Marwil J. Dávila-Fernández
Serena Sordi

Thirlwall's law: Binding-constraint or centre-of-gravity?
A possible Kaleckian solution

n. 853 – Aprile 2021
Thirlwall’s law: Binding-constraint or centre-of-gravity?
A possible Kaleckian solution*

Marwil J. Dávila-Fernández  Serena Sordi
Bucknell University  University of Siena

April 2021

Abstract

Thirlwall’s law is one of the most powerful empirical regularities among demand-led growth theories. In recent years, the challenges imposed by globalisation have led to a new wave of studies incorporating into this framework topics such as ecological sustainability, the complexity of innovation processes, the role of institutions, the composition of external imbalances, and gender issues. We notice some overlapping between two alternative interpretations: one that sees the law as a binding-constraint and another that adopts a centre-of-gravity perspective. It is argued that they are rather complementary. By means of a simple Keynesian multiplier model compatible with Harrodian instability, we show that assuming a balance-of-payments ceiling to growth gives rise to persistent and bounded fluctuations such that the external constraint works as an asymmetric centre-of-gravity. There is no need to impose a floor to output. The model is compatible with different sources of autonomous demand. Numerical simulations show the robustness of our results with respect to alternative scenarios.

Key words: Dynamic Harrod trade-multiplier; Growth; Open economies; Two-stroke oscillator

JEL: F43; O11; O40

*We are grateful to James Newell for his carefully reading and suggestions for improvement. The usual caveats apply.
1 Introduction

Frequently referred to as the dynamic Harrod trade-multiplier, the idea of a balance-of-payments constraint on growth stands as one of the most powerful empirical regularities among alternative theories of growth and distribution. On the assumption that countries trading in a foreign currency cannot sustain persistent and increasing current account imbalances, Thirlwall’s (1979) law states that the long-run performance of a given economy is well-approximated by the ratio between the income elasticity of exports over imports multiplied by the rate of growth of the rest of the world. This model offers a unique framework that combines demand and supply constraints to explain international growth rate differences and structural change.

Over the past four decades, the model has been generalised in a number of directions, incorporating capital flows, multisectoral issues, and adjustment dynamics. Textbook presentations that provide accessible, though less technical, treatments of the strengths and weaknesses of the model include Lavoie (2014, pp. 513-540) and Blecker and Setterfield (2019, pp. 425-512). It must be noted, however, that the challenges imposed by globalisation have led to a new wave of studies that incorporate into this framework topics such as ecological sustainability, the role of institutions and innovation, as well as gender issues. Old debates such as whether the balance-of-payment adjusts through prices, income or trade-elasticities (e.g. Krugman, 1989; Alonso and Garcimartin, 1998), or concerns that the law was merely expressing a tautology (see McCombie, 2011; Ros, 2013, pp. 223-245; Blecker, 2016) are giving way to a refreshed research agenda.

Among these recent developments, we identify some overlapping between two alternative interpretations. A more traditional view understands the rate of growth compatible with equilibrium in the balance-of-payments as a binding-constraint, resulting in a stable attractor. In the short-term, a country might grow faster or slower than this rate. In the long-term, nonetheless, the economy will tend towards it. This approach can be seen in theoretical contributions that explicitly or implicitly separate between time frames (Cimoli and Porcile, 2014; Ribeiro et al., 2017) as well as in empirical studies that either apply cointegration or Generalised Method of Moments (GMM) estimators (Gouvêa and Lima, 2010; Romero and McCombie, 2016). On the other hand, we have those that argue the long-run trend cannot be separated from what happens in short-period situations. In that case, Thirlwall’s law is seen as a centre-of-gravity of the economy (e.g. Garcimartin et al., 2016; Dávila-Fernández and Sordi, 2019). The system is never in a state of rest, continuously fluctuating around the demand-led trend.

While the difference might sound subtle, we believe it has important policy implications. As long as international conditions are relatively favourable, the first approach implies that, in the short-term, policy makers could treat the external constraint as a secondary issue. This is specially the case when the economy is confronted with more “urgent” needs such as, for example, high unemployment rates. However, the conclusion might be misleading, wrongly downplaying the importance of equilibrium in the current account. Alternatively, to conceive the rate of growth compatible with equilibrium in the balance-of-payments as always being satisfied on average might induce an underestimation of the role of other aggregate demand components.

This paper argues that the two views may be rather complementary. The proposed solution goes back to what we consider to be the deep roots of M. Kalecki’s thinking and his lifetime contribution to economics: first, the originality of his formulation of the principle of effective demand and, second, his understanding that the long-run has no independent
entity. Aggregate demand matters both in the short as well as in the long period. Modelling growth and cycles altogether is “the only key to the realistic analysis of the dynamics of a capitalist economy” (Kalecki, 1968, p. 263, emphasis added). This means that any satisfactory representation of a modern economic system must somehow contemplate the fact that it is never in a state of rest. Finally, we have the fundamental role of time-lags in investment decisions. Productive capacities are not created instantaneously. The period that elapses from the moment that firms make a decision to expand their capital until the corresponding additional plant or equipment is ready for production cannot be overlooked.

