_{t+1} = R_{p,t+1} = R_{u,t+1}$.

[9] Type H agents are patient, so we can assume their savings are positive. They possess logarithmic utility, so in the absence of labor income, they will consume a fraction $(1 - \beta_H)$ of their wealth and save the rest. Let us explain this. But, similarly, let us consider the Type L agents: Euler's inequality shows that the agent sums the present value of wages and financial wealth to calculate total wealth. Furthermore, they allocate a portion to finance minimum consumption $\bar{c}$.

- The model has no aggregate risk, so $E_0[w_t]$ and $E_0[R_t]$ are deterministic paths.

The solution with logarithmic utility, has a special property: the substitution effect (consuming less today when returns are high) and the income effect (consuming more today because high returns make you wealthier) exactly cancel out. This leads to a beautifully simple consumption rule that is a constant fraction of total wealth.

Case 1: No Labor Income ($w_t = 0$). As it may be noted, if the agent has no labor income, their problem is:

$$A_{t+1} = R_{t+1}(A_t - c_t) \tag{22}$$

The standard solution, derived from the Bellman equation, yields the consumption function:

$$c_t = (1 - \beta_H)A_t \tag{23}$$

The agent consumes the "annuity" value (or "interest") $(1 - \beta_H)$ of their financial wealth and saves the rest, $S_t = \beta_H A_t$.

Case 2: With Labor Income ($w_t > 0$). When we introduce labor income, the agent doesn't just consider their *financial* wealth; they consider their *total* lifetime resources. That is:

1. Human Wealth (H_t): We first define the agent's "human wealth" as the present discounted value of their entire future stream of labor income,

$$H_t = \sum_{j=0}^{\infty} \left(\frac{1}{\prod_{k=1}^{j} R_{t+k}} \right) w_{t+j} \tag{24}$$

   Since the model is deterministic, H_t is a known value at time t.

2. Total Wealth (W_t): The agent's total wealth is the sum of their financial wealth and human wealth:

$$W_t = A_t + H_t \tag{25}$$

3. The Intertemporal Budget Constraint: We can now write a new budget constraint in

terms of *total* wealth.

$$A_{t+1} + H_{t+1} = R_{t+1}(A_t + w_t - c_t) + H_{t+1} \tag{26}$$

By definition, $H_t = w_t + \frac{H_{t+1}}{R_{t+1}}$, which means $H_{t+1} = R_{t+1}(H_t - w_t)$. Substituting this in:

$$W_{t+1} = R_{t+1}(A_t + w_t - c_t) + R_{t+1}(H_t - w_t) \tag{27}$$

$$W_{t+1} = R_{t+1}(A_t + H_t - c_t) \tag{28}$$

$$W_{t+1} = R_{t+1}(W_t - c_t) \tag{29}$$

4. The Solution: This budget constraint for *total wealth* (W_t) is mathematically identical to the one in Case 1. Therefore, the consumption rule is also identical, i.e. the agent will consume a constant fraction $(1 - \beta_H)$ of their *total wealth*:

$$c_{H,t} = (1 - \beta_H)W_t = (1 - \beta_H)(A_t + H_t) \tag{30}$$

Hence, the Type H agent simply calculates their total financial wealth $A_t = R_t k_{p,t} + p_{u,t} k_{u,t}$ and their human wealth H_t (the present value of all future wages) and consumes a fraction $(1 - \beta_H)$ of the sum. In particular, note that:

- Patience and Savings. Because $\beta_H > \beta_L$, this agent consumes a *smaller* fraction of their wealth than the Type L agent. This is why they are the "patient" savers who accumulate capital in the long run.

- Link to "Crowding Out". This behavior is the precise micro-foundation of what we will show below, i.e.: i) When a bubble occurs, $p_{u,t}$ rises. ii) This directly increases the agent's financial wealth A_t. iii) This increases their total wealth W_t. iv) The agent responds by increasing their consumption: $c_{H,t} \uparrow = (1 - \beta_H)(A_t \uparrow + H_t)$. v)

This increase in consumption by the Type H agents reduces the total pool of savings available in the economy, "crowding out" investment in productive capital (K_p) and causing wages (w_t) to fall.

Analysis of Type L Agent Behavior. The Stone-Geary utility function $\log(c_L - \bar{c})$ leads to exactly the (constrained) Permanent Income Hypothesis (PIH) decision rule. Let's walk through the mathematical logic that confirms it.

Reformulating the Problem. The agent's problem is:

$$\max E_0 \sum_{t=0}^{\infty} \beta_L^t \log(c_{L,t} - \bar{c}) \tag{31}$$

Subject to:

$$A_{t+1} = R_{t+1}(A_t + w_t - c_{L,t}) \quad \text{and} \quad A_{t+1} \geq 0 \tag{32}$$

The key insight is to define a new variable, "discretionary consumption":

$$c_{d,t} \equiv c_{L,t} - \bar{c}$$

This is the amount of consumption *above* the subsistence level. The agent's actual consumption is therefore $c_{L,t} = \bar{c} + c_{d,t}$. Now, substitute this into the objective function and the budget constraint, i.e. the objective is:

$$\max E_0 \sum_{t=0}^{\infty} \beta_L^t \log(c_{d,t}) \tag{33}$$

and the budget constraint is:

$$A_{t+1} = R_{t+1}(A_t + w_t - (\bar{c} + c_{d,t})) \tag{34}$$

$$A_{t+1} = R_{t+1}(A_t + (w_t - \bar{c}) - c_{d,t}) \tag{35}$$

This transformation is powerful. The agent's problem is now mathematically identical to the Type H agent's problem, but with two differences:

1. They maximize utility over discretionary consumption ($c_{d,t}$).
2. Their "income" is discretionary income ($w_t - \bar{c}$).

The Unconstrained Decision Rule. Following the standard PIH solution (as we did for the Type H agent), the agent first calculates their total "discretionary" wealth, i.e.

