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WHAT CAPITAL THEORY CAN TEACH US, REVISED

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Abstract

This paper attempts a pedagogical explanation of what the debates in capital theory around Sraffa's results are about, and why they have profound implications for the theory of income distribution. First it is pointed out that the classical or surplus approach does not suffer from the inconsistencies it was accused of. Then the difference is stressed between traditional neoclassical theory which attempted the determination of long-period general equilibria, and the neo-Walrasian versions. A simple model illustrates the need for a given endowment of value capital for the determination of long-period equilibria; the impossibility of specifying this endowment is pointed out; the abandonment of that treatment of capital in the neo-Walrasian versions is shown to render those versions incapable of indicating the behaviour of actual economies.

JEL Classification B21 B51 D50

1. The classical conflict-based view of wage determination.

In 1953 Joan Robinson published an article criticizing the neoclassical treatment of capital as a single factor of production, arguing that the neoclassicals never clarify in what units capital is measured. This started a debate, that took renewed vigour after the publication in 1960 of Piero Sraffa's book *Production of Commodities by Means of Commodities*, in which Sraffa argued that a correct analysis of how normal prices and choice of technique depend on income distribution produces results that "cannot be reconciled with any notion of capital as a measurable quantity independent of distribution and prices". Subsequent contributions, in particular by Pierangelo Garegnani, argued that those results undermined the entire neoclassical approach to value and distribution; the neoclassicals denied it. The debate still continues, but nowadays generally it is not mentioned at all to economics students, who therefore

¹ In recent years this deeply revised and extended version of a lecture delivered to graduate students at Università di Pisa on March 27, 2015 and then published as Fabio Petri, "What Capital Theory Can Teach Us", in Gabellini, Gasperin, Moneta (eds.), *Economic Crisis and Economic Thought*, Routledge 2019, has been a basic reading for my Siena lectures to the Economics PhD program of the Tuscan Universities. This is the 2025 version. **MCM** stands for F. Petri, *Microeconomics for the Critical Mind*, Springer, 2021.

cannot make up their minds as to its relevance. It is my task here to supply the minimal elements needed to understand why the results in capital theory at the centre of this debate are very important. We will be able (i) to understand the central differences between the two main competing approaches to value distribution and growth at present dominating debates in economic theory: the classical or surplus approach, and the neoclassical or supply-and-demand approach; (ii) to grasp a fundamental change undergone after the 1930s by the neoclassical approach, with the abandonment of the method of long-period positions; (iii) to conclude that the surplus approach is clearly scientifically the stronger one; (iv) to understand the need for a non-neoclassical macroeconomics not based on Say's Law.

Capital theory studies the effects of the presence of capital goods (produced means of production) for the determination of prices and income distribution; in particular, it studies what determines the rate of return on capital (rate of profits or rate of interest). Historically, this question has received two very different answers, connected with the two main approaches to income distribution, employment, and growth that have dominated economic theory in succession: the classical or surplus approach of Adam Smith, David Ricardo and Karl Marx; and the neoclassical or marginal or supply-and-demand approach, which has dominated since the end of the 19th century. In the 1950s and 1960s Piero Sraffa brought the surplus approach back to the attention of economists, arguing that it was in fact superior to the neoclassical approach, on the basis of new results precisely in the theory of capital: these results show that an apparent weakness of the surplus approach in the determination of the rate of profits, long considered a proof of the untenability of the approach, is in fact surmountable, while the neoclassical approach comes out to encounter insurmountable difficulties. A consistent minority of economists has been convinced by his arguments, and has developed a non-neoclassical approach to value and distribution that combines the Keynesian role of effective demand with the views on income distribution of Marx, Ricardo, and Adam Smith.

Adam Smith next to Marx may surprise you, because he is often presented as a precursor of the neoclassical view of competitive markets as very efficient, owing to his mention of an 'invisible hand'; in fact, it suffices to turn to the issue that most sharply distinguishes the classical from the neoclassical approach – what determines real wages – to see that Smith is clearly not neoclassical, indeed he sounds Marxist before Marx, with his explicit acceptance of class conflict:

“The workmen desire to get as much, the masters [Smith's term for the employers, the capitalists] to give as little as possible. The former are disposed to combine in order to raise, the latter in order to lower the wages of labour.

“It is not, however, difficult to foresee which of the two parties must, upon all ordinary occasions, have the advantage in the dispute, and force the other into a compliance with their

terms. The masters, being fewer in number, can combine much more easily ... A landlord, a farmer, a master manufacturer, a merchant, though they did not employ a single workman, could generally live a year or two upon the stocks which they have already acquired. Many workmen could not subsist a week, few could subsist a month, and scarce any a year without employment. In the long-run the workman may be as necessary to his master as his master is to him; but the necessity is not so immediate. ...

“[The masters’ combinations] are frequently resisted by a contrary defensive combination of the workmen; who sometimes too, without any provocation of this kind, combine of their own accord to raise the price of their labour. ... But whether their combinations be offensive or defensive, they are always abundantly heard of. In order to bring the point to a speedy decision, they have always recourse to the loudest clamour, and sometimes to the most shocking violence and outrage. They are desperate, and act with the folly and extravagance of desperate men, who must either starve, or frighten their masters into an immediate compliance with their demands. The masters upon these occasions are just as clamorous upon the other side, and never cease to call aloud for the assistance of the civil magistrate ... The workmen, accordingly, very seldom derive any advantage from the violence of those tumultuous combinations, which, partly from the interposition of the civil magistrate, partly from the superior steadiness of the masters, partly from the necessity which the greater part of the workmen are under of submitting for the sake of present subsistence, generally end in nothing but the punishment or ruin of the ringleaders.” (Wealth of Nations, Bk. I, Ch. VIII, paragraphs xi-xiii)

To say that Smith was Marxist before Marx is a paradoxical way to stress that Marx did not invent the general picture of capitalism he was proposing, he was simply inheriting and developing a much older approach, born with William Petty, Cantillon and the Physiocrats; Marx makes the implications of this approach more explicit. These implications were perceived to be critical of the capitalist social structure much before Marx wrote, as made clear in 1831 by the (progressive!) member of Parliament Richard Poulett Scrope, who accused the diffusion of Ricardian theory of being a *crime* because it sapped the foundations of the “principles of sympathy and common interest” fundamental for social peace:

“Surely the publication of opinions taken up hastily upon weak, narrow and imperfect evidence – opinions which, overthrowing as they did the fundamental principles of sympathy and common interest that knit society together, would not but be deeply injurious even if true – does amount to a crime ... In their theory of rent, they have insisted that landlords can thrive only at the expense of the public at large, and especially of the capitalists; in their theory of profits, they have declared that capitalists can only improve their circumstances by depressing those of the labouring and numerous class; ... In one and all of their arguments they have studiously exhibited the interests of every class in society as necessarily at perpetual variance with every other class!” (quoted in M. Blaug, *Ricardian Economics*, 1958, pp. 149-50)

Sraffa in his Cambridge lectures of 1928-30 quoted an analogous passage from H. C. Carey, *The past, the present and the future*, London 1848:

“Mr. Ricardo’s System is one of discords ... its whole tends to the production of hostility among classes and nations ... this book is the true manual of the demagogue, who seeks power by means of agrarianism, war, and plunder ... The sooner they [the lessons which it teaches] shall come to be discarded the better will it be for the interests of landlord and tenant, manufacturer and mechanic and mankind at large”.

Marx explained this violent rejection of Ricardo as ideological, due to the need to fight the rising labour movement. Indeed these condemnations of Ricardo confirm that from Ricardo (i.e. from Smith) one derives a ‘Marxist’ picture of capitalism centered on class conflict. In the surplus or classical approach, the distribution between wages and profits, of what is left of the social product after paying land rents and reconstituting used-up capital goods, is determined by forces similar to those that determine the standard of living of serfs under feudalism. The reason, why feudal lords are able to extract an income from their control of land, is that peasants cannot subsist without access to land, and therefore the power to prohibit access to land gives the lord the power to pretend part of land’s produce. The lower the subsistence of serfs, the more is left for the landlord, but a lower limit to reducing the serfs’ subsistence is posed, first of all, by their need to eat (and to be protected from cold etc.) enough to be able to work and to reproduce, and secondly, by their need for those consumptions in excess of strict physical subsistence that have become indispensable for social respect and self respect in the given society, and thus for preventing the explosion of popular insurrections. Capitalists, as Marx particularly stresses but Smith had already admitted, have a power over labourers analogous to the power of feudal lords over peasants, the power to exclude them from earning a living by refusing to employ them; because it is the capitalist class that owns the means of production, and thus the firms and the control over the available jobs. (Note how Smith takes for granted the strong alliance among members of the same class, elsewhere he mentions that whenever a capitalist does not behave in the interests of his class he is immediately reproached and even ostracized by the other capitalists. This disappears in the marginal approach.) But their power to compress wages finds a limit in the danger of explosions of protests, which may be accompanied by “the most shocking violence and outrage”, if wages go below the level that custom has rendered the minimum deemed indispensable for a decent living. A persistent rise in real wages tends to change consumption habits and to make the new living standard as indispensable as the old one was: for example, being inserted in social life, even among the poorer sections of the working class, nowadays requires not to stink; not so

150 years ago. Wage bargaining starts from the level that past wage levels have made customary, and its outcome depends on bargaining power. Relative to the time of Adam Smith, the elements influencing the relative bargaining power of the two opposed classes of capitalists and wage labourers have become more complex, many more elements enter the picture, such as trade unions, the degree of unity of the working class, the enlargement of the right to vote, the greater role of state economic policies, the welfare state, the laws regulating the labour market (e.g. prohibitions of firing labourers without reasonable cause, or contrariwise prohibitions of trade unions). The level of unemployment remains particularly important: unemployment is a necessity for capitalism, to keep workers' demands in check (Kalecki 1943). See MCM 12-18.

2. *The neoclassical picture: capitalism as a fair and efficient cooperation of sacrifices.*

The picture of capitalism emerging from the marginal/neoclassical approach is very different; hence the importance of deciding which approach is correct. In order to understand how capital theory can help on this issue, we must be clear as to how the neoclassical approach determines income distribution. This approach conceives production as the cooperation of 'factors of production', and argues that if the free play of competition is not impeded, the economy tends toward the full employment of all factor supplies, with factors receiving rewards that reflect the contribution of each unit of each factor to social welfare. The idea is that the choices of firms and of consumers cause the demand for each factor to be a decreasing function of its *rental rate* (or simply *rental*, for brevity, that is the price of its services: the wage rate, in the case of labour; the rate of interest, in the case of capital). Then if the rental of a factor in excess supply decreases, the demand for it increases, and a tendency arises toward a full employment equilibrium. (I will leave aside land supposing it is overabundant and hence free, because land rent is less crucial for the purpose of highlighting the differences between classical and neoclassical approach.)

Let us remember how the approach derives the decreasing demand for a factor, with reference to labour. Suppose an economy where there is only one product, corn, produced in yearly cycles by two factors, labour L , and capital consisting of corn (seed) K . Land is overabundant and hence a free good. For simplicity, the supplies of labour and of capital are rigid. Capital is *circulating* capital, that is, used up in a single utilization. Production is performed by firms which, because of free entry and competitive elimination of less efficient producers, are price-takers and end up by all adopting the same technology, described by a production function $C=f(K,L)$ where C

is *net* corn product², with well-defined and smoothly decreasing net marginal products³. This technology exhibits constant returns to scale at least at the level of the industry (which in this case coincides with the entire economy), because, either there are constant returns to scale already at firm level, or at firm level returns to scale are first increasing and then decreasing, so there is an optimal firm dimension; in the latter case, competition requires that this optimal dimension be small relative to total demand, and then variations in the *number* of identical, optimally-sized firms ensures constant returns to scale for the entire industry. So we can treat the entire economy as a single giant firm with production function $C=F(K,L)$ with constant returns to scale. Then marginal products depend only on factor *proportions*, and factor payments equal to marginal products exhaust the product (Euler's Theorem).

So for each given real wage (in terms of corn), all firms tend to adopt the same L/K ratio, the one which renders the marginal product of labour equal to the real wage; this ratio increases if the real wage decreases. Assuming that the supply of corn-capital is fully employed, the L/K ratio determines aggregate labour demand, and the labour demand curve is – with the wage in ordinate – the marginal product curve of labour in the economy as a whole, see Fig. 1. The intersection with the vertical supply curve of labour determines a unique and stable equilibrium wage w_{eq} .

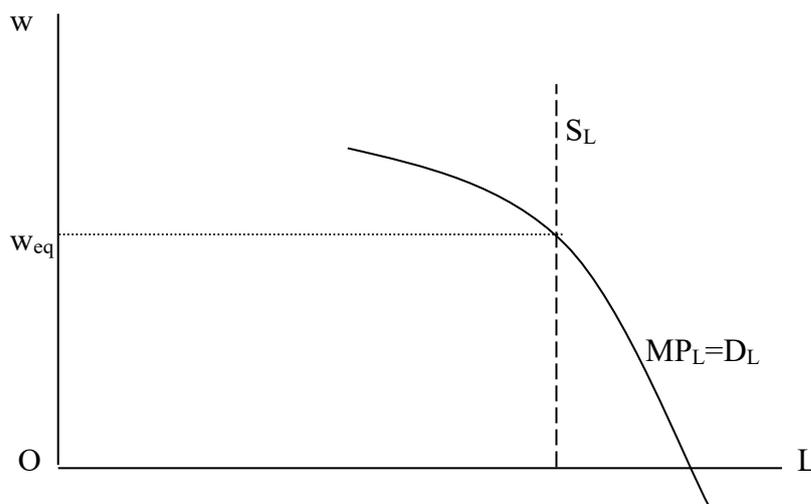


Fig. 1. The labour demand curve D_L coincides with the labour marginal product curve MP_L .

² Gross (that is, total) corn production will then be $C+K$.

³ The distinction between gross and net marginal product is needed for capital: the net marginal product of one more unit of capital is the increase in corn output, minus the one unit needed to replace the used-up unit of corn-capital.

Owing to our assumption that the supply of capital is fully employed, there is only one market on which equilibrium must be reached: the labour market, and if the real wage responds to excess demand, it tends toward the equilibrium level. (The product market is necessarily in equilibrium whatever the real wage, the moment it is assumed that all income goes to purchase the product, and that *only the employed factors have income to spend*.⁴)

This construction is symmetrically applied to the demand for corn-capital, assuming labour is fully employed. The price of capital services, i.e. the net rental of capital, is the rate of interest. With labour fully employed (i.e. supply=demand), one derives the curve of the (net) marginal product of capital for the economy as a whole, and, with the rate of interest in ordinate, this curve is the demand-for-capital curve, again regularly decreasing and therefore ensuring the stability of the capital market. The assumption, when one studies the demand curve for a factor, that the other factor is fully employed, reflects what, according to this theory, will be the final result anyway, because the demand curve for a factor is decreasing for any given

⁴ The italicized sentence points to an important difference between traditional marginalist discussions of the time-consuming adjustments argued to bring equilibrium about, and the contemporary analyses of stability based on the fiction of the ‘auctioneer’, a hypothetical institution that blocks the economy, proposes prices, invites each agent to communicate demand and supply promises at those prices, calculates excess demands on the several markets, and then, unless equilibrium has been reached on all markets, declares all promises void, proposes new prices (higher where excess demand had been positive, lower where negative), and collects new promises, repeating the procedure until general equilibrium is reached, with no exchange or production allowed until then (which means the adjustments must be assumed practically instantaneous). What interests us now is that in this ‘Walrasian tâtonnement’ it is assumed that, at each ‘round’, consumers have a spendable income equal to the value of the factors they plan to supply. But this can only be legitimate in the fairy-tale world of the auctioneer, where demand intentions are hypothetical and only to be respected in equilibrium. In the real world, only factor supplies which have found purchasers generate an income for their owners; this is particularly important in the labour market. Only by admitting this fact can the neoclassical economist determine the situation generated by a real wage kept persistently above its equilibrium level by law or trade unions: the unemployed labourers, having no income, have no purchasing power, hence cannot demand the product; the demand for the product equals the income earned by the employed factors, which is equal to the value of the product; so the product market is in equilibrium, the sole market not in equilibrium is the labour market. With this more realistic treatment of consumer incomes (which is the one considered obvious by traditional neoclassical economists and by Keynes) it is possible to have disequilibrium in only one market. “[W]hen, in a stationary closed community, the general level of real wages is raised, and maintained, at a height inconsistent with normal employment ... a final position must be reached which is precisely the same as that which would have occurred if there had been a direct reduction in the number of labourers available ... The final position thus reached is one of equilibrium, if the existence of the unemployed is left out of account.” (Hicks 1932, pp. 198-199). Thus the labour demand curve can be seen as indicating, at each wage, what the supply of labour should be in order to be fully employed.

employment of the other factor, and therefore the tendency toward equilibrium between supply and demand for a factor exists even when the other factor is not yet fully employed; so the simultaneous tendency toward full employment on either market sooner or later reaches full employment on at least one market, and then also on the second market⁵. Competition and entry will anyway ensure that whatever earnings do not go to one factor will go to the other factor, because entrepreneurial pure profits tend to zero.