By means of a simple Keynesian multiplier model in which the unique equilibrium solution is unstable, we show that assuming a balance-of-payments ceiling to growth gives rise to persistent and bounded fluctuations such that the external constraint works as an asymmetric centre-of-gravity. We show there is no need to impose a floor on output, nor is there need for any assumption concerning the behaviour of fiscal policy. In fact, the model is compatible with different sources of autonomous demand that may or may not grow at the same rate. Numerical simulations are performed showing the robustness of our results as well as their sensitivity to different scenarios.

The remainder of the paper is organised as follows. In the next section, we revisit the fundamentals of the dynamic Harrod trade-multiplier. We provide a brief overview of the main recent developments in the field differentiating between binding-constraint and centre-of-gravity perspectives. Section 3 presents a simple demand-led growth model that reconciles both views as well as a numerical example showing the emergence of periodic orbits. Some final considerations follow.

2 International trade and economic growth

As pointed out not long ago by Razmi (2016), what distinguishes Thirlwall’s law from other models is the role of the demand side in defining the nature of growth. Going back at least to Prebisch (1959), the fundamental assumption is that, in the long-run, a country that trades in foreign currency cannot sustain increasing balance-of-payments imbalances. McCombie and Thirlwall (1999, p. 49) argue that the term “balance-of-payments constraint” indicates that the performance of a country in international markets – from goods to services including the financial sphere – limits growth to a rate below the one internal conditions would warrant.

Abstracting from any price considerations, equilibrium in the balance-of-payments is approximated by equilibrium in trade, implying:

\[ X = M \] (1)

where \( X \) are exports and \( M \) stands for imports.

Suppose that the following traditional functions for exports and imports hold:

\[ X = X(Y^*), \quad X_{Y^*} > 0 \]

\[ M = M(Y), \quad M_Y > 0 \] (2)

where \( Y^* \) and \( Y \) correspond to foreign and domestic output, respectively.
Substituting (2) into Eq. (1) and log-differentiating, we have that the rate of growth compatible with equilibrium in the balance-of-payments, $g_{BP}$, is given by:

$$g_{BP} = \rho \frac{\dot{Y}^*}{Y^*}$$

(3)

where

$$\rho = \frac{\varphi}{\pi} \Rightarrow 1$$

corresponds to the ratio between the income elasticity of exports, $\varphi = \frac{\partial X}{\partial Y} \frac{Y^*}{X}$, and imports, $\pi = \frac{\partial M}{\partial Y} \frac{Y^*}{M}$. When $\rho > 1$, the economy is growing faster than its trade partners. In the case of a developing country, this means it is catching-up with the rest of the world. On the contrary, $\rho < 1$ suggests a process of falling-behind. Hence, $\rho$ can be understood as a measure of non-price competitiveness and has proved to explain effectively international growth rate differences.

In the past forty years, the model has been generalised in a number of directions, from incorporating capital flows (Thirlwall and Hussain, 1982; Moreno-Brid, 2003) to some initial assessments of technology and institutions (Cimoli, 1988; Fagerberg, 1988), and multisectoral issues (Araújo and Lima, 2007). The latter became the starting point of a fruitful empirical literature on the law. Building on a Pasinettian framework to explain uneven development (also see Araújo and Teixeira, 2004), the authors disaggregate trade into $n$ sectors such that non-price competitiveness responds to both the dynamics between and within different production activities. Higher growth rates can be achieved by moving the production structure towards sectors with higher income elasticity of exports relatively to imports as well as by changing the respective elasticities of each sector. In the first case, there is a progressive structural change in the composition of trade. In the second, the economy benefits from a broad process of technological upgrading which results from the assimilation and adaptation of new technologies.

2.1 A note on prices and the real exchange rate

By allowing exchange rates to influence trade performance, in the original presentation of the model $g_{BP}$ depends on the rate of change of relative prices. Under purchasing power parity, however, they cannot continuously appreciate or depreciate, thus justifying our choice to abstract from this element here. Still, one should recognise that the existing empirical evidence suggests a positive association between the level of the real exchange rate, RER, and economic growth, especially in developing countries. This correspondence appears to be such that an undervalued currency favours growth while overvaluation hurts it (e.g. Rodrik, 2008; for a literature review see Rapetti, 2020).

In fact, a number of scholars have formalised the hypothesis that $\rho$ directly responds to the level of relative prices (see Oreiro, 2016). Alternatively, authors such as Ribeiro et al. (2016), suppose non-price competitiveness mainly depends on technological and distributive variables, which in turn are a non-linear function of RER. Oreiro et al. (2020) adopt a similar route assuming that $\rho$ is crucially determined by the share of modern manufacturing industries in trade, the latter being a function of the exchange rate. The story is basically the same in Missio et al. (2017) who, in a disaggregated framework,

---

1For any generic variable $x$, the time derivative is indicated by $\dot{x}$, while $\dot{x}/x$ corresponds to its rate of growth. On the other hand, the derivative of any generic function $x(\tau)$ will be indicated as $dx/d\tau = x_\tau$. 


propose that the sectoral income-elasticities of exports and imports respond to relative prices. A differentiation between tradable and non-tradable sectors that formally addresses the problem of hidden unemployment is presented by Razmi et al. (2012).