1. Financial Wealth (A_t): $A_t = R_t k_{p,t} + p_{u,t} k_{u,t}$
2. "Discretionary" Human Wealth ($H_{d,t}$): This is the present value of all future *discretionary* income.
$$H_{d,t} = E_t \sum_{j=0}^{\infty} \left(\frac{1}{\prod R} \right) (w_{t+j} - \bar{c}) \tag{36}$$
3. Total Discretionary Wealth ($W_{d,t}$):
$$W_{d,t} = A_t + H_{d,t} \tag{37}$$

The optimal choice for $c_{d,t}$ is to consume the annuity value $(1 - \beta_L)$ of this total discretionary wealth:

$$c^*_{d,t} = (1 - \beta_L) W_{d,t} = (1 - \beta_L)(A_t + H_{d,t}) \tag{38}$$

The agent's total *actual* consumption is this amount plus the baseline subsistence:

$$c^*_{L,t} = \bar{c} + c^*_{d,t} = \bar{c} + (1 - \beta_L)(A_t + H_{d,t}) \tag{39}$$

This confirms the logic perfectly. We can rewrite $H_{d,t}$ as $H_{w,t} - H_{\bar{c},t}$ (where $H_{w,t}$ is the present value of *all* wages and $H_{\bar{c},t}$ is the present value of *all* future subsistence costs). This shows the agent "sets aside" the present value of their subsistence needs from their total wealth and consumes a fraction $(1 - \beta_L)$ of what remains.

The Borrowing Constraint. The rule above is the *unconstrained* solution. It implicitly assumes the agent can borrow against future discretionary income (i.e., A_{t+1} can be negative). However, the model imposes the borrowing constraint $A_{t+1} \geq 0$. This constraint is what is referred to as the "Euler inequality" ($u'(c_t) \geq \beta RE[u'(c_{t+1})]$), which only binds as an equality if the constraint is slack. The agent's total "cash-on-hand" (current financial wealth plus current income) is:

$$X_t = A_t + w_t$$

The constraint binds if the unconstrained optimal consumption $c^*_{L,t}$ is greater than the agent's cash-on-hand:

$$c^*_{L,t} > A_t + w_t$$

If this happens, the agent cannot consume their desired amount. They are constrained by their available resources and will simply consume all their cash-on-hand, leaving no savings.

$$c_{L,t} = A_t + w_t \quad \text{(if constraint binds)} \tag{40}$$

The Full Decision Rule. The agent's final, constrained decision rule is:

1. Calculate total cash-on-hand: $X_t = A_t + w_t$
2. Calculate total discretionary human wealth: $H_{d,t}$
3. Calculate unconstrained consumption: $c^*_{L,t} = \bar{c} + (1 - \beta_L)(A_t + H_{d,t})$
4. Choose the minimum of the two:

$$c_{L,t} = \min(c^*_{L,t}, X_t) \tag{41}$$

As we will see below, this behavior is is the heart of the poverty trap. When w_t is low (because K_p is low), the agent's $H_{d,t}$ is low (or negative). This makes $c^*_{L,t}$ low. Furthermore, their cash-on-hand X_t is low. They will be perpetually in or near the constrained region, consuming all their income just to get close to $\bar{c}$. Their savings rate, $s_L = (X_t - c_{L,t})/X_t$, endogenously falls to zero, trapping the economy in a low-capital, low-wage state.

3. Equilibrium Analysis

In the steady state, all aggregate and individual variables are constant over time:

$$c_{i,t} = c_i^*, \quad k_{i,t+1}^p = k_i^{p*}, \quad k_{i,t+1}^u = k_i^{u*}, \quad w_t = w^*, \quad r_t^p = r^{p*}, \quad p_t^u = p^{u*}.$$

Under stationarity and rational expectations, $\mathbb{E}_t[R_{p,t+1}] = R_p^*$ and $\mathbb{E}_t[R_{u,t+1}] = R_u^*$, so all expected returns are constant and equalized across assets: $R_p^* = R_u^* = \frac{1}{\beta_H} = \frac{1}{\beta_L}$, only if both types are unconstrained.

However, because $\beta_H > \beta_L$ and Type L may face binding consumption constraints, steady state equilibrium typically features: $R_p^* = \frac{1}{\beta_H} < \frac{1}{\beta_L}$, implying that only the high-patience dynasty (H) holds positive productive capital, while the low-patience dynasty (L) is either credit constrained or holds only unproductive assets. The Steady-State Euler Equations are: i) For Type H (log utility), $1 = \beta_H R_p^*$. ii) For Type L (Stone–Geary utility), $1 \geq \beta_L R_p^*$, with equality only if $c_L^* > \bar{c}$.

Aggregate Capital and Output. The aggregate steady-state capital stock and output satisfy the firm's first-order conditions, i.e. $r^{p*} = F_K(K^{p*}, 1) - \delta_p$, $w^* = F_L(K^{p*}, 1)$, and the resource constraint, $n_H c_H^* + n_L c_L^* + \delta_p K^{p*} = F(K^{p*}, 1)$.

Wealth Distribution in the Steady State. Given that only the patient type invests in productive assets, steady-state wealth is concentrated among H dynasties, i.e. $k_L^{p*} = 0$, $k_H^{p*} = \frac{K^{p*}}{n_H}$. Type L agents consume near their subsistence level, $c_L^* \approx \bar{c}$, while Type H agents accumulate productive wealth until the Euler condition binds with equality.

Therefore, we may notice that the steady state exhibits an endogenous stratification of wealth and portfolio composition. Patient dynasties (high β_H) accumulate productive capital and grow wealth over time, while impatient dynasties (low β_L) remain constrained near subsistence consumption and hold limited or no productive assets. This divergence is self-reinforcing: when the return on productive capital (r_p) exceeds the implicit return on unproductive assets (r_u), the wealthy dynasties continue to accumulate faster, whereas the constrained dynasties are unable to increase their wealth. Consequently, persistent differences arise in both income and welfare across agent types. Economically, this mechanism highlights how heterogeneity in patience and initial wealth, together with subsistence constraints, can generate long-run inequality endogenously. It also suggests that policies improving access to productive assets or relaxing subsistence constraints may help reduce the gap between dynasties and shift the economy toward higher aggregate outcomes

Let us analyze the different types of equilibria this model can exhibit. Particularly, i) **Fundamental Steady State (K_H^*)**, which consider a non-bubbling, stationary steady state where all aggregate quantities are constant: $K_p(t) = K_p^*,\ p_u(t) = p_u^*$. ii) **Poverty Trap Steady State ($K_L^* < K_p^*$)**, since the subsistence constraint $\bar{c}$ can create it, i.e. a low-level stable steady state. Moreover, let us consider that the aggregate law of motion for productive capital be $K_{p,t+1} = G(K_{p,t})$, where

$$G(K_p) = (1 - \delta_p)K_p + S(K_p)$$

and $S(K_p) = n_H S_H(K_p) + n_L S_L(K_p)$ is the aggregate savings allocated to productive capital (assuming no bubble, $\lambda = 1$). A steady state K^* satisfies $K^* = G(K^*)$, or equivalently $\delta_p K^* = S(K^*)$. Local stability is determined by the derivative $G'(K^*) = 1 - \delta_p + S'(K^*)$.