Behind both decreasing demand curves there is the fact that changes in relative factor rentals cause ‘well behaved’ *factor substitution* in the choice of production methods by firms, where ‘well behaved’ means in favour of the factor that becomes relatively cheaper, and therefore conducive to stability. Factor substitution was perhaps not immediately evident in the derivation of the demand curve in Figure 1, but in fact when the real wage decreases, it is the rise in the optimal L/K *ratio* – that is, the convenience partly to substitute labour for capital in the production of any given output – that, when combined with a given K, allows concluding that the demand for labour increases; symmetrically, when capital becomes relatively cheaper, it is the convenience partly to substitute capital for labour in the production of any given output that reduces the optimal L/K, which, if L is given, allows concluding that the demand for capital increases.

Note that consumer choice was not needed to determine the equilibrium and to argue a tendency toward it in this simple economy. But consumer choice *among produced consumption goods* does have an important role in the approach (and this explains why so much space is devoted to it in teaching): it strengthens the approach^[6] by supplying a reason, for decreasing demand curves for factors, *alternative* or *additional* to the mechanism of technological substitution among factors just illustrated. Let us assume our economy (still with labour and corn-capital the only factors) produces several consumption goods, and to make the role of consumer choice as clear as possible, let us assume fixed proportions between labour and capital in each industry, that is, no technical substitutability. Assume these fixed proportions differ as between industries. To determine the demand curve for labour let us assume, as before, that capital is fully employed. Suppose real wages decrease and the rate of

⁵ Note that the adjustment toward equilibrium here described does not assume the auctioneer-guided tâtonnement, it can be viewed in realistic terms; when a decrease in the rental of a factor induces firms to employ more of it, incomes increase by as much as output, so there is again equilibrium in the output market. The abandonment of this realistic picture in favour of the totally unrealistic auctioneer-guided tâtonnement has reasons that will be pointed out later.

⁶ On the contrary, consumer choice as to supply of factors is rather a cause of difficulties for the approach, since it can cause factor supply curves to be ‘backward-bending’, which can cause instability and multiplicity of equilibrium.

interest rises; this causes a relative cheapening of the goods produced with a high L/K ratio, and conversely a relative rise in price of the goods produced with a low L/K ratio. Then consumers, it is argued, will *plausibly* shift the composition of their consumption in favour of the goods that have become relatively cheaper, that is, the ones produced with higher L/K ratios. This causes an expansion of the industries that use more labour relative to capital, and a contraction of the other ones. Since capital is fully employed, this entails the transfer of some units of capital from industries with a lower L/K ratio to industries with a higher L/K ratio. For example some units of capital may have to leave an industry where each unit of capital is combined with 1 unit of labour, and move to an industry where it is combined with 4 units of labour: each such transfer of a unit of capital raises the demand for labour by 3 units. The decrease in wages raises the *average* proportion L/K in the entire economy, by causing a relative expansion of the industries that use more labour per unit of capital. This mechanism too can be seen as a mechanism of factor *substitution*, in the ‘production’ of utility: a given utility level can be obtained with different combinations e.g. of strawberries and meat, and more strawberries and less meat mean the use of more labour and less capital.[⁷] Since the change in overall L/K is indirectly caused by changes in consumer choices, this is called the *indirect* factor substitution mechanism, while the one based on changes of optimal production methods is called the technological or direct factor substitution mechanism.

We have illustrated the indirect factor substitution mechanism as an *alternative* basis of decreasing factor demand curves, operative even when the direct mechanism is paralyzed by the absence of technical substitutability; but of course the neoclassicals argue that generally both mechanisms are operative, and therefore a wage decrease raises labour demand both because consumers shift their demands in favour of ‘labour-intensive’ goods, and because firms shift to using more labour and less capital per unit of output. The sum of the two effects allows arguing that factor demand curves are not only decreasing but also, in all likelihood, rather elastic, which gives credibility to two theses important for the plausibility of the approach: first, that a ‘backward-bending’ supply curve of labour does not cause multiplicity and instability of equilibrium, because the demand curve is more elastic; second, that reaching equilibrium on the labour market does not require implausible reductions of real wages.

We perceive here one reason why the neoclassical approach was found plausible: it seems to rest on an indubitable *foundation in facts*: the existence of firms’ choice among alternative production methods, and of consumer choice among

⁷ See below, at the end of section 7, for a numerical example with strawberries and meat produced by corn and labour.

different consumption baskets. We will see that the facts are there but do not have the implications the neoclassical approach derives from them.

3. *Differences between the two approaches.*

So far land was assumed to be a free good, but the derivation of decreasing factor demand curves can be applied also with more than two factors; given the employments of all factors but one, for the last factor the two neoclassical factor substitution mechanisms allow the derivation of a decreasing demand curve, and then the market of that factor can be argued to be, in all likelihood, stable, and in equilibrium the factor receives its full-employment marginal product.

Note the symmetry: each factor receives its marginal product; the same law applies to all factors. Each unit of each factor receives its *contribution to production*, what society would lose if that unit withdrew from production. For capital, ‘withdrawal’ of a unit means that it is not reconstituted after being used up, the owner prefers to consume the unit of corn instead of saving it again and reallocating it to seed-corn. So what keeps capital existing, or increases it, is that its owners save, abstaining from consuming more. It becomes then possible to find a sacrifice behind each contribution: the wage rewards the sacrifice of unpleasant labour, the rate of interest rewards the sacrifice of abstinence from immediate consumption.

It becomes difficult, then, to argue that labour is exploited. Each labourer receives what she/he contributes to society, and each saver-capital owner too. Capitalism appears to be, not a society based on the extortion from labour of some of its product, but instead a society of *co-operation* of sacrifices of factor owners (cooperation, because more capital raises the marginal product of labour, and vice-versa): the harmonious picture Richard Poulet Scrope was looking for.

The evaluation of unemployment too is very different from the classical/Marxian one. Labour unemployment results from a real wage higher than the equilibrium level; the cure must consist in lowering the real wage; if unemployment persists, the responsibility is of what prevents wages from decreasing: the power of trade unions, usually; or minimum-wage laws; so the workers, whose support gives strength to trade unions, or to the political parties that passed the minimum-wage law, should ultimately blame themselves if unemployment persists. The classical approach views unemployment as largely due to political decisions opposing full employment.

The central analytical difference between the two approaches is the absence, in the classical approach, of the idea of factor substitution mechanisms, and therefore also of the idea of decreasing demand curves for labour or capital. This explains the different role of custom, social bonds, politics in the two approaches. In the neoclassical approach supply and demand, derived from consumer choices and firms’

choices, are capable of determining prices, quantities and income distribution on the basis of a very restricted set of institutions: essentially, competition and a general respect of contracts and of private property. In the classical approach, on the contrary, without a socio-political determination of the relative bargaining power of social classes (that has greatly fluctuated in the last two centuries) there would be no explanation of the level of wages; custom and politics are *indispensable*; thus an Adam Smith or a Marx is also naturally a sociologist and a political scientist. (Ricardo, a successful financial stock broker, was less interested in broader political aspects and appears less conscious than Smith of the importance of class coalitions and of the inferior bargaining power of workers vis-a-vis capitalists, he wrote: “Like all other contracts, wages should be left to the fair and free competition of the market” (1951–73, I: 105).)

The two approaches are very different, the resulting pictures of capitalism are radically different. The influence of the marginal approach on political thought and on sociology was immense, see Simon Clarke, *Marx, Marginalism and Modern Sociology*, Macmillan, 1982. How does capital theory help in assessing which approach is more solid?

4. *Surmounting a weakness of the classical approach.*

As briefly announced in section 1, Sraffa’s advances in capital theory have shown that a weakness of the classical or surplus approach in the determination of the rate of profits, a weakness which was long believed to destroy the consistency of that approach, is in fact surmountable; on the contrary, there are deficiencies in the marginalist/neoclassical treatment of capital, that do not appear surmountable.

The classical weakness was an inability to arrive at a fully satisfactory determination of the *rate of profits*, the average rate of return on the productive employment of capital. Classical authors knew that if income distribution between wages and profits changes, the normal relative prices of commodities change.⁸ Ricardo

⁸ Normal or long-period prices (certainly known to the reader as Marshallian long-period prices) are the relative prices that guarantee a uniform rate of return, or rate of profit, in all (efficient) investments; they depend on income distribution. They are the centers of gravitation of market prices. The gravitation results from investment being directed in greater proportion to the industries offering a higher rate of return; the increase of production in these industries reduces the product price, and thus the rate of return, which thus tends to become uniform (product prices are then equal to the respective minimum average costs inclusive of the normal rate of return on advanced capital). As long as the rates of return are not uniform, relative prices keep changing; as market prices gravitate toward their long-period values, the forces that tend to change them become weaker and finally disappear when the rate of profits is uniform, so normal prices can be treated as persistent, constant; the economy

for example knew that if two commodities are both produced by one unit of labour as sole input, but the first is sold immediately after paying the wage, while the second must ripen for a further year before it is sold, then competition (that pushes prices to equal average costs) causes the normal, long-period price of the first to equal the wage, w , while the normal price of the second will be $w(1+r)$ where r is the normal rate of profits, because the capital needed to advance the wage for a year must earn the normal rate of return, or the commodity will not be produced; the normal relative price of this second commodity with respect to the first is $(1+r)$, so it changes with r . Therefore there arises an apparent danger of vicious circle. The given real wage and the given production methods adopted in the economy in a certain period make it possible to determine which goods, out of the total production of the economy in that period, go to the employed workers as wage goods, and which capital goods have been used to produce the total output. The vector of produced goods, minus the wage goods paid to workers, and minus the replacement of the used-up capital goods, yields the vector of surplus goods appropriated by the capitalists, whose value is total profits. The *rate of profits* is the ratio of the normal value of this vector to the normal value of the vector of capital goods employed; since the two vectors have different composition, this ratio depends on relative prices, but these depend on the rate of profits, so one cannot determine relative prices without knowing the rate of profits, and one cannot determine the rate of profits without knowing relative prices. Dmitriev: “Thus, we are apparently trapped in a logical circle: profit must be known in order to determine value, but profit itself is dependent on value” (1974: 58).^[9]

To escape this vicious circle, Ricardo argued that the labour theory of value (relative prices equal to ratios of labours embodied^[10] – determinable without needing

is then in a normal or long-period position. Adam Smith called ‘natural prices’ these long-period prices, Marx called them ‘prices of production’. Up to the advent of the neo-Walrasian versions of the marginal approach, it was universally accepted that the theory of value must explain long-period prices.

⁹ This apparent circularity opened the way to this criticism by Walras: after taking the wage as given and stating that “the selling price of products is determined by their costs of production ..[that is, with P aggregate (net) product price, S total wages, I total interest or profits] $P=S+I$... the English economists tell us that ‘the amount of interest or profit is the excess of the aggregate price received for the products over the wages expended on their production’, in other words, that it is determined by the equation $I=P-S$[but] it is impossible for I to determine P at the same time that P determines I . In the language of mathematics one equation cannot be used to determine two unknowns.” *Eléments d’Economie Pure* (Jaffé Engl. edition, 1954, p. 425).

¹⁰ The labour embodied in a commodity is the live labour directly and indirectly employed to produce it, i.e. the sum of the live labour directly employed, plus the labour employed to produce the used-up capital goods which is equal to *their* labour embodied (this can in turn be seen as labour directly employed to produce them, plus labour directly employed to produce the capital goods used-up to produce those capital goods, plus ... and so on, obtaining an infinite but convergent series). Differently paid labour times are reduced to ‘simple labour’ time in proportion to wages; so the labour

to know the rate of profit!) was a *sufficient approximation* to actual normal relative prices; but it was a weak argument: in the example just given, the labour theory of value would indicate that the two commodities have the same price (both are produced by one unit of labour), which is false as long as r is positive. Marx too relied, in ways I cannot specify now, on the labour theory of value. Subsequent anti-Marxist economists could accuse Ricardo's and Marx's entire approach of being wrong because not based on a satisfactory theory of normal relative prices and therefore unable to explain them and the rate of profits.

Sraffa's equations that determine long-period prices have shown that this classical weakness was only apparent. It may help to start from the well-known Marshallian partial-equilibrium determination of the competitive long-period price of a commodity (produced good): free entry causes this price to tend to minimum average cost (inclusive of the normal rate of return on the capital employed, that neoclassical economists identify with the rate of interest). But there is a deficiency in Marshall's argument. In order to determine the minimum average cost of a product, all input prices must be given; but this may be impossible before the price of the product is determined. Take steel. It is produced with the use, among other inputs, of machines made of steel. If we allow time for the price of steel to tend to minimum average cost, during this time an analogous tendency toward minimum average cost will operate for the machines that produce steel; so consistency requires that, in determining the minimum average cost of steel, the cost of these machines be *their* minimum average cost. But this cannot be determined before we know the price of steel.

The solution is to determine *simultaneously* the minimum average cost, imposed by competition and free entry, of all produced commodities directly or indirectly entering the cost of steel, disaggregating the costs completely. Once one does it, one discovers that, if the real wage is given, the rate of profits is determined simultaneously with relative product prices. To show it, assume two goods only, corn and iron, produced by labour and by themselves (as *circulating* capital goods) in yearly production cycles. Assume, initially, that in both industries *technical coefficients* (efficient inputs needed per unit of output) are given. Land is free. Then 'price equal to minimum average cost' implies two equations, where a_{ij} ($i, j=1, 2$) is the given technical coefficient of input i in the production of good j , a_{Lj} is the given

embodied in a commodity is numerically equal to the *wages* directly and indirectly paid for its production. If the rate of profit is zero, cost coincides with wages embodied, and then the long-period equality of price and cost implies relative prices equal to relative wages embodied, see footnote 11 below. Unfortunately, as the example in the text shows, the proportionality between prices and labours embodied no longer holds (except in the very special case called 'equal organic composition of capital') the moment a positive rate of profits enters prices.

technical coefficient of labour in the production of good j , w is the wage rate (here assumed paid at the end of the production period, but one could also assume advanced wages), r is the normal rate of return on capital (rate of profits, or rate of interest if one neglects risk):

$$(1) \quad p_1 = (1+r)(a_{11}p_1+a_{21}p_2)+a_{L1}w$$

$$(2) \quad p_2 = (1+r)(a_{12}p_1+a_{22}p_2)+a_{L2}w.$$

The meaning of these equations is that to produce a unit of good 1, corn, the capitalist must anticipate money capital at the beginning of the year to buy quantities a_{11} of good 1 and a_{21} of good 2, so corn's price must allow replacing the used-up capital goods (reconstitute the money capital) and must also yield the normal rate of profits or of interest r on the capital employed (otherwise it would not be convenient to employ capital in that industry). The same holds for good 2, iron. The price of a good as input and as output is the same because we are determining *long-period* prices (Smith's natural prices), the *persistent* relative prices toward which competition and entry cause market prices to converge.

Once you write these Sraffian equations you see that the classical vicious circle is not there. Choose a numéraire, for example corn, $p_1=1$, and you have a system of two equations in three variables, the relative price of iron in terms of corn, the real wage in terms of corn, and the rate of profits. Take, with the classicals, the real wage in terms of corn as given, determined by the bargaining power of the opposed classes, $w =$ some given amount of corn, and you are left with two equations in two variables: the rate of profit is determined simultaneously with relative prices, and without recourse to the defective labour theory of value. More than two goods introduce no additional theoretical difficulty because for each added commodity you add one more variable, the price of the commodity, and one more equation, the condition that this price must equal 'average cost' (inclusive of the normal rate of return on capital).¹¹

¹¹ We can now check that if $r=0$, prices are proportional to labours embodied. Set $r=0$ in equations (1) and (2); they become $p_1=a_{11}p_1+a_{21}p_2+a_{L1}w$, $p_2=a_{12}p_1+a_{22}p_2+a_{L2}w$. Divide both sides of each equation by w , which means choosing the physical wage as numéraire, $w=1$; prices $p_1^*=p_1/w$, $p_2^*=p_2/w$ are now *labour-commanded prices*, indicating how many units of labour one can buy with a good. Now consider the equations determining the labours embodied in corn and in iron:

$$m_1 = a_{11}m_1+a_{21}m_2+a_{L1} ,$$

$$m_2 = a_{12}m_1+a_{22}m_2+a_{L2}.$$

E.g. the first one determines the labour m_1 embodied in a unit of corn as the sum of direct or live labour a_{L1} plus the labour embodied in the corn and iron employed as inputs. These equations are identical to the price equations for $r=0$ and $w=1$, with $p_1^*=p_1/w$ in place of m_1 , and $p_2^*=p_2/w$ in place of m_2 . So labour-commanded prices *equal* labours embodied; a different numéraire (and hence $w \neq 1$) maintains prices *proportional* to labours embodied. Indeed as said in the previous footnote, if $r=0$ then cost equals direct and indirect wages, hence price is proportional to direct and indirect labour.