2.2 External constraint or centre-of-gravity?

The challenges created by globalisation have led to a new and ongoing wave of studies that incorporate into this framework topics such as ecological sustainability (see Guarini and Porcile, 2016; Althouse et al., 2020), a deeper assessment of the role of technical change and innovation (e.g. Setterfield, 2011; Cimoli and Porcile, 2014; Cimoli et al., 2019), micro-foundations from an Agent-Based Modelling perspective (Dosi et al. 2019, 2020), and endogenous aspects of institutional change (Dávila-Fernández and Sordi, 2020; Porcile and Sanchez-Ancochea, 2020), as well as gender issues (as in Seguino, 2010; Seguino and Setterfield, 2010). In almost all cases, either the trade equations are modified to take into account a particular element of interest, or non-price competitiveness is directly endogenised with respect to a vector of explanatory variables.\(^2\)

Frequently, scholars have adopted a more traditional view in which the rate of growth compatible with equilibrium in the balance-of-payments is a binding-constraint that results in a stable attractor. Authors such Ribeiro et al. (2017) explicitly separate time frames. A certain model is assumed for the short-term, another for the medium-term, and finally, in the long-run, growth adjusts to the external constraint, which may or may not be endogenous to other macroeconomic variables (for example, see Porcile and Lima, 2010; Cimoli and Porcile, 2014). Some systems allow for a dynamic adjustment towards equilibrium in the balance-of-payments while others simply take it for granted as a long-run condition. This can also be seen in empirical contributions that either apply cointegration or Generalised Method of Moments (GMM) estimators (Gouvêa and Lima, 2010; Romero and McCombie, 2016; Romero and McCombie, 2018).

On the other hand, it has been argued that a comprehensive macroeconomic theory should contemplate the possibility of endogenous fluctuations. In the case of Spinola (2020, 2021) cycles have a decreasing amplitude. Nishi (2019) demonstrated the emergence of a stable orbit while Dávila-Fernández (2020) showed that fluctuations might be persistent and irregular. Besides the mechanisms underlying the interaction between labour market, goods market, and international trade – which always contemplate an endogenous natural rate of growth – the crucial element in these studies is a subtle reinterpretation of Thirlwall’s law as a centre-of-gravity. In that case, growth is not properly constrained by the balance-of-payments, but actually oscillates around \(g_{BP}\) without ever reaching a state of rest. This idea has also been explored from an empirical point of view by Garcimartin et al. (2016), who explicitly propose the development of a balance-of-payments constraint theory for business cycles (see Kvedaras et al., 2020).

The empirical contributions of Felipe et al. (2019) and Felipe and Lanzafame (2020) provide groundbreaking evidence in that direction. Their work refers to Indonesia and China, respectively, but comes with a distinct feature: the estimation of time-varying trade elasticities based on the Kalman filter. In this way, we end up with a measure of non-price

\(^2\)Thirlwall (2011) describes the historical antecedents of the model and revisits some of the main theoretical and empirical contributions. The rapid progress of computational tools and statistical packages has recently led to a large increase in the number of studies in the field that need to be properly assessed and systematised. Providing a comprehensive review of these developments, however, goes beyond the scope of this paper, and the task remains to be done in future research.
competitiveness that also changes over time. This opens up a large avenue of research on the determinants of $\rho$. Some preliminary results include the application of Bayesian Model Averaging (BMA) techniques to a large set of potential regressors. They show how the actual rate of growth permanently fluctuates around $g_{BP}$, which in turn also changes over time. We understand their findings to be in line with those that see Thirlwall’s law as a centre-of-gravity. The economy is never in a state of rest, continuously fluctuating around the demand-led trend.

Fig. 1 provides a graphical example of our previous discussion with $g_{BP} = 0.025$ indicated by the dotted blue line. Panel (a) depicts the more traditional scenario in which the rate of growth compatible with equilibrium in the balance-of-payments works as a stable attractor. In the very short-term, the economy might grow faster or slower than that rate. In the long-term, however, a country cannot sustain increasing current account imbalances, hence, growth will converge towards the rate determined by the external constraint. Panel (b) corresponds to the alternative scenario in which the economy fluctuates around $g_{BP}$, but only satisfies it on average. This difference has important theoretical and policy implications.

While scholars in both positions accept that international specialisation reflects the economic complexity of countries’ productive structures, those who adopt a centre-of-gravity approach might be overestimating the importance of trade performance. In this way, they run the risk of inducing policy makers to underestimate the role of other components of aggregate demand. On the other hand, the separation between time frames is also misleading. As long as international conditions are relatively favourable – or the country has enough foreign reserves – a binding-constraint approach implies policy makers do not have to worry about $g_{BP}$ when facing more “urgent” needs. This is because such a rate is more related to slow-motion processes of structural change. However, this conclusion may wrongly downplay the relevance for macroeconomic stability of equilibrium in the current account.
3  A possible Kaleckian solution

Our brief assessment of the recent literature on the balance-of-payments constrained growth model suggests some overlapping between two interpretations. While the difference might sound subtle, it has important implications for the way we understand the dynamics of the economy. In this Section, we argue that both views may actually be complementary rather than substitutes. The proposed solution goes back to what we consider to be the deep roots of M. Kalecki’s thinking and his lifetime contribution to economics: (i) the originality of his formulation of the principle of effective demand, (ii) his understanding that “the long-run trend is but a slowly changing component of a chain of short-period situations; it has no independent entity” (Kalecki, 1968, p. 263), and (iii) the fundamental role of time-lags in investment decisions.3