- If $|G'(K^*)| < 1$, the steady state is locally stable.
- If $|G'(K^*)| > 1$, the steady state is unstable.

Then, we can analyze the stability of the three potential steady states identified in the model: K_H^* (High-Income), K_L^* (Poverty Trap), and K_M^* (Unstable Threshold). The existence of these states depends on the model parameters, particularly the subsistence level $\bar{c}$.[10] Hence, we state the following propositions.

Proposition 1 (**Efficient Steady State**). *There exists a unique fundamental steady-state capital stock given by:*

$$K_H^* = \left(\frac{\alpha}{\delta_p}\right)^{\frac{1}{1-\alpha}} \tag{42}$$

This high-level equilibrium K_H^* is locally stable.

Proof. In a stationary steady state, the unproductive asset price is constant ($p_{u,t+1} = p_{u,t}$), yielding a gross return $R_u = 1$. The no-arbitrage condition (11) implies the gross return on productive capital must also be one, $R_p = 1$. This requires the net return to equal the depreciation rate:

$$r_p^* = \delta_p$$

Using the firm's optimality condition $r_p^* = \alpha(K_p^*)^{\alpha-1}$, we find the unique fundamental steady-state capital stock:

$$K_H^* = \left(\frac{\alpha}{\delta_p}\right)^{\frac{1}{1-\alpha}}$$

At this state, $w(K_H^*)$ is high, so $c_L > \bar{c}$. The constant price p_u^* is determined by the market clearing condition for $\bar{K}_u$ given the wealth distribution at K_H^*. The high-level equilibrium K_H^* is defined by $\delta_p K_H^* = S(K_H^*)$, where both agent types are saving ($w(K_H^*) > \bar{w}_{Thresh}$, the wage threshold implied by $\bar{c}$). It corresponds to the uppermost intersection of the $G(K_p)$ curve with the 45-degree line in the bifurcation diagram (Figure 1). Due to diminishing returns to capital in the production function ($\alpha < 1$), the wage function $w(K_p)$ is concave, and consequently, the aggregate savings function $S(K_p)$ becomes concave at high levels of K_p. Geometrically, this means the $G(K_p)$ curve approaches the 45-degree line from above. Mathematically, crossing from above implies that the slope of $G(K_p)$ at the intersection point

[10] We assume parameters are such that all three exist, consistent with Figure 1.

must be less than the slope of the 45-degree line (which is 1). Therefore,

$$G'(K_H^*) < 1$$

The derivative is $G'(K_H^*) = 1 - \delta_p + S'(K_H^*)$. For standard calibrations ensuring that savings do not decrease too rapidly with capital at the steady state (i.e., $S'(K_H^*) > -(1 - \delta_p)$), we also have $G'(K_H^*) > -1$. Typically, $S'(K_H^*) > 0$ holds due to income effects, ensuring $G'(K_H^*) > 0$. Thus, we have $0 < G'(K_H^*) < 1$, which implies $|G'(K_H^*)| < 1$. The high-income steady state K_H^* is locally stable. □

A non-bubbling steady state is a stationary equilibrium where $\dot{K}_p = 0$, $\dot{c}_i = 0$, and $\dot{p}_u = 0$.[11] For the High-Income Equilibrium (K_H^*), from $\dot{p}_u = 0$, the no-arbitrage condition gives: $r_p(K_H^*) - \delta_p = 0 \implies r_p(K_H^*) = \delta_p$. From $\dot{c}_H = 0$, the Euler equation (5) gives: $r_p(K_H^*) - \delta_p - \rho_H = 0 \implies r_p(K_H^*) = \delta_p + \rho_H$. However, these two conditions ($r_p = \delta_p$ and $r_p = \delta_p + \rho_H$) can only hold if $\rho_H = 0$, which is not standard. But, in a steady state with an unproductive asset, the asset's return $\dot{p}_u/p_u$ must be zero. For agents to hold both assets, the return on productive capital $r_p(K_p^*) - \delta_p$ must also be zero. This defines the Modified Golden Rule capital stock, $K_{MGR}^* = (\alpha/\delta_p)^{1/(1-\alpha)} = K_H^*$. That is, K_H^* represents the standard "modified golden rule" capital stock, the efficient level sustainable in the long run without bubbles. Moreover, agents will only choose this level of capital if their discount rate matches, i.e., $\rho_H = 0$. If $\rho_H > 0$, agents are too impatient to sustain K_{MGR}^*. They will decumulate capital until their marginal return equals their discount rate.

Corollary 1. *At this steady state, the wage $w(K_H^*) = (1-\alpha)(K_H^*)^\alpha$ must be sufficiently high*

[11] Notice that, in a standard growth model without a bubbly asset, the steady state K^* is defined by $\dot{c}_H = 0 \implies r_p(K^*) - \delta_p = \rho_H$. This gives the standard steady state:

$$K^* = \left(\frac{\alpha}{\rho_H + \delta_p}\right)^{\frac{1}{1-\alpha}}$$

In our model, the unproductive asset K_u can only be held in steady state if its return (0) equals the return on productive capital. Thus, the only possible steady state is $K_H^* = (\alpha/\delta_p)^{1/(1-\alpha)}$. For this to be an equilibrium, we must assume agents are patient enough to support it (e.g., $\rho_H = 0$).

such that $w(K_H^*) > \bar{c}$ (assuming parameters allow this state to exist), so both agent types are saving and consuming strictly above subsistence. The wealth distribution is determined by the difference in patience ($\beta_H > \beta_L$).

Let's assume the parameters (ρ_H, ρ_L) are such that the economy would converge to a high K_H^*. The poverty trap K_L^* exists as a second, separate steady state if the subsistence constraint binds. This is an equilibrium $K_L^* < K_H^*$ where $c_L(t) = \bar{c}$ and $\dot{c}_L = 0$. The poverty trap is not a different steady-state level, but rather a path-dependent outcome where the distribution of wealth is different (Type L agents are trapped at $\bar{c}$ while Type H agents hold all the capital).

Proposition 2 (Poverty Trap). *If the subsistence constraint $\bar{c}$ is sufficiently high relative to the wage generated at low capital stocks, there exists a stable steady state (low-level equilibrium) $K_L^* < K_H^*$. The poverty trap steady state (K_L^*) is locally stable.*

Proof. The poverty trap steady state K_L^* is the lowest positive steady state, defined by $\delta_p K_L^* = S(K_L^*)$, where Type L agents are constrained ($w(K_L^*) \leq \bar{w}_{Thresh}$). At this point, $S_L(K_L^*) = 0$ and the derivative $S_L'(K_L^*) = 0$ (assuming K_L^* is strictly below the threshold where $w = \bar{w}_{Thresh}$). Aggregate savings are solely determined by Type H agents: $S(K_L^*) = n_H S_H(K_L^*)$. That is, a steady state $K_L^* < K_H^*$ where the subsistence constraint binds for Type L agents, so $c_L(K_L^*) = \bar{c}$.