Note also: if you write the equations with three goods, but with the third one, diamonds, not used as input, you will note the irrelevance of the third equation for the determination of r . *Only the production conditions in the industries directly or indirectly producing the wage goods enter the determination of r .* Ricardo in his first attempt to surmount the vicious circle assumed that corn needs only corn (as capital and as wages) for its production, then its equation suffices to determine r (this is the case in the above equations if $a_{21}=0$); then competition and entry will cause the other industries to adjust their outputs and prices so as to reach the r determined in the corn industry - *as in the strawberries-meat example of section 7.* (For more on the ‘corn model’ see Petri Capital Theory 2016, or Microeconomics for the Critical Mind ch. 1.)

The theory can be extended to admit technical choice. One tool is the *envelope of wage curves*. For given technical coefficients, the above system of equations, once a numéraire is chosen, establishes a univocal connection between wage rate and rate of profits, a function $w(r)$, often called *wage curve*. We could derive it from equations (1) and (2) but I prefer to do it for a simpler case, an example of so-called ‘Austrian’ technology, a simpler technology that will reappear in an example later. For each commodity, as long as among its inputs there are produced goods (capital goods, all circulating), one can trace *their* inputs, and if these include produced goods one can go back to *their* inputs, and so on; if the technology is Austrian, in this backwards process from a good to its inputs one never finds the same commodity twice, and in a finite number of steps one finds only labour as an input (assume land is free); then the production of any commodity can be seen as started by unassisted labour, which produces some circulating capital good which the next period, together with more labour, produces another circulating capital good, which the subsequent period... (after as many more similar steps as necessary) finally produces the final commodity. An example could be corn (good 1) produced by labour alone, and iron (good 2) produced by corn and labour; the production of iron can be seen as started by unassisted labour which produces a circulating capital good (corn), which the next period together with more labour produces iron. Let us determine the wage curve. Assume (to make the equations a bit less simple) that wages are *advanced*, and that now *iron* is the wage good (difficult to conceive? then reinterpret ‘iron’ to mean bread) and the numéraire ($p_2=1$; w now is a quantity of iron, or of bread):

The thing is very intuitive, this explains why, even without advanced mathematics, classical authors clearly saw that if $r=0$ then prices are proportional to labours embodied, and then Ricardo could argue that as long as r is rather small, relative prices cannot deviate much from relative labours embodied (in one point he says, not more than 7%).

$$(3) \quad p_1 = (1+r)wa_{L1}$$

$$(4) \quad p_2 = 1 = (1+r)(p_1a_{12}+wa_{L2}).$$

The second equation can be rewritten $1 = (1+r)[(1+r)wa_{L1}a_{12}+wa_{L2}]$, that implies

$$(5) \quad w = \frac{1}{(1+r)^2 a_{L1}a_{12} + (1+r)a_{L2}}.$$



This is the equation of the wage curve for this particular example; it shows that as r rises w decreases, and that (in this case) w remains positive for ever, tending only asymptotically to zero as r tends to infinity. The normal case, however, is not an ‘Austrian’ technology but one in which some commodities are directly or indirectly required as inputs in their own production and indeed in the production of *all* commodities; they are then called *basic* commodities; this is the case for both corn and iron in the corn-iron economy if corn needs iron as an input and iron needs corn as an input. The other commodities are called non-basic.

When there is at least one basic commodity, then r cannot rise indefinitely, because at a certain point w becomes zero¹² and would become negative if r increased further; and economic meaningfulness requires that w does not become negative. The study of the formal properties of the resulting system of equations is greatly aided by mathematical results on nonnegative matrices, in particular by the *Perron-Frobenius theorem*, that I cannot explain here but whose relevance is briefly illustrated in Appendix. The Perron-Frobenius theorem allows proving that the wage curve associated with given technical coefficients is continuous, decreasing, and (as long as the economy is capable of producing a physical surplus if wages are zero, and technology is not ‘Austrian’) has positive intercepts on both axes. If one changes the production method of even only one industry, the wage curve changes.

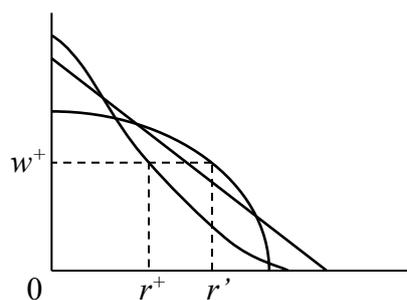


Fig. 2. Three wage curves in a non-Austrian economy. If the economy is initially at (r^+, w^+) , because of Okishio’s Theorem firms are pushed by cost minimization to change the production methods until the economy reaches the outermost wage curve; if w is fixed at w^+ , the rate of profits will rise to r' .

¹² This is easily seen in the simple case of corn the sole product, produced by labour and corn seed. Assume the production of 1 unit of corn at the end of the year requires 0.2 units of corn as seed at the beginning of the year, and 4 labour-years. Corn is the numéraire. Then the rate of profits, with the yearly wage w paid at the end of the year, is determined by $1 = (1+r) \cdot 0.2 + 4w$. The maximum rate of profits, corresponding to $w=0$, is 400%.

Given the alternative production methods of each industry, and having chosen a common numéraire, for each possible *technique* (combination of production *methods*, one per industry)¹³ one can derive the corresponding wage curve $w(r)$, and one can draw all these curves in the same diagram. Their North-East or ‘outer’ envelope indicates the maximum real wage obtainable for each rate of profit. We meet now a very important result (*Okishio’s Theorem*). The theorem applies to an economy without joint production (all capital goods are circulating) and where land is free or the changes in quantities produced and in production methods do not alter the no-rent land. (It can be generalized to production with durable capital.)

Assume the economy is using a certain technique, and that income distribution corresponds to point (r^+, w^+) on the wage curve of that technique, with associated the vector of product prices \mathbf{p}^+ determined by the system of equations “price=average cost inclusive of normal r ” (eqs. (1)-(2) for two goods; for the case of n commodities, equations (8) in Appendix) based on that technique’s production methods; assume that, at the ruling prices and income distribution (\mathbf{p}^+, r^+, w^+) , for one of the ‘wage-relevant’ commodities¹⁴ there is a production method, different from the one currently adopted, that would allow producing that commodity at lesser cost and therefore allows extraprofits; if this method replaces the current one, the theorem states that in at least a neighbourhood of r^+ the new wage curve is to the North-East of the previous one, that is, $w(r^+)$ on the *new* wage curve is higher than w^+ (and if one keeps the wage unchanged at w^+ then on the new wage curve r is higher than r^+). Conversely, given the technique the economy is using and the associated wage curve, if there is a different combination of the available methods (a different technique) that generates a wage curve which passes to the North-East of point (r^+, w^+) (that is, such that for the given w^+ it determines a higher r and for the given r^+ it determines a higher w), then, for at least one commodity entering the numéraire in terms of which the wage is measured or its means of production, there is a production method belonging to the second technique but not to the first which, at the ruling prices and income distribution (\mathbf{p}^+, r^+, w^+) , allows production of that commodity at lesser cost, and therefore allows extraprofits for the firm adopting it. The implication is that the firms that produce that commodity will tend to adopt this more profitable method and thus the economy will change technique; the extra profits won’t last very long because entry will push prices

¹³ Each industry is assumed to produce a single good, so joint production is excluded.

¹⁴ That is, a commodity entering the basket (the numéraire) in terms of which the real wage is measured, or directly or indirectly necessary for the production of that basket: wage-relevant commodities are those whose production method influences the shape of the wage curve.

to equal the new supply prices¹⁵ in that industry and in all industries, the economy will move to a more ‘outside’ wage curve, and r will rise if w stays fixed at w^+ , or w will rise if r stays fixed at r^+ . With nobody consciously wanting it, the repeated adoption of the cost-minimizing methods imposed by competition and entry will bring the economy to the outer envelope of wage curves.¹⁶

Thus even with technical choice, a given real wage determines relative long-period prices and the rate of profits univocally; the wage curves allow us to see which technique is chosen^[17]. The classical conflict-based approach to real wages *is* capable of determining relative long-period product prices and rate of profits.^[18]

5. *Capital goods are different from lands.*

Neoclassical economists might still claim that their approach is superior because it recognizes something that escaped classical economists: the fact that the indubitable existence of firms’ choices among alternative production methods, and of consumers’ choices among different consumption baskets, implies the factor substitution mechanisms from which the neoclassical approach derives its supply-and-demand-equilibrium determination of real wage and rate of profits (identified with the rate of

¹⁵ ‘Supply price’ means the minimum price needed in order for a good or service to be supplied: for a produced commodity it is the minimum price capable of covering costs (inclusive of the normal rate of profit, or of interest, on the required capital advance) and thus inducing the production of the commodity.

¹⁶ The comparison of two wage curves associated with different techniques (and of course the same numéraire in terms of which the real wage is measured) allows concluding as to which technique is tendentially imposed by competition (the one with the outer wage curve at the given real wage or given rate of profits) only when the alternative regards how to produce a ‘wage-relevant’ commodity. If two techniques differ only in the method of a wage-irrelevant commodity, a non-basic commodity (e.g. diamonds) not part of the numéraire basket, the two wage curves coincide (because unaffected by how that commodity is produced), so they do not indicate which method for the production of that non-basic commodity is the cost-minimizing one. What wage curves can teach becomes further complicated when there is joint production but we cannot study this here.

¹⁷ The tendency of technical choice to the outer envelope of wage curves shows that Marx was wrong in a famous belief, the *tendency of the rate of profit to fall* due to mechanizing technical progress, see Petri, ‘Capital Theory’ (in Faccarello Kurz 2016), or more extensively MCM 66-69: Okishio’s Theorem shows that whenever a new process is introduced because cost-minimizing at the prevailing prices, it will leave real wage and profit rate unaffected if it concerns a wage-irrelevant commodity, otherwise it will create room for a rise of the rate of profit *or of the real wage* – which helps to explain the historical evidence of a secular rise of real wages with no reduction of profit rates.

¹⁸ If there are scarce natural resources, one also needs the quantities produced, in order to determine land rents; see MCM 24-29, and Garegnani (1983) for the different role of demand in the neoclassical and in the classical approach. Also see MCM 904-911.

interest¹⁹). However this claim founders on the impossibility to extend to economies with *heterogeneous* capital goods the picture illustrated above, in section 2, where capital was physically homogeneous.

The tendency toward the neoclassical equilibrium illustrated in section 2 was based on the possibility to draw, for each factor, a supply curve and a *decreasing* demand curve. Let us stop on what these curves require. Beside the preferences of consumers, the supply curve requires a *given endowment* of the factor, on whose basis consumer choice can determine the supply of the factor to firms ²⁰ (in section 2 this supply was assumed rigid). The demand curve requires, for each given value of the rental of the first factor, the determination of its demand as *the level of its supply that would cause the general equilibrium to generate that rental*, and this, besides given preferences and given technological possibilities, requires (i) that the *other* factors be fully employed, and this again requires *given endowments, now of these other factors*, and (ii) that when the first factor becomes relatively cheaper, there is substitution (direct and/or indirect) in its favour so that the demand for the factor increases. Both requirements—given factor endowments, and ‘well-behaved’ substitution—have been shown to be untenable for capital when capital is heterogeneous. Let us see why, starting with the first requirement.

One might think that each capital good could be treated in the same way as the different types of land: as a distinct factor with its given endowment. But the amounts in existence of capital goods differ from the amounts of the several types of land, in that capital goods are continuously produced and used up in production, and their amounts can be quickly altered by differences between the flow of their production

¹⁹ An important change of the meaning of ‘profits’ occurs with the advent of the marginal/neoclassical approach. In it, labour and capital (and land) are just factors of production, with rentals determined by the same mechanism; this symmetry makes it natural for the marginalists to consider both wages and interest as part of production costs; then profit is redefined to mean the revenue of the entrepreneur in excess of costs that *include* interest payments; while, for the classicals, interest payments are part of profits. So zero-profit prices for the marginalists are prices that cover costs inclusive of a rate of return on capital equal to the rate of interest; a classical author would call them prices such that the rate of profits is equal to the rate of interest. In this paper, the meaning of ‘profit’ is the classical one; the neoclassical meaning is rendered by ‘entrepreneurial pure profit’. Neglecting risk, rate of profits and rate of interest can be treated as equal. But to help readers accustomed more to Marshallian than to classical language, I have used ‘cost’ of a product neoclassically as including also the normal rate of return on capital, which the classicals did not treat as a cost but rather as an addition to cost, a surplus of price over cost appropriated by the capitalist.

²⁰ Since consumers differ in tastes and endowments, unless one assumes rigid factor supplies the supply curve of a factor depends on how the total endowment of that factor is distributed among consumers, but I will neglect this dependence. The problem I intend to point out concerns total endowments of capital goods and would arise even if consumers were all identical in tastes and in factor endowments.

and the flow of their productive consumption by firms; the endowment (= the amount present in the economy) of a capital good will tend to adapt to the demand by firms, and will stop changing quickly only when it has adjusted to the needs of firms, with the production of the capital good compensating its consumption by firms. *Any change in the demand for a capital good, be it due to change in production methods or in the outputs of industries that utilize that capital good, will quickly alter the amount of the capital good present in the economy, as its production and its normal inventories adjust to the changed demand.* This imposes a clear requirement on the determination of the data on whose basis the equilibrium is determined, if equilibrium prices are conceived as the normal prices toward which the economy gravitates owing to time-consuming adjustments, and therefore as having the same role as the classical normal prices, of indicating the average of market prices.²¹ Thus suppose in a nation there is labour immigration and one wants to determine the neoclassical equilibrium to which the economy will converge on the basis of the new labour supply. Because of the increased labour supply, wages will start to decrease, the theory argues; this wage decrease prompts technical substitution inside firms, and substitution in consumer choices; both substitutions alter the demands for most capital goods, possibly drastically (the demand for some capital goods will go to zero, if they are required only by production methods that are no longer convenient). So after a short while the amounts in existence of most capital goods will have changed. Now, the time required for the change in wages to reach an equilibrium between supply and demand in the labour market cannot but be considerable. Especially the change in labour demand cannot but take considerable time to manifest itself: after a wage change, it takes time for product prices to change, for consumers to change their demands, for productive capacity in consumption goods industries to adjust so that prices go back to average costs, for the productive methods in all industries to adapt to the changed real wage. Clearly all these adjustments take months at least; and yet, the new labour demand determined by the changed wage may still not be equal to labour supply, requiring further changes of the wage and further adjustments taking more months. During this time the amounts in existence of most capital goods will change. So in order to determine the equilibrium corresponding to the changed labour supply, the endowments of the several capital goods cannot be considered *given*, they must be considered *variables endogenously determined by the equilibrium itself*.

²¹ I am illustrating here the *traditional* marginalist conception of equilibrium, the one found in all founders and early developers of the marginal/neoclassical approach, Walras included (before the 4th edition of his *Eléments*). As I explain later, the currently dominant *neo-Walrasian* versions of general equilibrium theory are not compatible with this conception of the role of equilibrium, as shown by the recourse to the fairy-tale of the ‘auctioneer’ to explain how equilibrium is reached.

6. *Capital the single factor of variable 'form', and long-period equilibria.*

Up to at least the 1930s *the conclusion reached in the previous paragraph was admitted* by marginalist/neoclassical economists, and *it was not viewed as a problem* because the several capital goods were conceived to be embodiments of a *single* factor “capital”, capable of changing ‘form’, that is, of embodying itself in different vectors of capital goods representing the same “quantity of capital”; and it was the total “quantity of capital”, the endowment of this single factor “capital”, that was treated as given and in equilibrium rendered equal to the demand for it by an appropriate rate of interest, its rental. The *composition* of capital, i.e. the endowments of the several capital goods, was left to be determined by the equilibrium, as required by time-consuming adjustments, on the basis of the demand for the capital goods most convenient at the equilibrium prices for the production of the demanded final goods (this will become clearer with a model illustrated below). The factor “capital” was conceived as a quantity of exchange value ‘embodied’ at each moment in the existing capital goods, and capable, by replacing the used-up capital goods with different capital goods of same value, of taking in time the ‘form’ best adapted to the amount of labour with which it was to cooperate, and to the goods to be produced. The idea, taken from the spontaneous consciousness of business men, was that when certain capital goods are used up, the depreciation funds destined to repurchase them can also be used to pay for different capital goods; the factors that might reproduce the used-up capital goods will then be used to produce different capital goods, whose value will be the same because produced by the same resources; the total value of capital will not change, but will be now ‘embodied’ in a different ‘form’ i.e. a different capital vector. (See Petri 2004 p. 29 for quotes illustrating this conception.) Thus in 1898 Knut Wicksell wrote that though “the forms of capital change, its total value remains unchanged, since, in place of the consumed capital goods, new ones of equivalent value enter successively”.²² The general equilibrium based on this notion of capital,

²² *This* is the treatment of capital – i.e. as a single factor – that justifies the assumption of smooth substitutability in production functions. If capital goods physically specified are treated as distinct factors in production functions, the cases when one can vary the proportion between the *same* factors are very rare; generally a different production method requires *different* capital goods. With capital conceived as a quantity of value of variable ‘form’, and with given prices, isoquants with labour and capital as inputs can be derived by asking, for each amount of labour, what is the minimal “quantity of capital” (value of capital goods) that allows producing the given output: implicitly, different points of the same isoquant will be associated with different vectors of capital goods. But this derivation of the isoquants is based on *given* prices of the several capital goods; and this destroys the usefulness of these isoquants, because if the rate of interest or the real wage changes, all relative prices change and isoquants shift, so we cannot use the old isoquants to determine the change in capital-labour proportion, nor is the way isoquants shift easily ascertainable. How the capital-labour proportion will change can

since it determined the ‘form’ of capital endogenously, aimed at determining the tendential result of time-consuming adjustments including changes in the composition of the vector of capital goods; general equilibrium then required that this composition had reached an equilibrium, and this required the absence of reasons for wanting to alter this composition, that is, a *uniform rate of return* upon investing in any capital good; so this type of general equilibrium included the *traditional condition of a uniform rate of return on the supply price of capital goods* (equivalent to the classical condition of uniform rate of profits). Relative to taking the endowment of each capital good as given (a procedure these authors considered unacceptable for the reasons explained in section 5), the additional degrees of freedom appearing, in the general equilibrium equations, owing to the treatment of the endowments of the several capital goods as variables, were eliminated by that condition, plus the condition of equilibrium between supply and demand for total “capital”^[23]. The prices determined

be ascertained via different tools, and what emerges, as explained below in the text, is that things can change in ways contrary to neoclassical beliefs.

