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Growth led by government expenditure and exports: public and external debt stability in a supermultiplier model

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Abstract

The Sraffian supermultiplier model revealed the role of autonomous demand in economic growth. Nevertheless, the long-run sustainability of a growth process driven by autonomous demand requires the stability of the financial stocks behind it. Growth led by government expenditure and exports is thus stable if both public and external debts present convergent dynamics. Thus, in this paper, we develop a supermultiplier model for an open economy with government to assess the stability of growth led by government expenditure and exports. We analyze the stability conditions for public debt and foreign debt ratios. Public debt-to-income ratio stability requires that the interest rate is smaller than the output growth rate. Foreign debt-to-exports ratio stability requires that the international interest rate is smaller than the growth rate of exports. The external constraint may appear as a restriction to external indebtedness, imposing an upper limit to growth. Nonetheless, the presence of a domestic autonomous expenditure may relax the external constraint. The model allows for two demand-led growth regimes: balance of payments constrained and policy constrained. A fiscal policy rule is proposed to keep the foreign debt ratio below an upper limit. Simulations of five cases show the conditions for stability of debt ratios, and the outcomes of the fiscal policy rule and a structural change policy. In the simulations, fiscal policy successfully reduces the equilibrium foreign debt-to-export ratio by decreasing the share of government expenditures in autonomous demand. Successful industrial policies that increase exports' growth keep the foreign debt ratio below the threshold with a higher growth rate than the fiscal policy rule. Altogether, the model provides stability conditions for growth in an open economy paying its international liabilities in foreign currency.

Keywords: Sraffian supermultiplier; Thirlwall's Law; demand-led growth; public debt; external debt.

1 INTRODUCTION

Post-Keynesian Economics assigns a prominent role to government and foreign trade as drivers of demand. Government spending and exports are typical examples of "external markets" (Kalecki, 2016), fundamental for the absorption of growing output in capitalist economies. External to the capitalist income circuit, these expenditures can be defined as autonomous demand (Cesaratto, 2015). In the Sraffian supermultiplier class of models, autonomous expenditures are the ultimate drivers of demand and growth. Because of this key role, they must be carefully appreciated. Thus, we investigate two dimensions of the sustainability of foreign trade and government spending. The first one concerns the balance of payments and the external constraint, and the second, the long-run stability of public finance.

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The Sraffian supermultiplier provides a useful framework for analyzing the consequences of autonomous demand to economic growth (Freitas and Serrano, 2015; Serrano, 1995). The supermultiplier literature has discussed growth models led by different sources of autonomous demand (Allain, 2015; Hein, 2018; Lavoie, 2016; Nah and Lavoie, 2017). However, growth in the supermultiplier depends on the long-run sustainability of the financial stocks behind the autonomous components of aggregate demand (Hein and Woodgate, 2020; Pariboni, 2016). Steady growth in an open economy with government thus requires the stability of the financial imbalances of the public and the external sectors.

This paper aims to analyze the sustainability of growth led by exports and public expenditure. Hence, we examine the stability of external and public debt ratios in a supermultiplier model with exports and public expenditure. Debt ratios are stable when the growth rates (of output or exports) are larger than the interest rates on the (public or external) debt service. Nevertheless, the growth of public expenditures may threaten the stability of the external debt in the presence of international financial constraints. Therefore, we propose a fiscal policy that contains foreign indebtedness. Five simulated experiments support the analytical conclusions of the model and compare policy alternatives.

We contribute to the Sraffian supermultiplier literature by modeling an open economy with government and assessing the sustainability of its growth path. We also contribute to the literature on growth and the external constraint (Barbosa-Filho, 2001; Moreno-Brid, 1998; Thirlwall, 1979) by discussing how this constraint emerges in an open economy supermultiplier model. Thirlwall's Law reappears as an upper limit to growth in the long run. Nonetheless, introducing a domestic autonomous expenditure implies that the external constraint is not always binding given that demand is not driven only by exports.

The remainder of this paper is structured as follows. Section 2 presents a supermultiplier model with exports and public expenditure as components of autonomous demand. Section 3 introduces debt dynamics in the model, defining the equilibrium values for public and external debt ratios and the conditions for their stability. This section also proposes a fiscal policy rule and compares the model's results for external debt with Thirlwall's external constraint. Section 4 presents five alternative simulations of the model and discusses policy implications. A final section concludes that growth can be feasibly led by autonomous public expenditure and exports.

2 THE SRAFFIAN SUPERMULTIPLIER WITH GOVERNMENT EXPENDITURE AND EXPORTS

The literature on the Sraffian supermultiplier has explored different sources of autonomous demand. The original framework studied autonomous consumption (Serrano, 1995). The sustainability of debt (or the wealth stock) financing autonomous consumption motivated relevant discussions (Fiebiger and Lavoie, 2019; Mandarino et al., 2020; Pariboni, 2016). With different objectives, Freitas and Christianes (2020) and Hein and Woodgate (2020) studied the interaction between autonomous consumption and public expenditure. Those are among the few works that model more than one source of autonomous demand in the supermultiplier model. Public expenditures have been studied by (Allain, 2015; Hein, 2018). Exports have been discussed in the supermultiplier approach by (Dejuán, 2017; Nah and Lavoie, 2017). Analyses of exports as an autonomous expenditure are also found in Thirlwall's growth models tradition (McCombie, 1985; Thirlwall, 1979).

We build on the Sraffian supermultiplier model to study growth in an open economy with government.[1] We introduce two autonomous expenditures: exports and public expenditure. Both expenditures are autonomous once they are neither financed out of the current income nor directly caused by production decisions. Exports depend on foreign demand, not on domestic demand. Sequentially, government counts with several degrees of freedom to run deficits and expand (or compress) public expenditures independent of the current income and taxation. This is particularly true for countries issuing public debt in their sovereign currency (Cesaratto, 2016; Wray, 2015).

Let us start deriving the equilibrium condition for the goods market. Notation is standard.

$$Y_t = C_t + I_t + G_t + X_t - M_t \tag{1}$$

Consumption is fully induced by the current level of disposable income, according to a constant marginal propensity to consume (c) and a constant income tax rate (τ).

$$C_t = c(1 - \tau)Y_t \tag{2}$$

[1] See Freitas and Serrano (2015) for a detailed exposition of the Sraffian supermultiplier and the analysis of the stability condition.

Imported goods are either inputs to domestic production or part of the induced consumption. Imports are therefore proportional to the level of income with a constant propensity to import (m).

$$M_t = mY_t \tag{3}$$

Along the lines of the capital stock adjustment principle, investment depends on both propensity to invest and current income.

$$I_t = h_t Y_t \tag{4}$$

The level of output is determined by total autonomous demand (Z) and by the supermultiplier, as in equation 5.

$$Y_t = \left(\frac{1}{1 - c(1-\tau) - h_t + m} \right) Z_t \tag{5}$$

The investment function follows a flexible accelerator (Freitas and Serrano, 2015). Propensity to invest thus slowly adjusts whenever current capacity utilization differs from normal capacity, as in equation 6; γ stands for the sensitivity of the investment share to deviations of capacity utilization concerning normal capacity, being larger than 0 and lower than 1; u_n represents the normal degree of capacity utilization, and u, the current capacity utilization.

$$\dot{h}_t = h_t \gamma (u - u_n) \tag{6}$$

Finally, we can determine output growth (g) according to the growth of aggregate demand.[2]

$$g_t = \frac{h_t \gamma (u_t - u_n)}{1 - c(1-\tau) - h_t + m} + g_{Zt} \tag{7}$$

Output growth depends on the growth of autonomous demand and changes in the propensity to invest. If the growth rate of autonomous expenditures (g_Z) remains stable for a long period, the degree of capacity utilization converges toward the normal capacity ($u = u_n$). Thus, the investment share stabilizes ($\dot{h} = 0$), and the growth rate of output is solely determined by the growth rate of autonomous expenditures (Freitas and Serrano, 2015; Serrano, 1995) (as in 8).

$$g = g_Z \tag{8}$$

[2] We obtain equation 7 by taking log and derivatives from equation 5, and then substituting equation 6.

We can now discuss the role of each autonomous expenditure on total autonomous demand. An explicit treatment of this point is part of the contribution of the paper.