- At this point, $S_L(K_L^*) = 0$.
- The capital stock is sustained only by Type H agents.
- The Type H Euler equation in steady state is: $u_H'(c_H^*) = \beta_H u_H'(c_H^*) \cdot R_p(K_L^*)$
- This simplifies to $R_p(K_L^*) = 1/\beta_H$.

Substituting the return function:

$$\alpha(K_L^*)^{\alpha-1} + (1 - \delta_p) = 1/\beta_H$$

Solving for K_L^*:

$$K_L^* = \left(\frac{\alpha}{1/\beta_H - (1-\delta_p)}\right)^{\frac{1}{1-\alpha}}$$

This second stable steady state creates the S-shaped law of motion. For K_L^* to be a stable trap, the $G(K_p)$ curve must cross the 45-degree line from above at this point. This requires $G'(K_L^*) < 1$. The derivative is:

$$G'(K_L^*) = 1 - \delta_p + S'(K_L^*) = 1 - \delta_p + n_H S_H'(K_L^*)$$

Stability ($G'(K_L^*) < 1$) requires $n_H S_H'(K_L^*) < \delta_p$. Can this condition hold? The savings function $S_H(K_p)$ depends on the wage $w(K_p)$ and the return $r_p(K_p)$ at K_L^*. While the Inada conditions imply $r_p(K_p) \to \infty$ as $K_p \to 0$, the steady state K_L^* occurs at a positive capital level. At K_L^*, $r_p(K_L^*)$ is finite but typically high, and $w(K_L^*)$ is low. The response of Type H savings $S_H'(K_L^*)$ depends on the interplay between the income effect (higher K_p raises w) and the substitution effect (higher K_p lowers r_p). For standard preferences like log-utility, the income effect often dominates at low capital levels, making $S_H'(K_L^*) > 0$. However, if Type H agents are sufficiently patient (high β_H), their savings function will be relatively flat, especially if α is low (low capital share). If n_H is small (few Type H agents), the condition $n_H S_H'(K_L^*) < \delta_p$ is likely to hold. Geometrically, the existence of three intersections as depicted in Figure 1, with K_L^* being the lowest positive one, requires the $G(K_p)$ curve to cross from above at K_L^*. Therefore, assuming parameters consistent with the existence of such a trap, we must have $0 < G'(K_L^*) < 1$. Thus, K_L^* is locally stable. □

This poverty trap equilibrium, K_L^*, has the following properties:

1. The wage $w(K_L^*) = (1-\alpha)(K_L^*)^\alpha$ is low enough that Type L agents are at or bindingly close to their subsistence constraint $c_{L,t} = \bar{c}$.

2. The marginal utility for Type L agents is effectively infinite, $u'(c_L) \to \infty$. Their Euler equation (5) breaks down as an equality, and their optimal savings rate endogenously

falls to zero (they consume all income to meet $\bar{c}$).

3. The aggregate productive capital stock K_L^* is maintained solely by the savings of the Type H agents. It is implicitly defined by the Type H Euler equation ($1/c_H = \beta_H(1/c_H)R_p$) evaluated at K_L^* and the goods market clearing condition ($Y(K_L^*) = n_H c_H + n_L \bar{c} + \delta_p K_L^*$).

Notice that, the economy exhibits path dependence. An unstable threshold K_M^* separates the basins of attraction of K_L^* and K_H^*. Economies starting below K_M^* converge to the poverty trap, characterized by low output, low wages, and extreme inequality (Type L agents hold zero wealth and consume only $\bar{c}$).

Proposition 3. *The Middle Steady State (K_M^*) is unstable.*

Proof. The middle steady state K_M^* is the intermediate intersection, located between K_L^* and K_H^*. It typically occurs near the capital level where the wage $w(K_p)$ crosses the threshold $\bar{w}_{Thresh}$ required for Type L agents to start saving. Just below K_M^*, $S_L(K_p) = 0$. Just above K_M^*, $S_L(K_p) > 0$ and is increasing rapidly ($S_L'(K_p) > 0$ is large). This rapid increase in savings from the large population n_L causes the aggregate savings function $S(K_p) = n_H S_H(K_p) + n_L S_L(K_p)$ to become sharply convex and steep around K_M^*. Geometrically, the $G(K_p)$ curve must cross the 45-degree line from below at K_M^*. This implies that the slope of $G(K_p)$ must be greater than the slope of the 45-degree line at this point.

$$G'(K_M^*) > 1$$

The derivative is $G'(K_M^*) = 1 - \delta_p + S'(K_M^*)$. The condition $G'(K_M^*) > 1$ requires $S'(K_M^*) > \delta_p$. Given that $S'(K_M^*)$ captures the sharp "kick-in" of savings from the large group n_L as they overcome the subsistence constraint, this condition will generally hold for parameters that generate the S-shaped curve and multiple equilibria. Since $G'(K_M^*) > 1$, the middle steady state K_M^* is unstable. It acts as a threshold separating the basins of attraction for K_L^* and K_H^*. □

3.1. Rational Bubbles, Crowding Out, and Wealth Inequality Dynamics

The model allows for non-stationary equilibria where the price of the unproductive asset grows indefinitely.

Rational Bubbles with Infinite Horizons. A central feature of our analysis is the existence of a rational bubble. At first glance, this contradicts the canonical result that rational bubbles are ruled out in economies with infinitely-lived, optimizing agents (e.g., Kamihigashi, 2017). This standard no-bubble theorem, however, relies critically on the assumption of complete markets and the resulting aggregate transversality condition (TVC). Our model deviates from this setup. The subsistence constraint ($\bar{c}$) on the impatient (Type L) agents, combined with a no-borrowing constraint ($A_{L,t+1} \geq 0$), creates a powerful and persistent form of endogenous market incompleteness. Type L agents are unable to borrow against their future labor income to smooth consumption. This market incompleteness is precisely what invalidates the standard aggregate TVC and permits a bubble to exist, as formalized in the following proposition (which builds on the class of models exemplified by Martin and Ventura, 2012).

