DIPARTIMENTO DI ECONOMIA POLITICA E STATISTICA — DEPS

unisi

QUADERNI DEPS 925/2025

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March 2025

UNIVERSITÀ DI SIENA 1240

Working Paper Series of the Department of Economics and Statistics of the University of Siena
Piazza San Francesco 7-8, 53100, Siena | Italy
deps.unisi.it

ISSN: 17209668

Structural change and public debt dynamics*

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March 7, 2025

Abstract

This paper presents a behavioural macro-dynamic model to study the relationship between informality, structural change, and public debt. Building on a structuralist framework, I innovate by using discrete choice theory to address the probability of workers being formal or informal. The formal sector combines manufacturing and business activities, while informality refers to the non-business low-productivity sector. It is shown analytically and through numerical simulations that when capital accumulation (g) is greater than interest rates (i), the unique equilibrium point is stable and formalisation implies higher debt. Reducing informality and public debt is possible only when $i < g$. However, in this case, the equilibrium becomes unstable as the economy becomes prone to debt spirals. Numerical experiments using BRICS data show Russia, India, and China belong to the first case, while Brazil and South Africa might correspond to the second. Introducing a production "chain effect" makes the model compatible with multiple equilibria. A closer look at India suggests it is in a low-debt, high-informality trap. Overcoming this requires careful consideration of government consumption composition between sectors and realising that a more formal economy requires accommodating higher public debt.

Keywords: Structural change; Dual economies; Informality; Public debt; Fiscal policy

JEL: O11, 017, E62, 041, E6

*A previous version of this paper was presented at the Structural Transformation and Contemporary Late Development Workshop, Manchester, United Kingdom; the Groningen Growth and Development PhD Workshop, Groningen, Netherlands; the 23rd Pontignano PhD-Economics Annual Meeting, Siena, Italy; the 12th MDEF Workshop, Urbino, Italy; the MACRO-HET Workshop, Siena, Italy; the 4th SDMRG Workshop, Brasilia, Brazil; and at the Centre for Development Studies Seminar Series, Kerala, India. I am extremely grateful to Marwil J. Davila-Fernandez for his constant support in bringing this idea to life. I am also thankful to the participants, particularly to P. Skott, S. Bowles, U. Pagano, G. Thakur, C. Bhattacharyya, R. Biswas, L. Abad, and G. de Vries for their comments and suggestions for improvement. Usual caveats apply.

1 Introduction

Our modern world is the product of the Industrial Revolution, which marked a transition from agriculture to manufacturing activities. Most advanced economies have long moved into a new, post-industrial phase where workers are increasingly allocated to modern *business* services. Developing countries, in contrast, have experienced an analogous process of structural change with two critically distinct features. First, the industrialization phase has been short-lived or has not happened. Second, there is intrinsic heterogeneity in the composition of the tertiary sector, as a significant portion of the workforce ends up in low-productivity *non-business* services. While such a process diverges from the classical narratives in Lewis (1954) and Kuznets (1955), contemporary studies have provided a comprehensive characterization of such a pattern of structural change (Rodrik, 2016; Alisjahbana et al., 2022; McMillan and Zeufack, 2022). In this context, it has been argued that fiscal policy might be a critical determinant or facilitator of growth and development (Vera, 2009; Dao, 2012; Skott and Gómez-Ramırez, 2018).

My research question lies at the intersection between these two major themes. We know little about how the composition of government expenditures and the correspondent tax structure affect informality. We also have limited knowledge about the implications of structural change on public debt dynamics. The present paper develops a macro-dynamic model to study the interaction between these two dimensions. Building on the structural framework in Skott (2021), I innovate by endogenizing agents' probability of being (in)formal using discrete-choice theory (Lux, 1995; Brock and Hommes, 1997; for a review, see Franke and Westerhoff, 2017).[1] Skott's model provides a natural environment for our exercise because it is mainly concerned with functional finance in economies where modern and traditional sectors coexist. Still, and similar to other studies in the field (e.g. Sen, 2023; Kruse et al., 2023; Aggarwal, 2018), the behavioural aspects behind the probability of whether to enter or leave informality are relegated as secondary. The main novelty of this paper is to provide a tractable framework to address such a problem.

I will argue that public debt and informality in economies undergoing structural transformation are determined simultaneously. My model proposes a narrative that combines the role of public debt and structural change, addressing emerging macro-behaviours as the sum of agents' micro-interactions. The informal sector corresponds mainly to non-business services and minor labor-intensive manufacturing enterprises, using labor as a single production factor. Its output is thus distributed within that sector and does not pay taxes. Formal activities combine most manufacturing and business services, employing labour and capital, while paying taxes from wages and profits. Government spending is allocated to both sectors. The higher the share going to the formal, the higher the probability of workers belonging to this sector. Taxes, on the other hand, work in the opposite direction. This interaction is expected to smooth structural change and feedback on fiscal policy.

Numerical experiments with Brazil-Russia-India-China and South-Africa (BRICS) data allow me to provide a more concrete visualization of the system's properties. I document the existence of a trade-off between informality, public debt, and (in)stability. When capital accumulation

[1] This approach is part of a literature where a large population of agents face, most of the time, a binary decision. They may choose between optimism and pessimism, extrapolation or static expectations, etc. Recent applications include credit and stock market interactions (Kubin et al., 2019; Flaschel et al., 2018), environmental attitudes (Dávila-Fernández et al., 2024; Sordi and Dávila-Fernández, 2023; Cafferata et al., 2021), electoral cycles (Di Guilmi et al., 2023), and monetary policy (De Grauwe, 2011; De Grauwe and Foresti, 2023).

(g) exceeds the interest rate on public bonds (i), the equilibrium is stable, but formalization implies higher debt. A simultaneous reduction of informality and public debt is possible only when $i > g$, though the equilibrium becomes unstable. Introducing a production "chain effect" makes the model compatible with multiple equilibria and is particularly useful for addressing the Indian case. Such a component is critical in this country because its caste-based networks often determine access to resources, employment opportunities, and trust within communities, influencing individuals' choices regarding informal or formal sector participation. These dynamics can reinforce existing disparities, as certain caste groups may have stronger ties to informal networks or face systemic barriers to entering the formal sector. Most studies in the field have treated informality and public debt separately, both in theoretical and empirical settings. I will consider recent advancements in structural change literature to study the interconnections between the two. Understanding these social and structural influences is crucial to designing policies that effectively promote formalization and inclusive economic growth.

The remainder of the paper is structured as follows. In the next section, I present a brief literature review of the major themes leading to my research question. I also provide empirical insights regarding stylised facts of the informal employment and public debt relationship. Section 3 introduces the modelling framework, followed by the study of the resulting dynamic system. A series of numerical experiments using BRICS data appears in Section 4. The model is extended to consider production "chain effects" referring to the Indian context, discussing "big-push" options a la Rosestein-Rodan. Some final considerations follow.

2 Literature review and empirical insights

2.1 A brief literature review

Lewis (1954) provides a compelling view of growth resulting from reallocating labour and resources across sectors. The model emphasises structural transformation processes as key to economic growth, a perspective that has gained renewed attention even today. Economics dualism narratives and assessments have evolved and shaped over time by incorporating various perspectives, such as labour market integration (Rauch, 1991), productivity growth (Diao and McMillan, 2018), to environmental quality (Oliveira and Lima, 2020). The co-existence of formal and informal firms or sectors portrays a vast part of the literature with the recent insights of exclusion and exit views in determining the choice of transition (La Porta and Shleifer, 2014), tradable and non-tradable sectors (Razmi et al., 2012), extensive and intensive margins of informality (Ulyssea, 2018) to name a few. Informality remains a complex and persistent challenge in developing economies.

In general, economies with higher informality also face the stage of structural transformation but with varying phases and heterogeneity. It is also true that, to a great extent, no single structural transformation path works for all developing countries (Bah, 2011). Recent analyses, including those conducted by Erumban et al. (2019), have underscored the potential for positive structural change to be achieved through the formalisation of the economy. At the same time, studies by Leon-Ledesma and Moro (2020) stress the service sector expansion and reveal that the shift from goods to services leads to significant macroeconomic changes, including higher rates of real investment and lower rates of real interest.

But this is not somehow aligned with the interpretation of Di Meglio et al. (2018), as they provide evidence in favour of the Kaldorian argument that manufacturing and business services make substantial contributions to overall productivity growth. Nevertheless, these changes or transition processes require modifications in fiscal policy to promote or sustain long-term economic growth. The theoretical framework proposed by Thakur (2023) argued for service-led growth in dual economies given the dimension of structural transformation. The recent empirical evidence by Fan et al. (2023) unequal effects of service-led growth in India identifies productivity growth in non-tradable consumer services as a significant driver of structural transformation.

Given the dimensions and pathways of structural transformation, it is important to understand the role and effectiveness of fiscal policy in smoothing structural change. In a dual economy setting, addressing credit constraints alone is insufficient for promoting formal sector growth and requires a broader set of policies (Skott & Gómez-Ramırez, 2018). Additionally, it is important to acknowledge that these low-income and developing economies frequently encounter greater fiscal constraints, ultimately resulting in elevated public debt levels. Following this, the macro model developed in Ribeiro and Lima (2019) indicates that a fiscal rule limiting government spending but not including interest payments may not ensure a non-explosive trajectory of the public debt-to-output ratio. A recent analysis by Chatterjee and Turnovsky (2023) discovered that simultaneous increases in government consumption and investment result in a gradual decrease in employment and output within the informal sector over time. So, it is quite clear that the link between structural change and fiscal policy is intricate over time, with fiscal policy having a pivotal role in fostering and adjusting to these transformations.