Since Z is the sum of G and X, g_Z is given by the average of the growth rates of exports and public expenditure, weighted by the share of each expenditure on Z:

$$g_Z = \sigma_t g_{Gt} + (1 - \sigma_t) g_{Xt} \tag{9}$$

Where,

$$\sigma_t = \frac{G_t}{Z_t} \tag{10}$$

The share of government expenditure in autonomous demand (σ_t) changes whenever the growth rates of exports and public expenditure differ.[3] If public expenditure grows faster (slower) than exports, then σ_t continuously increases (decreases).[4]

$$\dot{\sigma}_t = \sigma_t (1 - \sigma_t)(g_{Gt} - g_{Xt}) \tag{11}$$

The fully adjusted position of the supermultiplier model requires a persistently stable growth rate of autonomous expenditures (Freitas and Serrano, 2015). That occurs, in the model with two autonomous expenditures, when both expenditures grow persistently at the same rate. In this case, each autonomous expenditure keeps a constant share in total autonomous demand. Hence, σ must be constant in the fully adjusted position.[5]

To analyze the steady state, it is necessary to assume that both public expenditures and exports grow at the same rate. We maintain this assumption in sections 2.1 and 3.3. Nonetheless, we allow for different growth rates and changes in the share of each expenditure in autonomous demand in the simulated cases presented in section 4, exploring the dynamics out of the fully adjusted position. We also show that the share

[3] Departing from equation 10, we can take logs and derivative with respect to time, obtaining the following equation:

$$\dot{\sigma}_t = \sigma_t (g_{Gt} - g_{Zt})$$

Finally, by substituting the definition for g_{Zt} given in equation 9, we can define the rate of change of σ as follows in equation 11 in the text.

[4] We can theoretically conceive that one of the expenditures keeps growing faster than the other until σ converges to one of the extreme positions — that is, either $\sigma = 0$ or $\sigma = 1$. In that case, one of the autonomous expenditures dominates the explanation of the growth of autonomous demand. However, as modern economies usually present positive demand from exports and public expenditures, we argue that this condition does not usually hold.

[5] The Appendix (available upon request) shows that a constant growth rate for the total autonomous demand can only be obtained either if both expenditures constantly grow at the same rate or under a very specific growth pattern. The second alternative requires that the growth rate of each autonomous component varies in time, exactly compensating from movements in the other and from changes in σ to keep a constant value for g_Z. We do not explore this possibility in this paper.

of each expenditure on autonomous demand matters for the equilibrium value of both public and external debt ratios.

The first step, however, is to analyze the equilibrium of the model.

2.1 *Fully adjusted position*

The equations below summarize the dynamics of the model. The presence of two autonomous expenditures requires adding one equation (14) to the original model.

$$\dot{u}_t = u_t \left[\sigma_t g_{Gt} + (1-\sigma_t) g_{Xt} + \frac{h_t \gamma (u_t - u_n)}{1 - c(1-\tau) - h_t + m} - \left(\frac{h_t}{v}\right) u_t + \delta \right] \tag{12}$$

$$\dot{h}_t = h_t \gamma (u_t - u_n) \tag{13}$$

$$\dot{\sigma}_t = \sigma_t (1-\sigma_t)(g_{Gt} - g_{Xt}) \tag{14}$$

The model achieves the fully adjusted position when $\dot{h}_t = \dot{u}_t = \dot{\sigma}_t = 0$. The condition $\dot{\sigma}_t = 0$ gives us that $g_Z^{**} = g_{Gt} = g_{Xt}$. According to equation 14, when both autonomous expenditures grow at the same rate, σ achieves a constant value. The equilibrium can be achieved at any level of σ. Then, g_Z is sufficiently persistent to lead the propensity to invest toward equilibrium and capacity utilization toward normal capacity ($u^{**} = u_n$, as in 13). Finally, equation 12 gives us an expression for the equilibrium propensity to invest: $h^{**} = \frac{v}{u_n}(g_Z + \delta)$.

3 Public expenditure, exports, and debt stability

The tradition of the balance of payments constrained growth models highlighted exports as a source of autonomous demand (McCombie, 1985; Thirlwall, 1979). This perspective acknowledges the twofold role of exports in economic growth. On the one hand, exports constitute part of aggregate demand, affecting output and employment, and growth. On the other hand, exports provide foreign currency, allowing a country to pay for imports of inputs and capital goods required for economic growth (Kaldor, 1966). On the basis of this premise, Thirlwall (1979) argued that the balance of payments represents the dominant constraint to demand expansion in open economies. In this view, demand and income changes bring the balance of payments into equilibrium rather than changes in exchange rates or relative prices.

Thirlwall (1979) introduced a balanced trade account condition, meaning that nominal exports must be equal to nominal imports, as a long-run constraint to demand-led growth. Abstracting from changes in terms of trade, the growth rate compatible with balanced trade is given by the growth rate of exports divided by import's income elasticity. This result is known as Thirlwall's Law.

A few contributions introduced capital flows in the balance of payments constrained growth models (Moreno-Brid, 1998, 2003; Thirlwall and Hussain, 1982). Capital flows can finance trade account deficits in nonexplosive dynamics, keeping a stable ratio between capital inflows and income (Moreno-Brid, 1998). Growth is still constrained by the balance of payments but under different conditions. Moreno-Brid (2003) further developed this model, including net interest payments.[6]

Barbosa-Filho (2001) suggested framing the balance of payments constraint in terms of the long-run sustainability of the external debt rather than as a constant deficit or balanced trade condition. The external debt-to-income ratio is stable whenever the income growth rate is larger than the interest rate on the external debt service. Building on this approach, Bhering et al. (2019) argued that the foreign debt should be normalized to the level of exports rather than domestic income. The currency mismatch between domestic income and foreign debt implies that the value of exports is the proper measure of solvency of a country whose liabilities are taken in foreign currency.

Within the supermultiplier literature, exports have been less explored in comparison with other autonomous expenditures. Dejuán (2017) included exports as the only autonomous expenditure to study an open economy supermultiplier model, although the author focuses on stability concerns. Nah and Lavoie (2017) included exports in a supermultiplier model, further assuming that export growth rate is given by an exogenous trend plus a negative effect of the wage share. The authors focus on the relation between growth and distribution in the "traverse" to the fully adjusted position. Neither of these works introduces a second source of autonomous demand nor discusses the relation of the model with the balance of payments constraint.

Unlike the mentioned contributions, we simultaneously account for exports and public expenditure as sources of autonomous demand. We discuss how the model presented in this paper relates to the balance of payments constraint to economic growth. We also define conditions for the long run stability of both public and external debt ratios. The convergence of debt ratios to stable values supports the feasibility of growth being led by public expenditure and exports in the long run.

[6] Caldentey and Moreno-Brid (2019) emphasized the effect of long-run trends in terms of trade, as the previous models abstracted from the impact of changes in the terms of trade in either tightening or loosening the external constraint.

The stability of public debt has long been a concern in theoretical debates. Domar (1944) concluded *inter alia* that the public debt-to-income ratio converges to an equilibrium level in the long run if the growth rate is larger than the interest rate. We refer to this result as the Domar stability condition.

Being rather independent of taxation, public expenditures' growth can be considered exogenous to current income and firm's production decisions, thereby being an autonomous expenditure. Indeed, countries with monetary sovereignty can finance deficits with taxes, debt issues, and monetary emissions (Cesaratto, 2016; Wray, 2015). Allain (2015) modeled public expenditure as an autonomous expenditure in a Kaleckian supermultiplier model, showing public expenditure may act as a stabilizer of economic growth.[7] Nevertheless, the author does not account for the public deficit and the public debt since he assumes the public budget is kept balanced by tax rate adjustments. Hein (2018) introduced public expenditure, debt dynamics, and related distributive concerns in a Kaleckian supermultiplier model. Hein (2018) concluded that primary public deficit (or surplus) and public debt converge to stable values as long as the growth rate of public expenditure is larger than the interest rate on debt service and that the supermultiplier's stability condition holds.