Proposition 4 (Existence of a Rational Bubble Equilibrium)**.** *Assume an economy with Type H (β_H) and Type L ($\beta_L, \bar{c}$) agents, with $\beta_H < 1$. If the parameters ($\bar{c}, n_L, \beta_L$) are such that in a steady state K^*, Type L agents are permanently constrained (i.e., $A_{L,t+1} = 0$ because their desired unconstrained consumption $c^*_{L,t}$ exceeds their cash-on-hand X_t), then the aggregate economy is characterized by market incompleteness. This incompleteness allows for a stationary rational bubble equilibrium ($K^*_{bub}, p_{u,t}$) defined by:*

1. ***A constant, constrained productive capital stock*** *K^*_{bub} defined by the patient (Type H) agent's Euler equation:*

$$R_p(K^*_{bub}) = \alpha(K^*_{bub})^{\alpha-1} + (1 - \delta_p) = \frac{1}{\beta_H} \tag{43}$$

2. ***A positive, non-fundamental price path*** *$p_{u,t} > 0$ for the unproductive asset, which*

grows at the economy's (high) fundamental rate of return:

$$\frac{p_{u,t+1}}{p_{u,t}} = R_p(K^*_{bub}) = \frac{1}{\beta_H} \tag{44}$$

Proof. An economic intuition. The standard no-bubble proof (Kamihigashi, 2017) relies on backward induction from infinity, which shows that no agent would be the "last" holder of a worthless asset. This argument fails in our model for two reasons:

1. Persistent Capital Scarcity: The subsistence constraint on the large n_L population group prevents them from saving. This "under-saving" starves the economy of aggregate capital, keeping K_p permanently *below* the dynamically efficient (Modified Golden Rule) level. Because capital is scarce, its marginal product $r_p(K_p)$ is high.

2. High Required Return: The equilibrium return on productive capital is therefore pinned down by the patient, unconstrained Type H agents' Euler equation. They are the only agents saving, and they will accumulate capital until its return equals their required rate of time preference: $R_p(K^*) = 1/\beta_H$. Since $\beta_H < 1$, this means the fundamental return of the economy is $R_p > 1$.

3. Sustaining the Bubble: This permanently high rate of return is what sustains the bubble. The Type H agents are the only ones wealthy enough to hold assets. They face a no-arbitrage condition between K_p and K_u. Since K_p offers a high fundamental return of $1/\beta_H$, they are perfectly willing to hold the "bubbly" asset K_u as long as its price is also expected to grow at that same rate ($p_{u,t+1}/p_{u,t} = 1/\beta_H$).

In essence, the market incompleteness caused by the constrained Type L agents ensures that the fundamental return R_p never falls to 1, thus the bubble's growth is rational and sustainable. The unconstrained Type H agents satisfy their own transversality condition, but the aggregate TVC fails, allowing the bubble to exist as a store of value held by the patient agents. □

A rational bubble equilibrium is a sequence of prices and allocations satisfying the equilibrium conditions where $p_{u,t} \to \infty$ as $t \to \infty$, and the no-arbitrage condition (12) holds along the path. Such bubbles can exist if the economy's growth rate in the fundamental steady state is less than the rate of return on capital (dynamic efficiency, $R_p >$ Growth Rate).

A non-stationary equilibrium where $p_{u,t} \to \infty$. The bubble price $B_t = p_{u,t}$ must satisfy the no-arbitrage condition:

$$E_t[B_{t+1}] = B_t \cdot E_t[R_{p,t+1}] = B_t \cdot E_t[\alpha K_{p,t+1}^{\alpha-1} + (1 - \delta_p)].$$

A rational bubble is a non-stationary equilibrium where $p_u(t) \to \infty$ but all equilibrium conditions hold. No-Arbitrage, i.e. the bubble's price $p_u(t)$ must grow at the same rate as the net return on capital:

$$\frac{\dot{p}_u(t)}{p_u(t)} = r_p(K_p(t)) - \delta_p$$

Bubble-Sustained State (K^*_{bub}): Can a bubble exist in a steady state (i.e., $\dot{K}_p = 0$)? If $\dot{K}_p = 0$, then $K_p(t) = K^*_{bub}$. This implies $r_p(K^*_{bub})$ is constant. The no-arbitrage condition becomes $\frac{\dot{p}_u(t)}{p_u(t)} = r_p(K^*_{bub}) - \delta_p = g_p$ (a constant growth rate). This is a standard "bubble path" where $p_u(t) = p_u(0)e^{g_p t}$. The economy's dynamics are driven by the Type H agents' Euler equation: $\frac{\dot{c}_H}{c_H} = r_p(K^*_{bub}) - \delta_p - \rho_H$. For K_p to be constant, aggregate consumption C must grow at the same rate as output Y (which is 0). Thus, $\dot{c}_H = 0$. This implies $r_p(K^*_{bub}) - \delta_p = \rho_H$. This defines the steady state $K^*_{bub} = (\alpha/(\rho_H + \delta_p))^{1/(1-\alpha)}$.

Therefore, a bubble can grow at rate $g_p = \rho_H$, sustained by the standard steady-state capital stock K^*_{bub}.

Proposition 5 (Speculative Crowding Out)**.** *The existence of a rational bubble on the unproductive asset causes a permanent reduction in the steady-state level of productive capital compared to the fundamental steady state ($K^*_{bub} < K^*_H$).*

The mechanism (Endogenous Wealth Effect) is characterized by:

1. **Wealth Effect**. In a bubble equilibrium, $p_{u,t}$ is high and growing. This increases the total wealth $W_{i,t} = [r_{p,t} + (1 - \delta_p)]k_{p,i,t} + p_{u,t}k_{u,i,t}$ of agents holding the unproductive asset.

2. **Optimal Response**. According to the optimizing agents' Euler equations (5), a higher current wealth (relative to future expected income) leads to higher current consumption $c_{i,t}$ (consumption smoothing). From $u_i'(c_{i,t}) = \beta_i E_t[...]$, agents with diminishing marginal utility smooth this wealth increase by consuming more today. Thus, $c_{i,t} \uparrow$.

3. **Resource Constraint**. Higher aggregate consumption $C_t = n_H c_{H,t} + n_L c_{L,t}$ implies, through the economy's resource constraint ($Y_t = C_t + I_t$), lower aggregate investment $I_t = K_{p,t+1} - (1 - \delta_p)K_{p,t}$.