One of the closest studies on public debt and dualism in developing economies is the two-sector and three-sector modelling framework by Skott (2021), which refers to functional finance tools to stabilize demand at levels that align with the growth objectives of the modern sector. He envisages a similar line of interpretation to the notion of public finance. This model links public debt dynamics smoothly and the informal-formal transition process. Still, its main limitation is that it lacks the behavioural elements behind the probability of whether to enter or leave informality. In the present paper, I propose to integrate discrete choice theory (Brock & Hommes, 1997) to tackle the issue, resulting in a novel and tractable behavioural heterogeneous agents model in which informality and public debt are simultaneously determined.

2.2 Some empirical insights

An informative way to display the sectoral composition of the labour force is to take the share of those in the formal sector minus those in informality. This procedure results in an index $x \in [-1, 1]$, where $x = 1$ indicates full formality while $x = -1$ represents complete informality. Fig. 1 reports x on the horizontal axis using self-employment as a proxy of the informal sector. Data on informality comes from (Elgin et al., 2021). The vertical axis shows public debt normalised by the capital stock (b) obtained from the IMF statistics and the PWT 10. Different colours mark per capita Gross Domestic Product (GDP) levels. Low-income countries have relatively higher self-employment or informal employment and moderate or high debt levels. Regarding public debt, there is also higher dispersion across low-income countries, which decreases as income rises and formality becomes more prevalent.

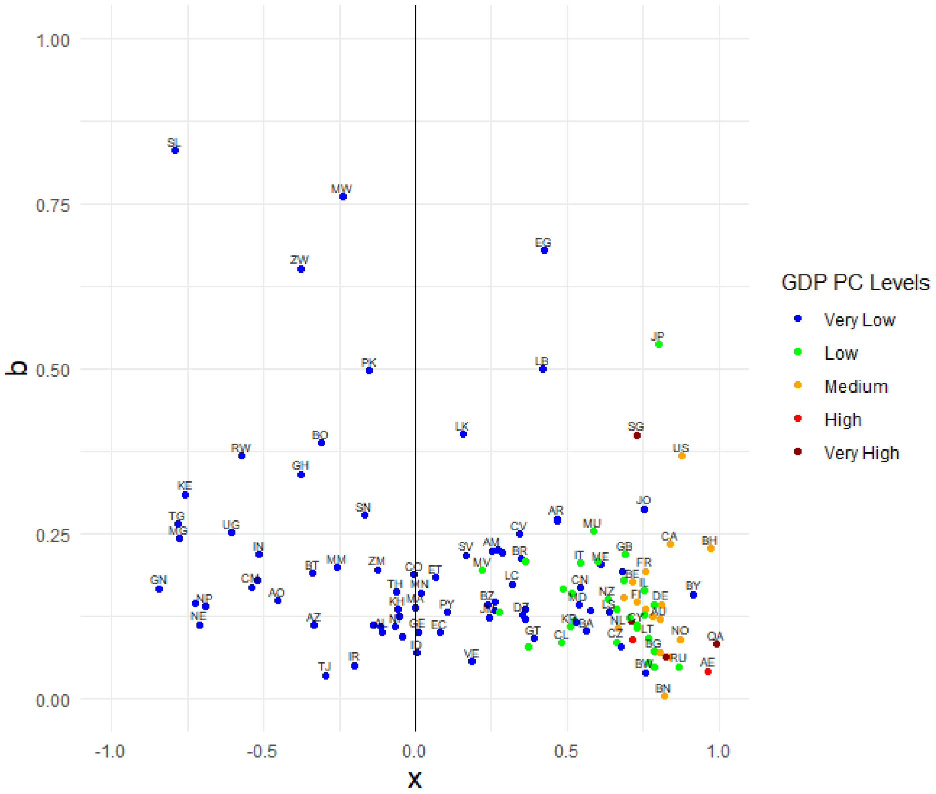
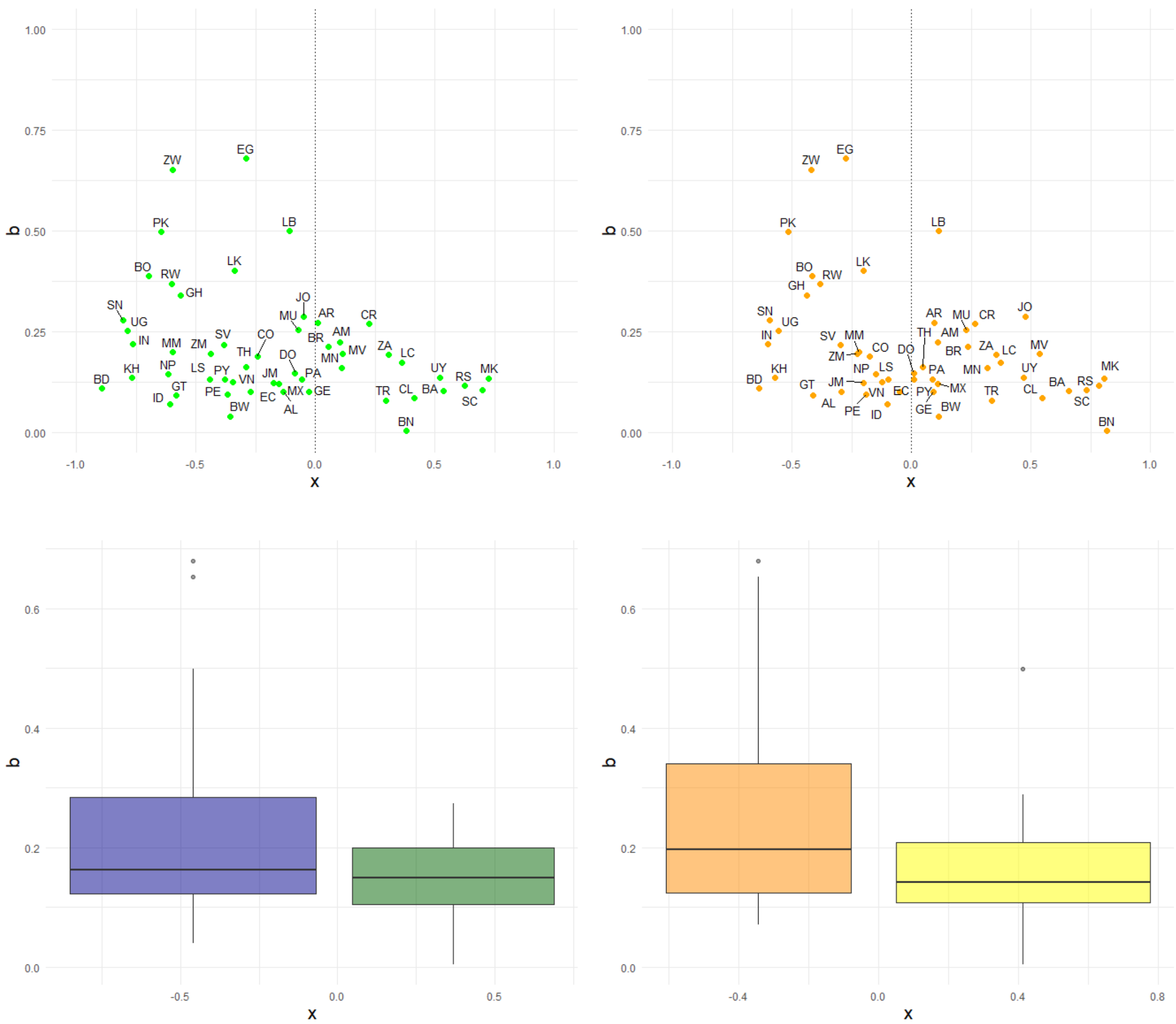


Figure 1: Self-employment to debt index

Focusing on developing countries, Fig. 2 shows the scatterplot between informal employment or informal economy size against public debt. They portray a similar picture in which countries with higher informality have greater debt dispersion. As economies formalize, public debt appears to have a skewed distribution with lower levels. Zooming in at the regional level, Fig. 3 disaggregates the correspondence between informal employment and public debt in Africa, Asia, and Latin America. Higher informal employment is related to lower debt only in Africa. Still, there is a variation in its levels across countries in the region. Such a clear-cut negative curve is not observed in Asia or Latin America. The relationship appears flat in the latter, indicating relatively uniform public debt across informality degrees. Such trends are robust to other informality indicators, as shown in Fig. 4.

These observations emphasise the importance of the connection between the evolving structure of the labour force and debt levels, particularly in the context of informal employment. More than 60 per cent of the global workforce is involved in informal employment (ILO, 2018), with the majority found in emerging economies (67%) and developing economies (90%). The informal economy encompasses 80% of all firms worldwide (UNDP, 2022). Determining the pattern of structural transformation that facilitates a desirable transition in economies characterised by dualism is crucial. Dual economies, in particular, need to be analysed independently, mainly because of the fundamental differences in their employment and growth structures. They are known for a huge share of informal activities with small size and low productivity (La Porta & Shleifer, 2014). The informal economy shrinks as the economy grows (La Porta & Shleifer, 2008), and micro evidence shows that both extensive and intensive margins of informal firms decline as the firms grow (Ulyssea, 2020).