These contributions consider government expenditure as the only source of autonomous demand. By contrast, Freitas and Christianes (2020) and Hein and Woodgate (2020) analyzed growth led by autonomous public expenditures and autonomous consumption. Freitas and Christianes (2020) focused on the steady state. They show that assuming the Domar stability condition holds, the growth rate of autonomous demand and the composition of autonomous demand (the share of each expenditure) affect the equilibrium level of the public debt-to-income ratio and the equilibrium primary deficit-to-income ratio. Freitas and Christianes (2020) also discussed the relation between the level of debt and the impact of interest rate and taxation on distribution, showing that changes in distribution have a feedback effect on the level of output (but not in its equilibrium growth rate).[8] In turn, Hein and Woodgate (2020) accounted for both autonomous consumption and government expenditure in a supermultiplier model framed within the Kaleckian perspective. The authors focused on the relation between the consumption out of the financial income of the debt creditors and Harrodian instability.

[7] Allain (2015) argued that autonomous demand had not been properly accounted for in Kaleckian growth literature yet, even in the case of expenditures that may be typically considered autonomous as public expenditure and exports.

[8] This result is consistent with previous conclusions of the supermultiplier growth model (Freitas and Serrano, 2015).

After revisiting these contributions, we can introduce the dynamics of public and foreign debt in the model.

3.1 *Public debt*

Let us start considering the case of public debt. Our main reference here is Freitas and Christianes (2020). However, unlike these authors, we restrict the assumption of a constant government share in autonomous demand to the steady-state analysis.

In equation 15, the change in total public debt ($\dot{B}$) is described. Public debt changes according to government expenditure (G_t), taxes — given by a proportional tax rate (τ) times current income level (Y_t) — and the debt service — depending on the interest rate (i) and the total debt (B_t).

$$\dot{B}_t = G_t - \tau Y_t + iB_t \tag{15}$$

The relevant economic variable for the analysis of public debt is the ratio between debt and income. We can denote it as b, defined as $b_t = \frac{B_t}{Y_t}$. The rate of change in the debt-to-income ratio follows equation 16.

$$\dot{b}_t = \sigma_t z_t - \tau + ib - g_t b_t \tag{16}$$

Remember that σ_t stands for the share of government expenditure in total autonomous expenditures. In turn, z_t stands for the ratio of autonomous expenditures to total output. As shown by Freitas and Christianes (2020), the term $\sigma_t z_t - \tau$ corresponds to the primary government deficit (or surplus) to output ratio.

3.2 *Foreign debt*

The analysis of foreign debt's long-run stability follows that of Barbosa-Filho (2001), although we focus on the debt-to-exports ratio as that of Bhering et al. (2019). We assume the balance of payments is composed of the trade balance, factor income balance, and capital account. We also assume the factor income balance is solely composed of external debt service payments. Hence, trade balance plus external debt service constitute the current account. Following Bhering et al. (2019), we assume that capital flows (F) are just sufficient to cover the deficit in the current account, as in equation 17. We relax this assumption in section 3.4.

$$F_t = M_t - X_t + R_t \tag{17}$$

In equation 17, the result of the balance of payments is equal to 0 so that the country neither accumulates nor loses reserves.

R stands for the external debt service payments, being proportional to the international interest rate (r) and the external debt (D). For simplicity, we assume r to be fixed in time.

$$R_t = rD_t \tag{18}$$

The change in external debt will be given by the net entrance of capital to finance the current account deficit. Thus,

$$\dot{D}_t = F_t \tag{19}$$

Finally, gathering the previous relations in equations 17, 18, and 19, we get

$$\dot{D}_t = M_t - X_t + rD_t \tag{20}$$

Let us define d as the ratio between external debt and exports: $d_t = \frac{D_t}{X_t}$. Thus, d_t changes according to equation 21.

$$\dot{d}_t = \frac{m}{(1-\sigma_t)z_t} - 1 + (r - g_{Xt})d_t \tag{21}$$

Bhering et al. (2019) showed that $r < g_X$ is a necessary condition for the stability of the external debt-to-exports ratio. In other words, foreign debt stability requires an export growth rate larger than the international interest rate. The condition is analogous to Domar's stability condition for public debt.

Equation 21 also shows that the path of foreign debt to exports ratio diverges as σ_t approaches 1. An increasing share of government in autonomous demand implies an increase in imports, owing to the increase in domestic demand, without a counterpart in exports.

3.3 *Fully adjusted position and debt stability*

We can summarize the system in five equations. Equations 12, 13, and 14 come from the supermultiplier growth model with two autonomous expenditures. The other two equations describe public and foreign debt dynamics.

$$\dot{b}_t = \sigma_t z_t - \tau + ib - g_t b_t \tag{22}$$

$$\dot{d}_t = \frac{m}{(1-\sigma_t)z_t} - 1 + (r - g_{Xt})d_t \tag{23}$$

From our previous discussion, we know that the fully adjusted position requires that both autonomous expenditures grow at the same rate ($g_X = g_G$). Hence, σ remains constant in the equilibrium, which can happen at any level of σ. We denote this equilibrium value as σ^*. In section 4, we analyze changes in σ by simulating the dynamics of the model out of the steady-state.

Given a persistently stable growth rate of autonomous demand (g_Z), the economy converges to the fully adjusted position as in the baseline supermultiplier model. In the equilibrium, the growth rate converges to the growth rate of autonomous demand, propensity to invest stabilizes, and capacity utilization converges to normal capacity. The equilibrium positions for output growth, propensity to invest and capacity utilization coincide with the ones presented in section 2.1.

Here, we discuss the equilibrium levels for the public debt-to-income ratio and foreign debt-to-exports ratio. By setting equation 22 equal to 0, we obtain the equilibrium level of the public debt-to-income ratio, expressed in the equation below:

$$b^* = \frac{\sigma^* z^* - \tau}{g_Z - i} \tag{24}$$

Note that z^* emerges from the system of equations describing the supermultiplier growth model. z^* is given by the inverse of the supermultiplier, as shown in equation 25.

$$z^* = 1 - c(1-\tau) - (g_Z + \delta)\frac{v}{u_n} + m \tag{25}$$

The public debt-to-income ratio converges to a positive value if the public budget is in deficit in the fully adjusted position, that is, if $\sigma^* z^* > \tau$. The greater the share of government expenditure in autonomous demand (σ^*), the greater the equilibrium public debt-to-income ratio, as obtained by Freitas and Christianes (2020). Nevertheless, the stability of the public debt ratio does not depend on the value of σ^*. In turn, Domar's stability condition ($g^* > i$) is crucial for the stability of the public debt-to-income ratio (as in Hein (2018) and Freitas and Christianes (2020)).

Now, consider the case of foreign debt. By setting equation 23 equal to 0, we can obtain the equilibrium level for the foreign debt-to-exports ratio:

$$d^* = \left[\frac{m}{(1-\sigma^*)z^*} - 1\right]\left(\frac{1}{g_X - r}\right) \tag{26}$$

Equation 26 shows the country will accumulate foreign debt if, in equilibrium, the term within brackets is positive. This occurs when the economy presents trade deficits in the equilibrium, given the propensity to import and the equilibrium share of exports in output ($(1-\sigma^*)z^*$). Otherwise, *i.e.*, if the equilibrium generates trade surpluses, the economy will accumulate foreign credit rather than external debt.

The stability of d requires that the growth rate of exports is larger than the interest rate on foreign debt. If this condition does not hold, then the growth of exports is not enough to offset the increase in the foreign debt caused by the debt service, leading to an increasing foreign debt-to-exports ratio. Moreover, equation 26 shows that if the ratio between government expenditure and income is equal to one (*i.e.*, $\sigma^* = 1$), the debt-to-exports ratio diverges to infinity. If government expenditure dominates the determination of autonomous demand growth, domestic income growth will continuously increase imports while the economy does not export. Thus, the economy accumulates trade deficits, which leads to an increasing amount of foreign debt in each period.

3.4 *Constraint to external debt*

Till now, we have assumed that creditors will finance any level of foreign debt. However, international institutions may impose restrictions on indebted countries. Alternatively, international capital markets may operate with a threshold, meaning that above a certain limit, capital inflows cease. We therefore introduce an upper limit to the debt-to-exports ratio — similarly to Bhering et al. (2019). We denote the bound to foreign debt ratio as $\bar{d}$.