²³ A proof of this point is supplied by the model to be illustrated shortly. So the treatment of the capital goods as representing a given total endowment of ‘capital’ of variable ‘form’ is indispensable to determine long-period equilibria; it is not an approximate ‘simplification’ useful for rough empirical estimations and more rigorously replaceable with a vector of given endowments of the several capital goods, as neo-Walrasian theorists believe.

The proof can be generalized (to the case of circulating capital goods) as follows (the remainder of this footnote is optional reading), starting from the equations of the general equilibrium of a non-capitalistic economy that produces with labour and m types of land, and no rate of interest. Initially reinterpret $n \leq m$ of the lands as n kinds of capital goods but (as in Walras before the chapters on ‘capitalization’) with given endowments, non-depreciating and not produced: nothing changes in the equilibrium equations because in fact these capital goods are lands. The equilibrium determines their rentals (the prices of their services) as if they were lands. Now turn to assuming these capital goods are circulating capital goods, with endowments to be endogenously determined, quantities produced to be determined, selling prices to be determined, and rentals they earn at the end of the period to be determined (assume savers buy the capital goods at the beginning of each production cycle and loan them to firms, obtaining the gross rental on the capital good at the end of the period). These capital goods are embodiments of the single factor ‘capital’, in equilibrium there is equality between supply and demand for this factor, whose ‘price’ is the rate of interest, hence firms are satisfied with employing the amount of ‘capital’ they are employing, and in the ‘form’ appropriate to their needs, so they only want to replace the capital goods they consume, the economy is stationary. Then we are adding $3n+1$ unknowns to the system of equilibrium equations: the n endowments of capital goods at the beginning of each period; the n quantities produced of capital goods; the n prices of the capital goods; and the rate of interest. The n rentals are not additional new unknowns because they were already present in the first general equilibrium system, the one with the capital goods treated like lands. So we need $3n+1$ additional equations. The condition of price equal to minimum average cost holds for the capital goods too and supplies n more equations; the stationary condition supplies n equations stating that for each capital good production equals endowment; the condition of uniform rate of return on their supply prices adds n further equations, that state that the rate of return on investing in the purchase of each capital good must equal the rate of interest r . There remains one degree of freedom,

by this kind of equilibrium were long-period prices, analogous to the classical natural prices, associated with a uniform rate of profits (rate of interest on supply price). Accordingly this kind of general equilibrium can be called a *long-period* general equilibrium. Knut Wicksell's *Lectures* contain a clear formulation of such an equilibrium (a simplified version is contained in my 1978 article "The difference between long-period and short-period ..." in *Australian Economic Papers*; another one is in Petri (2020)). We will see later (§8) that contemporary general equilibrium theory is different. Now I present a simple model of traditional general equilibrium, restricted for simplicity to two consumption goods and two capital goods.

A common current mistake in interpreting traditional marginalist authors is to take the presence in them of capital treated as a single factor as indicating the use of an Aggregate Production Function which specifies total output of the economy, treated as a single good, as a function of labour, and of capital described as a single factor homogeneous with output,^[24] as in Solow's growth model. In fact the use of APFs, born only in the 1930s as a construct hopefully able to give rough statistical indications, was never conceived as capable of being a foundation for the marginal approach; indeed no one among the founders or the second generation of marginalist economists used it, they were all clear that the approach had to be capable of dealing with heterogeneous capital and many consumption goods. The misunderstanding derives from not grasping that the notion of general equilibrium of these traditional marginalist economists did not at all aggregate goods, but needed nevertheless an endowment of capital specified as a given single quantity of variable 'form' because it was a long-period equilibrium which left the amounts (the 'endowments') of the several capital goods to be *endogenously* determined by the equilibrium, hence unknowns, determined by assuming that the given 'quantity of capital' is 'embodied' in the several capital goods, *constraining their amounts so that more of one capital good means less of some of the others*, and that the equilibrium composition of capital is such as to equalize the convenience of investing into any capital good.

which in order to be 'closed' requires the equation establishing equality between the total endowment of 'capital' the single factor, and the demand for it. Without this equation the model would not determine whether the economy is richly or poorly endowed with capital goods, their amounts would be indeterminate. The given endowment of 'capital' constrains their amounts, more of a capital good will require less of some other ones.

²⁴ Capital is treated by traditional marginalist authors as an amount of exchange value, hence, in this aspect, homogeneous with aggregate output which is also an amount of exchange value, and this helped the statisticians' belief that it was possible to conceive them as different quantities of the same 'stuff', as assumed in Aggregate Production Functions. The basic mistake of the traditional neoclassical approach is particularly clear here, the dependence of normal relative prices on income distribution, and of market relative prices on market conditions, is neglected, as if relative prices could be treated as fixed.

The logic of a *long-period* marginalist equilibrium can be grasped by considering a simple example, starting from an economy without capital goods and then seeing how the model needs to be changed to accommodate capital goods.

Assume first that the economy uses labour and two types of land, called goods 1 and 2, to produce two consumption goods, called goods 3 and 4. Production of both consumption goods takes 1 period. Each product is produced according to a (different) CRS production function with smooth substitutability between the three factors, and firms choose the factor proportions that minimize average cost once the three factor rentals, w , v_1 , v_2 , paid at the end of the period, are given. Competition with entry brings product prices to equal minimum average cost. Cost minimization determines the efficient technical coefficients (per unit of output) of labour and of the two types of land in the production of good 3, a_{L3} , a_{13} , a_{23} , and in the production of good 4, a_{L4} , a_{14} , a_{24} , as *functions* of w , v_1 , v_2 , but for brevity I do not indicate this functional dependence explicitly. Good 3 is the numéraire. Given the three factor rentals, the equilibrium prices of the two consumption goods must obey (MinLAC stands for minimum long-period average cost):

$$(1) P_3 = \text{MinLAC}_3 = a_{13}v_1 + a_{23}v_2 + a_{L3}w$$

$$(2) P_4 = \text{MinLAC}_4 = a_{14}v_1 + a_{24}v_2 + a_{L4}w$$

$$(3) P_3 = 1 \text{ numéraire equation (other numéraire equations are possible).}$$

The quantities produced q_3 and q_4 must equal the consumers' total demands for them, that I indicate as $D_3(\cdot)$, $D_4(\cdot)$, again for brevity not explicitly indicating that they are functions of prices and rentals, determined by the equilibrium choices of consumers on the basis of their given utility functions and factor endowments.

$$(4) q_3 = D_3(\cdot)$$

$$(5) q_4 = D_4(\cdot)$$

Once the factor rentals are given, prices are given and then quantities demanded are given too, so one can derive the demand for each factor as the sum of its demand in each industry, given by technical coefficient times quantity produced, and one can write the equilibrium condition of equality between this demand, and the supply decided by consumers and indicated as $L(\cdot)$, $K_1(\cdot)$, $K_2(\cdot)$:

$$(6) L(\cdot) = a_{L3}q_3 + a_{L4}q_4$$

$$(7) K_1(\cdot) = a_{13}q_3 + a_{14}q_4$$

$$(8) K_2(\cdot) = a_{23}q_3 + a_{24}q_4.$$

Assuming sufficient substitutability, these conditions will be satisfied as equalities, i.e. no factor remains partially unemployed. (Easy to add factors and goods.)

These are 8 equations but 7 independent equations because one of the supply=demand equations can be derived from the other ones owing to the Corollary of Walras' Law ('if all markets but one where price is positive are in equilibrium, then

the last market too where price is positive must be in equilibrium, owing to all consumers and firms having balanced budgets'.) The variables are 7: three factor rentals, two product prices, two quantities produced. The equilibrium equations are so few because I am omitting (assuming they are satisfied) the conditions imposing cost minimization by firms, and utility maximization by consumers, and considering only their results. So the conditions for general equilibrium without capital goods are quite simple. Factor rentals are crucial. (IGE only reinterprets.)

Now assume these lands are in fact capital goods, but indestructible, *and not produced* because consumers do not wish to perform savings nor is it necessary to replace the used-up capital goods since these do not depreciate: then nothing changes in the equations. But this is only an initial step to make it easier to proceed to the assumption that lands 1 and 2 are in fact capital goods but not indestructible, actually circulating capital goods, so the economy must replace them entirely at the end of each production cycle (each year) if the capital stock is not to decrease. Now gross savings are indispensable for the economy to continue to operate, but assume for simplicity that consumers do not wish to perform *net* savings, i.e. they save just enough to replace the used-up capital goods. The equilibrium is then stationary, it describes an economy not changing in time; every period the equilibrium is repeated unaltered. This requires that it be equally convenient to invest savings in either capital good, so we must introduce the rate of return (rate of interest, rate of profit) r , which must be the same in both uses of savings; otherwise the composition of the demand for capital goods would change. Because of competition capital good 1 will have an equilibrium sales price equal to its minimum average cost, indicate it as P_1 , and the same holds for capital good 2 and its price P_2 :

$$(9) \quad P_1 = \text{MinLAC}_1 = a_{11}v_1 + a_{21}v_2 + a_{L1}w$$

$$(10) \quad P_2 = \text{MinLAC}_2 = a_{12}v_1 + a_{22}v_2 + a_{L2}w.$$

Of course now the technical coefficients indicate the quantities of *capital goods* 1 and 2 and of labour L employed per unit of output of each capital good. Assume that the capital goods just produced are bought by savers and lent to firms at the beginning of the period, and at the end of the period the firms pay rentals v_1 and v_2 . With circulating capital, the condition of equal rates of interest implies

$$(11) \quad v_1 = (1+r)P_1,$$

$$(12) \quad v_2 = (1+r)P_2.$$

This is how the condition of Uniform Rate of Return on Supply Price, URRSP, appears in this model: *supply price* means the minimum price necessary to induce a positive supply of a product, hence equal to minimum average cost. Equivalently, one can assume that savers lend their savings to firms, at rate of interest r ; firms use these savings to buy the capital goods at prices P_1 and P_2 at the beginning of the period, and

must now include in their cost what they must give back to the lenders, which includes interest too as indicated by equations 11 and 12. Note that in equations 1, 2, 9, 10 we can replace the rentals v_1, v_2 with the right-hand sides of equations 11, 12, and in this way we reach the familiar expressions for the price=cost conditions adopted in the Sraffian price equations:

$$P_1 = \text{MinLAC}_1 = (1+r)(a_{11}P_1 + a_{21}P_2) + a_{L1}w$$

and analogously for capital good 2;

$$P_3 = \text{MinLAC}_3 = (1+r)(a_{13}P_1 + a_{23}P_2) + a_{L3}w$$

and analogously for good 4.

Equations 4 and 5 continue to apply (except that now among the prices influencing consumer choice and determining the demand for consumption goods D_3 and D_4 there is also the rate of interest); equations 6, 7, 8 continue to hold with the same modification, plus that now the demands for the factor endowments must include the factors needed to produce the capital goods (see equations 6', 7', 8' below).

Note that so far we have added three new variables, the prices of the two capital goods and the rate of interest, and four additional equations, equations 9, 10, 11 and 12; but the replacement of the two lands with two circulating capital goods adds 4 further unknowns: the 2 endowments of capital goods at the beginning of each production cycle, indicate them as K_1 and K_2 , which must be *endogenously* determined by the equilibrium; and the 2 quantities *produced* each period of these capital goods, indicate them as k_1 and k_2 . So we need 3 further equations. The condition of stationary equilibrium gives us two more equations stating that for each capital good, production equals the endowment:

$$(13) \quad K_1 = k_1$$

$$(14) \quad K_2 = k_2.$$

In accord with fn. 23, reinterpreting the $n=2$ lands as capital goods we add $3n+1=7$ new variables, and so far only 6 additional equations. *There remains one degree of freedom*, and this is eliminated by the condition of demand for capital the single factor equal to its given total endowment K^* , necessarily an amount of exchange value as proved below. Let us then recapitulate the equilibrium equations of this economy but adopting, for the price=cost conditions, the Sraffian form which reduces the number of equations by eliminating from the equations the explicit indication of the capital rentals v_1, v_2 :

price = minimum average cost equations with URRSP, and a numéraire equation:

$$1' \quad P_1 = (1+r)(a_{11}P_1 + a_{21}P_2) + wa_{L1}$$

$$2' \quad P_2 = (1+r)(a_{12}P_1 + a_{22}P_2) + wa_{L2}$$

$$3' \quad P_3 = (1+r)(a_{13}P_1 + a_{23}P_2) + wa_{L3}$$

$$4' \quad P_4 = (1+r)(a_{14}P_1 + a_{24}P_2) + wa_{L4}$$

$$5' \quad P_3 = 1 \quad (\text{or also e.g. } P_3 + P_4 = 1) \quad \text{numéraire equation}$$

supply = demand equations for capital endowments, labour, consumption outputs:

$$6' \quad L = a_{L1}k_1 + a_{L2}k_2 + a_{L3}q_3 + a_{L4}q_4$$

$$7' \quad K_1 = a_{11}k_1 + a_{12}k_2 + a_{13}q_3 + a_{14}q_4$$

$$8' \quad K_2 = a_{21}k_1 + a_{22}k_2 + a_{23}q_3 + a_{24}q_4$$

$$9' \quad q_3 = D_3(\cdot)$$

$$10' \quad q_4 = D_4(\cdot)$$

production of new capital goods under static stationariness assumption^[25]:

$$11' \quad K_1 = k_1$$

$$12' \quad K_2 = k_2$$

total demand for 'capital' (= value of desired endowments of capital goods) equals the given endowment K^* of 'capital':

$$13' \quad P_1k_1 + P_2k_2 = K^*.$$

One of the equations demand=supply is not independent, because of *Walras' Law*. So 12 independent equations in the 12 unknowns $P_1, P_2, P_3, P_4, w, r, K_1, K_2, q_1, q_2, k_1, k_2$. Without the last equation, 13', that establishes equality between demand for, and the given endowment K^* of, 'capital' conceived as a single factor of variable 'form' (an amount of value), there would not be a determinate solution.

The given endowment of capital treated as a single factor of variable 'form' is indispensable. Without it the quantities of capital goods remain indeterminate, nothing determines whether the economy is abundantly or poorly endowed with capital goods. The given K^* puts a constraint on what quantities of the several capital goods the economy can afford: more of one will mean less of some other one.

²⁵ I.e. zero net savings. This assumption should have been carefully justified by traditional marginalist economists, given that generally net savings are not zero in actual economies, but was not; the justification that appears derivable from the logic of long-period marginalist equilibrium and the marginalist treatment of capital as a single factor would seem to be as follows. The rate of interest determined by a long-period equilibrium is the one inducing the economy to employ, and to be content with going on employing, the existing stock of capital K ; but such a rate of interest would induce firms to persist with the existing K/L ratio, hence not to absorb net savings which, if accepted, imply a desire for new plants with an average K/L ratio *different* from the current one. So the actual rate of interest which induces the absorption of nonzero net savings must be at least a bit different from the net marginal product of the given K . But the high elasticity of the demand for capital implicitly assumed by the neoclassical theory of income distribution implies that net savings require only a very small alteration of the rate of interest relative to stationariness, so the neoclassical economist can argue that the zero-net-savings equilibrium determines income distribution sufficiently correctly even in economies which are slowly moving away from that equilibrium owing to non-zero net savings. (For more on this issue, cf. GECM ch. 4 and Petri 2015.)

(The proof that a *Wicksellian* model of long-period equilibrium too needs a given scalar endowment of capital (an amount of exchange value) is in Petri (1978).)