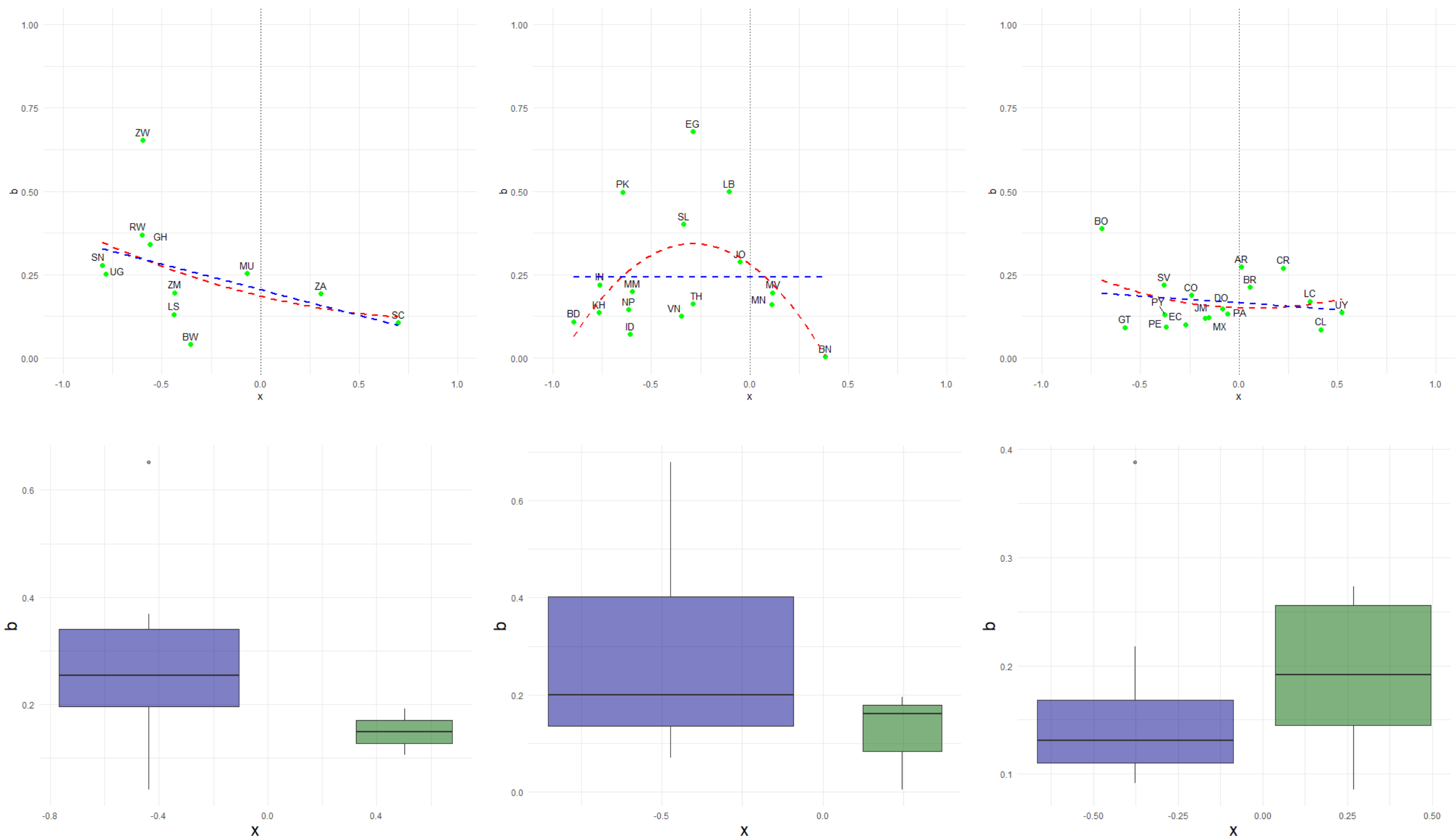
Figure 2: Informality and public debt in developing economies



(a) Informal employment-debt index

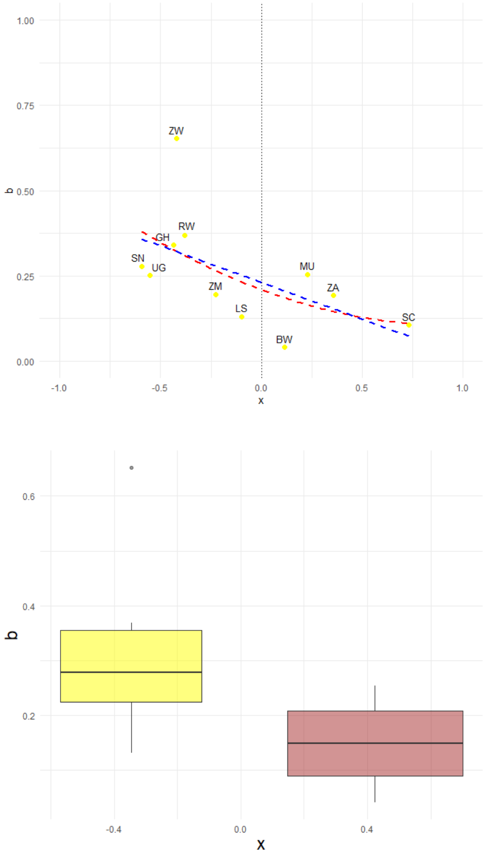
(b) Informal size-debt index

Figure 3: Informal employment to debt relationship by region

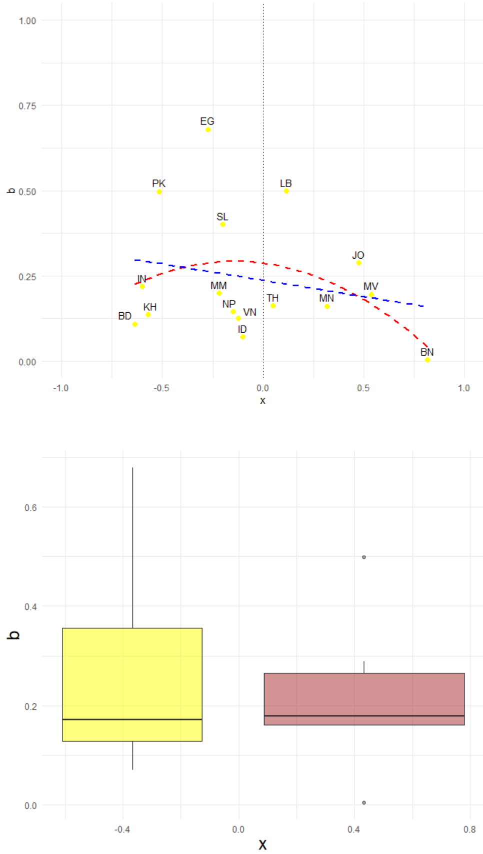


(a) Africa (b) Asia (c) Latin America

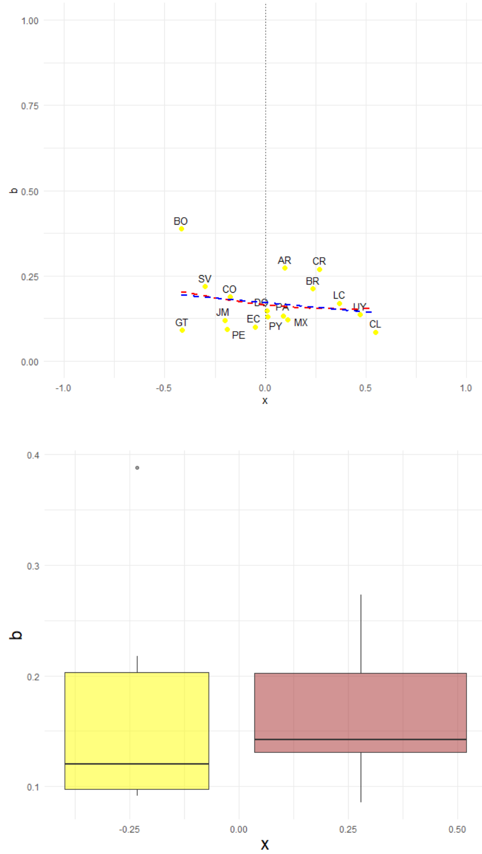
Figure 4: Informal size to debt relationship by region



(a) Africa



(b) Asia



(c) Latin America

Thus, moving a highly populated and unproductive population out of informality calls for a new challenge in developing economies. Traditional agriculture economies need to shift to formal manufacturing and business services activities rather than to informal labor-intensive manufacturing and non-business services, as the fiscal burden and economic outcomes may differ as the transition choice changes. Thus, in the next section, we present a behavioral macrodynamic model that allows studying the interaction between structural change and fiscal policy. My critical innovation will be to motivate the probability of being formal or informal by relying on discrete-choice theory.

3 Model

Consider a dual economy, primarily in agriculture, that is going through structural change. On the one hand, it can transit towards an informal non-business sector that uses only labor and has constant productivity. We could also include minor labor-intensive manufacturing enterprises here. On the other hand, it can move to a formal sector mainly comprising proper manufacturing industries and business services. Now, let us introduce fiscal policy, as the government can play a crucial role in fostering the structural change process. My main narrative will distinguish between two main forces that play a role in opposite directions. First, only the formal sector pays taxes, while taxation, to some extent, encourages informality as the average agent would prefer not to pay them. Second, fiscal authorities can allocate public spending to formal and informal sectors. Building on Skott (2021), this paper aims to investigate the role of public debt in dual economies when informality and public debt are simultaneously determined. This approach addresses emerging macro-behaviours as the sum of micro-interactions in informal economies. Agents will face a probability of being formal or informal based on discrete-choice theory (Brock & Hommes, 1997). The model is divided into three main blocks of equations. Fig 5 summarises its main transmission channels.

3.1 Being formal or informal

Suppose the labour force is equal to the population (N), being constant and divided between those either informally (L_I) or formally (L_F) employed:

$$N = L_I + L_F \tag{1}$$

We introduce a formality index (x) to represent the sectoral composition of the population, defined as:

$$x = \frac{L_F}{N} - \frac{L_I}{N} \tag{2}$$

where $x \in [-1, 1]$. The binary decision mechanism used here follows binary choice models like Brock and Hommes (1997); Lux (1995) and Franke and Westerhoff (2017). Here, when $x = -1$, all workers are informal; on the other hand, $x = 1$ indicates a situation in which a perfect or complete transition to formality. Thus, $x = 0$ accounts for the equality between the share of the population employed in formal and informal activities.