Naturally, this constraint does not raise concerns if it is larger than the equilibrium level of foreign debt. Hence, let us focus on the case in which $\bar{d} < d^*$. In this case, the variables determining the equilibrium external debt-to-exports ratio (that is, the variables on the right-hand side of equation 26) must be changed to meet such a constraint, pushing the equilibrium level of d toward $\bar{d}$.

Suppose the current foreign debt-to-exports ratio is sufficiently below $\bar{d}$. In that case, there is room for structural change policies that may affect the composition of output, the propensity to import, and even stimulate export growth as technical progress increases the income elasticity of exports.[9] In terms of the model's parameters, industrial and innovation policies inducing structural change may reduce m, increase

[9] Araujo and Lima (2007) showed that structural change affecting the composition of output and exports towards a larger share of industries with a larger demand income-elasticity can loosen the external constraint, increasing the growth rate compatible with the balance of payments constraint. Cimoli et al. (2009) argued that some developing economies performed better than others in terms of growth because

the share of exports in output (reducing σ), or increase the growth of exports (g_X). Those policies affect the equilibrium foreign-debt-to exports ratio, contributing to keeping it within the upper limit. Since these policies take time to affect the parameters, their implementation requires that the economy is still able to maintain increasing indebtedness before their results appear.

By contrast, current indebtedness may be too close to $\bar{d}$ or policymakers may reject structural change policies. In that case, fiscal policy can change the value of σ^*. Fiscal policy can keep the growth rate of public expenditure below the growth rate of exports until σ reaches the value of $\bar{\sigma}$, as shown in equation 27: $\bar{\sigma}$ corresponds to the ratio between public expenditure and income that is compatible with the upper limit to foreign debt-to-exports ratio, given the growth rate of exports, the propensities to import and consume, the tax rate, and the international interest rate. Putting it differently, fiscal policy can keep $g_G < g_X$, which leads to a decreasing σ. This regime must last long enough, allowing σ to reach $\bar{\sigma}$. When this happens, the growth rate of public expenditure can return to the steady state value (equal to g_X). Finally, the economy achieves a new steady-state compatible with the constraint to external indebtedness.

$$\bar{\sigma} = 1 - \left(\frac{m}{z^*(1+\bar{d})(g_X - r)} \right) \tag{27}$$

Hence, a long-term limit to external indebtedness constrains the growth of public expenditures. This result is consistent with the balance of payments constrained growth models tradition. This view has emphasized that corrections in the balance of payments happen through changes in aggregate income rather than exchange rates or relative prices (Thirlwall, 1979). The equilibrium compatible with the balance of payments constraint can be achieved using demand management, that is, fiscal policy (Barbosa-Filho, 2001).

The external constraint is a financial constraint to an economy that relies on imported inputs or final goods purchased with an internationally accepted currency. In our model, this currency can be obtained either by exporting or by accepting foreign liabilities. If the availability of capital flows depends on the level of the foreign debt-to-exports ratio, growth is also affected by the debt ratio. When the foreign debt-to-exports ratio exceeds the acceptable upper level, capital flows cease, interrupting the availability of imported inputs and final goods required for production and growth to take place.

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of structural change in favor of more technological and efficient sectors which allow for a larger output growth within the external constraint.

The sustainability of growth in the long term thus requires the economy to remain within the external debt limit. Thus, the threshold for the foreign debt ratio sets a boundary for the growth of domestic autonomous expenditures (in this model, public expenditure). This result is in line with Hein and Woodgate (2020), who, in a different context, suggested the assessment of how financial constraints affect the sustainability of autonomous expenditures.

3.5 *Fiscal policy rule*

Fiscal policy management can prevent the convergence of the debt-to-exports ratio toward an excessively high ratio. We define a rule for fiscal policy according to which the growth rate of public expenditure is compatible with the limit to foreign indebtedness. This rule requires evaluating the necessary change in σ in each period and adjusting the growth rate of public expenditures accordingly. Knowing $\bar{d}$ and g_X allows us to obtain $\bar{\sigma}$ as in equation 27. Note that z^* is also known since we are assuming a constant g_X. The export growth rate gives us the steady-state value for h, the only time-varying parameter in the determination of z^*. However, defining both σ_t and $\bar{\sigma}$ is insufficient to establish the rule for the growth rate of public expenditures. We must still find the speed of adjustment of σ_t that is compatible with the limit in the foreign-debt-to exports ratio. This speed is proportional to the distance between the current foreign-debt-to export ratio and the ratio's upper limit. In other words, the sooner the current path of debt tends to achieve $\bar{d}$, the faster must be the convergence of σ_t toward $\bar{\sigma}$.

Let us further explain this mechanism with a brief example, shown in Figure 1. Assume that according to current trends, the economy will converge to a steady-state compatible with the limit to the foreign debt-to-export ratio ($\bar{d}_0$). Furthermore assume that an international shock in global financial liquidity reduces the upper limit to external indebtedness from $\bar{d}_0$ to $\bar{d}_1$ in period t_0. The new upper limit for d is no longer consistent with the equilibrium ratio. Thus, fiscal policy must bring public expenditure share in autonomous demand toward a value compatible with the new limit to foreign debt-to-income ratio.

The speed of this adjustment depends on the period in which debt would surpass the threshold $\bar{d}_1$. An approximation for this period can be obtained via a Taylor

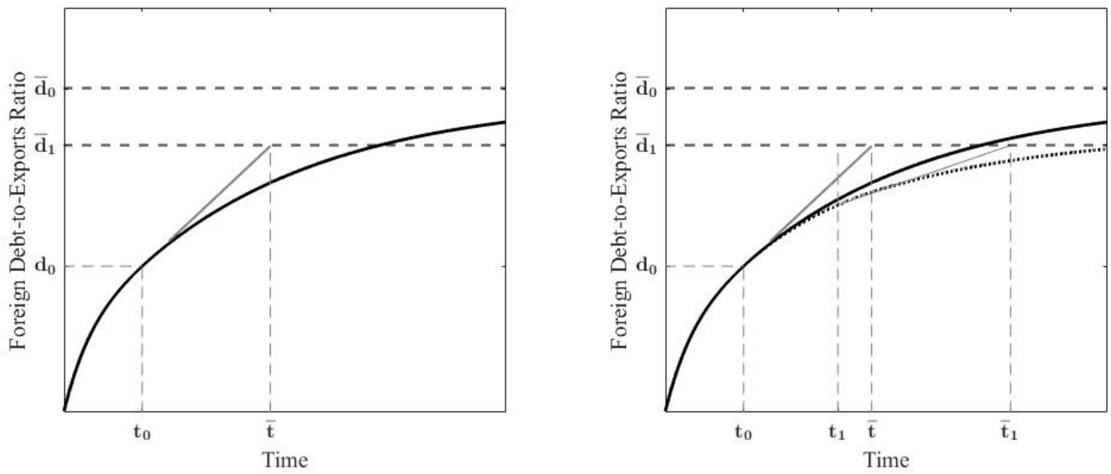

Figure 1: *Foreign Debt-to-Exports Ratio and Shock in $\bar{d}$ before (left) and after fiscal policy (right)*

expansion of the foreign debt-to-exports ratio. We denote this linear approximation as $\bar{t}$.[10] Evaluating $\dot{d}$ in the period of the shock gives us the expression below.

$$\bar{t} = t_0 + \frac{\bar{d}_1 - d_0}{\dot{d}_0} \tag{28}$$

where $\dot{d}_0$ corresponds to the change in d evaluated in period t_0.

$$\dot{d}_0 = \frac{m}{(1-\sigma_0)z_0} - 1 + (r - g_X)d_0 \tag{29}$$

In equation 28, we define the time horizon in which fiscal policy can act before the debt-to-exports ratio achieves its upper limit. The difference $\bar{t} - t_0$ defines the time interval within which such changes must occur. We can, thus, define that the share of government expenditure in autonomous demand changes continuously at a rate given by equation 30. This equation sets the proportional change in σ, happening in each instant of time, required to avoid that d exceeds $\bar{d}$. Thus, this fiscal policy rule provides a gradual adjustment in σ since the change in this variable is distributed through several periods.

$$\dot{\sigma}_0 = \frac{\bar{\sigma} - \sigma_0}{\bar{t} - t_0} \tag{30}$$

[10] A higher-order Taylor expansion can provide a more precise estimate. We use a linear approximation in order to keep the analysis simple and emphasize the theoretical argument.