In an open economy, the expenditure identity is given by:

\[ Y = C + I + G + X - M \]

where \( C \) stands for consumption, \( I \) is investment, and \( G \) corresponds to government expenditures. For our purposes, \( Y \) can be divided into two main components:

\[ Y = Q(Y) + Z, \quad 0 < Q_Y < 1 \]  \hspace{1cm} (4)

that is, induced demand, \( Q(\cdot) \), and non-induced expenditures, \( Z \). Scholars more aligned with N. Kaldor’s ideas may argue that exports are the only truly autonomous component of demand. The so-called “neo-Kaleckians”, on the other hand, would perhaps assume that entrepreneurs’ “animal spirits” make investment partially non-induced. Others have made the case that government consumption is somehow semi-autonomous. For our purposes, the model is compatible with different sources of induced and non-induced demand that may or may not grow at the same rate.

Applying the implicit function theorem, Eq. (4) can be rewritten as:

\[ Y = H(Z), \quad H_Z > 0 \] \hspace{1cm} (5)

where \( H(\cdot) \) incorporates the Keynesian multiplier effect. This is a well-known textbook result that does not require a lengthy explanation. The level of demand is an increasing function of its non-induced component.

Log-differentiating Eq. (5), we have:

\[ \frac{\dot{Y}}{Y} = \varepsilon \frac{\dot{Z}}{Z} \]

where \( \varepsilon = \frac{\partial H}{\partial Z} \frac{Z}{H(Z)} > 0 \) is the elasticity of aggregate demand with respect to its autonomous component, standing for the dynamic Keynesian multiplier. Notice the similarities with Eq. (3). Indeed, if none of the domestic absorption components is autonomous, the economy

---

3Given that this is not a paper devoted to the history of economic thought, we shall not go into the details of the works of the Polish economist. For this purpose, the reader is invited to see the collection edited by Sebastiani (1989) that includes the views of R. Goodwin, N. Kaldor, D. Patinkin, and J. Steindl, among others. For a more recent assessment, see the book by López and Assous (2010) as well as Franke (2018). One should also notice that, despite using a similar “label”, our approach is completely different to what is frequently referred to as the “neo-Kaleckian” model for open economies (as in Blecker, 1989, and those that came afterwards).
will be export-led. To simplify notation, redefine $\dot{Y}/Y = g$ and $\dot{Z}/Z = z$. The expression above becomes:

$$g = \varepsilon z$$  \hspace{1cm} (6)

Depending on which assumptions we make regarding the dynamic multiplier, it is possible to accommodate the concept of Harrodian instability, which corresponds to the case $\varepsilon > 1$. Doing that, this elasticity can be written as:

$$\varepsilon = 1 + \phi$$  \hspace{1cm} (7)

where $\phi \geq 0$ captures the magnitude of the accelerator effect.\footnote{As an example that has gained some attention among alternative theories, Eq. (7) is compatible with Freitas and Serrano (2015) and the so-called “super-multiplier”. After imposing a large set of linear functional forms, they basically suppose $\varepsilon$ is equal to one plus variations in the investment-output ratio. Of course, the Hicksian multiplier-accelerator model with a “floor” and/or a “ceiling” continues to be the most successful mechanism generating persistent fluctuations (e.g., Gallegati et al., 2003; Sordi, 2006; Sushko et al., 2010). As it will become clear later, our approach has several similarities with this last set of contributions, even though it is conceived for an open economy.}

We understand $\phi$ to be further divided into two main forces. On the one hand, we recognise that firms see their prospects improve when demand is accelerating, and thus plan new investment projects, $A$. In modelling growing economies, the determination of investment refers to relative rather than absolute changes of the capital stock. On the other hand, one should take into account that, \textit{ceteris paribus}, growth implies a cost of adjusting the current stock of machinery and equipment, $B$. Accordingly:

$$\phi = A - B$$  \hspace{1cm} (8)

While firms respond to an increase in the rate of growth of demand by increasing capital accumulation, this process does not continue indefinitely. As the rate of growth accelerates, there is an increase in import requirements that, \textit{ceteris paribus}, results in a trade-deficit. Given that, in the long-run, a country that trades in foreign currency cannot sustain increasing balance-of-payments imbalances, $g_{BP}$ defines the relevant ceiling. When international conditions are favourable, the external constraint is less stringent and the economy can handle greater trade deficits. In formal terms:

$$A = \min\{v \dot{g}, \gamma g_{BP}\}$$  \hspace{1cm} (9)

where $v > 1$ is the response of the piecewise-linear function $A(\cdot)$ to variations in the rate of growth of demand before reaching the ceiling. The stringency of international credit markets is captured by $\gamma > 0$, with a high value of this parameter standing for periods of high liquidity.