[optional reading] One might doubt that this model reflects the marginalist views, because price equations 1' to 4' look 'Sraffian'; but in fact they are also perfectly Walrasian! This is seldom recognized but is important. Walras too aimed at determining long-period equilibrium prices, and the notion of long-period price is the same in Walras and in Sraffa, the thing is only a bit obscured by differences in assumptions about durability and depreciation of capital goods. Let us follow Walras, who assumes capital goods are durable and that for each, depreciation is a constant percentage of its value. With P_k the selling price or supply price of capital good k when new, equal to cost of production, and with p_k the price of the services of capital good k i.e. the gross rental rate it earns at the end of each period, i the rate of interest, μ_k the depreciation charge, and v_{k+} the insurance premium, Walras writes that in equilibrium for all capital goods it must be (see Walras/Jaffé p. 269):

$$(*) \quad P_k = p_k / (i + \mu_k + v_{k+}).$$

This is a condition of uniform rate of return on supply price, URRSP, that takes into account gradual depreciation, and compensation for risk. For ease of comparison with Sraffa let us assume that in Walras too all capital is circulating capital and production is in yearly production cycles for all goods, so $\mu_k = 1$, and let us neglect risk i.e. insurance premiums, $v_{k+} = 0$, so rate of interest i and rate of profit r can be treated as equal and p_k is the same as the gross rentals v_1 and v_2 in equations 11 and 12. Indeed equation (*) becomes $P_k = p_k / (1 + r)$ which can be re-written as:

$$p_k = P_k \cdot (1 + r) \quad (\text{the same as my equations 11 and 12}).$$

This is exactly what is implicitly assumed in Sraffa's equations and more explicitly in my equations 11 and 12. This shows that equilibrium in Walras is associated with a notion of price which is the same notion of natural price or long-period price or price of production as in Ricardo or Marx. (Probably Walras's complete disaggregation opened the way to Dmitriev's understanding and solution to Ricardo's problem of a rate of profits depending on relative prices depending on the rate of profits.) So Walras's price equations would have allowed him to realize that in fact the circular reasoning of which he accused Ricardo was not there, once the real wage is taken as given. But Walras did not understand it, he was totally within the frame of mind that income distribution is determined by supply and demand, so he could not accept the idea of a given real wage determined by class conflict.

The same equivalence with Sraffa's prices can be shown for Wicksell (Petri 1978). There is no difference between the product prices that classical economists, and *traditional* marginalist economists, Walras included, were trying to determine: long-period prices. The difference is in the theory of income distribution.

There is a logical necessity behind the measurement of this factor "capital" of variable 'form' as an amount of exchange value. Proof: All units of a factor tend to earn the same rental rate. Thus take two lands A and B of same quality and different

surface; if land A earns, as *total* rent payments, twice as much as land B, this must mean that land A has a surface twice the surface of B. Now consider two different capital goods A and B; assume capital good A earns as net rental twice as much as capital good B; we want to see this as due to the productive contribution of a single factor ‘capital’ embodied in them; then we must conclude that A contains twice as many units of ‘capital’ as B. But the net rental earned by a capital good is the interest on its value, and the rate of interest is uniform, so the value of A is twice the value of B. Hence the “quantity of capital” ‘embodied’ in different capital goods is necessarily proportional to the value of those capital goods; hence “quantity of capital” is necessarily *value* of capital; nor is there any other, physical way to specify this quantity that respects this proportionality: weight or volume of capital goods have no univocal connection with productivity or earnings.²⁶

Then the capital endowment of an economy is the value of the capital goods present in it, and traditional marginalist economists thought it legitimate to take this total value as given, as one of the data for the determination of equilibrium. But consider again the economy with labour immigration. When the wage rate starts to decrease, the composition of capital changes, and the neoclassical argument is that “in place of the consumed capital goods, new ones of equivalent value enter successively”; but the composition of capital changes because demands change, causing changes in prices, so the revenues of firms will not generally allow setting aside the same depreciation funds as before; furthermore, many capital goods are durable so they do not disappear quickly, and their value (like the value of land) can change very considerably if their prospective earnings change because of a change in income distribution. The value of capital goods at any given time will also depend on accidental transitional events such as a recession causing under-utilisation of fixed capital, a new invention that renders some existing plants obsolete, a stock exchange crash, prices not yet brought by competition to equal average costs, and so on. So the value of the capital goods existing in an economy does not have the *persistence* and *independence from the variables that the equilibrium should determine*, needed for legitimate inclusion among the data of equilibrium. One of the best marginalist economists, Knut Wicksell, was very honest on the issue. He wrote (*Lectures*, 1902-

²⁶ Therefore the answer to a question like “Why was capital traditionally measured as a quantity of value by marginalist economists?” must include, first, the reason why capital had to be conceived as a single factor of variable ‘form’ (answer: adjustments take time, therefore the adjustments that according to this approach bring to equilibrium include changes in the endowments of the several capital goods, therefore these endowments must be treated as variables, and then, without this given quantity of ‘capital’ the single factor constraining these endowments, the equilibrium is indeterminate, the system of equations remains with a degree of freedom, see fn. 19); and second, the reason why this single factor had to be measured in value terms (answer: just given in the text).

1928, Engl. ed. 1934, p. 202 of vol. 1): “But it would clearly be meaningless – if not altogether inconceivable – to maintain that the amount of capital is already fixed before equilibrium between production and consumption has been achieved. Whether expressed in terms of one or the other, a change in the relative exchange value of two commodities would give rise to a change in the value of capital” (p. 202). A few lines later Wicksell admits that this implies an “indeterminateness” of the endowment of capital.[²⁷] But then, since there is no way to determine the given capital endowment, long-period general equilibria are indeterminable.

7. Reswitching.

The problem explained in Section 6 prompted some neoclassical economists to propose a different neo-Walrasian approach (see section 9), which in recent decades has become the universally accepted one in general equilibrium theory. But up to at least the 1960s the general attitude was to disregard the problem and to consider it legitimate to treat capital as somehow a single factor, with a demand negatively elastic with respect to the rate of interest because a lower rate of interest would cause substitution in its favour both in firms’ choices and in consumer choices. A clear example of such a treatment of capital is Solow’s 1956 growth model. This treatment is still unproblematically used in all applied mainstream analyses, e.g. macro, international economics, labour economics (and as argued in section 9, inevitably so).

But in 1960 an example in Sraffa’s book *Production of commodities by means of commodities* showed that this notion of capital was indefensible. We can use a simpler example due to Paul Samuelson (1966). It is based on ‘Austrian’ technology, which because of the absence of basic commodities often permits simpler analyses. Consider an economy that produces two goods, champagne, and whiskey. Production is in periodic cycles, *period t* goes from *date t* (moment *t* on the continuous time line) to *date t+1*, outputs of production processes carried out during period *t* are sold at date *t+1*, wages of labour employed in period *t* are also paid at date *t+1*. Production of 1

²⁷ Note that the treatment of capital as an amount of exchange value implies that, when determining the supply and demand choices of consumers, the endowment of capital of each consumer must be an amount of exchange value, the value of the capital goods owned by the consumer. The indeterminateness admitted by Wicksell also applies to this capital endowment of each consumer: any price change would change that endowment (by changing the prices of capital goods) and hence the consumer’s choices even if the capital goods she owned did not change, so the illegitimacy of a given total value endowment of capital before equilibrium prices are known also implies *an indeterminateness of consumer choices*; so the functions $D_3(\cdot)$, $D_4(\cdot)$ in the model above were actually as illegitimate as the given K^* , they were based on the mistaken treatment of quantities of capital as well defined independently of prices and of the composition of capital.

unit of champagne requires the payment of 7 wages *two* periods before the sale of the product (i.e. if the product is sold at date t which is the beginning of period t , the 7 units of labour were utilized during period $t-3$ and paid at date $t-2$). Production of 1 unit of whiskey requires the payment of 2 wages three dates before, and of 6 wages one date before the date when the product is sold. Then long-period prices p_c of champagne and p_w of whiskey must satisfy:

$$p_c = 7w(1+r)^2$$

$$p_w = 2w(1+r)^3 + 6w(1+r).$$

Put $w=1$, that is, choose labour as numéraire, i.e. measure prices in *labour commanded* (how many units of labour one can ‘command’ i.e. purchase, have at one’s command, with the value of the good), then

$$(6) \quad p_c = 7(1+r)^2$$

$$(7) \quad p_w = 2(1+r)^3 + 6(1+r).$$

As r rises, both prices rise, so the *real* wage decreases in terms of both goods. Easily checked:

$$p_c = p_w \text{ for } r=50\% \text{ and for } r=100\% ;$$

$$p_w < p_c \text{ for } 0.5 < r < 1 ;$$

$$p_c < p_w \text{ for } 0 < r < 0.5 \text{ and for } r > 1.$$

At $r=0$ it is $p_c < p_w$ because cost consists only of wages and a unit of champagne requires the payment of 7 wages, against 8 for a unit of whiskey; but the price difference decreases as r increases, and is reversed as r becomes greater than 50%, because interest costs increase faster in the production of champagne than of whiskey; however, as r increases further, compound interest causes a greater increase of the cost of whiskey, so the price of whiskey starts approaching the price of champagne and the ratio between the two prices is reversed again as r becomes greater than 100%. This shows that the ratio between the normal price of two commodities need not vary monotonically when the rate of profits (or rate of interest) rises.

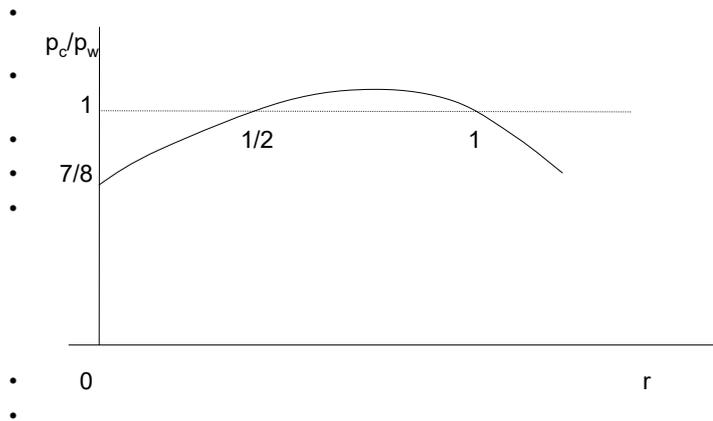


Fig. 3. The relative price of champagne to whiskey as a function of the rate of profit (or rate of interest), in Samuelson's 1966 example.

Sraffa in his 1960 book, where he produced the first example proving this result, commented: "The reversals in the direction of the movement of relative prices, in the face of unchanged methods of production, cannot be reconciled with *any* notion of capital as a measurable quantity independent of distribution and prices." (p. 38).

Indeed, if it were possible to ascertain independently of income distribution the capital/labour proportion in the production of the two goods, as capital becomes relatively more expensive the good with the higher K/L ratio should *always* become more expensive relative to the other one; the price ratio p_c/p_w should be a monotonic function of income distribution, as when the factors are labour and land. The reversals in the direction of the movement of relative prices mean that the K/L of the two techniques is changing with distribution in spite of no physical change in the methods utilized. This destroys the legitimacy of conceiving production as employing a definite quantity of a factor 'capital' measurable independently of what that quantity should contribute to determine, i.e. income distribution and prices. The factor 'capital' does not exist. Any paper that treats capital as a single productive factor is wrong.

This also affects technical choice. In the previous example, now interpret p_c and p_w as the unit cost (in labour commanded) of producing the *same* good, a consumption good (the sole one the economy produces), say meat, with two different alternative

techniques²⁸ which, to continue with the previous symbols, we may distinguish as ‘C’ and ‘W’. Now the wage is measured in meat; the maximum wage payable by each technique for a given r is the reciprocal of the labour-commanded price determined by equations (6) and (7). For each given r , the technique that competition imposes is the one that allows paying the higher wage, because the producers who adopt it can undersell the producers using the other technique since, at any common real wage, they have lower costs. Technique ‘C’ is the more convenient one, the cost-minimizing one, at $r=0$; it remains the more convenient one as r rises up to $r=0.5$; as r increases further, producers find it convenient to switch to technique ‘W’, but find it convenient to ‘reswitch’ to technique ‘C’ if r becomes greater than 100%. This is one case of the famous ‘reswitching’. So with heterogeneous capital goods the picture of the available techniques as arrangeable in order of decreasing capital-labour ratio, and of rises in the rate of interest (and decreases of the real wage) as always prompting changes toward using more labour and less capital, is impossible to sustain. The notion itself that it is possible to establish which one, of two techniques, uses more capital per unit of labour in some ‘technological’ sense is destroyed.

8. Reverse capital deepening. Subsystems. Meaning of $p=wL+rK$.

Reswitching destroys the logical legitimacy of conceiving capital as a single factor of production; one of the effects is that it reinforces the conclusion that one cannot formulate a general equilibrium model in which there is a given endowment of this single factor ‘capital’. This can be seen as a supply-side criticism of neoclassical theory in its traditional versions, in that it destroys the right to take as given one of the endowments this theory needs to determine equilibria. But reswitching also has demand-side critical implications: it undermines the assumption of decreasing demand curves for factors. In particular, it undermines the conception of aggregate investment as a decreasing function of the rate of interest. Once one gets rid of many confusions in the neoclassical theory of investment, that I cannot discuss here (see Petri 2003; 2004, ch. 7; 2015), it becomes clear that the real foundation of the mainstream view of investment as negatively interest elastic is the connection between investment and the neoclassical demand curve for capital. The argument runs as follows. One must distinguish two types of gross investment. First, replacement of used-up circulating capital (raw materials, component parts to be assembled into final products) for

²⁸ As the determination of the value of capital associated with each technique (see below) illustrates, each way of producing the consumption good includes several processes in succession, that is, presumes the existence of several industries each one with its production method, therefore it is a distinct technique.

continuation of production in existing plants: the needed circulating capital is conditioned by the machinery utilized and therefore generally no variability exists in technical coefficients unless the durable machinery is changed; so this part of investment allows nearly no variability of capital-labour proportions. Second, replacement of used-up durable capital (possibly with different durable capital), or as I put it for brevity, building of *new plants*: it is here that capital-labour proportions can be varied. Once the durable capital goods are chosen, technical coefficients are very rigid (some limited variability exists almost only in agriculture, where irrigation and fertilizers can be varied; but a car needs one engine and four tyres, and amounts of labour determined by how mechanized the factory is). In other words, technical choice can only be exercised when one invests in new plants; it is only in new plants that income distribution can determine the desired K/L ratio (see Petri, 2004, p. 31 fn. 30, and §4.3, and Petri, 2015, for a better grasp of the reasoning). So imagine all durable capital in an economy lasts 10 years and each year 1/10 of all plants is scrapped and 1/10 of the labour force is re-employed in new plants. It is in these new plants that firms are free to choose the K/L ratio. The neoclassical idea is that the interest rate determines the K/L ratio adopted in new plants. Once the full employment of labour is assumed, the denominator of this fraction is given, it is the fraction of the labour force ‘freed’ every year by the closing down of the oldest plants, 1/10th in our example; then the desired K/L ratio determines the K to be combined with labour in the new plants, that is investment; if the desired capital/labour ratio rises because the rate of interest decreases and prices adapt to the new income distribution, firms will want to combine more capital with labour in the new plants, so investment increases.

The capital demand curve is the K to be combined with the entire labour force once the rate of interest determines the optimal average K/L ratio, and it is considered decreasing because according to the neoclassicals the optimal K/L ratio is a decreasing function of the rate of interest; investment is a decreasing function of the interest rate because it is a reduced-scale copy of the demand-for-capital curve, in that it reflects the application of the optimal K/L ratio, not to the entire labour force, but only to the fraction of the labour force to be employed every period in new plants²⁹ - assuming full employment is maintained.

There is more than one problem with this argument; here I illustrate the one evidenced by reswitching (another, see Petri 2015, is the illegitimacy of *assuming* that labour is fully employed — this should result from the analysis, on the contrary the

²⁹ If all capital were circulating capital, entirely renewed each period, (gross) investment and demand for capital would *coincide* because each period the entire labour force would be combined with entirely new capital goods. This is the case in Samuelson’s example interpreted as applying to choice of techniques, see below.

analysis must assume it otherwise investment is indeterminate because a given K/L ratio in new plants does not determine the ‘capital’ employed in new plants since the denominator is not given). This problem is, that there is no guarantee that the demand curve for value capital (per unit of labour) is a decreasing function of the interest rate. To show it, we again refer to Samuelson’s example, interpreted as applying to meat producible with the two alternative techniques ‘C’ and ‘W’. Meat, the sole consumption good, is the numéraire. For each technique, we must assume the economy produces meat as net product; prices and industry dimensions are fully adjusted to income distribution and to the given technique; we must enquire which capital goods will be present in the economy per unit of labour, and with which value.

With technique ‘C’, if one unit of meat is produced every period then in each period the economy produces one unit of meat as net product and furthermore replaces the used-up capital goods, hence the following *three* production processes go on simultaneously in each period:

- 7 units of unassisted labour produce, at the end of the period, as output a capital good that we can indicate as $c1$
- $c1$ alone, unassisted by labour, ripens into capital good $c2$
- $c2$ alone, unassisted by labour, ripens into one unit of meat.