We can now substitute $\dot{\sigma}_0$ in equation 14 after isolating g_{Gt}. Equation 14 determines $\dot{\sigma}$ according to the current level of the parameter and the difference between g_{Gt} and g_{Xt}. We obtain the following fiscal policy rule:

$$g_{Gt} = g_X + \frac{\dot{\sigma}_0}{\sigma_0(1-\sigma_0)} \tag{31}$$

In equation 31, we define the growth rate of public expenditure consistent with the aim of reducing the share of public expenditure in autonomous demand. It is given by the growth rate of exports minus the required change in the share for each period (that is, $\dot{\sigma}_0$) divided by the product $\sigma_0(1-\sigma_0)$. The second term of the equation is negative since we need that σ decreases. The problem of the foreign debt ratio appears when $\bar{\sigma} < \sigma_0$, which, in turn, means that $\dot{\sigma}_0$ must be negative.

Nonetheless, we can conceive an opposite case in which the economy converges toward a d smaller than $\bar{d}$. If this is the case, fiscal policy can lead σ to a larger value of $\bar{\sigma}$ by setting the growth rate of public expenditures temporarily above the growth rate of exports. The rule can follow the same principle presented above, with the opposite sign. Such a policy leads the economy to a permanently higher output than if it had not been executed.

Returning to our example, the rule in equation 31 defines a growth rate for public expenditure smaller than the growth rate of exports. This policy decreases the share of government in autonomous expenditures (σ). Nonetheless, each instant σ decreases, the whole curve d is shifted downward, as shown in Figure 1. In turn, this shift changes the estimate of the period in which the d achieves the limit. Now, this period corresponds to $\bar{t}_1$. Considering this change, the process described above can be repeated periodically, making the adjustment of σ even smoother.

The fiscal policy rule therefore can be updated periodically, changing g_{Gt} according to the shifts in the d curve. This policy alternative is very gradual, avoiding a sudden interruption on the growth path and its economic and social consequences. However, such policy requires framing the issue of external debt as a matter of long-term equilibrium, which is possible when the short-term liquidity in foreign currency is guaranteed.

3.6 *External debt and the external constraint*

In section 3.3, the foreign debt-to-exports ratio converges to a stable value under two conditions. First, the growth rate of exports must be larger than the interest rate on foreign debt. Second, the share of public expenditures in autonomous demand must be smaller than 1. However, introducing a limit to the foreign debt-to-exports ratio brings back the external constraint to economic growth. Bhering et al. (2019, p. 491) argued that if there is a ceiling to foreign debt ratio, the growth rate compatible with the external constraint in demand-led growth models is given by Thirlwall's Law.

In fact, Thirwall's Law reapers in our model as the balance of payments constrained growth rate in the long-term. The external constraint thus imposes an upper limit to growth. However, it does not determine the output growth rate, which depends on the growth rates of both components of autonomous demand. Thus, the introduction of a domestic autonomous expenditure implies the constraint is not necessarily binding.

Nonetheless, the economy can grow faster than the rate defined by Thirlwall's Law and still keep a constant foreign debt-to-exports ratio. That results from the difference between the exports growth rate and the international interest rate, which relaxes the external constraint in the short run.

Let us analytically demonstrate these results, departing from the expression for the change in the foreign debt-to-exports ratio (equation 23). Since we assume that the propensity to import is constant, imports grow at the same rate of output. Thus, we can rewrite the current value of imports as the initial value multiplied by the accumulated output growth. The same procedure can be done with exports' initial value and exports growth rate. By these means, we obtain equation 32. The initial values of imports and exports are denoted by the subscript 0.

$$\dot{d}_t = \left(\frac{M_0 e^{\int_0^t g.dt}}{X_0 e^{\int_0^t g_X.dt}} \right) - 1 + (r - g_{Xt}) d_t \tag{32}$$

Suppose that we must keep a constant ratio of foreign debt to exports. In that case, we set $\dot{d} = 0$. We keep the assumption that $r < g_X$, otherwise d would be inherently unstable. With simple operations in the last equation, after making $\dot{d} = 0$, we obtain the expression below.

$$\int_0^t g.dt = \int_0^t g_{Xt}.dt + \ln\left(\frac{X_0}{M_0}\right) + \ln[1 - (r - g_{Xt}) d_t] \tag{33}$$

The integral of the growth rates over the period $[0, t]$ must be equal to the average growth rates in the same period times t. Hence, we can substitute both integrals by their averages as in equation 34. The superscript μ denotes the average over the period $[0, t]$.

$$g^{\mu} t = g_X^{\mu}.t + \ln\left(\frac{X_0}{M_0}\right) + \ln[1 - (r - g_{Xt}) d_t] \tag{34}$$

In this case, the average output growth rate that is compatible with a constant d is given by equation 35.

$$g^{\mu} = g_X^{\mu} + \frac{1}{t}\{\ln\left(\frac{X_0}{M_0}\right) + \ln[1 - (r - g_{Xt})d_t]\} \tag{35}$$

Thus, the average growth rate compatible with a constant d is equal to the average growth rate of exports plus a term depending on the initial trade balance, and another term depending on the difference between the international interest rate, the exports growth rate, and the debt ratio. Suppose that the economy presented a trade deficit in period 0 (with $X_0 < M_0$). In that case, the term within braces is positive whenever the inequality 36 holds. Thus, the average output growth for the interval $[0, t]$ can exceed the average growth of exports if the initial trade account deficit normalized by exports is smaller than the relief in the foreign debt ratio caused by the excess of exports growth with respect to the international interest rate in period t.

$$\frac{M_0 - X_0}{X_0} < (g_{Xt} - r)d_t \tag{36}$$

The difference between exports growth and the international interest rate has a negative impact on the change in the foreign debt-to-exports ratio. Inequality 36 holds when this effect more than compensates the impact of the initial trade deficit on external indebtedness, which relaxes the external constraint. In this case, the larger the difference $(g_X - r)$, the more relaxed the external constraint.

According to this condition, domestic expenditures can grow faster than exports without affecting the foreign debt-to-exports ratio. This result provides further degrees of freedom to g_G since the growth of output (g) depends on the growth rate of autonomous demand, which is a weighted average of the growth of the two autonomous expenditures. In other words, public expenditures are allowed to grow even faster if the output can grow faster than exports. The smaller the σ, the larger the possible difference between the growth rate of the two autonomous expenditures.

However, equation 35 also shows that the average output growth rate that keeps a constant d approaches the average exports growth rate in the long run. As t approaches $+\infty$, g^{μ} converges to g_X^{μ}. That result is not different from Thirlwall's Law. According to Thirlwall's Law, the balance of payments constrained growth rate (g_b) is given by the growth rate of exports (g_X) divided by the income elasticity of imports (π) (as in equation 37).

$$g_b = \frac{g_X}{\pi} \tag{37}$$

In the present case, the income elasticity of imports is constant and is equal to 1 since we assume the propensity to import to be constant.[11] Equation 37 therefore implies that the growth rate of output must be equal to the growth rate of exports.

Thus, the introduction of a ceiling for the foreign debt-to-exports ratio brings back the external constraint to economic growth. The growth rate compatible with the external constraint (g_b) represents an upper limit to growth. For large values of t, g^{μ} gets closer to g^{μ}_X, which, for a unitary propensity to import, corresponds to the growth rate known as Thirlwall's Law (Thirlwall, 1979). That rate appears as the long-term constraint to economic growth when the foreign debt-to exports ratio must be kept constant.

Introducing a domestic autonomous expenditure means that the external constraint may not be binding, since the growth of demand is no longer led solely by exports. If public expenditures persistently grow less than exports, the economy does not reach the growth rate given by g_b. In that case, the foreign debt-to-exports ratio tends to decrease. Thus, the growth path does not necessarily coincide with the balance of payments constrained growth rate. The external constraint imposes an upper limit to growth. By contrast, Thirlwall's Law determines the long term growth rate if exports are the only source of autonomous demand.[12]

We conclude that the model allows for two different growth regimes. As stated by Freitas and Dweck (2013, p. 168), the supermultiplier "theoretical framework conceives two possible growth regimes: a balance of payments constrained demand-led growth process, and a policy constrained (or pure) demand-led growth process".