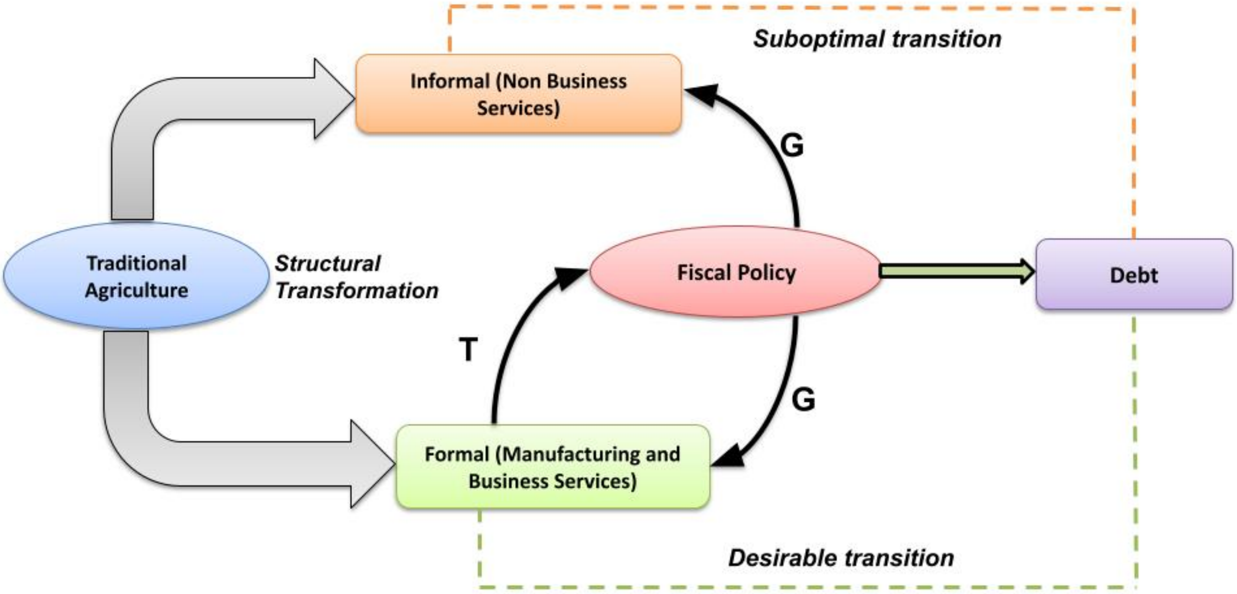


Figure 5: Structural change and public debt dynamics.

Taking the time derivatives of Eq. (2), we get:[2]

$$\dot{x} = \frac{\dot{L_F} - \dot{L_I}}{N} \tag{3}$$

Variations in the sectoral composition of the labour force are given by the difference between the probability of belonging to either sector. Thus, we have:

$$\begin{aligned} \dot{L_F} &= L_I p^F - L_F p^I \\ \dot{L_I} &= L_F p^I - L_I p^F \end{aligned} \tag{4}$$

where p^F is the probability of transiting to formality, whereas p^I is the probability of ending in the informal sector.

Following the discrete choice literature (Brock and Hommes, 1997; Franke and Westerhoff, 2017), assume:

$$p^F = \frac{\exp\left(\beta V^F\right)}{\exp\left(\beta V^F\right) + \exp\left(\beta V^I\right)} \tag{5}$$

$$p^I = \frac{\exp\left(\beta V^I\right)}{\exp\left(\beta V^I\right) + \exp\left(\beta V^F\right)} \tag{6}$$

where $\beta > 0$ is referred to as the intensity of choice parameter. When $\beta = 0$ or close to zero, both p^F and p^I become nearly identical. Alternatively, when β is large or goes to infinity, the

[2] For any generic variable y, we indicate as $\dot{y}$ its time derivative (dy/dt), while $\dot{y}/y$ corresponds to the respective growth rate. Super or subscripts F and I are used to indicate formal and informal sectors.

two probabilities tend to be 1 and 0, respectively. In our context, it shows how responsive or elastic the transition attains formalisation with its employment trajectory.

Assuming the value attached to the probability of becoming formal (V^F) is the same as not becoming informal ($-V^I$), I write:

$$V^F = -V^I$$

which is just saying that all variables that affect the probability of going to the formal sector also influence the chance of becoming informal, but with the opposite sign. I distinguish between two forces. First, and quite intuitively, higher taxes likely reduce formalisation incentives. Second, by spending either on formal or informal activities, the government can also influence structural change. Thus:

$$V^F = \underbrace{\alpha}_{\text{Gov. Exp. Comp.}} - \underbrace{\frac{T}{K}}_{\text{Tax effect}} \tag{7}$$

where α is the share of government expenditures in the formal sector. Therefore, a higher α increases the probability of being formal because the government directly creates opportunities through its expenditures. Finally, we normalise the tax effect by the capital stock (T/K) to get as close as possible to Skott (2021), from whom we adopt the public debt dynamics. Eq. (7) provides a behavioural rationale for the transition dynamics.

Dividing the two expressions in (4) by N, the change in the share of the population which is formally and informally employed is expressed as:

$$\begin{aligned} \frac{\dot{L}_F}{N} &= p^F - \frac{L_F}{N} \\ \frac{\dot{L}_I}{N} &= p^I - \frac{L_I}{N} \end{aligned} \tag{8}$$

Subtracting the second from the first in Eq. (8), and using the definition of x, we have:

$$\dot{x} = p^F - p^I - x \tag{9}$$

Further substituting Eqs. (5) and (6) into (9), recalling that $V_F = -V_I$, gives:

$$\dot{x} = \tanh\left(\beta V^F\right) - x \tag{10}$$

Finally substituting Eq. (7) into (10), we obtain a dynamic relation capturing the link between structural change and fiscal policy:

$$\dot{x} = \tanh\left(\beta\left(\alpha - \frac{T}{K}\right)\right) - x \tag{11}$$

so that that the sectoral composition of workers in the economy responds to taxation and how the government spends.

3.2 Production Technology

3.2.1 Informal Sector

The non-business services sector, i.e. our informal sector, only uses labour (L_I) and has a constant technology co-efficient ($\bar{\lambda}_I$). Therefore:

$$Y_I = \bar{\lambda}_I L_I \tag{12}$$

Output produced (Y_I) is distributed within the industry, where workers receive wages (w_I) based on their average productivity. Thus, the informal sector's distribution is as follows:

$$Y_I = w_I L_I \tag{13}$$

Eqs. (12) and (13) imply that wages are equal to labour productivity and constant, given by

$$w_I = \bar{\lambda}_I$$

3.2.2 Formal sector

The production function in the formal sector, which includes manufacturing and business services, is assumed to be Leontief. So output (Y_F) is given by:

$$Y_F = \min\{\sigma K; \lambda_F L_F\} \tag{14}$$

where σ is the output-capital ratio, λ_F shows the labour productivity in the formal sector. This function utilizes capital (K) and labour (L). Their effective use varies, but these variations occur around average values that can be considered close to the targeted levels (Skott, 2021). Therefore, efficiency conditions imply:

$$Y_F = \sigma K = \lambda_F L_F \tag{15}$$

In the short run, the capital stock is fixed and predetermined, while employment depends on it and the level of labour productivity. The capital coefficient remains constant, but labor productivity gradually rises as time passes.

Taking log derivatives of (15), capital accumulation determines the output growth in the formal sector:

$$\frac{\dot{Y}_F}{Y_F} = \frac{\dot{K}}{K}$$

and abstracting from depreciation for simplicity, $\dot{K}/K = I/K$. From (15), it also follows that formal employment is such that:

$$L_F = \frac{\sigma K}{\lambda_F} \tag{16}$$

This implies that those workers incapable of securing employment in the formal part of the economy transit to the informal sector. Recalling that $N = L_I + L_F$, then from Eq. (16), we have:

$$\frac{L_I}{N} = 1 - \frac{L_F}{N}$$
$$= 1 - \frac{\sigma K}{\lambda N}$$

The formal sector fuctional income distribution is determined by the real wage paid on labour (w_F) and the rate of return on capital (r), implying:

$$Y_F = w_F L_F + rK \tag{17}$$

The profit share (π) is defined by $\pi = rK/Y$. Therefore, we can rearrange Eq. (17) so that real wages are determined by workers' productivity and profit share in the formal sector:

$$w_F = \lambda_F (1 - \pi)$$

Labour productivity growth ($\dot{\lambda}/\lambda$) is contingent upon the rate of accumulation, which is equivalent to the rate of increase of production in the formal sector. That is, using a Kaldor-Verdoorn type specification, we can say that labour productivity growth depends on the rate of capital accumulation and is defined as:

$$\frac{\dot{\lambda}}{\lambda} = \rho_0 + \rho_1 \frac{\dot{K}}{K}$$

3.3 Aggregate demand

Aggregate consumption is determined by expenditures out of wage income (C_w) and capital income (C_π), including a marginal wealth effect (μ). The informal sector consumes completely and only from wage income and does not pay any tax. On the other hand, the formal sector consumes after paying taxes on wages (τ_w) and capital (τ_π). Thus:

$$C = C_w + C_\pi$$

such that

$$C_w = w_I L_I + (1 - \tau_w) w_F L_F$$
$$C_\pi = c_\pi (1 - \tau_\pi)(\pi Y_F + iB) + \mu (K + B)$$

where c_π is the marginal propensity to consume out of capital income, and i is the interest rate on public bonds (B).

To keep our narrative as simple as possible, capital accumulation (g) is supposed to be exogenously given and constant:

$$\frac{I}{K} = g$$

so that, at least for the moment, we avoid discussions on the determinants of invesment decisions.