Indeed, one of the immediate concerns regarding the original formulation of Thirlwall’s law was by how much growth could deviate from $g_{BP}$ as in Eq. (3). Thirlwall and Hussain (1982) address the issue in a rather elegant way, but it was Moreno-Brid (2003) who then imposed a long-term constraint defined as a constant ratio of the current account deficit to income. Most studies interested in the behaviour of current account deficits have followed a similar route. Minor variants can be found in Bhering et al. (2019), who redefine the external constraint in terms of the capacity to export. Our approach is related to this
Figure 2: A piece-wise linear function for the accelerator effect with only an external constraint ceiling. In this example, we assume parameter values $v = 1.5$, $\gamma = 2$, and $g_{BP} = 0.03$.

literature to the extent that we take the constraint as a multiple of $g_{BP}$.$^5$

One could also think of $\gamma$ as related to foreign exchange reserves and the level of confidence in financial markets. High reserves limit external vulnerability to shocks during times of crisis or when access to borrowing is curtailed. In this way, they make it possible to have a less-binding ceiling on accumulation. Fig. 2 provides a graphical representation of Eq. (9). For values of $\dot{g} \leq \gamma g_{BP}/v$, $A(\cdot)$ is a linear increasing function of $\dot{g}$ with a slope $v$. After the ceiling is reached, $\dot{g} > \gamma g_{BP}/v$, external conditions impose a constraint on capital accumulation. For example, $\gamma = 2$ implies international markets allow growth to accelerate in principle up to twice the size of the rate compatible with equilibrium in the balance-of-payments.

On the other hand, the cost of adjusting the capital stock to growth is simply:

$$B = \beta \dot{g}$$  \hspace{1cm} (10)

where $0 < \beta < 1$ guarantees that there is actually an accelerator effect after all.

The idea of a lag in investment decisions goes back to Kalecki (1935) and was later taken up by Goodwin (1951) in his nonlinear accelerator model (for a modern revisitation, see Sordi, 2006; Franke, 2018). Indeed, productive capacities are not created instantaneously. The period that elapses from the moment that firms make a decision to expand their capital until the respective additional plant or equipment is in place and ready for production corresponds to the implementation lag, $0 < \theta < 1$. To use Kalecki’s terminology, investment goes through a “gestation period”. This means that $A(\cdot)$ depends on decisions taken at time $t - \theta$ while $B(\cdot)$ happens in $t$. Substituting Eqs. (9) and (10) into Eq. (8), and taking

$^5$The recent European debt crisis has demonstrated that when internal imbalances are out of hand, they may constrain output and have implications for economic growth in a severe way. In this respect, Soukiazis et al. (2012, 2014), building on Thirlwall’s law, have presented a growth model that takes into account both internal and external imbalances. The crucial step consists in disaggregating imports into the various components of aggregate demand. For an assessment of the interplay between capital flows, terms-of-trade and the exchange rate, see Pérez-Caldentey and Moreno-Brid (2019). A comparison between wage-led and debt-led regimes in a balance-of-payments constrained economy is investigated by Pérez-Caldentey and Vernengo (2017).
account of the previous discussion:

$$\phi (t; \theta) = \min \{v \dot{g} (t - \theta), \gamma g_{BP}\} - \beta \dot{g} (t)$$  \hspace{1cm} (11)

In order to show that Harrodian instability is not being artificially tamed by the inclusion of lags, we impose:

$$v > \beta + \theta / z$$

so that firms strongly respond to demand signals.

Substituting Eq. (11) into (7), we obtain:

$$\varepsilon (t; \theta) = 1 + \min \{v \dot{g} (t - \theta), \gamma g_{BP}\} - \beta \dot{g} (t)$$  \hspace{1cm} (12)

such that the dynamic Keynesian multiplier is procyclical with a time-lag. An acceleration of growth produces a similar response from capital accumulation that magnifies the multiplier effect up to the point at which the external constraint binds. The respective adjustment costs are always paid after the decision to add a new unit of capital is made.

Introducing Eq. (12) into (6) and rearranging, it follows that:

$$z \beta \dot{g} (t + \theta) + g (t + \theta) - z \min \{v \dot{g} (t - \theta), \gamma g_{BP}\} = z$$

from which, expanding the two leading terms \( \dot{g} (t + \theta) \) and \( g (t + \theta) \) in a Taylor series, while dropping all but the first two terms in each, and dividing the resulting expression by \( z / \beta \theta \), we obtain a generalisation of the so-called Lord Rayleigh-type equation:

$$\ddot{g} + F(\dot{g}) + G (g) = \frac{1}{\beta \theta}$$  \hspace{1cm} (13)

where

$$F(\dot{g}) = \left( \frac{1}{\theta} + \frac{1}{z \beta} \right) \dot{g} - \frac{\min \{v \dot{g}, \gamma g_{BP}\}}{\beta \theta}$$

and

$$G (g) = \frac{g}{\beta \theta z}$$

For simplicity, given that Eq (13) only depends on variables in \( t \), we omit it from the expression. Sordi (2006) provides a complete assessment of the mathematical properties of this family of equations with an application to economics.