4 SIMULATIONS

We rely on numerical simulations to analyze five different experiments. All of them start from a steady-state position, in which both components of autonomous demand grow at a 4% rate. Initially, the degree of capacity utilization is equal to the normal degree, and the propensity to invest is equal to its steady-state value. We discuss the

[11] Total imports are given by $M = mY$. Therefore, we can calculate the income elasticity of imports as follows:

$$\pi = \frac{dM}{dY}\frac{Y}{M} = \frac{m}{m} = 1$$

[12] Indeed, models arriving at this conclusion assumed exports are the only source of autonomous demand. Thirlwall (1997, p. 378) claimed the balance of payments constrained growth model build on the same assumptions as Harrod's foreign trade multiplier, "namely, that exports are the only component of autonomous demand, that trade is balanced, and the terms of trade remain unchanged". Although McCombie (1985) introduced domestic autonomous expenditures, he provided a limited analysis of their implication to growth in the long term. The assumption that exports are the only autonomous expenditure is often associated with Kaldor's remarks on growth (Kaldor, 1978, p. 146).

evolution of foreign debt and public debt ratios, and growth of autonomous demand and output in each of the five cases.

First, we present a baseline case in which both foreign debt and public debt ratios converge to a stable positive value. Foreign-debt-to exports ratios converges to a positive value because $[m/(1-\sigma^*z^*)]-1>0$. In other words, the economy accumulates external debt because it maintains trade account deficits in the equilibrium. Foreign debt-to-exports ratio converges to a stable value because $g_X > r$ in the baseline case. In turn, the public debt-to-income ratio converges to a positive value because the public budget presents primary deficits in the equilibrium ($\sigma^*z^* - \tau > 0$). Convergence of public debt-to-income ratio results from the fact that output growth rate is larger than the interest rate ($g > i$).

The other four cases explore different features within the same framework. They keep most of the parameters of the baseline case introducing a change in one particular attribute to examine its effect. Results for key variables are summarized in table 1.[13]

Table 1: Summary of the Results of the Simulations

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
Name	Baseline	Explosive d	Explosive b	Fiscal policy	Structural change
Change	-	$r = 0.05$	$i = 0.045$	Policy rule (p.30)	$g_X = 0.045$ (p.40)
g^*	0.04	0.04	0.04	0.04	0.045
b^*	1.0126	1.0126	$+\infty$	0.8022	0.3777
d^*	4.2944	$+\infty$	4.2944	2.5000	2.3927
h^*	0.2029	0.2029	0.2029	0.2029	0.2118
z^*	0.4096	0.4096	0.4096	0.4096	0.4007
σ^*	0.4138	0.4138	0.4138	0.4039	0.3969

Figure 2 compares the baseline case with cases 2 and 3. Case 2 reveals the explosive dynamics for the foreign debt-to-exports ratio, obtained when the interest rate on foreign debt service (r) is higher than the growth rate of exports. Case 3 shows the divergent pattern for the public debt-to-income ratio. Similar to the previous case, the interest rate on the public debt (i) is higher than the output growth rate.

The other variables of cases 2 and 3 converge toward equilibrium values, as shown in Table 1. In fact, they converge toward the same equilibrium as the baseline case. Nonetheless, this conclusion depends on the assumption that the fast increase in debt ratios does not affect the parameters. This may not be the case in reality. A fast increase in public debt, as in case 3, may cause a decrease in the growth rate of public expenditures, changing the pattern observed in the simulation. Rather, it may motivate

[13] The parameters of the simulations are presented in a table in the Appendix (available upon request). Simulation exercises applied the software MATLAB 2019b. The code is available upon request.

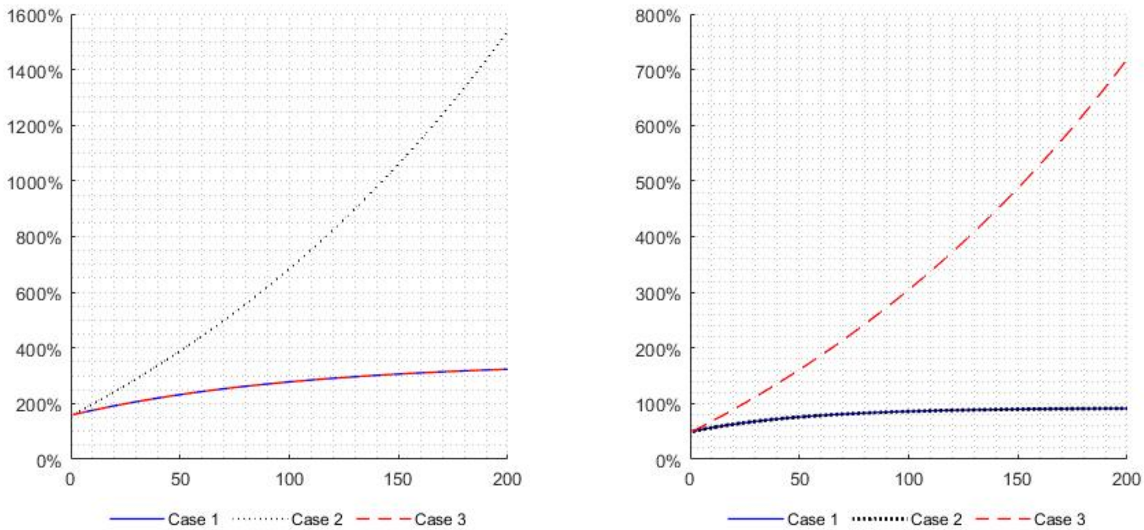

FIGURE 2: *Evolution of the foreign debt-to-exports ratio (right) and the public debt-to-income ratio (left) for cases 1, 2, and 3.*

monetary authorities to reduce i, pushing the public debt-to-income ratio to a stable path. If foreign debt-to-exports is the one following a divergent path, as in case 2, then above a threshold value for d, capital inflows may cease. The economy becomes unable to pay for its external liabilities. Consequently, it may run for the support of international organizations. Sometimes that means a debt renegotiation with a lower interest rate. Sometimes financial support implies the imposition of large real exchange rate devaluations and cuts in public expenditures. If the real devaluation is successful, it reduces the propensity to import. Because of its regressive impact on the distribution, it also reduces the propensity to consume. In any case, these different scenarios show that parameters do not tend to remain still while debt ratios follow and explosive dynamics. Nevertheless, we abstract from those complex feedback effects since they do not arise from any mechanical change in the parameters but are contingent outcomes from policy decisions. We take those parameters as given, not assuming any policy response in these cases.[14] We can, thus, move forward to the policy choices presented in cases 4 and 5.

Case 4 describes the case in which there is a ceiling to the foreign debt-to-exports ratio, and fiscal policy follows the rule presented in section 3.4. This experiment builds on the same parameters as the baseline case. The only difference is that the fiscal policy rule is executed from period 30 onward. Figure 3 compares the performance of both

[14] Modeling mechanical adjustment in the parameters due to an explosive debt dynamics would not be accurate. Future research can explore alternative scenarios of economic policy in the face of increasing debt ratios.

debt ratios in the baseline case with case 4. It shows that fiscal policy intervention is successful in keeping the foreign debt-to-exports ratio below the threshold of $\bar{d}$. The objective of fiscal policy is to reduce the share of public expenditures on autonomous demand by temporarily slowing the pace of public expenditures' growth. The decrease in the steady-state value for σ implies also a decrease in the equilibrium public debt-to-income ratio. In the baseline case, b converges to 1.01, whereas after the intervention of fiscal policy it goes to 0.80.