Overall, government expenditure depends on two main components. The first is independent of the structural composition of the economy ($\bar{\gamma}$). The second shows the percentage of government consumption from the formal sector (α). Thus, $1-\alpha$ is the share of government consumption from the informal part of the economy. It is to be noted as a critical assumption that $\alpha > 1/2$ as larger government contracts come with some degree of law enforcement:

$$\frac{G}{K} = \bar{\gamma} + \gamma \left[\alpha \frac{L_F}{N} + (1-\alpha) \frac{L_I}{N} \right] \tag{18}$$

where γ is a scaling parameter. It is not difficult to show that $L_F/N = (1+x)/2$ and $L_I/N = (1-x)/2$, a demonstration that is left in the Appendix A.1. Substituting this last result into Eq. (18) gives us:

$$\frac{G}{K} = \bar{\gamma} + \gamma \left[x \left(\alpha - \frac{1}{2} \right) + \frac{1}{2} \right] \tag{19}$$

The government receives all of its revenue from the formal sector through income tax share (τ_w) and corporate tax share (τ_π). The latter comes partly from capital and interest income in the formal sector. Therefore, taxes are expressed as:

$$\begin{aligned} \frac{T}{K} &= \frac{\tau_w w_F L_F + \tau_\pi (\pi Y_F + iB)}{K} \\ &= \tau_w (1-\pi) \sigma + \tau_\pi (\sigma \pi + ib) \end{aligned} \tag{20}$$

where $b = B/K$ is a normalisation that simplifies the model's presentation and is frequently used in the structuralist literature.

Adhering to the notion of functional finance (Lerner, 1943; Skott, 2016) public debt evolves:

$$\dot{B} = G + iB - T \tag{21}$$

which indicates that government expenditures, including interest payments, must be equal to revenues, either from taxes or new debt. Normalizing Eq. (21) by the capital stock, then substituting Eqs. (19) and (20) into the resulting expression, gives us:

$$\dot{b} = \bar{\gamma} + \gamma \left[x \left(\alpha - \frac{1}{2} \right) + \frac{1}{2} \right] + ib - [\tau_w (1-\pi) \sigma + \tau_\pi (\sigma \pi + ib)] - gb \tag{22}$$

This second dynamic equation shows a direct and positive relationship between the share of government expenditure in the formal sector and the public debt dynamics. Similarly, higher capital accumulation is also a key factor leading to a stable debt trajectory.

3.4 Dynamic System

Substituting Eq. (20) into (11) gives us the transition dynamics between formal and informal sectors. Finally, Eq. (22) corresponds to the dynamics of public debt. These two variables, x and b, are simultaneously determined. Our 2-dimensional non-linear dynamic system is given by:

$$\dot{b} = \bar{\gamma} + \gamma \left[x \left(\alpha - \frac{1}{2} \right) + \frac{1}{2} \right] + ib - \left[\tau_w (1-\pi) \sigma + \tau_\pi (\sigma\pi + ib) \right] - gb = f_1(b, x)$$
$$\dot{x} = \tanh \left(\beta(\alpha - \tau_w (1-\pi) \sigma - \tau_\pi (\sigma\pi + ib) \right) - x = f_2(b, x) \tag{23}$$

Notice that formalization implies a fiscal pressure provided that $\alpha > 1/2$. Interest rates net of taxes also lead to an increase in public debt, an effect that is counterbalanced by capital accumulation. The feedback from fiscal policy to structural change occurs through the probability functions from the discrete choice skeleton.

In steady state, $\dot{b} = \dot{x} = 0$. Therefore, the respective equilibrium conditions are:

$$\bar{\gamma} + \gamma \left[x \left(\alpha - \frac{1}{2} \right) + \frac{1}{2} \right] + ib - \left[\tau_w (1-\pi) \sigma + \tau_\pi (\sigma\pi + ib) \right] - gb = 0$$
$$\tanh \left(\beta(\alpha - \tau_w (1-\pi) \sigma - \tau_\pi (\sigma\pi + ib) \right) - x = 0 \tag{24}$$

We can state and prove the following Proposition regarding the existence and stability of a unique equilibrium point:

Proposition 1 *Provided that $g > i$, the dynamic system (23) admits a unique locally stable solution (x^*, b^*) that satisfies:*

$$b^* = \theta_0 + \theta_1 x^*$$

$$x^* = \tanh(\omega_0 + \omega_1 b^*)$$

where

$$\theta_0 = \frac{-\frac{\gamma}{2} + \tau_w (1-\pi) \sigma + \tau_\pi \sigma\pi - \bar{\gamma}}{i(1-\tau_\pi) - g} \lesseqgtr 0$$

$$\theta_1 = \frac{\gamma \left(\frac{1}{2} - \alpha \right)}{i(1-\tau_\pi) - g} > 0$$

$$\omega_0 = \beta[\alpha - \tau_w (1-\pi) \sigma - \tau_\pi \sigma\pi] \lesseqgtr 0$$

$$\omega_1 = -\beta\tau_\pi i < 0$$

Proof. See Mathematical Appendix. ■

4 Numerical experiments

To provide a more concrete view of the dynamic properties of the model, I calibrate it with BRICS data. This is an interesting group of developing countries, as they have experienced significant economic changes over the years, marked by varying debt levels and structural transformation paths. I present a numerical simulation exercise to illustrate their employment transition and public debt dynamics. The remaining implicit parameters of the model are adjusted to provide economically meaningful outcomes. It is to be emphasised that since we are not calibrating a "real economy", the interpretation of this exercise continues to be mainly qualitative. My calibration strategy follows the sources reported in Table 1. The parameter defining the share of government consumption in the formal sector (α) is taken at 0.9 uniformly for all the countries. Similarly, the intensity choice (β) is set at 0.3 in our baseline scenario. Meanwhile, $\bar{\gamma}$, which defines government consumption out of capital independent of structural composition, is adjusted to the model for more economically intuitive results. Table 2 reports the adopted values of each country.

The values of g and i are critical for the stability of the equilibrium point. Our exercise reveals a curious distinction between Brazil and South Africa with respect to the other BRICS countries. Figs. 6 and Fig. 7 contrast both cases. The black dots in the diagrams correspond to annual data from 1990 to 2018. Our simulations suggest a trade-off between reducing informality, public debt, and instability. In India, China, and Russia, $g > i$ implies that a larger formal sector is necessarily accompanied by higher public debt. The rationale for this result is the following. Because the government mainly spends in the formal sector ($\alpha > 1/2$), reducing informality creates fiscal pressure. As the economy is growing faster than interest payments, higher debt is required to sustain a larger x. Still, public debt does not explode as the economy is growing strong. In contrast, when $i > g$, reducing informality reinforces the initial fiscal pressure already caused by relatively high interest rates. In the absence of economic growth, the only way to equilibrate the system is by forcing a reduction in public debt. Still, low growth implies such equilibrium is unstable, as any small deviation results in a debt spiral.

4.1 Introducing a "chain effect"

The model developed in the preceding section defines the relationship between an agent's probability of sectoral transition and the resulting impact on debt dynamics within the economic system. Various factors could potentially influence this process, apart from the role of fiscal policy, which was discussed before. One among them is that individuals do not develop their views and decisions about the formalization process merely in a close environment. In reality, agents are moulded by the opinions of individuals in their immediate vicinity. Decisions to enter the formal sector are influenced by the prevailing norms within a community. If formal employment is associated with stability and social benefits, individuals are more likely to pursue it. At the same time, positive experiences with formalization, such as access to healthcare and social security, may encourage others to formalize (Fields, 2004; ILO, 2022). Conversely, where informal work is more common and socially accepted, people may resist formalization due to fears of losing flexibility or income (Perry, 2007; Maloney, 2004).

If social networks are largely informal, individuals are likely to remain informal. Caste plays a significant role in shaping economic networks in India, influencing access to resources, opportunities, and social mobility. Caste-based networks act as critical sources of support and trust, facilitating employment, credit, and entrepreneurship within communities (Munshi, 2011; Munshi, 2015). However, these networks also perpetuate exclusivity, marginalizing lower-caste groups and restricting their entry into formal markets. Access to finance and business opportunities is often limited for marginalized castes, while dominant castes benefit from systemic advantages and well-established networks (Borooah, 2005). Labour market segmentation and migration patterns further underscore caste-based disparities, with marginalized groups confined to informal, precarious roles (Thorat and Neuman, 2012). These entrenched networks reinforce economic inequality, making caste a critical factor in economic behaviour and decision-making in India (Munshi, 2019).

Parameter	Definition	Source
α	Share of government consumption in the formal sector	....
γ	Real Government consumption out of capital given formal and informal share	Penn World Table, 2019
$\bar{\gamma}$	Government consumption out of capital autonomous of structural composition	....
τ_w	Tax on wage income	Worldwide Tax Summaries - PwC
τ_π	Tax on Capital Income	Worldwide Tax Summaries - PwC
π	Profit share	Penn World Table, 2019
σ	Output-Capital ratio	Penn World Table, 2019
g	Capital accumulation rate	Penn World Table, 2019
i	Real rate of interest on bond	IMF Debt database, 2022
β	Intensity of choice	....