4. **Insights:** The economy converges to a new, lower "bubble-sustained" equilibrium K^*_{bub} where the lower level of investment is just sufficient to cover depreciation $\delta_p K^*_{bub}$. This K^*_{bub} is necessarily lower than K^*_H. That is, given Y_t, an increase in C_t must be offset by a decrease in I_t, i.e. $I_t \downarrow \implies K_{p,t+1} \downarrow$. The bubble crowds out productive capital. Thus,

$$\dot{K}_p(t) = K_p(t)^\alpha - (n_H \rho_H a_H(t) + n_L c_L(t)) - \delta_p K_p(t)$$

   The key insight is that the total wealth $a_H(t)$ held by the saving agents includes the bubble component $p_u(t)k_{u,H}(t)$. A larger bubble (higher $p_u(t)$) means $a_H(t)$ is larger for any given $K_p(t)$. Since $c_H(t)$ is proportional to $a_H(t)$, a larger bubble leads to higher consumption $c_H(t)$. From the $\dot{K}_p(t)$ equation, higher $C(t)$ leads to lower net investment $\dot{K}_p(t)$. Therefore, the existence of the term $p_u(t)\bar{K}_u$ in the aggregate wealth of saving agents creates a wealth effect that increases consumption and "crowds out" investment in productive capital, leading to a lower steady-state K^*_p than would exist without the bubble.

Speculation is not neutral. By creating fictitious wealth, it endogenously reduces the economy's propensity to save, directly "crowding out" real investment in factories, technology, and infrastructure. This leads to a permanently lower level of real output and real wages. Therefore, the interaction between bubbles and heterogeneity starkly impacts wealth distribution. That is:

1. **Direct Wealth Effect:** Since patient agents (Type H) endogenously hold a larger share of total wealth (due to $\beta_H > \beta_L$), they also hold a larger share of the unproductive asset $\bar{K}_u$ in any equilibrium. A bubble disproportionately increases their wealth through capital gains on $p_{u,t}$.

2. **Indirect Wage Effect:** The bubble crowds out K_p, leading to a lower K^*_{bub}. This reduces the marginal product of labor, causing the real wage $w(K^*_{bub})$ to fall. This directly harms the constrained agents (Type L) who rely more heavily on wage income.

3. **Trap Deepening:** The fall in wages pushes Type L agents closer to their subsistence constraint $\bar{c}$. A large enough bubble can trigger a collapse from the vicinity of K^*_H into the basin of attraction of the poverty trap K^*_L, dramatically increasing inequality and reducing social mobility.

Corollary 2 (Bubble-Induced Collapse). *The consequence of a temporary bubble, which can push the economy from the K^*_H basin of attraction to the K^*_L basin, causing a permanent collapse.*

To illustrate the model's implications across different levels of economic development, we calibrate its parameters to represent stylized High-Income (HIC), Middle-Income (MIC), and Low-Income (LIC) economies. This allows us to analyze how structural differences affect the prevalence of poverty traps and the vulnerability to speculative bubbles.

4. Calibration and Simulation Analysis: Income Group Comparison

We base our calibration on empirical regularities observed across countries at different development stages, drawing on academic literature and international databases. The core

DGE model structure remains the same, but key parameters are adjusted as follows:

- **Capital Share (α):** Often observed to be higher in developing countries due to relative factor scarcities and measurement issues (Gollin, 2002).

- **Depreciation Rate (δ_p):** Tends to be higher in lower-income countries reflecting capital quality, infrastructure, and faster obsolescence (data from Penn World Table, e.g., PWT 10.0, Feenstra et al., 2015).

- **Discount Factors (β_H, β_L):** Lower patience (higher discount rates) are typically associated with lower income levels due to factors like higher mortality risk, weaker institutions, and less developed financial markets. We assume a constant gap between β_H and β_L.

- **Subsistence Level ($\bar{c}_{rel}$):** We define the subsistence threshold $\bar{c}$ relative to the potential high-income steady-state output $Y(K_H^*)$ achievable with the country's technology (α, δ_p). This relative threshold is significantly higher in poorer countries (World Bank Poverty Lines).[12]

- **Share of Constrained Agents (n_L):** The fraction of the population living near or below poverty lines, or facing binding liquidity constraints, is substantially higher in LICs and MICs (World Bank PovcalNet).[13]

- **Unproductive Asset Supply ($\bar{K}_{u,rel}$):** The value of assets like land and housing relative to GDP varies. We assume a higher ratio in MICs due to rapid urbanization pressures (Knoll et al., 2017b).

[12] See: https://datatopics.worldbank.org/world-development-indicators/themes/poverty-and-inequality.html, https://www.worldbank.org/en/news/factsheet/2025/06/05/june-2025-update-to-global-poverty-lines

[13] See: https://pardeewiki.du.edu/index.php?title=PovcalNet_Online_Poverty_Analysis_Tool, https://pip.worldbank.org

Table 1 summarizes the parameter sets used.[14]

Table 1: Calibration Parameters by Income Group

Parameter	HIC	MIC	LIC	Justification/Source
Capital Share α	0.33	0.36	0.4	Increasing with lower development (Gollin, 2002)
Depreciation δ_p	0.06	0.08	0.1	PWT data, higher rates in developing economies
High Patience β_H	0.97	0.95	0.93	Implies real rates$\approx$ 3%, 5%, 7%
Low Patience β_L	0.94	0.92	0.9	$\beta_L = \beta_H - 0.03$
Subsistence (% of Y_H^*) $\bar{c}_{rel}$	10%	30%	50%	Higher relative poverty thresholds in LICs (World Bank)
Unprod. Assets (% of Y_H^*) $\bar{K}_{u,rel}$	300%	400%	200%	Land/housing value relative to GDP (Knoll et al., 2017b)
Share Constrained n_L	0.4	0.65	0.85	Reflects poverty/liquidity constraint data (World Bank)
Share Unconstr. n_H	0.6	0.35	0.15	$n_H = 1 - n_L$

We use the simplified OLG-style mapping for visualization, derived from the calibrated parameters. $K_{p,t+1} = (1-\delta_p)K_{p,t} + \lambda_t[n_H S_H(w_t, K_{p,t}) + n_L S_L(w_t, K_{p,t})]$.

Simulation 1: Bifurcation Analysis Across Income Groups. We compare the aggregate law of motion $K_{p,t+1} = G(K_{p,t})$ for each economy type (assuming no bubble, $\lambda_t = 1$). Figure 1 plots the curves against the 45-degree line.

[14]Note: Absolute subsistence $\bar{c}$ is calculated for each type based on its $\bar{c}_{rel}$ and its specific $Y(K_H^*)$. $\bar{K}_u$ is similarly scaled.