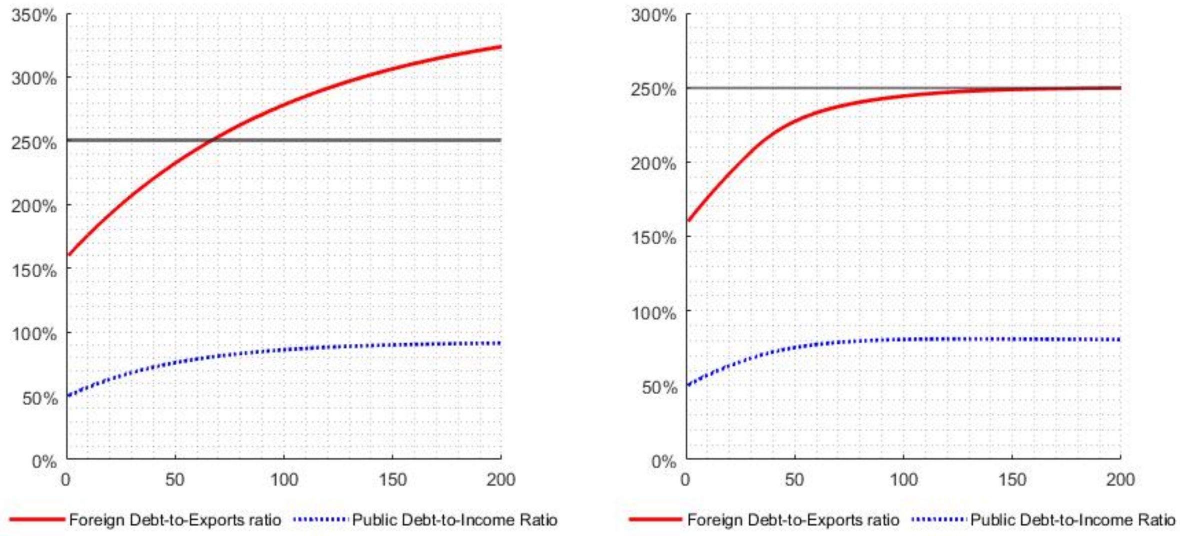

FIGURE 3: *Debt ratios in the baseline case (left) and the fiscal policy case (right)*

Figure 4 shows the continuous decrease in the share of public expenditures in autonomous demand (σ) after the period in which fiscal policy starts. This share slowly converges to $\bar{\sigma}$, which corresponds to the share of public expenditure in autonomous demand that is compatible with the ceiling to the foreign debt-to-exports ratio.

The graph at the bottom-right of Figure 4 shows the evolution of the growth rates of output and each component of autonomous demand. Public expenditures' growth rate decreases immediately after the intervention of fiscal policy. Output growth also decreases in the same period, but less than public expenditures since exports keep growing at the same rate. Both public expenditures and output growth rates converge to the growth rate of exports. This convergence happens as fiscal policy brings σ closer to $\bar{\sigma}$, implying the stabilization of the level of d. The picture also shows this adjustment is very gradual. Despite the decrease in the public expenditures' growth rate, it remains close to the growth rate of exports. The adjustment in σ is also shown to be progressive and slow. This is a desirable property of this policy rule.

Nonetheless, policymakers may reject a contractionary fiscal policy, preferring to promote industrial policies that foster structural change. This alternative is simulated

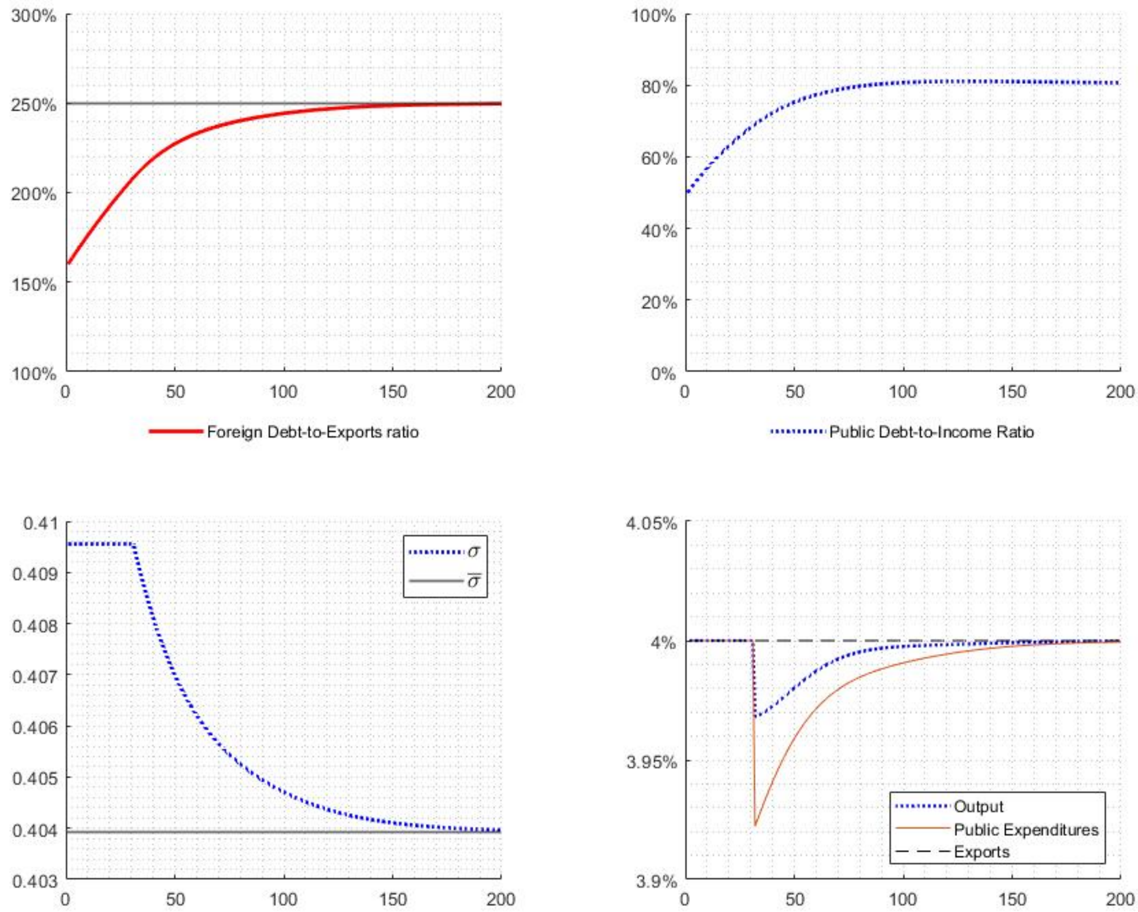

FIGURE 4: *Evolution of debt ratios, share of public expenditure in autonomous demand, and growth rates for case 4.*

in case 5, summarized in Figure 5. Suppose that the government executes this type of policy in period 30. If industrial policy succeeds, then the growth rate of exports accelerates a few periods after their implementation.[15] Assume that the industrial policies do work, affecting the economy after 10 periods. In that case, from period 40 onward, the growth rate of exports accelerates. In the experiment that means increasing the growth rate of exports from 4% to 4.5%. In line with the balance of payments constrained growth tradition, higher growth of exports may be associated with an increase of the income elasticity of exports. That may happen as the economy

[15] Naturally, the relation among industrial policies, structural change, and growth is complex. We cannot ensure the effectiveness of these policies. In fact, the simulation of case 5 indicates the extent of the boost in exports required to avoid fiscal austerity and still achieve stability of the debt ratios. In practice, policymakers can choose among many different combinations of the two pure cases (4 and 5) compared here.