Table 1: Calibration strategy

Parameter	Brazil	Russia	India	China	South Africa
α	0.90	0.90	0.90	0.90	0.90
γ	0.075	0.077	0.040	0.080	0.073
$\bar{\gamma}$	0.006	0.05	0.06	0.06	0.015
τ_w	0.30	0.26	0.175	0.25	0.315
τ_π	0.15	0.20	0.275	0.25	0.28
π	0.43	0.47	0.48	0.42	0.43
σ	0.265	0.23	0.0282	0.261	0.28
g	0.018	0.028	0.061	0.049	0.014
i	0.042	0.012	0.03	0.014	0.039
β	0.3	0.3	0.3	0.3	0.3

Table 2: Parameter values BRICS

Figure 6: Public Debt and Structural Change when $g > i$

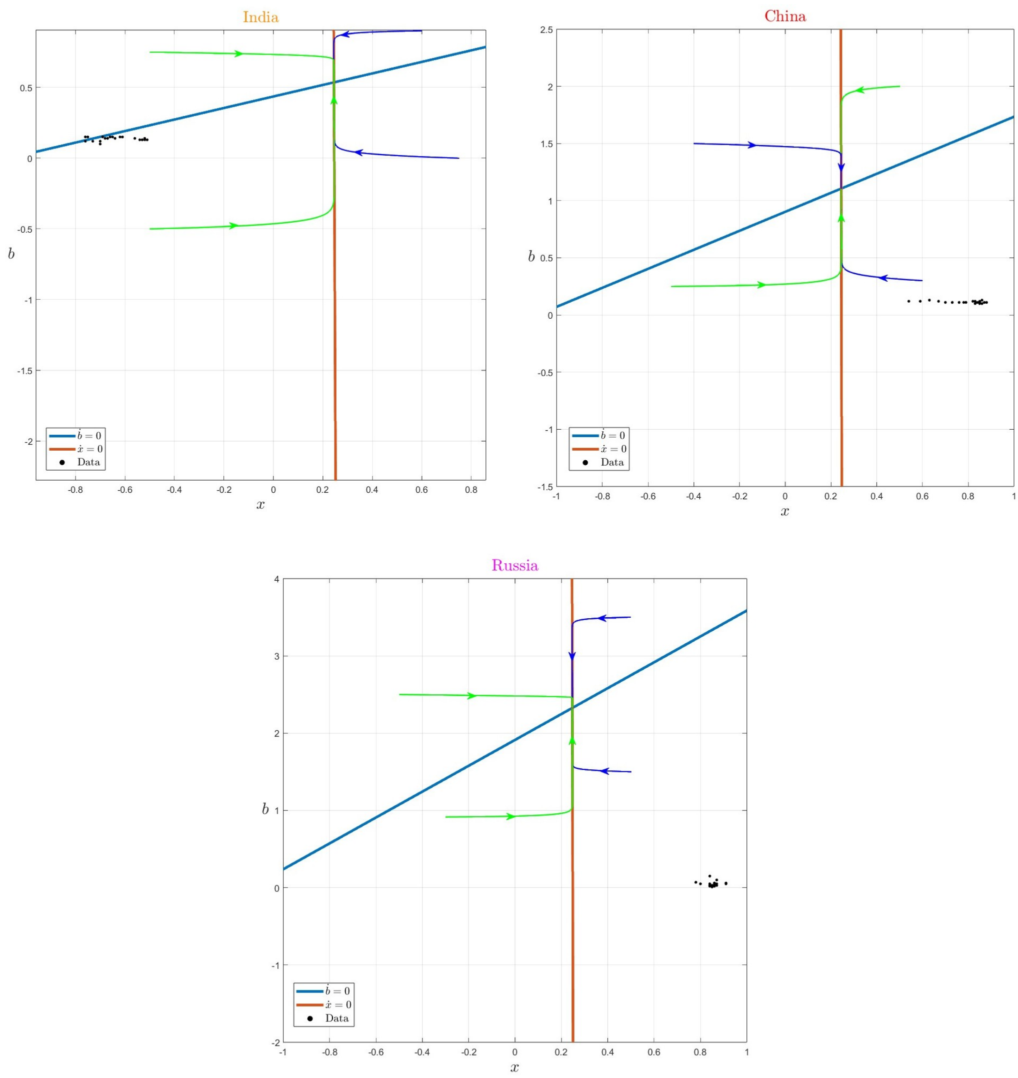
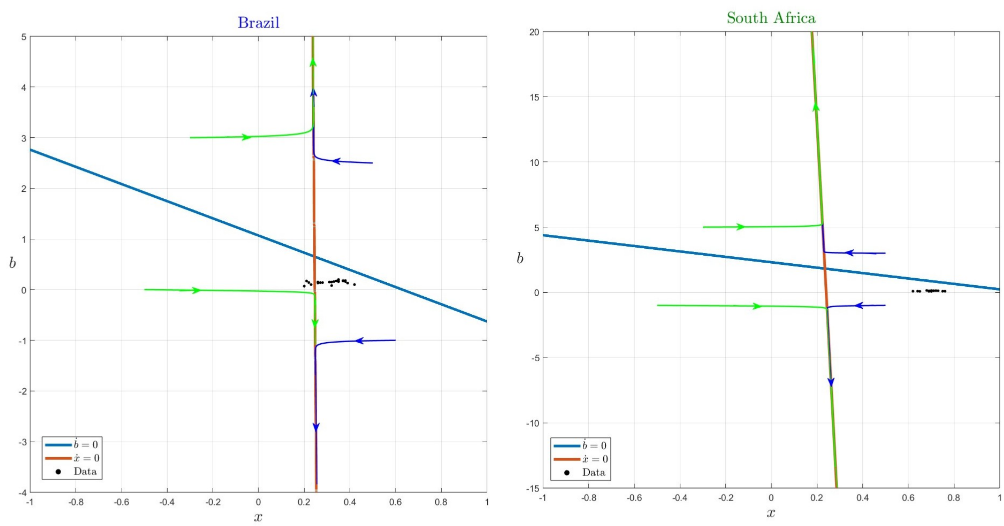


Figure 7: Public Debt and Structural Change when $i > g$



A simple way to incorporate some of these insights into my model is to introduce a "chain effect" into Eq. (7).[3] We rewrite it as:

$$V^F = \underbrace{\alpha}_{\text{Gov. Exp. Comp.}} - \underbrace{\frac{T}{K}}_{\text{Tax effect}} + \underbrace{\phi x}_{\text{Chain effect}} \tag{25}$$

where $\phi > 0$ captures the strength of such a channel. That is, if more formal peers surround a worker, then her or his choice of transition is influenced positively, which could lead to a higher formalization in the economy and vice versa. A higher value ϕ implies that individuals or agents in the economy are more significantly influenced by the formal sector around them, enhancing their likelihood of transitioning from informal to formal activities. When individuals are surrounded by a thriving formal sector, they are likely to perceive the benefits of formalization, such as better access to credit, technology, training, and stable employment (De Mel et al., 2013).

The adjusted probability functions are now expressed as:

$$p^F = \frac{\exp\left(\beta(\alpha - \frac{T}{K} + \phi x)\right)}{\exp\left(\beta(\alpha - \frac{T}{K} + \phi x)\right) + \exp\left(-\beta(\alpha - \frac{T}{K} + \phi x)\right)} \tag{26}$$

$$p^I = \frac{\exp\left(-\beta(\alpha - \frac{T}{K} + \phi x)\right)}{\exp\left(\beta(\alpha - \frac{T}{K} + \phi x)\right) + \exp\left(-\beta(\alpha - \frac{T}{K} + \phi x)\right)} \tag{27}$$

Notice that the social interaction effect is very weak for ϕ values closer to zero. In the limit, we return to the original case.

Our new 2-dimensional non-linear dynamical system is given by:

$$\begin{aligned} \dot{b} &= \bar{\gamma} + \gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + ib - \left[\tau_w\left(1-\pi\right)\sigma + \tau_\pi\left(\sigma\pi + ib\right)\right] - gb = f_1(b, x) \\ \dot{x} &= \tanh\left(\beta\left[\alpha - \tau_w\left(1-\pi\right)\sigma - \tau_\pi\left(\sigma\pi + ib\right) + \phi x\right]\right) - x = f_2(b, x) \end{aligned} \tag{28}$$

Therefore, the new equilibrium conditions are:

$$\begin{aligned} \bar{\gamma} + \gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + ib - \left[\tau_w\left(1-\pi\right)\sigma + \tau_\pi\left(\sigma\pi + ib\right)\right] - gb = 0 \\ \tanh\left(\beta\left[\alpha - \tau_w\left(1-\pi\right)\sigma - \tau_\pi\left(\sigma\pi + ib\right) + \phi x\right]\right) - x = 0 \end{aligned} \tag{29}$$

We can state and prove the following Proposition on the existence of multiple equilibria.

Proposition 2 *Define the auxiliary parameters υ and u, given by:*

$$\upsilon = \theta_1\omega_1 + \beta\phi$$

$$u = \omega_0 + \theta_0\omega_1$$

[3] The chain effect is closely related to the notion of herd behavior. For applications of such a concept to different economic problems, see Banerjee (1992), Cipriani and Guarino (2014); a reference to group effects appears in Cafferata et al. (2021), Charness and Sutter (2012) or Gillet et al. (2009); Roychowdhury (2019) and Lewbel et al. (2022) use instead peer effects.

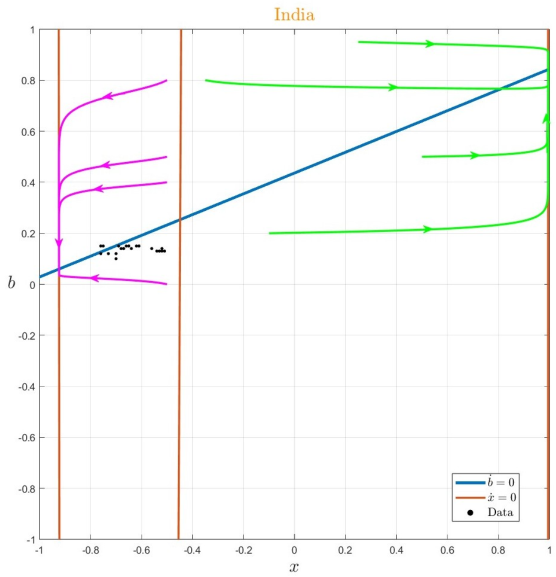


Figure 8: Multiple equilibria

Provided that $v \leq 1$ while u is arbitrarily small, the dynamic system (28) has a unique equilibrium solution (x^*, b^*) that satisfies (29). As we increase v above a certain threshold, a Pitchfork bifurcation occurs. The model admits two additional equilibria, one with $x^* > 0$ and the other such that $x^* < 0$.