This can be illustrated graphically as follows (t-3, t-2 etc. are the dates when the corresponding period starts):

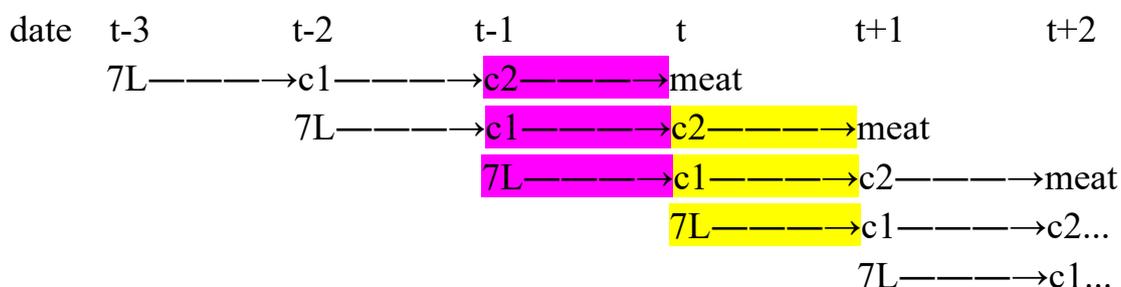


Fig. 3bis

Thus at the beginning of each period t the economy’s stock of capital consists of one unit of $c1$ and one unit of $c2$. The value of $c1$ is $7w$, the value of $c2$ is $7w(1+r)$. Hence the value of capital is $K_c = 7w + 7w(1+r)$ per unit of consumption good. Since a net output of one unit of consumption good implies the employment of 7 units of labour, the value of capital *per unit of labour* is $K_c/7 = w + w(1+r)$.

With technique ‘W’, each period the following production processes go on simultaneously (**draw the analogous graphical illustration!**):

- 2 units of unassisted labour produce as output at the end of the period a unit of a capital good that we can indicate as $w1$

- $w1$ unassisted by labour ripens into capital good $w2$
- $w2$ together with 6 units of labour produces capital good $w3$
- $w3$ unassisted by labour ripens into one unit of meat.

At the beginning of each period the economy's stock of capital consists of capital goods $w1$, $w2$ and $w3$. Their respective values are $2w$; $2w(1+r)$; $2w(1+r)^2+6w$. The value of capital per unit of consumption good is $K_w = 8w+2w[(1+r)+(1+r)^2]$, and per unit of labour it is $K_w/8$.

These values of capital are the normal 'demands' for capital per unit of labour implied by each technique. Let us calculate them at $r=100\%$, where w is $1/28$ for both techniques. It is $K_c/7 = 3/28 = 3w$, and $K_w/8 = w+12w/8 = 5w/2$, which is less than $3w$. So as r rises from a little less to a little above 100% , technical choice is in favour of technique 'C' which implies a *higher*, and not a lower, value of capital per unit of labour: '**reverse capital deepening**'. Thus *a rise of r causes an increase of investment!* This possibility, confirmed by many other examples, shows that there is no guarantee that as the rate of interest decreases, investment increases³⁰. The mechanism on which the neoclassical approach relies to argue that investment tends to adjust to full-employment savings comes out to have been based on a false analogy between capital the value factor, and factors measurable in 'technical' units such as labour or land.

This is very relevant, because the neoclassical tendency toward full employment requires that investment adjusts to full-employment savings, and the adjustment relies on the capacity of decreases of the rate of interest to raise investment if the latter is less than full-employment savings. The same supposed interest-elasticity of investment has been extremely important in the debates on Keynes, allowing the criticism based on the 'Keynes effect', namely, that reductions of the money wage do raise employment by causing a decrease of the price level, hence of the demand for money, hence of the rate of interest, with consequent increase of investment and hence of aggregate demand and of employment; the conclusion was that full employment is not reached because money wages are not downward flexible. Reverse capital deepening undermines this criticism, strengthening the thesis that capitalist economies have no spontaneous tendency toward full labour employment.

The value of capital per unit of labour can also be determined graphically from wage curves, see Fig. 4. Assume the net output is a quantity of a consumption good or basket of goods, which is the numéraire. Suppose the economy is on a wage curve at point $(r, w(r))$; the vertical intercept of the wage curve measures the net output per unit

³⁰ Note also that as r rises above 100% (and the wage decreases), the switch is from technique 'W', that uses 8 units of labour per unit of net output, to 'C' that uses 7 units of labour: the wage decrease induces the use of *less* labour *per unit of net output*: again, 'substitution' can work the opposite of neoclassical beliefs.

of labour, because if $r=0$ all net output goes to wages. Indicate the value of this net output per unit of labour as y . (If net output is a single good, y is a physical quantity.)

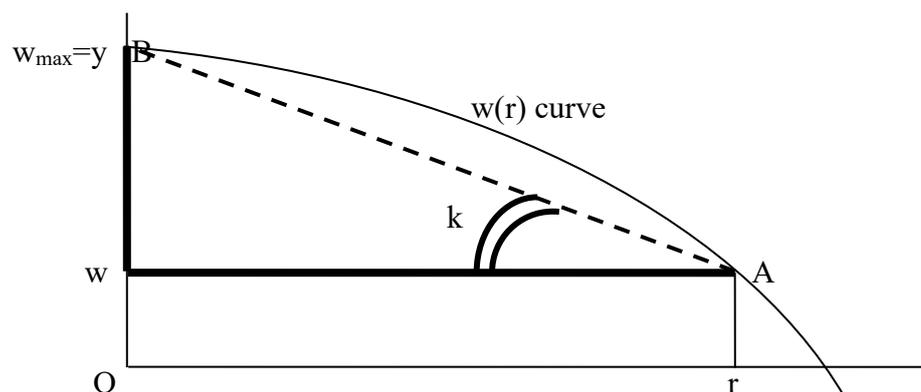


Fig. 4. Derivation of k , the normal value of capital per unit of labour, from the wage curve. With a composite commodity with the same composition as the given net output as numéraire, let y be (the value of) *net* output per unit of labour; it is equal to w when $r=0$. From $y = w + rk$ it follows that $k = (y - w)/r$, which is the absolute slope of segment AB from the point on the wage curve corresponding to the given r , to the vertical intercept $y = w_{\max}$ of the wage curve.

When $r > 0$, y is divided between one wage, and profits (or interest) at rate r over the value of capital per unit of labour, which we indicate as k ; hence $y = w + rk$, that implies $k = (y - w)/r$. Now, $y - w$ is the vertical distance from the vertical intercept of the wage curve to the given value of w ; and r is the length of the segment in abscissa from the origin to the given value of r . Therefore k is the trigonometric tangent of the angle formed by the horizontal line through (r, w) and the segment connecting point (r, w) on that wage curve to its vertical intercept, see the figure. If changes of r cause switches to another technique, y changes, and one finds the value of capital from the new wage curve, see the Figure on p. [26]. If the technique does not change, but its $w(r)$ curve is not linear, then k changes with r because of relative price changes.³¹

Then one can introduce technical choice, the same net output producible with two different techniques differing in only one method, hence different wage curves; these will differ in shape and in intercepts, e.g. the vertical intercepts differ because the amount of labour needed to produce the given net output depends on the technique. Suppose the two wage curves cross at r^* ; then (the converse of) Okishio's Theorem says that for r a little less than r^* the economy prefers a different technique from the one preferred for r a little greater than r^* , so a rise of r causes the economy to *switch*

³¹ If the $w(r)$ curve is strictly concave, along it the value of capital increases with r . This is not given great importance by neoclassical economists because as r rises it is argued that generally the optimal technique changes frequently, and then the important thing is how the value of capital changes owing to technical change. But the change in the value of the *existing* capital goods if income distribution changes destroys the right to consider the endowment of 'capital' as given.

technique, and in time the value of capital per unit of labour will tend from the value associated with one wage curve to the value associated with the other wage curve: r^* is called a switchpoint, and if there is reswitching and r^* is the higher of the two values of r at which the wage curves cross, r_2 in Fig. 5, the jump in the value of capital is ‘anti-neoclassical’ (r rises and the switch is to a higher k). This is ‘reverse capital deepening’. With many alternative techniques and many switch points on the envelope of wage curves, one can have a behaviour of k as in Fig. 6.³² Clearly, *even if* the supply of capital could somehow be determined and treated as given, in no way the curve in the right-hand diagram in Figure 6, with its several instances of reverse capital deepening (increases of r associated with increases of k), could be viewed as a demand-for-capital curve determining a unique and stable intersection with the capital supply curve. More concretely, the neoclassicals argue: the supply of capital manifests itself as a succession of supplies of (gross) savings; in every period the excess of full-employment gross national income over expenditure on consumption goods is the income earned by the resources *not* used to produce consumption goods; these, if employed, allow producing capital goods for a value K^\wedge (equal to the value of gross savings); equilibrium requires that this supply of savings be absorbed by an amount of gross investment $I=K^\wedge$; this will happen if the rate of interest induces the adoption of such an average capital-labour ratio K^\wedge/L^\wedge in new plants — where L^\wedge is the flow of labour to be re-employed in new plants because of the closing down of old plants —, that firms demand new capital goods for a value K^\wedge , which absorbs the resources not used to produce consumption, and hence employs the flow of gross savings; the idea is that if investment is less than savings, the excess savings will cause the rate of interest to decrease, then firms will want a higher K/L ratio in new plants, and K^\wedge (gross investment) will rise. But reverse capital deepening shows that the K/L ratio need not be a decreasing function of the rate of interest, hence investment too cannot be considered a regularly decreasing function of the interest rate. The critics conclude that there is no reason to believe that the rate of interest is capable of bringing investment to adjust to full employment savings.

Let us briefly indicate the implications of the above for the notion of a *decreasing demand curve for labour* and hence of the need for real wages to decrease if labour employment is to increase. In a long-period framework, this curve is indeterminable because to determine the marginal product of a factor one needs given amounts (that is, for the neoclassicals, fully employed amounts) of other factors; so one needs to specify the endowment of capital the single value factor, and this endowment is indeterminable. In a Marshallian short-period framework where only

the durable plants are given, the given plants mean that a gradually decreasing marginal product of labour has no plausibility because the given plants render the labour per unit of output rigid, more labour requires using durable capital for more hours per week, with no fall in labour productivity; so no fall in wages is needed to make it convenient to employ more labour if demand increases, and if demand does not increase a fall in wages does not make it convenient to employ more labour. Even if one admitted a decreasing marginal product of labour, any increase in labour employment, entailing an increase in aggregate income and hence in savings, would need an increase in investment to be sustainable, and we have just criticized the adjustment of investment to savings as implausible. Implication: employment depends not on wages but on aggregate demand and whatever determines it.

intl 1

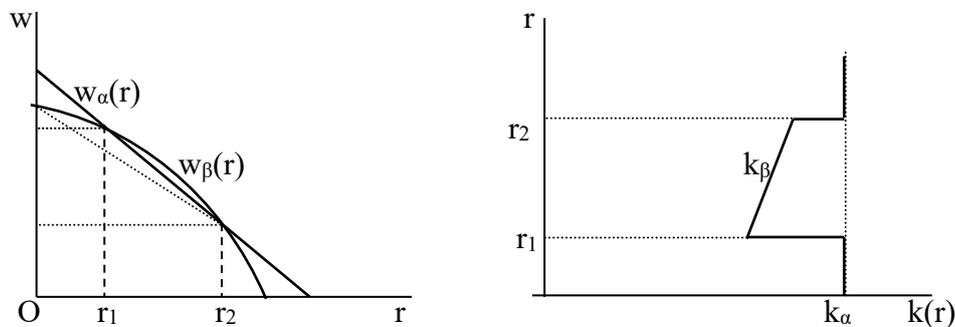


Fig. 5 Reswitching as cause of reverse capital deepening. There are two techniques, α and β , with wage curves $w_\alpha(r)$ and $w_\beta(r)$ that cross twice. Same net product and numéraire. In the right-hand diagram r (the 'price' of capital) is in ordinate, the value of capital k per unit of labour is in abscissa; for each level of r , k is determined as in Fig. 4 by the wage curve on the outer envelope. As r changes, if the technique does not change the value of capital changes as shown in Fig. 4 and is constant for technique α whose wage curve is a straight line. At switchpoints, k is indeterminate between its values with either technique because techniques are equally profitable so they can co-exist and can be used in variable proportion. The change in k when r increases from a little less to a little more than r_1 is downwards as neoclassicals expect, but at r_2 the opposite is the case, as r rises from a little less to a little more than r_2 , k rises, as in Samuelson's example. So at r_2 there is reverse capital deepening. Fig. 6 applies the same procedure to many wage curves.

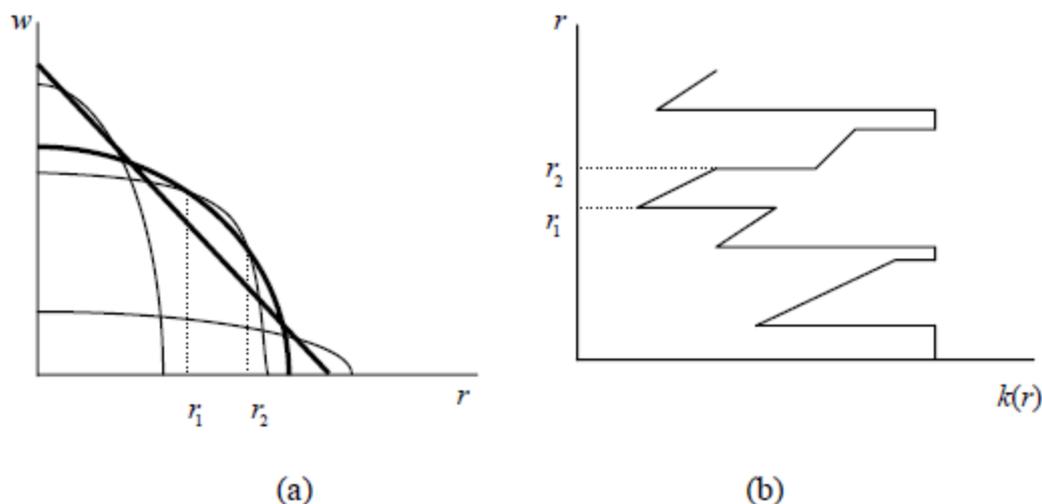


Fig. 6. An example of possible behaviour of the value of capital per unit of labour as the technique changes with income distribution. The reader should be able to derive (b) from (a) following the procedure used in Figs. 4 and 5. Clearly a relationship between r and value of capital per unit of labour as in this Figure cannot support a view of the interest rate as ensuring uniqueness and stability of equilibrium in the savings-investment market.

It is opportune now to explain the notion of **subsystem** and how it helps to understand the determination of the demand for capital once the net product and the cost-minimizing production methods are given. This notion will also help to understand the neoclassical traditional preference for representing the cost-causation of the price p_j of a good produced by labour and capital as $p_j = wL_j + rK_j$, which differs from the $p_j = (1+r) \cdot (\text{value of capital goods}) + wa_{L_j}$ of the Sraffa-type equations: the L_j in $p_j = wL_j + rK_j$ is not the same as the a_{L_j} of the Sraffian price equation.

If a unit of good j is produced by a yearly process with the quantity L of labour and T of land (with β rent per unit of land), paid at the end of the year, then the normal price of the good is given by $p_j = wL_j + \beta T_j$, and the labour and land employed are there again at the end of the year, ready to be employed in another production cycle. Labour and land have contributed their *services* to the production of the good, and now they are still there, ready to contribute their services again next period. If the good is produced by labour and capital goods and neoclassical theory wants to view it as produced by labour L and a quantity K of a single factor ‘capital’ in perfect analogy with production with labour and land, ‘capital’ being a factor whose services are repaid by the rate of interest r , then the price of the good must be representable as $p_j = wL_j + rK_j$. But then the quantity of capital employed *cannot* be simply the value of the capital goods employed in the *direct* production of good j (e.g. if good j is iron, good 2 in the corn-iron economy, it cannot be only the capital value used in its direct production, $a_{12}p_1 + a_{22}p_2$), the quantity of capital must be so conceived that it too at the end of the year is there again, in unchanged quantity, ready to contribute the same

services in the next production cycle, which is what was assumed for labour and for lands. This requires that the ‘production’ of the unit of good j be conceived as also including a production of capital goods that replace the used-up capital goods, so that at the end of the year the amount K of capital is again there; then K must also include the value of the capital goods used to produce the reintegration of the used-up capital goods. Let me illustrate with the corn-iron economy.

Let x_1 and x_2 be the total productions per year respectively of corn and of iron. Let us determine the *net* products y_1 and y_2 of corn and of iron, that is, the excesses over the consumption of corn-capital and of iron-capital as inputs. These are determined by (for a general treatment with matrices see MCM Online App. to Ch. 2):

$$y_1 = x_1 - (a_{11}x_1 + a_{12}x_2)$$

$$y_2 = x_2 - (a_{21}x_1 + a_{22}x_2).$$

Assume we want to obtain 1 unit of net output of iron and zero net output of corn, i.e. $y_1=0, y_2=1$; this will mean such total productions x_1 and x_2 that at the end of the year we are left with one unit of iron after replacing the used-up capital goods; x_1 and x_2 are determined by the above system under $y_1=0, y_2=1$. This is the *subsystem* (ensemble of industries, each one producing a certain output) associated with 1 unit of iron as *net* product. If we put $y_1=1, y_2=0$, the solutions x_1 and x_2 give us the *subsystem* associated with 1 unit of corn as net product. Year after year, if the net product is (y_1, y_2) , the economy produces the two net products and reproduces the capital goods it starts with and uses up; in imagination it can be seen as two subsystems run one next to the other, one producing $(y_1, 0)$ and the other $(0, y_2)$ as net products. One can enlarge and talk of the subsystem of any net product (y_1, y_2) .