In steady-state, \( \ddot{g} = \dot{g} = 0 \). The equilibrium rate of growth of the economy, \( \bar{g} \), is determined by the rate of growth of autonomous demand:

$$\bar{g} = z$$

If we accept that exports are the only truly non-induced component of aggregate demand and that there is equilibrium in the current account, then \( \bar{g} = g_{BP} \). The model is, nonetheless, perfectly compatible with other non-capacity generating autonomous sources, that do not need to grow at the same rate, so that \( \bar{g} \gtrless g_{BP} \). Moreover, as demonstrated in the appendix, the equilibrium point is unstable, in line with the Harrodian principle of instability.
3.1 A numerical example

We rely on numerical simulations to better appreciate the economic properties of the model and the robustness of our results to different scenarios. As a note of caution, it is important to highlight that we are not calibrating a real economy. Parameter values were chosen using a trial-and-error approach, making adjustments so as to obtain trajectories that are economically meaningful:

\[ \beta = 0.65, \ \theta = 0.01, \ z = 0.03, \ v = 1.5 \]

while the rate of growth compatible with equilibrium in the balance-of-payments was supposed to be lower than the rate of growth of demand:

\[ g_{BP} = 0.025, \ \gamma = 2 \]

such that by all means there is a binding external constraint.

Fig. 3 shows the emergence of a clockwise periodic oscillator in the \((g, \dot{g})\) phase space. The dotted blue line corresponds to Thirlwall’s law while the red one is the rate of growth of the autonomous components of aggregate demand. Panel (a) brings a surprising result. Fluctuations are asymmetric with respect to the equilibrium solution. In fact, \(g_{BP}\) works as if it were the centre-of-gravity of the economy with a clear orbit around it. We know that by construction this is not the case. The economy never comes to a state of rest, and neither \(g_{BP}\) nor \(z\) can be considered stable attractors. Plotting the time series, as in panel (b), reveals the role of the ceiling in “constraining growth”.

Suppose instead:

\[ g_{BP} = 0.03 \]

that is, the rate of growth compatible with equilibrium in the balance-of-payments is equal to the actual growth rate of non-induced demand. As previously discussed, this is equivalent to assuming that there is equilibrium in the current account, domestic absorption is fully induced and only exports are autonomous. Panels (c) and (d) illustrate such case. Given that \(z = g_{BP}\), red and blue dotted lines overlap. Still, the oscillator is there, giving rise to asymmetric persistent fluctuations. An external observer would have the impression that growth is limited by the external constraint, though in fact it is also the centre-of-gravity of the economy.

A final example is the case in which, for instance:

\[ g_{BP} = 0.035 \]

We are acknowledging again that other components of demand such as government expenditures, for example, might be non-induced. However, \(z < g_{BP}\) implies that the country in question is accumulating reserves in foreign currency. In panel (e) we have that Thirlwall’s law is completely outside the orbit. In fact, time-series reported in panel (f) show \(g_{BP}\) is significantly above the peak of the cycle. Notice how the dotted blue line appears now on

---

\(^6\)A careful analysis of the underlying second order differential equation reveals that the emerging periodic attractor corresponds to the so-called two-stroke oscillator. Providing a complete mathematical assessment of its properties goes beyond the scope of this paper. In an ongoing research project, we present a graphical and analytical treatment of this issue. Here, it is enough to highlight that it is sufficient to have a ceiling in the piece-wise accelerator function to obtain the asymmetric cycle. For a formal presentation of this family of oscillators, the reader is invited to see Sordi (2006).
Figure 3: Two-stroke clockwise oscillator when $z > g_{BP}$, panels (a)-(b), $z = g_{BP}$, panels (c)-(d), and $z < g_{BP}$, panels (e)-(f).
the right of the red one. While we can discuss how sustainable this process is, the model indicates trade-cycles emerge only because of the existence of the external constraint, without the need of a floor to control for Harrodian instability.

One may wonder what is the impact of an increase in investment implementation time-lags. We know $\theta$ has no effect over the equilibrium rate of growth. Nonetheless, a first intuition from Eq. (11) is that higher gestation periods are related to business-cycles of lower amplitude. This basically comes from the fact that accumulation costs are paid at time $t$ while the decision to invest is made at time $t - \theta$. Fig. 4 compares the size of the orbits when

$$\theta = 0.01$$

in green, and

$$\theta = 0.015$$

in magenta, confirming our previous insights. For convenience, we only present the case in which $z = g_{BP}$ but the result is fundamentally the same for $z \geq g_{BP}$. The intersection between the dotted black and red lines corresponds to the equilibrium point.

From an economic point of view, it makes sense that more complex projects require additional care in preparation and execution, which in turn are related to a higher $\theta$. Such a characteristic brings resilience in the form of higher inertia to the system. Besides $\rho$, this is a fundamental difference between developed and developing economies, and helps to explain why growth is more volatile in the latter – or to use the Financial Times terminology, the occurrence of “chicken flight” growth episodes. The combination of $\rho < 1$ and a relatively high $\theta$ results in an economy that is highly volatile and falls behind, even when the level of confidence of international financial markets is high.

The issue of the adjustment of $z$ towards $g_{BP}$ remains open and goes beyond the scope of this article. Still, it is not difficult to present a basic structure for the supply-side of the economy, showing a simple way in which labour markets and labour productivity might guarantee that actual and natural rates of growth do not fall apart. Let us consider the following Leontief production technology:

$$Y = \min \left\{ \frac{K}{\theta}, qNe \right\}$$

(14)
where $K$ stands for the capital stock, $\theta$ is the capital-output ratio, $q = Y/L$ corresponds to labour productivity, with $L$ indicating the number of workers employed, $N$ is the labour force, and $e = L/N$ is the participation rate.