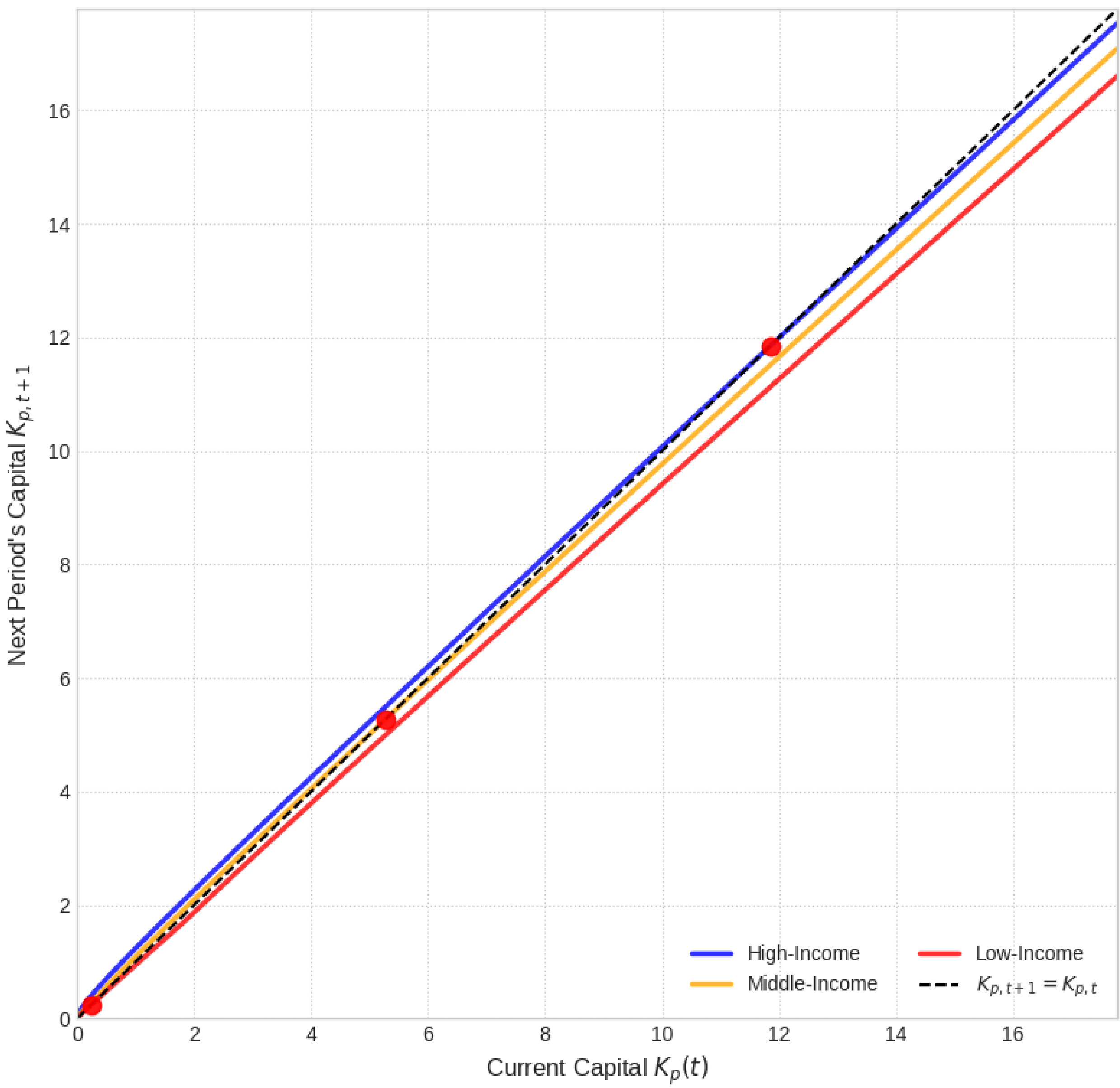

Figure 1: Bifurcation Diagrams by Income Group. It shows the $G(K_p)$ curves for HIC (Blue), MIC (Orange), and LIC (Red). The S-shape becomes more pronounced, the poverty trap (K_L^*) becomes larger and more dominant, and the high-income state (K_H^*) becomes lower or vanishes entirely as income level decreases.

A feasible interpretation of Figure 1 is as follows:

- HIC (Blue): The S-shape is mild. The subsistence constraint affects a smaller part of the population ($n_L = 0.4$) and the relative threshold ($\bar{c}_{rel} = 10\%$) is low. It has a stable high-income state K_H^* and potentially a very low, stable poverty trap K_L^* and an unstable K_M^*. Escaping the trap (if it exists) is relatively easy.

- MIC (Orange): The S-shape is more pronounced ($n_L = 0.65, \bar{c}_{rel} = 30\%$). The poverty trap K_L^* is higher and more stable (basin of attraction is larger). The high-income state K_H^* still exists but may be slightly lower than the HIC due to higher depreciation/lower patience. The gap between K_L^* and K_H^* widens.

- LIC (Red): The S-shape is very strong ($n_L = 0.85, \bar{c}_{rel} = 50\%$). The poverty trap K_L^* is high, stable, and has a large basin of attraction. Due to the high share of constrained agents, high subsistence, higher depreciation, and lower patience, the high-income state K_H^* may vanish entirely (or become very difficult to reach). The economy is highly likely to be stuck in the poverty trap. This illustrates a severe poverty trap scenario.

Simulation 2: Bubble Impact Across Income Groups (Bubble Simulation: Crowding Out). We simulate a temporary (e.g., 10-period) speculative bubble ($\lambda_t = 0.5$ for $t \in [20, 30]$) starting from each economy's respective high-income steady state K_H^* (or a hypothetical point above K_M^* if K_H^* doesn't exist for the LIC). Figure 2 shows the results.