Proof. See Mathematical Appendix. ■

4.2 The Indian story

Considering the socio-economic complexity and prevalence of higher levels of informality, India seems to be a special case within the BRICS. It also points out a critical limitation of our model without "chain effects". As shown in Fig. 6, our calibration strategy indicated India is mostly a formal country, which is untrue. We are ready to assess whether the extended version of the model overcomes such a limitation. Maintaining almost all original parameters, I assume now $\phi = 3.4$ and $\beta = 0.7$. Fig. 8 shows that we now have the coexistence of two stable attractors. As before, equilibrium points correspond to the intersection between blue and brown lines. Depending on initial conditions, the economy might end up in a virtuous state with high formality and public debt or a trap with negative x and low b. There are three equilibrium points: the stable outer points and the saddle central.

Despite being highly stylised, the model fairly represents India's case and the possibility of a development trap. The data clustering near low x values indicates a structural trap, where the economy struggles to transition naturally. Multiple equilibria suggest different potential economic paths, each with distinct implications for public debt and the productive structure. I mark in magenta the initial conditions leading to the high informality and low debt point, while in green, we have those leading to a formal economy with higher public debt. On the left-hand side of the phase diagram, b is lower because the informal sector imposes fewer fiscal demands on the government despite not paying taxes. On the right-hand side, formality is partly sustained precisely by higher public expenditures financed through debt. Still, the economy is stable as capital accumulation is stronger than interest rates.

The central unstable equilibrium is a tipping point where small deviations can push the economy toward contrasting situations. Fig. 8 allows us to discuss the feasibility of at least two "big push" development strategies analogous to the notions of Rosenstein-Rodan (1943) (see also Murphy et al., 1989; Skott and Ros, 1997). Suppose you are currently in a high-informality trap. One option consists of increasing government expenditures and, consequently, public debt first, trying later to improve formality. That would correspond to shocking the economy along the vertical axis and later along the horizontal axis. A second strategy would be to focus on changing the productive structure, leaving public expenditure to increase later naturally. This alternative implies a big push along the horizontal axis first to achieve the attracting region of the equilibrium point with low informality. The second strategy might be preferable as it requires only one shock instead of two.

5 Final Considerations

This paper develops a behavioural macro dynamic model of public debt in a dual economy with formal and informal sectors. The decision to transition from informal agriculture to the formal or informal sector is not merely the independent action of agents; rather, it is likely to also depend on the dynamics of fiscal policy. I rely on discrete choice theory to provide behavioural micro-foundations to the probability of being in the (in)formal sector. Analysing the debt dynamics given the pathway of structural transformation is a key addition to the existing structural change literature.

My setup provides a coherent narrative in which the sectoral composition of workers and public debt are simultaneously determined. It shows that a stable equilibrium occurs when the rate of capital creation is higher than the interest rate paid on debt. Still, in this case, higher formalization implies higher debt. A simultaneous reduction of informality and debt is possible only under high interest rates. However, in such a scenario, debt spirals threaten macroeconomic stability. A "chain effect" is introduced to further refine the model, capturing the role of social networks and economic interactions in shaping transition choices. This is particularly relevant in the Indian context, where caste-based networks significantly influence access to resources, employment, and economic opportunities. The modified framework reveals the existence of multiple equilibria, indicating that India could be in a low-debt, high-informality trap. Overcoming this requires a coordinated policy effort to shift workers into the formal sector while accommodating higher public debt levels.

Overall, the findings underscore the importance of designing fiscal policies that align with structural transformation goals. In economies undergoing significant labor reallocation, public expenditure should be strategically directed toward the formal sector, complemented by institutional mechanisms that lower entry barriers to formality. The study highlights that achieving a more formalized economy is not merely about reducing informality but about recognizing and managing the accompanying fiscal implications. Future research should explore the role of financial constraints, technological change, and institutional factors in shaping the dynamics of informality and public debt. Additionally, expanding the model to incorporate international trade and capital flows could provide further insights into the macroeconomic complexities of structural transformation in developing economies.

A Mathematical Appendix

A.1 Formal and informal shares

Recall from Eq. (2) that:

$$x = \frac{L_F}{N} - \frac{L_I}{N}$$

By scaling both sides of the equation, we get:

$$1 + x = \frac{N}{N} + \frac{L_F - L_I}{N}$$

Now, dividing by 2 gives us:

$$\frac{1+x}{2} = \frac{N}{2N} + \frac{L_F - L_I}{2N} = \frac{L_F + L_I + L_F - L_I}{2N}$$

That is:

$$\frac{1+x}{2} = \frac{L_F}{N} \tag{A.1}$$

Analogously, from Eq. (2), we can write instead:

$$-x = \frac{L_I}{N} - \frac{L_F}{N}$$

So that:

$$1 - x = \frac{N}{N} + \frac{L_I - L_F}{N}$$

Dividing by 2 gives us:

$$\frac{1-x}{2} = \frac{N}{2N} + \frac{L_I - L_F}{2N} = \frac{L_F + L_I + L_I - L_F}{2N}$$

Therefore:

$$\frac{1-x}{2} = \frac{L_I}{N} \tag{A.2}$$

A.2 Proof of Proposition 1

From (23), we can write:

$$ib - \tau_\pi\, ib - gb = -\gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}$$

Combining and reordering;

$$b(i - \tau_\pi i - g) = -\gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}$$

$$b = \frac{-\gamma\left[x(\alpha - \frac{1}{2}) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}}{i - \tau_\pi i - g}$$

$$b = \frac{-\gamma\left[x(\alpha - \frac{1}{2}) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}}{i(1-\tau_\pi) - g} \tag{A.3}$$

We can rewrite Eq. (A.3) as:

$$b = \theta_0 + \theta_1 x \tag{A.4}$$

where

$$\theta_0 = \frac{-\frac{\gamma}{2} + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}}{i(1-\tau_\pi) - g}$$

and

$$\theta_1 = \frac{\gamma\left(\frac{1}{2} - \alpha\right)}{i(1-\tau_\pi) - g}$$

Similarly, from the second equilibrium condition, it follows:

$$x = \tanh\left(\beta(\alpha - \tau_w\,(1-\pi)\,\sigma - \tau_\pi\,(\sigma\pi + ib)\right) \tag{A.5}$$

Thus, the same could be expressed as:

$$x = \tanh(\omega_0 + \omega_1 b) \tag{A.6}$$

where

$$\omega_0 = \beta[\alpha - \tau_w\,(1-\pi)\,\sigma - \tau_\pi\sigma\pi]$$

and

$$\omega_1 = -\beta\tau_\pi i$$

Thus, from equations (A.2.1) and (A.6), we can say that;

$$x = \tanh\,(\omega_0 + \theta_0\omega_1 + \theta_1\omega_1 x) \tag{A.7}$$

A.2.1 Existence of equilibria

Define $y(\cdot)$ and $z(\cdot)$ as functions of x, such that:

$$y = x$$
$$z = \tanh\,(j + hx)$$

where

$$j = \omega_0 + \theta_0\omega_1$$
$$h = \theta_1\omega_1$$

When $g > i$, it follows that $\theta_1 > 0$. Recalling that $\omega_1 < 0$, we have that $h < 0$. From the properties of the hyperbolic tangent, z is decreasing for all $x \in (-1, 1)$. Given that y is increasing in x, there exists a unique value of x for which

$$x = \tanh\left(j + hx\right)$$

Therefore, a unique x^* exists for which Eq. (A.7) is satisfied. Finally, from Eq. (), we can easily obtain b^*. The point (x^*, b^*) is the unique equilibrium solution of the system.

A.2.2 Local Stability

Recalling the dynamic system (23) given by

$$\dot{b} = \bar{\gamma} + \gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + ib - \left[\tau_w\left(1-\pi\right)\sigma + \tau_\pi\left(\sigma\pi + ib\right)\right] - gb = f_1(b,x)$$
$$\dot{x} = \tanh\left(\beta\ \left(\alpha - \tau_w\left(1-\pi\right)\sigma - \tau_\pi\left(\sigma\pi + ib\right)\right) - x = f_2(b,x)$$

We first derive the respective characteristic equation. We linearize the dynamic system around the internal equilibrium point to obtain:

$$\begin{bmatrix}\dot{b}\\ \dot{x}\end{bmatrix} = \underbrace{\begin{bmatrix}J_{11} & J_{12}\\ J_{21} & J_{22}\end{bmatrix}}_{J^*}\begin{bmatrix}b-b^*\\ x-x^*\end{bmatrix}$$

The elements of the Jacobian matrix J^* are given by:

$$J_{11} = \left.\frac{\partial f_1(b,x)}{\partial b}\right|_{(b^*,x^*)} = i(1-\tau_\pi) - g \lesseqgtr 0$$
$$J_{12} = \left.\frac{\partial f_1(b,x)}{\partial x}\right|_{(b^*,x^*)} = \gamma(\alpha - \frac{1}{2}) \lesseqgtr 0$$
$$J_{21} = \left.\frac{\partial f_2(b,x)}{\partial b}\right|_{(b^*,x^*)} = (1-x^2)\beta\tau_\pi i \lesseqgtr 0$$
$$J_{22} = \left.\frac{\partial f_2(b,x)}{\partial x}\right|_{(b^*,x^*)} = -1 < 0$$

The characteristic equation can be written as:

$$\lambda^2 - trJ\lambda + \det J = 0$$

Trace and determinant of the Jacobian are as follows:

$$tr(J) = i(1-\tau_\pi) - g - 1 \lesseqgtr 0 \tag{A.8}$$

$$\det(J) = i(1-\tau_\pi) + g) - \gamma(\alpha - \frac{1}{2})(1-x^2)\beta\tau_\pi i > 0 \tag{A.9}$$

The necessary and sufficient conditions for the stability of the equilibrium point (b^*, x^*) are that all roots of the characteristic equation have negative real parts, i.e. $tra(J) < 0$ and