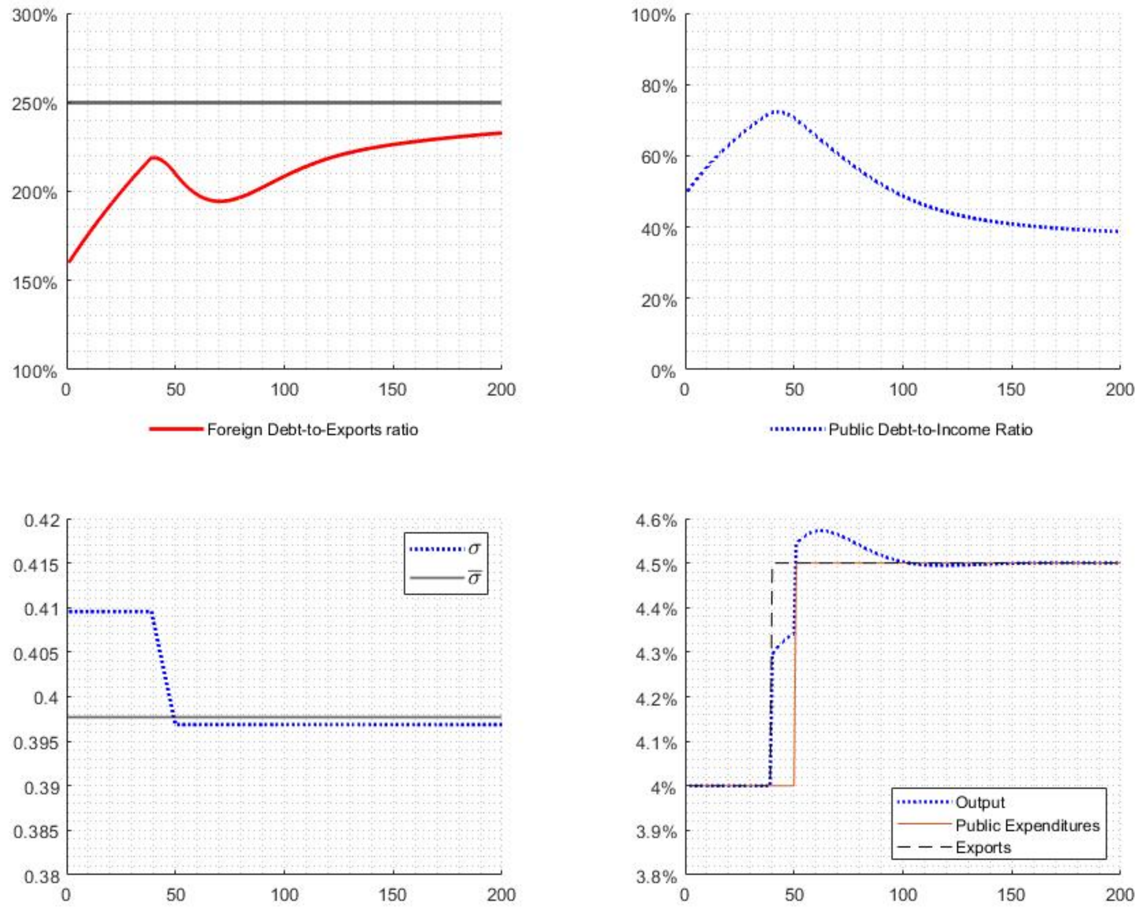

Figure 5: *Evolution of debt ratios, share of public expenditure in autonomous demand, and growth rates for case 5.*

develops production in more technologically intensive industries (Araujo and Lima, 2007; Cimoli et al., 2020, 2009).

The higher growth rate of exports for the same $\bar{d}$ leads to a new $\bar{\sigma}$ (value for the share of public expenditure compatible with the ceiling to the foreign debt-to-exports ratio). Note that the $\bar{\sigma}$ of case 5 is smaller than the $\bar{\sigma}$ of case 4. A new steady state can only be achieved if the public expenditures growth rate accelerates, reaching the new growth rate of exports. Nevertheless, that can only happen after σ becomes smaller or equal to $\bar{\sigma}$. Thus, the economy converges to a new steady-state.

Initially, the output growth rate follows the original steady-state. It accelerates after the increase in the growth rate of exports. In the following periods, it keeps accelerating, as σ decreases, increasing the impact of exports on output. Finally, when the public expenditures growth rate adjusts to exports growth, output growth converges to a new stead state. It becomes higher than the growth rate of autonomous demand because of the temporary increase in the propensity to invest, which is necessary to restore normal utilization after an acceleration of growth. Finally, output growth approaches the growth rate of autonomous expenditures in a new fully adjusted position.

The foreign debt-to-exports ratio also converges and remains below $\bar{d}$ in case 5. In turn, the public debt-to-income ratio converges to a lower value than in case 4. Lower b^* in case 5 results from the higher steady-state growth rate of output, the lower share of public expenditure in autonomous demand (σ^*), and the lower ratio of autonomous demand to total output (z^*). The lower z^* is associated with a higher h^*. Table 1 shows the equilibrium propensity to invest is higher in case 5, in which the economy presents a higher output growth rate than in the other cases. In sum, if the chosen industrial policies successfully affect exports growth, then case 5 provides better performance in terms of output growth than the fiscal policy executed in case 4. The growth rates of these two cases are compared in Figure 6.

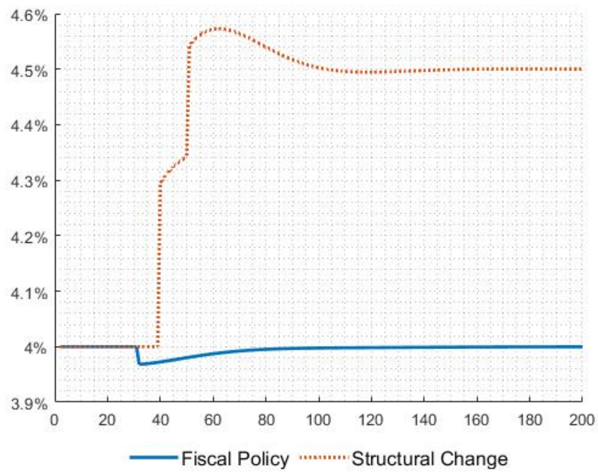

FIGURE 6: *Output growth rate in case 4 (Fiscal Policy) and case 5 (Structural Change).*

The simulated experiments support the theoretical results presented in this paper. An interest rate smaller than the growth rate is crucial for the long-run stability of public debt. An international interest rate smaller than exports growth has the same importance for the external debt. The fiscal policy proposed in section 3.4 is successful in keeping the foreign debt-to-exports ratio below $\bar{d}$ by adjusting the share of public expenditures in autonomous demand. After the adjustment in σ is finished, public expenditure can follow the same pace as exports, keeping a constant σ and allowing the economy to converge to a steady-state compatible with the limit to external debt. The decrease in σ also implies a decrease in the equilibrium level of the public debt-to-income ratio in the scenario with the fiscal policy. Figure 4 shows the adjustment in σ is slow and gradual. The impact of the fiscal policy on output growth also reveals a gradual adjustment of this variable. The last scenario shows that debt ratios can converge under a better economic performance when structural policies are successfully applied.

Different authors have emphasized the importance of these policies to relax the external constraint by increasing the income elasticity of exports (Araujo and Lima, 2007; Cimoli et al., 2020, 2009). Such policies can improve economic performance. Indeed, case 5 presented a higher output growth rate than previous cases. It also ensured convergence of the foreign debt-to-export ratio to a value below $\bar{d}$. The public-debt-to income ratio converged to a smaller value than in the other cases since output growth was larger and both σ and z^* were smaller.

Overall, the five experiments show that there is no *a priori* obstacle to the long-run sustainability of growth led by autonomous demand as determined by public expenditures and exports.

5 FINAL REMARKS

Growth in open economies depends fundamentally on the demand from government and foreign trade. However, complex interactions emerge from integrating government and trade in a growth process. We approached this issue in a supermultiplier model in which exports and public expenditure compose autonomous demand. This framework allows for examining the long-run stability of public and foreign debt ratios, and, thus, the sustainability of growth led by public expenditure and exports. The convergence of the public debt-to-income ratio requires that the output growth rate is larger than the interest rate on debt service. In turn, the convergence of the foreign debt-to-exports ratio depends on an analogous condition, *i.e.*, that exports growth is larger than the interest rate on foreign debt service. These conditions define the long-run sustainability of growth led by public expenditures and exports.

The model also revealed that the equilibrium values for debt ratios vary with the share of each expenditure in total autonomous demand. These shares therefore can be managed through fiscal policy, bringing debt ratios toward desirable levels. The external constraint to growth appears in this model as a limit to foreign indebtedness, associated with constraints in international liquidity. Thus, we proposed a fiscal policy rule on the basis of the moderation of government demand to meet a ceiling in the external debt-to-exports ratio. The fiscal policy rule successfully keeps the external debt ratio below the upper threshold in a gradual path. However, this policy has a cost in terms of the average growth rate. Such a cost may be mitigated by industrial policies that increase the income elasticity of exports. Although industrial policies may not affect growth and structural change as expected, in case of success, they ensure the convergence of debt ratios with better growth performance. Simulations support this conclusion by comparing alternative policy scenarios. Future research may include other properties in the model, providing additional insights for policy choices in open economies.

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