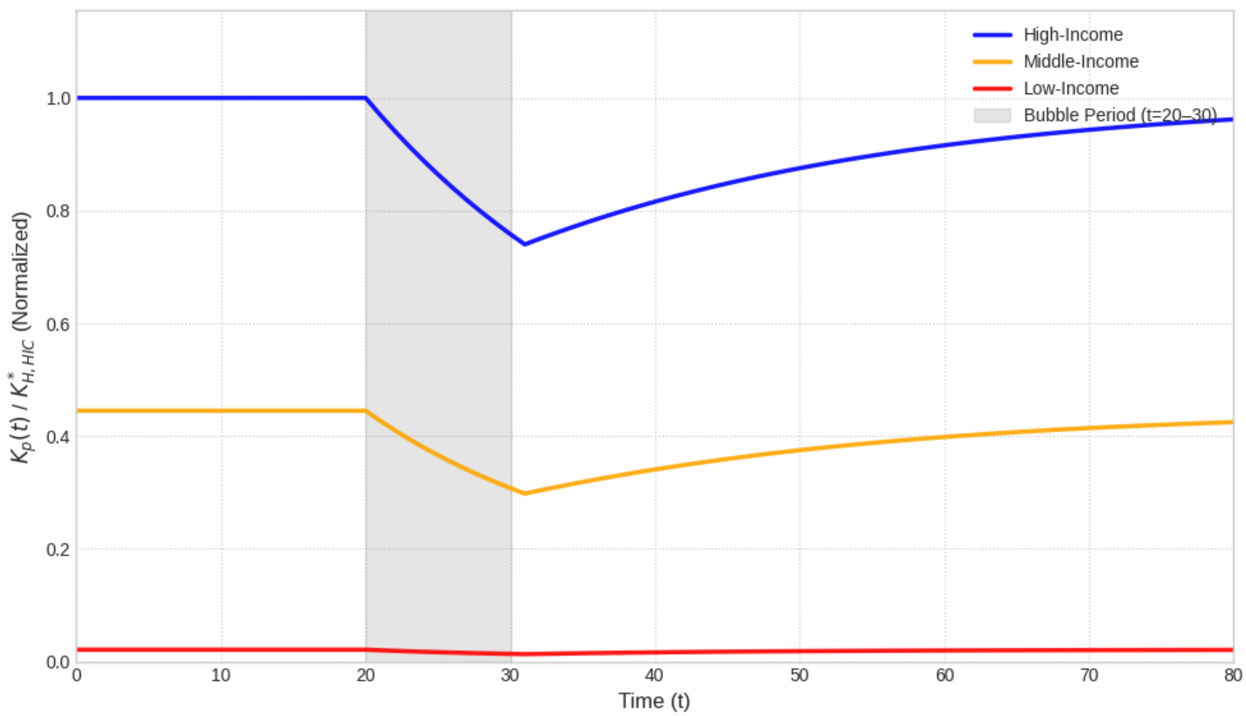

Figure 2: Time-Series Simulation of Bubbles by Income Group. Note: Shows $K_p(t)$ trajectories for HIC (Blue), MIC (Orange), and LIC (Red) when hit by a bubble. K_p axis shows capital relative to the HIC's K_H^*. The bubble causes a temporary dip in HIC, a collapse to the trap in MIC, and reinforces the trap in LIC.

A feasible interpretation of Figure 2 is as follows:

- HIC (Blue): The bubble causes a noticeable drop in productive capital K_p. However, the economy starts far from its unstable threshold K_M^*. After the bubble bursts (λ_t returns to 1), the economy recovers relatively quickly back to its high-income state K_H^*. Bubbles cause recessions but likely not permanent stagnation.

- MIC (Orange): The bubble induces a much deeper fall in K_p. The economy starts closer to its threshold K_M^*. The bubble is strong enough to push the capital stock below K_M^*. That is, the bubble pushes K_p below K_M^*. Even after the bubble bursts, the economy cannot recover to K_H^* and collapses into the poverty trap K_L^*. Even after the bubble bursts, the economy is caught in the poverty trap's basin of attraction and collapses towards K_L^*. Speculation can trigger a permanent development reversal.

- LIC (Red): If starting hypothetically above its K_M^*, the LIC collapses extremely rapidly back towards its dominant poverty trap K_L^*. If already in the trap, the bubble has little effect on K_p (as savings are already minimal) but would still cause wealth fluctuations and potentially worsen consumption for the constrained poor if asset prices affect subsistence costs indirectly. That is, if the LIC starts in a poverty trap K_L^*, a bubble might temporarily raise wealth but won't fundamentally change the equilibrium (as savings are already minimal). If we imagine an LIC temporarily pushed above K_M^* (e.g., by aid) and then hit by a bubble, it would immediately collapse back into the trap K_L^*. The economy shows extreme vulnerability and lack of resilience.

This comparative calibration strongly suggests that the same economic model produces vastly different outcomes depending on structural parameters associated with the level of development. For instance:

1. Endogenous Traps Vary: Poverty traps are not just a theoretical possibility but a likely outcome for economies with high subsistence needs, large constrained populations, low patience, and high depreciation, consistent with characteristics of many LICs.

2. Vulnerability Correlates with Income: Middle-income countries appear most vulnerable to suffering permanent damage from financial speculation, as they may have escaped the worst poverty but remain close enough to the threshold for a bubble-induced savings drop to trigger a collapse. High-income countries seem more resilient due to larger buffers above subsistence.

3. Policy Implications Differ: Anti-bubble policies (e.g., macroprudential regulation, transaction taxes) seem most critical for MICs to prevent reversals. For HICs, they might be more about managing volatility. For LICs, policies addressing the fundamental drivers of the trap (boosting productivity, improving institutions to raise patience/lower risk, increasing n_H via education/mobility) are paramount.

5. Concluding remarks

This paper develops a general equilibrium model with optimizing, heterogeneous agents that provides a unified explanation for the persistence of poverty traps, sluggish real growth, and rising wealth inequality. By replacing exogenous behavioral assumptions with standard micro-foundations (subsistence constraints and portfolio optimization), we show:

1. **Poverty traps** are a robust equilibrium outcome arising endogenously from subsistence needs, which halt savings for low-income agents when wages are low.

2. **Rational speculative bubbles** in unproductive assets can exist and are sustained by the agents' optimizing portfolio decisions.

3. These bubbles generate a negative wealth effect on aggregate savings, which permanently crowds out productive capital, leading to lower real wages and output.

4. Bubbles disproportionately increase the wealth of patient, high-wealth agents while simultaneously harming impatient, low-wealth agents by suppressing wages, thus providing a direct mechanism linking financial speculation to real economic stagnation and rising inequality.

The model suggests that the composition of national wealth matters significantly for real economic outcomes. Financial speculation, even when "rational," can impose significant negative externalities on the productive economy. This provides a strong theoretical rationale for considering policies aimed at curbing unproductive asset bubbles, such as targeted capital gains taxes, land value taxes, or financial transaction taxes. Such policies may not only improve wealth distribution but could also enhance real economic growth by breaking the "crowding out" mechanism and redirecting savings towards productive investment. Future research could explore the quantitative magnitude of these effects using more sophisticated numerical techniques and examine the optimal design of such policies in stochastic environments or with richer asset structures.

A fruitful avenue for future research would be to empirically test this prediction. Using the state-of-the-art Pareto extrapolation framework developed by Émilien Gouin-Bonenfant and Toda (2023) to construct reliable time-series data on top wealth shares, researchers could test whether periods of asset bubbles, as identified in this paper, are indeed followed by a suppression of real wage growth and an acceleration of wealth concentration.

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