If every year the economy produces *gross* quantities x_1 and x_2 , then it uses the total amount of labour $L = a_{L1}x_1 + a_{L2}x_2$. If the net product is positive for only one good, say one unit of iron, the services of the labour utilised, i.e. employed in the subsystem of that net product, can be seen as having entirely gone to produce that good without changing the amount of capital available for further production. And if one wants to see the vector of capital goods employed, $k_1 = a_{11}x_1 + a_{12}x_2$ and $k_2 = a_{21}x_1 + a_{22}x_2$, as embodying an amount K of a single factor ‘capital’, then the services of K can also be seen as having entirely gone to produce the one unit of iron without changing the amount of capital.

The notion of subsystem is independent of the theory of distribution one adopts. But if one follows the neoclassical approach, where goods are seen as produced by the services of factors of production labour and capital, then the net output of a subsystem is seen as produced by the services of the labour the subsystem employs, and by the *net* services of the capital K it utilizes and reintegrates, equal to the *value* of the capital goods the subsystem employs. The price p_2 of one unit of iron must then be equal to

the cost of the services of the labour and of the net services of the capital the subsystem employs, $p_2 = wL_2 + rK_2$, where L_2 is the total labour employed by the subsystem and K_2 is the value of the capital goods the subsystem employs (and reconstitutes). With $p_1 = 1$ the price of corn (numéraire) and p_2 the price of iron, K_2 is not the value of the inputs in the *direct* production of a unit of iron, it is the value of the capital goods employed in the entire subsystem, because *that* is the value of capital that must be employed so that the production of a unit of iron does not cause reduction of the stock of capital, and the same holds for L_2 . If the subsystem were entirely owned by a firm, in order to make zero extraprofits the firm should fix the price of iron as equal to $wL_2 + rK_2$. Indeed when prices equal average costs the firm starts with circulating capital goods of value K_2 , and its total revenue at the end of the year, that is the total value of the gross production of the subsystem, is $wL_2 + (1+r)K_2$; the re-purchase of the used-up capital goods costs K_2 ; the remainder, $wL_2 + rK_2$, is the revenue from the sale of the net product, one unit of iron. So zero extraprofits implies that the firm sets the price of iron as $p_2 = wL_2 + rK_2$.

This clarifies that when in the neoclassical approach one views the production of a net product vector as implying a long-period demand for labour and for ‘capital’, these demands are the *total* ones associated with the subsystem that produces that vector as net product. These are the factor demands associated with the economy’s net product, that in equilibrium must be equal to factor endowments. The factor demand for ‘capital’ in Figures 4-6 is the value of the capital goods used by the subsystem of the given net product per unit of labour, determined by the technique whose wage curve is under study. Fig. 3bis shows in the same colour the three processes activated each period in the subsystem producing a unit of meat with technique C.

The indirect substitution mechanism is based on how *these* long-period demands for labour and for ‘capital’ change with changes in the composition of consumption; having fixed the total demand for labour as equal to the given supply, it is the demand for ‘capital’ so determined that will change with the change in the composition of consumption and that must, in equilibrium, be equal to the total endowment of ‘capital’. And the price of each consumption good must be equal to the wages plus interest(=profit) to be paid in the respective subsystem.

This is shown, for an economy that produces corn and meat with the use of corn and labour, in §3.10.3 of MCM. It is a simple case worth studying, it shows that the fact that corn needs corn to be produced reduces the extent of substitutability between the two consumption goods relative to the substitutability from their direct methods.

A useful slightly more complicated example is the following. Suppose the economy produces corn, strawberries and meat, and corn is only used as a capital good and is the sole capital good. The net product of the economy consists of strawberries

and meat (i.e. corn production simply replaces the used-up corn-capital; the economy is stationary). Production is in yearly cycles. Assume production of a unit of corn (which is the numéraire) requires 0.2 units of corn-capital and 1 unit of labour; then with wages paid at the end of the year the price=cost for a unit of corn obeys $1=0.2(1+r)+w$. The wage is measured in corn, so this equation suffices to determine r once w is given, the rate of profits is a physical or material ratio (as in the first Ricardo, see above pp. 14-15); the prices of strawberry and of meat cannot affect r , competition will force them to yield the same rate of profit r as in the production of corn (strawberries and meat are non-basic goods). Production of a unit of strawberries requires 1 unit of corn-capital and 5 units of labour; production of a unit of meat requires 6.2 units of corn-capital and 1 unit of labour. If $w=0.7$ then rate of interest or rate of profits in the production of corn, and hence in the other two industries too, is $r=50\%$; the normal or long-period price of strawberries is $p_s=5=1(1+50\%)+5\cdot 0.7$, and of meat is $p_m=10$ (check that you are clear as to why). If the real wage goes down to $w=0.6$, then $r=100\%$, $p_s=5$, $p_m=13$: so meat, the more capital-intensive good, rises 30% relative to strawberries. Plausibly, the neoclassical economist argues, the composition of consumption changes in favour of strawberries and against meat ('plausibly', because *this result is not guaranteed*, owing to income effects: if strawberries are demanded only or nearly only by workers, the fall of w causes the demand for strawberries to decrease – a further weakness of the neoclassical approach).

The prices of strawberries and meat thus determined can also be obtained with the subsystems-based procedure, which allows to represent prices as $p_j=wL_j+rK_j$: let corn be good 1, strawberry good 2, meat good 3; indicate the net output of corn in the strawberries subsystem as $y_{1\text{strawb}}$, and in the meat subsystem as $y_{1\text{meat}}$; the respective gross outputs of corn are indicated as x_1 and x_1^* ; the equations determining the subsystems producing one unit of strawberries or one unit of meat are:

$$\begin{aligned} y_{1\text{strawb}} &= x_1 - (a_{11}x_1 + a_{12}x_2) = 0 & y_{1\text{meat}} &= x_1^* - (a_{11}x_1^* + a_{13}x_3) = 0 \\ y_2 &= x_2 = 1 & y_3 &= x_3 = 1 \end{aligned}$$

The result is that the corn-capital employed in the strawberries subsystem is 1.25 units, and labour employment is 6.25; if $w=0.7$ and $r=1/2$, the price of strawberries is $p_2 = 0.7\cdot 6.25 + 0.5\cdot 1.25 = 5$; the corn-capital employed in the meat subsystem is 7.75, and labour employment is 8.75; so the price of meat is $p_3 = 0.7\cdot 8.75 + 0.5\cdot 7.75 = 10$; the reader can do herself the calculations for $w=0.6$.

One further observation is important. The traditional neoclassical treatment of capital as a factor analogous to land, and of price = cost represented as $p = rK + wL$, can imply strange things about what the production function means. We have represented as typically neoclassical the case of corn C produced in yearly production cycles by

capital K and labour L according to a smooth differentiable production function, e.g. a Cobb-Douglas $C=K^\alpha L^{1-\alpha}$, where capital is corn too, a circulating capital good, and corn-capital and labour receive their marginal products. C is the net product of the economy, which goes to profits (interest) and to wages. But we know that total production of corn must be more than C , if C is the *net* product of corn. Call G the gross production of corn. Then C is G minus used-up capital, hence $G = K^\alpha L^{1-\alpha} + K$. *This* is the actual inputs \Rightarrow output production function implied by the usual net-product Cobb-Douglas. It is not Cobb-Douglas, and it is very strange because it means that production is possible even with no input of labour: if $L=0$ but $K>0$, gross output is not zero, it is equal to K , which is difficult to make sense of. You ought to pay close attention, when studying neoclassical models where capital is treated as a single factor, to how the difference between net and gross product is specified.

9. *The shift to neo-Walrasian equilibria.*

Why so much insistence on the deficiencies of the neoclassical conception of capital as a single factor and of well-behaved capital-labour substitution, if anyway the endowment of such a factor is indeterminable and accordingly long-period equilibria cannot be determined? Because a continuing faith in the traditional treatment of capital as a single factor, and in capital-labour substitution, is the hidden reason for the support nowadays enjoyed by the contemporary versions of general equilibrium theory, where capital the single factor, a quantity of value, seems to be absent.

Nowadays general equilibrium theory, in order to avoid the problem with the endowment of value capital pointed out in Section 6, takes as given the endowment of *each* capital good³³: precisely what was argued in Section 5 to be illegitimate according to the traditional neoclassical authors themselves. The resulting intertemporal or temporary equilibria are very-short-period equilibria, in that they do not give the composition of capital the time to change and adapt itself to the normal needs of firms; equilibrium must be determined on the basis of whatever amounts of the several capital goods are available at the moment one considers. This creates three new problems (Garegnani, 1990) that confirm the illegitimacy argued in Section 5:

- *Impermanence problem*: since adjustments take time, the endowments of capital goods change during disequilibrium, and this changes the equilibrium too; so

³³ Thus taking up again Walras's specification of the capital endowment, which had remained isolated among the founders of the marginal approach; but Walras's numerous grave inconsistencies, that we cannot discuss here, suggest that he was still trying to determine a long-period equilibrium; see Petri (2004 ch. 5; 2016; 2020; 2021 ch. 7). On the contrary the originators of the neo-Walrasian approach (Lindahl, Hayek, Hicks) knew that a long-period equilibrium must determine the composition of capital endogenously, and that they were moving to a different notion of equilibrium.

we do not know where disequilibria take the economy^[34]. The unrealistic assumption of the auctioneer in the study of the stability of equilibrium is made precisely to prevent changes of the data of equilibrium, and therefore of the equilibrium itself, during disequilibrium adjustments. But to *assume* a problem away does not mean to surmount it. The actual path of economies cannot be the one based on instantaneous adjustments. But how large will be the distance between actual path and equilibrium path? The equilibrium path cannot tell us; and it cannot exclude an increasing divergence. One consequence is that comparative statics becomes impossible, but then the effects of policies are indeterminable.

- *Price-change problem*: since equilibrium relative prices cannot be assumed persistent (because the arbitrary initial endowments of capital goods will be generally quickly altered), one must allow agents to take into account, in their optimization, expectable price changes; but only two equally indefensible roads are available: intertemporal equilibria require either the absurdity of complete futures markets (with yet-to-be-born consumers present at these markets!) or the absurdity of perfect foresight of both *all possible future states of the world* and of prices in each one of them (but how can future scientific discoveries, for example, be correctly predicted?); temporary equilibria without perfect price foresight (anyway nowadays abandoned) suffer from an *indefiniteness problem* because the single equilibrium as well as the path followed by a sequence of temporary equilibria come to depend upon arbitrary assumptions on unknowable initial expectations and on how these evolve over time.

- *Substitutability problem*: there is nearly no substitutability among factors when capital goods are treated as distinct factors; different production methods generally require different capital goods, not the same goods in different proportions (see footnote 14); thus the short-period demand for labour cannot but be nearly vertical, and the equilibrium real wage can easily be totally implausible, e.g. close to zero, and therefore very different from the real wage that will rule in the economy.

Implication: the working of actual economies cannot be correctly described by these equilibria, since these: i) cannot even be *defined* without assumptions that grossly violate reality, like perfect foresight or complete futures markets; ii) would need instantaneous adjustments in order for the data determining them not to change

³⁴ For the determination of the demand curve for labour, the impermanence problem means that it is illegitimate to try to determine this curve on the basis of a given vector of endowments of capital goods, because this vector will quickly change during disequilibrium and therefore cannot be the one on the basis of which one can determine the effect of different levels of the real wage. So the capital endowment, without which a labour demand curve cannot be determined, is indeterminable both in the long-period and in the neo-Walrasian versions of the neoclassical approach, because neither the value of capital, nor the vector of capital goods, can be taken as given.

significantly before the equilibrium is approached; and iii) can determine very unrealistic equilibrium values of capital goods' rentals and of the real wage.

Since actual economies certainly behave differently, these equilibria do not tell us how close or how similar the *actual* path followed by the real economy can be presumed to be to the equilibrium path. Older neoclassical equilibrium theorizing, based on persistent data (because it treated a given endowment of value capital as legitimate), could argue that disequilibrium actions did not alter the equilibrium itself, and then could argue that learning, error correction, repetition of transactions would ensure a gradual tendency toward equilibrium. With very-short-period general equilibria this way of reasoning becomes impossible, because the equilibrium itself is altered by disequilibrium actions (possibly *very* relevantly, since the actual economy will certainly behave very differently from the equilibrium prediction owing to the absence of perfect foresight and to the substitutability problem), and in directions the theory cannot predict because it only gives us the equilibrium path; so *one does not know where disequilibria will take the economy*. The theory is *totally silent* about the tendency of the actual economy. (Note that this criticism arises independently of reswitching and reverse capital deepening.)

And yet, as one must expect (otherwise the neoclassical approach would have to admit total failure), these equilibria are attributed explicative value, that is, are considered indicative of actual paths.³⁵ This can only be due to a *prior* belief that the qualitative characteristics of the equilibrium path traced by intertemporal equilibria or sequences of temporary equilibria are similar to those traced by actual paths. This belief cannot be based on the theory of the equilibrium path itself, because the latter says nothing on the actual path; so it must be based on something like the following type of reasoning:

“The traditional time-consuming marginalist disequilibrium adjustments in the labour and capital markets do exist and operate, and they maintain the economy close, on average, to the path traditionally traced by long-period neoclassical analyses, the path nowadays represented in simplified form by Solow-type growth models; now, disaggregated intertemporal equilibrium paths are qualitatively similar to Solow paths: they too trace a full-employment path, with income distribution determined by marginal products; it becomes then possible to argue that intertemporal equilibria too indicate sufficiently correctly the qualitative trend of the economy.”

³⁵ Specialists in general equilibrium theory are much more cautious than neoclassical macroeconomists and applied economists. A major one, Alan Kirman, has totally rejected the theory. Here at the end of the Appendix you find a passage by another specialist, H. Herings, who admits the dire state of the theory (note: he seems unaware of the substitutability problem and of the savings-investment problem).

But then it is a continuing faith in traditional neoclassical adjustments based on capital-labour substitution where capital is the single value factor, and therefore a faith in the capacity of old neoclassical analyses based on long-period tendencies to indicate the trends of market economies, that allows modern GE theory not to be laughed at as totally unbelievable³⁶. But such a faith is groundless, because of Sraffa's results that undermine that conception of capital-labour substitution.

In particular, the theory of intertemporal equilibria *defines* equilibrium as a situation of continuous *full* resource employment, but supplies no reason why this assumption should correspond to how actual economies behave. In these equilibria, the equality between investment and full-employment savings is not justified by acceptable adjustments; the declared abandonment of the conception of capital as a single factor prevents openly using traditional arguments to justify the stability of the savings-investment market. But recourse to the auctioneer-guided tâtonnement is no help, both because it is ridiculous nonsense if conceived for intertemporal equilibria (because then it requires the existence, already to-day, of all futures markets, with yet-to-be-born consumers present to-day to announce their future demands and supplies), and because, as explained in Petri (2017), the tâtonnement actually *assumes* a (very criticisable) version of Say's Law. As to the alternative assumption of perfect foresight, it is both unjustified (how can one have learned all the needed information?) and logically impossible (how can one predict future advances in scientific knowledge, or new creative ideas e.g. new fashions? a brain cannot predict what new thoughts it will elaborate in the future, but if one cannot predict future states of the environment one cannot predict the prices in them). As to temporary equilibria, everything depends on expectations, which might be anything; again the impossibility to rely on traditional adjustments renders the equality between investment and full-employment savings an unjustified *assumption*. Thus to the impermanence, substitutability and price-change problems one must add a fourth crucial problem of contemporary general equilibrium theory, and of the current mainstream macro models which claim it as their microfoundation: the absence of any justification for the assumed equality of investment and full-employment savings. To justify this equality, in the back of their minds the advocates of these models most probably have the kind of reasoning described above in inverted commas, which, as explained, is indefensible and in fact is *not* openly argued. The usual procedure is simply to assume equilibrium, and not to allow critics who reject the assumption openly to challenge it in the main journals.

³⁶ See Dvoskin and Petri, 2017, and section 3 of Petri, 2022, for evidence that this is indeed how modern neo-Walrasian economists reason. Applied mainstream analyses, e.g. international trade theory, or labour economics, confirm the absence of any doubts about treating capital as a single factor.

10. *Alternatives.*

In view of the above, it can be concluded that capital theory shows the need to abandon the neoclassical approach, and to search for an alternative theory, logically consistent and more capable of making sense of the empirical evidence. The modern classical-Keynesian approach mentioned in the first Section seems to be precisely such a theory³⁷.

For the determination of aggregate output and employment, this approach gives central role to aggregate demand. This will appear obvious, once one becomes fully aware of an aspect of real economies seldom stressed in teaching. Firms do not utilize their durable capital goods 24 hours a day all days at maximum speed; they can easily increase production if demand increases, and if extra labour is available (and nearly always it is). In most firms an increase of production flows by even a considerable percentage is no problem at all, it only requires extra hours and overtime labour or extra hirings. For this reason, aggregate production levels are highly flexible not only downwards, but also upwards. Given its flexibility, production adapts to demand. It is therefore aggregate demand (determined in turn by its autonomous components and by the Keynesian multiplier) that determines production and hence employment. (The recent reduction in production and employment in many European nations, owing to austerity policies imposed by the euro treaties, is as clear a proof of this fact as one can desire.) Growth is not supply-side determined, it is demand-constrained, the supply-side potential is almost never fully exploited.