Dynamic efficiency requires:

$$\begin{align*}
\frac{\dot{\theta}}{\theta} &= \frac{\dot{K}}{K} - g \\
\frac{\dot{e}}{e} &= g - \frac{\dot{q}}{q} - \frac{\dot{N}}{N}
\end{align*}$$

so that equilibrium in the goods and labour markets result in:

$$\begin{align*}
\frac{\dot{K}}{K} &= g \\
\frac{\dot{q}}{q} + \frac{\dot{N}}{N} &= g
\end{align*}$$

(15)

Capital accumulation directly follows the rate of growth of aggregate demand, as described throughout the paper. Persistent fluctuations in accumulation result from the interaction between the external constraint and the dynamic Keynesian multiplier. Moreover, we have that the so-called natural rate of growth, $\dot{q}/q + \dot{N}/N$, is equal to the actual rate of growth.

The profession has long recognised that, to a large extent, new technologies are capital embodied either in the form of tangible or intangible assets. We could think of such a mechanism in terms of the evolution of hardware and software components in computers. This means that, as the economy expands, the presence of dynamic economies of scale allows a faster increase in labour productivity through a process of learning-by-doing. Such a mechanism is frequently referred to as Kaldor-Verdoorn’s law. We shall also acknowledge that a tight labour market has a positive effect on $\dot{q}/q$. An increase in the bargaining power of workers leads to increases in real wages relative to labour productivity, reducing profitability. Firms respond accordingly by increasing their search for labour saving techniques. Hence, the vector of explanatory variables includes but is not limited to:

$$\frac{\dot{q}}{q} = P(g,e), \quad 0 < P_g < 1, \quad P_e > 0$$

(16)

where $P_g$ stands for Kaldor-Verdoorn’s coefficient.\footnote{McCombie and Spreatico (2016) demonstrated that if a linear form of Eq. (16) is adopted, “the intercept cannot and should not be interpreted as the separate contribution to economic growth of the rate of exogenous technical change” while “the Verdoorn coefficient also should not be interpreted as a measure of increasing returns to scale per se” (p. 1131, emphasis added).}

Substituting Eq. (16) into (15), and assuming the labour force grows at an exogenous rate, $n$, we obtain:

$$P(g,e) + n = g$$

(17)

Applying the implicit function theorem, it is easy to see:

$$\frac{de}{dg} = \frac{1 - P_g}{P_e} > 0$$
Figure 5: Participation rates and the rate of growth of labour productivity when $z = g_{BP} = 0.03$ and $\theta = 0.01$.

and we recover the positive correspondence documented in the literature between economic activity and participation rates.

Fig. 4 provides a graphical representation of persistent fluctuations in $e$ and $\dot{q}/q$ when Eq. (17) is always satisfied. We suppose a linear specification for the rate of growth of labour productivity:

$$P(g, e) = -\alpha_0 + \alpha_1 g + \alpha_2 e$$

and the following parameter values:

$$\alpha_0 = 0.05, \quad \alpha_1 = 0.5, \quad \alpha_2 = 0.06, \quad n = 0.01$$

which are in line with studies in the field (e.g. McCombie and Sprefico, 2016; Tavani and Zamparelli, 2017). A stagnant economy that does not employ a significant share of its population will actually experience a reduction in $q$, i.e. $P(0, 0) < 0$. This is in accordance with the idea that under persistently high unemployment rate there could be a deterioration in human capital and in the capacity for learning-by-doing.

Oscillations in the $(\dot{g}, e)$ and $(\dot{g}, \dot{q}/q)$ phase space are anti-clockwise oriented, as shown in Fig. 5. During the initial increasing part of the business cycle, output accelerates leading to higher labour market participation. Such a process does not go on forever. On the one hand, the external constraint imposes a limit to capital accumulation. On the other hand, strong workers continue pushing $q$ upwards. At a certain point, growth slows down eventually reducing $e$. Depending on the strength of the Kaldor-Verdoorn effect, the rate of growth of labour productivity will fall accordingly, as depicted in panels (a) and (b).

We should emphasise that the description just provided depends on Eq. (17) being always satisfied. Since we are not taking into account the dynamic adjustment of employment to the difference between actual and natural rates of growth, the picture remains somehow incomplete. At this stage, there are no “feedback” effects from the labour market or income distribution to aggregate demand, which is also an important limitation. While recognising the importance of these issues, we leave the task of building a full model to future research (for attempts in that direction see Perrotini-Hernández and Vázquez-Muñoz, 2019; or Dávila-Fernández, 2020). For the moment, it is sufficient to note the message that the external binding-constraint might be the centre-of-gravity of the economy.
4 Final considerations

Thirlwall’s law has proven to be a useful tool not only to explain uneven development but also to search for answers to the new challenges facing modern economies. Old debates are giving way to a new research agenda in the field and the balance-of-payments constrained growth model has been updated accordingly. This includes considerations on climate change, the complexity of innovation processes, the role of institutions, the composition of external imbalances, and gender issues.