$\det(J) > 0$. Both are satisfied when:

$$g > i \quad \alpha > \frac{1}{2}$$

A.3 Proof of Proposition 2

From the first expression in (29), we have:

$$ib - \tau_\pi\, ib - gb = -\gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}$$

Now combining and reordering:

$$b(i - \tau_\pi i - g) = -\gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}$$

$$b = \frac{-\gamma\left[x(\alpha - \frac{1}{2}) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}}{i - \tau_\pi i - g}$$

$$b = \frac{-\gamma\left[x(\alpha - \frac{1}{2}) + \frac{1}{2}\right] + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}}{i(1 - \tau_\pi) - g} \tag{A.10}$$

Eq. (A.10) can be rewritten as:

$$b = \theta_0 + \theta_1 x \tag{A.11}$$

where

$$\theta_0 = \frac{-\frac{\gamma}{2} + \tau_w\,(1-\pi)\,\sigma + \tau_\pi\sigma\pi - \bar{\gamma}}{i(1 - \tau_\pi) - g}$$

and

$$\theta_1 = \frac{\gamma\left(\frac{1}{2} - \alpha\right)}{i(1 - \tau_\pi) - g} \tag{A.12}$$

Similarly, from the second equilibrium condition in (29), it is easy to see that:

$$x = \tanh\left(\beta\left[\alpha - \tau_w\,(1-\pi)\,\sigma - \tau_\pi\,(\sigma\pi + ib) + \phi x\right]\right) \tag{A.13}$$

Thus, the same could be expressed as:

$$x = \tanh(\omega_0 + \omega_1 b + \beta\phi x) \tag{A.14}$$

where

$$\omega_0 = \beta[\alpha - \tau_w\,(1-\pi)\,\sigma - \tau_\pi\sigma\pi]$$

and

$$\omega_1 = -\beta\tau_\pi i \tag{A.15}$$

Substituting Eq. (A.11) into (A.14), it follows:

$$x = \tanh\left[\omega_0 + \theta_0\omega_1 + (\theta_1\omega_1 + \beta\phi)x\right] \tag{A.16}$$

A.3.1 Existence of equilibria

Rewrite Eq. (A.16), such that:

$$x = \tanh(u + vx) \tag{A.17}$$

where

$$u = \omega_0 + \theta_0\omega_1$$
$$v = \theta_1\omega_1 + \beta\phi$$

Suppose $u = 0$. Then, from the properties of the hyperbolic tangent, for $v \leq 1$, the system has a unique equilibrium point at $x^* = 0$. When $v > 1$, a Pitchfork bifurcation occurs, and the model admits two additional equilibria, one with $x^* > 0$ and the other $x^* < 0$. By continuity, this last result holds for arbitrarily small values of u.

A.3.2 Local Stability

Recalling the dynamic system (28) given by:

$$\dot{b} = \bar{\gamma} + \gamma\left[x\left(\alpha - \frac{1}{2}\right) + \frac{1}{2}\right] + ib - \left[\tau_w(1-\pi)\sigma + \tau_\pi(\sigma\pi + ib)\right] - gb = f_1(b,x)$$
$$\dot{x} = \tanh\left(\beta\left[\alpha - \tau_w(1-\pi)\sigma - \tau_\pi(\sigma\pi + ib) + \phi x\right]\right) - x = f_2(b,x)$$

We linearize it around the internal equilibrium point, obtaining:

$$\begin{bmatrix}\dot{b}\\ \dot{x}\end{bmatrix} = \underbrace{\begin{bmatrix}J_{11} & J_{12}\\ J_{21} & J_{22}\end{bmatrix}}_{J^*}\begin{bmatrix}b - b^*\\ x - x^*\end{bmatrix}$$

The elements of the Jacobian matrix J^* are given by:

$$J_{11} = \left.\frac{\partial f_1(b,x)}{\partial b}\right|_{(b^*,x^*)} = i(1-\tau_\pi) - g \lesseqgtr 0$$
$$J_{12} = \left.\frac{\partial f_1(b,x)}{\partial x}\right|_{(b^*,x^*)} = \gamma(\alpha - \frac{1}{2}) \lesseqgtr 0$$

$$J_{21} = \left.\frac{\partial f_2(b,x)}{\partial b}\right|_{(b^*,x^*)} = -\beta i\tau_\pi\left[1 - \tanh^2\left(\beta\left(\alpha - \tau_w(1-\pi)\sigma - \tau_\pi(\sigma\pi + ib) + \phi x\right)\right)\right] \leq 0$$
$$J_{22} = \left.\frac{\partial f_2(b,x)}{\partial x}\right|_{(b^*,x^*)} = \beta\phi\left[1 - \tanh^2\left(\beta\left(\alpha - \tau_w(1-\pi)\sigma - \tau_\pi(\sigma\pi + ib) + \phi x\right)\right)\right] - 1 \lesseqgtr 0$$

The characteristic equation can be written as:

$$\lambda^2 - trJ\lambda + \det J = 0$$

Trace and determinant of the Jacobian are as follows:

$$tra(J) = \beta\phi\left[1 - \tanh^2\left(\beta\left(\alpha - \tau_w(1-\pi)\sigma - \tau_\pi(\sigma\pi + ib) + \phi x\right)\right)\right] - g - i\tau_\pi + i - 1 \lesseqgtr 0 \quad \text{(A.18)}$$

$$\begin{aligned}\det(J) &= \{\beta\gamma i\tau_\pi(\alpha - \frac{1}{2}) + [i(1-\tau_\pi) - g]\beta\phi\} \cdot \left[1 - \tanh^2(E)\right] - i(1-\tau_\pi) + g \lesseqgtr 0 \\ &= \beta\gamma i\tau_\pi(\alpha - \frac{1}{2})\left[1 - \tanh^2(E)\right] + [g - i(1-\tau_\pi)]\{1 - \beta\phi\left[1 - \tanh^2(E)\right]\} \quad \text{(A.19)}\end{aligned}$$

where

$$E = \beta\left[\alpha - \tau_w(1-\pi)\sigma - \tau_\pi(\sigma\pi + ib) + \phi x\right]$$

Case 1: $u = 0,\ v \leq 1$

Given that in equilibrium $x^* = 0$, the elements in the main diagonal of the Jacobian simplify to:

$$\begin{aligned}J_{11} &= i(1-\tau_\pi) - g \\ J_{22} &= \beta\phi(1 - \tanh^2(E)) - 1\end{aligned}$$

so that

$$tra(J) = \beta\phi - 1 + i(1-\tau_\pi) - g$$

For stability, we need $tra(J) < 0$. This condition will be satisfied provided that $\beta\phi$ is arbitrarily small and $g > i$.

Moving on to the determinant, since $\tanh^2(E) = 0$ at $x^* = 0$, it simplifies to:

$$\det(J) = \beta\gamma i\tau_\pi(\alpha - \frac{1}{2}) + [g - i(1-\tau_\pi)]\,(1 - \beta\phi)$$

given

$$g > i \quad \alpha > \frac{1}{2}$$

and $\beta\phi$ sufficiently small, they ensure $\det(J) > 0$.

Case 2: $u = 0,\ v > 1$

When $v > 1$, a Pitchfork bifurcation occurs. The system now admits three equilibria, one central with $x^* = 0$, and two additional solutions for which $x^* \neq 0$. Starting with the case $x^* = 0$, the elements in the main diagonal of the Jacobian matrix simplify to:

$$\begin{aligned}J_{11} &= i(1-\tau_\pi) - g \\ J_{22} &= \beta\phi[1 - \tanh^2(E)] - 1\end{aligned}$$

so that the trace will be

$$tra(J) = \beta\phi - 1 + i(1-\tau_\pi) - g$$

Since $v = \theta_1\omega_1 + \beta\phi > 1$ and $\theta_1\omega_1 < 0$, this means $\beta\phi > 1 - \theta_1\omega_1$. Thus, the trace still might be positive or negative.

Moving on to the determinant, given that $\tanh^2(E) = 0$, it simplifies to:

$$\det(J) = \beta\gamma i\tau_\pi(\alpha - \frac{1}{2}) + [g - i(1-\tau_\pi)]\,(1 - \beta\phi)$$

Local stability requires a positive determinant, so that:

$$\beta\gamma i\tau_\pi\left(\alpha - \frac{1}{2}\right) + [g - i(1-\tau_\pi)]\,(1 - \beta\phi) > 0$$
$$\beta\gamma i\tau_\pi\left(\alpha - \frac{1}{2}\right) + [g - i(1-\tau_\pi)] - [g - i(1-\tau_\pi)]\,\beta\phi > 0 \tag{A.20}$$

Recalling that

$$\beta\phi > 1 - \theta_1\omega_1$$

we can rewrite $\beta\phi$ as

$$\beta\phi = 1 - \theta_1\omega_1 + \varepsilon \tag{A.21}$$

where $\varepsilon > 0$. Substituting Eq. (A.21) into condition (A.20), we have:

$$\beta\gamma i\tau_\pi\left(\alpha - \frac{1}{2}\right) + [g - i(1-\tau_\pi)] - [g - i(1-\tau_\pi)]\,(1 - \theta_1\omega_1 + \varepsilon) > 0$$
$$\beta\gamma i\tau_\pi\left(\alpha - \frac{1}{2}\right) + [g - i(1-\tau_\pi)] - [g - i(1-\tau_\pi)] + [g - i(1-\tau_\pi)]\,\theta_1\omega_1 - [g - i(1-\tau_\pi)]\,\varepsilon > 0$$

Substituting (A.12) and (A.15) into the expression above, it follows that:

$$-\,[g - i(1-\tau_\pi)]\,\varepsilon > 0$$

Therefore, the inequality is only satisfied for $i > g$. This means that when growth is greater than the interest rate, the equilibrium with $x^* = 0$ is unstable.

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