An implication of the classical-Keynesian perspective is, that unemployment is not so difficult to eliminate, it can be reduced by public policy that stimulates aggregate demand, and with no need to reduce wages (if there are balance-of-payments problems, it is unclear that it must be wages that must be reduced). If governments do not do it, the reason is political: as Marx said, and Kalecki confirmed (“Political aspects of full employment”, 1943, a beautiful short article), governments *want* some unemployment, and will intervene to *increase* unemployment (Margaret Thatcher!) when they feel the working class is too strong.

³⁷ Joan Robinson and John Eatwell, *An Introduction to Modern Economics* (1973), Samuel Bowles and Richard Edwards, *Understanding Capitalism* (1985), Malcolm C. Sawyer, *The Challenge of Radical Political Economy* (1989), are elementary introductions to this approach, useful to start grasping how one reasons inside such an approach. An attentive reading of K. Bharadwaj and B. Schefold (1990) introduces to most facets of the approach and debates within and around it. A good broad survey of non-neoclassical perspectives is Marc Lavoie (2014). Garegnani (2007) is the most detailed explanation of his view of the difference between the classical and the neoclassical approach. Stephen Marglin and Juliet Schor (1990), and Philip Armstrong, Andrew Glyn and John Harrison (1991) give a taste of how the approach allows a different perspective on the evolution of modern capitalism. Kalecki (1943) remains fundamental and strikingly simple.

TO READ MORE ON CAPITAL THEORY

The issues concisely discussed in this lecture are complex; further reading is recommended. Petri (2016b) is a short historical introduction. Petri (2021) is a broad textbook in advanced microeconomics useful for references and clarifications. Petri (2004) presents more extensively the arguments summarized here, insisting on the great difference between long-period analysis and neo-Walrasian models. Garegnani (1990) is absolutely recommended. Harcourt (1972) is the most detailed survey so far of the Cambridge debates, but it was written before Garegnani's clarifications. Garegnani (2012) is his last assessment of the state of the debate, published posthumously (he died in 2011). Petri (2015) points out those other problems with the neoclassical theory of investment, alluded to but left unexplained in section 8. Dvoskin and Petri (2017) confirms the argument advanced here in section 9 and clarifies the relevance of the Marshallian short period. Petri (2022) confirms the secret continuing reliance of neoclassical macro and growth theory on the officially rejected traditional capital-labour substitution. For more on long-period price analysis (and its extension to land rent, fixed capital, joint production) one can turn, in order of increasing complexity, to Mainwaring (1984), Pasinetti (1977), Kurz and Salvadori (1995).

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APPENDIX

APPENDIX to What Capital Theory Can Teach Us, revised November 2025

Perron-Frobenius theorem.

Let A be a square non-negative indecomposable matrix. Then:

(i) A has a real eigenvalue $\lambda^ > 0$, not repeated, and dominant (that is, not smaller in modulus than any other eigenvalue), and to it and only to it is associated a*

real non-negative, and in fact positive, eigenvector^[38] x^* ; for each other eigenvalue λ of A , it is $\lambda^* \geq |\lambda|$, and $\lambda^* > |\lambda|$ if A is positive^[39];

(ii) $(\rho I - A)^{-1} > 0$ (where ρ is a real scalar) if and only if $\rho > \lambda^*$;

(iii) λ^* is an increasing function of each element a_{ij} of A ;

(iv) if s is the smallest, and S the greatest, of the sums of the elements of a row of A , then $s < \lambda^* < S$, unless $s = S$ in which case $s = \lambda^* = S$; the same holds for the sums of column elements of A .

(If A is decomposable these results are weakened, cf. MCM p. 100)

Supplementary Property: Let A be a square matrix. It is $(I - A)^{-1} = \sum_{t=0}^{\infty} A^t = I + A + A^2 + A^3 + \dots$ iff $\lim_{t \rightarrow \infty} A^t = 0$, and it is $\lim_{t \rightarrow \infty} A^t = 0$ iff all eigenvalues of A are less than 1 in modulus. (This result holds for any square matrix.)

Implication: Let A be a non-negative matrix with $0 < \lambda^* = 1/(1+R) < 1$. Multiply A by $(1+r) < (1+R)$, then $(1+r)Ax = (1+r)\lambda^*x$, the dominant eigenvalue of matrix $[(1+r)A]$ is $(1+r)\lambda^* = (1+r)/(1+R) < 1$, hence $(I - (1+r)A)^{-1} = I + (1+r)A + (1+r)^2 A^2 + (1+r)^3 A^3 + \dots$

Let us reformulate price equations (1)-(2), generalized to any number of commodities, via matrices. The equations are:

$$p_1 = (1+r)(a_{11}p_1 + a_{21}p_2 + \dots + a_{n1}p_n) + a_{L1}w$$

$$p_2 = (1+r)(a_{12}p_1 + a_{22}p_2 + \dots + a_{n2}p_n) + a_{L2}w$$

.....

$$p_n = (1+r)(a_{1n}p_1 + a_{2n}p_2 + \dots + a_{nn}p_n) + a_{Ln}w.$$

There are n commodities, produced using labour and themselves (as *circulating* capital goods) in production cycles all lasting one period. Given production methods, one per industry (choice of technique is assumed to have already brought to cost-minimizing methods); no joint production; no land.

A semipositive matrix of technical coefficients of produced inputs, with a_{ij} input i per unit of output j ; a_{Lj} technical coefficient of labour in the production of commodity

³⁸ We do not specify whether it is a right or left eigenvector because the statement, although traditionally intended for right eigenvectors, in fact applies to either, since a left eigenvector is a right eigenvector of A^T , the transpose of A , again a square non-negative indecomposable matrix, and with the same eigenvalues as A .

³⁹ A matrix having a non-repeated eigenvalue of modulus strictly greater than the modulus of all other eigenvalues is called *primitive*. The last result in (i) states: every *positive* indecomposable matrix is primitive. But a matrix of technical coefficients is generally not positive, it contains zeros. An imprimitive non-negative indecomposable matrix has a particular form, but I skip the topic.

j. With \mathbf{p} the *row* vector of commodity prices (I omit the transpose symbol), the price equations are (capital inputs bought at the beginning of period, wages paid at the end):

$$(8) \quad \mathbf{p} = (1+r)\mathbf{p}\mathbf{A} + \mathbf{w}\mathbf{a}_L.$$

(Each *column* of \mathbf{A} a different industry; \mathbf{p} , \mathbf{a}_L are row vectors.) Check you remember what it means. Start from vector-matrix multiplication with *column* vectors, try $\mathbf{x}=(1+g)\mathbf{A}\mathbf{x}$. You obtain $x_i = (1+g)\mathbf{a}_i\mathbf{x}$, with \mathbf{a}_i the i -th row of \mathbf{A} ; in (8) $p_j = (1+r)\mathbf{p}\mathbf{a}^j + \mathbf{w}\mathbf{a}_{Lj}$, with \mathbf{a}^j j -th column of \mathbf{A} .

Choose any numéraire, for example $p_1=1$, or $\sum_i p_i=1$. When $w=0$, equation (8) becomes $\mathbf{p}=(1+R)\mathbf{p}\mathbf{A}$; define $\lambda=1/(1+R)$ and obtain $\mathbf{p}\mathbf{A}=\lambda\mathbf{p}$. This is a *left-eigenvector problem*: λ is an eigenvalue (real or complex) of square matrix \mathbf{A} , and \mathbf{p} is the associated left eigenvector. Let us assume that all commodities appearing in \mathbf{A} are basic^[40]; this means \mathbf{A} is *indecomposable* (or *irreducible*), then the Perron-Frobenius theorem states that \mathbf{A} has a non-repeated real eigenvalue λ^* which is dominant (that is, not smaller in modulus than any other eigenvalue) and to it and *only to it* are associated *real* non-negative, and in fact positive, right-eigenvector and left-eigenvector (for a compact proof see e.g. Heal, Hughes and Tarling, *Linear Algebra and Linear Economics*, 1974 p. 120). So an economically acceptable solution of $\lambda^*\mathbf{p}=\mathbf{p}\mathbf{A}$ exists *and is unique*, it determines the maximum rate of profits R as $\lambda^*=1/(1+R)$, and positive prices. A positive maximum rate of profits, $\lambda^*<1$, implies \mathbf{A} is capable of generating a positive net product (then the economy is *viable*).

This is evidenced by $\mathbf{x}=(1+g)\mathbf{A}\mathbf{x}$, where $g=R$ and \mathbf{x} is the vector of (gross) quantities produced: if $R=g>0$ then when wages are zero the economy is capable of *uniform positive growth*, hence of a positive net product.

Now note that, if wages are *advanced*, and physically specified, we can add them to the technical coefficients and obtain a matrix \mathbf{C} as the matrix of needed inputs inclusive of needed wage advances;⁴¹ that is, if the real wage allows purchasing on average z_1 units of good 1, z_2 units of good 2, etc., then $\mathbf{w}=\mathbf{p}\mathbf{z}$ (with \mathbf{z} a column vector), and $\mathbf{p}=(1+r)\mathbf{p}\mathbf{A}+(1+r)\mathbf{p}\mathbf{z}\mathbf{a}_L$; if now in industry j we replace each coefficient a_{ij} with $c_{ij}=a_{ij}+z_i a_{Lj}$ then

$$(9) \quad \mathbf{p}=(1+r)\mathbf{p}\mathbf{C} \text{ where } \mathbf{C}=\mathbf{A}+\mathbf{z}\mathbf{a}_L$$

(where $\mathbf{z}\mathbf{a}_L$ is a matrix) determines the same \mathbf{p} and r as the equation

⁴⁰ If there are nonbasic commodities, leave them temporarily out of the price equations and of the choice of numéraire, and determine prices and income distribution without them; then the costs of all the inputs to nonbasic commodities are determined, and one easily determines their prices.

⁴¹ This \mathbf{C} is not to be confused with the \mathbf{C} which in section 2 of PetriWhatCap indicated the net production of corn in the corn-labour economy.

$$\mathbf{p} = (1+r)\mathbf{p}\mathbf{A} + (1+r)\mathbf{w}\mathbf{a}_L \quad \text{where } \mathbf{w}=\mathbf{p}\mathbf{z};$$

now this r is the R of \mathbf{C} and is the normal rate of profits corresponding to the given *advanced* real wage; it is positive iff $\lambda_C^* < 1$ (viable economy), then the associated eigenvector is positive and yields us the relative product prices. It is easy to show a decreasing (sometimes called ‘inverse’) relationship between r and real wage. A rise of the real wage means an increase of some coefficient of \mathbf{z} and hence of \mathbf{C} ; the Perron-Frobenius theorem, *property (iii)*, states that λ^* is an increasing function of each element of \mathbf{C} , so R decreases.

With wages paid not in advance but in *arrears* (the opposite of ‘in advance’, i.e. at the end of the year) as in (8), R is the rate of profit associated with $\mathbf{w}=0$ in eq. (8), positive for $\lambda^* < 1$; for $r < R$ one has (MCM p. 114) that, having chosen a numéraire column vector \mathbf{v} , hence $\mathbf{p}\mathbf{v}=1$, eq. (8) implies $\mathbf{p}[\mathbf{I}-(1+r)\mathbf{p}\mathbf{A}] = \mathbf{w}\mathbf{a}_L$ that is $\mathbf{p} = \mathbf{w}\mathbf{a}_L[\mathbf{I}-(1+r)\mathbf{p}\mathbf{A}]^{-1}$ and post-multiplying both sides by \mathbf{v} the left-hand side becomes 1 and we obtain:

$$\mathbf{w} = 1 / [\mathbf{a}_L(\mathbf{I}-(1+r)\mathbf{A})^{-1}\mathbf{v}].$$

So once r and \mathbf{v} are given, \mathbf{w} is uniquely determined if $[\mathbf{I}-(1+r)\mathbf{A}]$ is invertible: this is guaranteed for $r < R$ by Perron-Frobenius property (ii): $(\rho\mathbf{I}-\mathbf{A})^{-1} > 0$ (where ρ is a real scalar) if and only if $\rho > \lambda^*$.

To see why, put $\rho = 1/(1+r)$; then $(1+r)[\rho\mathbf{I}-\mathbf{A}] = \mathbf{I}-(1+r)\mathbf{A}$. If $\rho\mathbf{I}-\mathbf{A}$ is invertible, so is $(1+r)[\rho\mathbf{I}-\mathbf{A}] = \mathbf{I}-(1+r)\mathbf{A}$. And it is invertible, because $r < R$ implies $\rho > \lambda^*$. Property (ii) also implies that $(\mathbf{I}-(1+r)\mathbf{A})^{-1} > 0$, so $\mathbf{w} > 0$, and since (8) can be re-written $\mathbf{p} = \mathbf{w}\mathbf{a}_L[\mathbf{I}-(1+r)\mathbf{A}]^{-1}$, prices are positive.

The proof that the real wage is a decreasing function of r for this case of \mathbf{w} paid post factum can be obtained in an interesting way. Back to (8), choose labour as numéraire, that is, $\mathbf{w}=1$; now prices measure *labour commanded*, the price of good i now means p_i/\mathbf{w} , it indicates how many units of labour one can purchase (or ‘command’, have at one’s command) with a unit of good i . Then equation (8) becomes $\mathbf{p}=(1+r)\mathbf{p}\mathbf{A}+\mathbf{a}_L$, and replacing \mathbf{p} on the right-hand side with $(1+r)\mathbf{p}\mathbf{A}+\mathbf{a}_L$, you obtain

$$\mathbf{p} = \mathbf{a}_L + (1+r)[(1+r)\mathbf{p}\mathbf{A} + \mathbf{a}_L]\mathbf{A} = \mathbf{a}_L[\mathbf{I} + (1+r)\mathbf{A}] + (1+r)^2\mathbf{p}\mathbf{A}^2;$$

repeat the replacement of \mathbf{p} on the right-hand side with $\mathbf{a}_L + (1+r)\mathbf{p}\mathbf{A}$, and reiterate this replacement again and again; you obtain a sum of infinite terms:

$$(10) \quad \mathbf{p} = \mathbf{a}_L [\mathbf{I} + (1+r)\mathbf{A} + (1+r)^2\mathbf{A}^2 + (1+r)^3\mathbf{A}^3 + \dots].$$

On the right-hand side, for each good there is a sum of *dated quantities of labour* each one increased by the profit required by the interval between their payment and the sale of the product. The Perron-Frobenius theorem aided by the

Supplementary Property shows that for $r < R$ the expression in square brackets converges to a finite positive matrix, $[I - (1+r)A]^{-1}$. Reassured on this, let us note that apart from the first, all elements inside the square brackets on the right-hand side of (10) increase as r increases, hence all elements whose sum determines \mathbf{p} are increasing functions of r , that is, all prices rise relative to the wage as r increases, which means that the purchasing power of the wage decreases in terms of *any* good as r increases, that is, the real wage unambiguously decreases in terms of any good or basket of goods. The reduction to dated costs can be performed for the price of any single or composite commodity;⁴² if we choose as such commodity the real wage basket, whose price is 1 since we choose to put $w=1$, then the equation corresponding to (10) (but restricted now to the single or composite good ‘real wage’) has 1 on the left-hand side, the only unknown is r , which shows in another way (Dmitriev) that a given real wage suffices to determine the rate of profit.

From a 2004 review by Herings of a 2002 book by Villanacci and others on General Equilibrium: The importance of a thorough understanding of the idealized model of general equilibrium with time and uncertainty that results from the application of these three principles [methodological individualism, maximization, consistency] cannot be overestimated. It does, however, typically lead to too limited attention for a number of important issues. The consistency principle focuses attention on equilibrium states. This ignores the pressing question of whether an economy out of equilibrium reaches any equilibrium state, if so, which equilibrium state is attained, and how exactly the equilibrating process operates. The maximization principle.... involves individuals with consistent intertemporal preferences that have common expectations on possible future states of nature, future preferences and availability of future technologies. Individuals are then assumed to be able to compute equilibrium prices, and, moreover, are able to coordinate jointly on one particular system of equilibrium prices. The authors do not contribute to the solution of these important shortcomings of modern general equilibrium theory. It would be unfair to blame them for this, as this simply reflects the state-of-the-art in the majority of general equilibrium research.

⁴² For a composite commodity i.e., a column vector \mathbf{v} of commodities, with price $p_v = \mathbf{p}\mathbf{v}$, the reduction becomes $p_v = \mathbf{p}\mathbf{v} = [(1+r)\mathbf{p}\mathbf{A} + \mathbf{a}_L]\mathbf{v} = \mathbf{a}_L[\mathbf{I} + (1+r)\mathbf{A} + (1+r)^2\mathbf{A}^2 + (1+r)^3\mathbf{A}^3 + \dots]\mathbf{v}$. (A single commodity is a basket \mathbf{v} with all elements zero except one equal to 1.) If basket \mathbf{v} is chosen as numéraire, on the left-hand side the equation has $\mathbf{p}\mathbf{v}=1$ and the sole unknown in the equation is r .