There is a certain convergence in terms of policy insights indicating the need for articulating a set of industrial policies targeting R&D investments, the development of firms’ innovative and adaptive capabilities, and a positive attitude towards change. Additional elements include the role of green-innovation as a window of opportunity for a new technological paradigm. Price concerns matter and the benefits of a competitive exchange rate have been properly documented. Still, the main message remains that non-price competitiveness is the crucial element. In this respect, the recent use of state-space models to estimate the law creates a unique opportunity to investigate its determinants between and within productive sectors.

As a way of concluding, we notice some overlapping of two alternative interpretations: one that sees the law as a binding-constraint and another that adopts a centre-of-gravity perspective. It is argued that they are rather complementary. By means of a simple Keynesian multiplier model compatible with Harrodian instability, we showed that assuming a balance-of-payments ceiling to growth gives rise to persistent and bounded fluctuations such that the external constraint works as an asymmetric centre-of-gravity. There is no need to impose a floor on output. The model is compatible with different sources of autonomous demand. Numerical simulations showed the robustness of our results to alternative scenarios.

A Appendix

The model’s underlying second order differential equation is given by:

\[ \dot{g} + F(\dot{g}) + G(g) = \frac{1}{\beta \theta} \] (A.1)

Define the auxiliary variables \( x_1 \) and \( x_2 \), such that:

\[
\begin{align*}
    x_1 &= g \\
    x_2 &= \dot{g}
\end{align*}
\]

which implies:

\[ x_2 = \dot{x}_1 \]

Hence, (A.1) can be rewritten as a two-dimensional first-order dynamic system:

\[
\begin{align*}
    \dot{x}_1 &= x_2 = h_1 (x_2) \\
    \dot{x}_2 &= -G(x_1) - F(x_2) + \frac{1}{\beta \theta} = h_2 (x_1, x_2)
\end{align*}
\]
In equilibrium, $\dot{x}_1 = \dot{x}_2 = 0$. We thus have:

$$G(\bar{x}_1) + F(\bar{x}_2) = \frac{1}{\beta \theta}$$

$$\bar{x}_2 = 0$$

Recall that $A(\cdot)$ is a piece-wise function with two regimes.

- In the first regime, where $\dot{\theta} \leq g_{BP}/v$, the system admits a unique equilibrium solution, $P = (\bar{x}_1, \bar{x}_2)$, defined and given by:

$$\begin{align*}
\bar{x}_1 &= z \\
\bar{x}_2 &= 0
\end{align*}$$

Linearising the dynamic system around $P$, we obtain:

$$\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
J_P
\begin{bmatrix}
x_1 - \bar{x}_1 \\
x_2 - \bar{x}_2
\end{bmatrix}$$

(A.2)

where

$$\begin{align*}
\dot{j}_{11} &= \frac{\partial h_1(x_2)}{\partial x_1} \bigg|_P = 0 \\
\dot{j}_{12} &= \frac{\partial h_1(x_2)}{\partial x_2} \bigg|_P = 1 \\
\dot{j}_{21} &= \frac{\partial h_2(x_1, x_2)}{\partial x_1} \bigg|_P = -G_{x_1} < 0 \\
\dot{j}_{22} &= \frac{\partial h_2(x_1, x_2)}{\partial x_2} \bigg|_P = -F_{x_2} \gg 0
\end{align*}$$

so that

$$|J|_P - \lambda I = \begin{vmatrix} -\lambda & 1 \\
-G_{x_1} & -F_{x_2} - \lambda \end{vmatrix} = \lambda (F_{x_2} + \lambda) + G_{x_1} = \lambda^2 + F_{x_2} \lambda + G_{x_1}$$

The characteristic equation is

$$\lambda^2 + \left(\frac{1}{\theta} + \frac{1}{\beta z} - \frac{v}{\beta \theta z} \right) \lambda + \frac{1}{\beta \theta z} = 0$$

and eigenvalues

$$\lambda_{1,2} = \frac{-\frac{1}{\theta} - \frac{1}{\beta z} + \frac{v}{\beta \theta} \pm \sqrt{\left(\frac{1}{\theta} + \frac{1}{\beta z} - \frac{v}{\beta \theta z} \right)^2 - \frac{4}{\beta \theta z}}}{2}$$

Given that

$$v > \beta + \frac{\theta}{z}$$

the unique equilibrium point is either an unstable node or an unstable focus.
In the second regime, where \( g > g_{BP}/v \), the singular point becomes:

\[
\begin{align*}
\bar{x}_1 &= (1 + \gamma g_{BP}) z \\
\bar{x}_2 &= 0
\end{align*}
\]

The Jacobian matrix is similar to (A.2) while the characteristic equation is equal to

\[
\lambda^2 + \left( \frac{1}{\theta} + \frac{1}{\beta z} \right) \lambda + \frac{1}{\beta z}
\]

with eigenvalues

\[
\lambda_{1,2} = \frac{-\frac{1}{\theta} - \frac{1}{z\beta} \pm \sqrt{\left(\frac{1}{\theta} + \frac{1}{z\beta}\right)^2 - 4\frac{1}{\beta z}}}{2}
\]

so that

\[
\begin{align*}
\lambda_1 &= -\frac{1}{\beta z} \\
\lambda_2 &= -\frac{1}{\theta}
\end{align*}
\]

Since both eigenvalues are real and negative, the equilibrium for this regime is a stable node.

References


