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Maurice Allais on Equilibrium and Capital in some of his 1940s Writings

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Abstract - The article discusses M. Allais' contributions on equilibrium and capital during the 1940s. While in his Traité (1943) Allais formalizes for the first time an intertemporal general equilibrium (IGE) in a finite-horizon economy, he subsequently abandons this notion, and in the Économie (1947) resumes, instead, the more traditional method based on the notion of stationary equilibrium. The article argues: i) that Allais' reasons to leave the IGE framework behind, of which the most important turn round his misgivings about the sufficiently correct foresight entailed by that notion, and that reflect the impossibility to establish a correspondence between observations and theory by means of the IGE method, are well-justified; ii) that his shift to the method based on the notion of stationary equilibrium to connect the results of neoclassical theory with observations cannot be accepted, since a notion of stationary equilibrium that would make this correspondence possible must face an insurmountable difficulty in the treatment of the factor capital.

Keywords: Allais, Intertemporal Equilibrium, Stationary Equilibrium, Perfect Foresight, Centre of Gravitation

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I. INTRODUCTION.

As is well known, neoclassical price and distribution theory is currently based on the twin notions of temporary equilibrium (TGE) and intertemporal general equilibrium (IGE).[1] What is probably less well known is that, among the scholars who presented these notions for the first time during the 1930s and '40s, Maurice Allais, with his *Traité d'économie pure* ([1943] 1994, *third ed.*, hereafter *Traité*), gives the earliest formalization of an intertemporal equilibrium in a finite-horizon economy.[2] Indeed, although several commentators have highlighted the richness of Allais' early writings[3], current microeconomic textbooks only remember his contributions on decision theory -the well-known 'Allais' paradox'-, and secondary literature devoted to his work on neoclassical price and distribution theory during the 1940s is actually quite narrow.[4] All this is indeed surprising, if for no other reason, because some general equilibrium specialists, notably Debreu and Malinvaud, have both acknowledged the influence of Allais in their own work.[5]

In view of the little attention that Allais' early thought has received so far, this article attempts to partially fill this gap by examining the *method of analysis* pursued by Allais along his most important writings on price-and-distribution theory during the 1940s, namely the *Traité*, but also the sequel of that work, *Économie et intérêt* ([1947] 1998, *sec. ed.* hereafter, *Économie*).[6] I attempt to show

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[1] Actually, given that a TGE may easily not exist (*cf.* Ravagnani, 2010), the sole rigorous version of modern general equilibrium theory is IGE theory.

[2] *Cf.* Belloc and Moreaux (2008, p. 1117).

[3] *Cf. e.g.* Grandmont (1989), Munier (1995) and Arena (2000), among others.

[4] In this respect, Allais' contributions to economic theory during the 40s are barely mentioned by some well-known contemporary books devoted to the history of economic analysis (see e.g. Screpanti and Zamagni, 2005) or not mentioned at all (see e.g. Blaug, 1985); moreover, his early contributions have neither received the deserved attention of those studies specially devoted to the history of general equilibrium analysis (see e.g. Ingrao and Israel, 1990 and more recently Tieben, 2009). Finally, only few articles are devoted to Allais' early writings on capital and equilibrium theory: see *e.g.* Weintraub (1991) and Lenfant (2005), who examine Allais's (1943) analysis of dynamic stability; Malinvaud (1995), who analyses the overlapping generations model developed in Allais (1947), and specially Béraud (2013), who compares Allais' IGE model with Hicks' TGE model, in particular in their respective analyses of stability and optimality.

[5] *Cf.* e.g. Krueger's (2003, p. 182) interview to Malinvaud for the latter's position. For Debreu's, *cf. e.g.* Dréze (1989, p. 12).

[6] Indeed, the *Traité* and *Économie* must be conceived as a part of one major treatise in economic theory, as Allais (1994, p. 19) acknowledges in the introduction to the third edition of the *Traité*. In fact, in that work, Allais usually refers to chapter VII as the one that will

that, if any, only a thin line of continuity exists between Allais' works and those contributions that, during the second half on the 20th century, developed and perfected the notion of intertemporal equilibrium. Indeed, we shall see that, while on the one hand in the *Traité* Allais presents an IGE model *tout court,* in *Économie,* on the other hand, to specifically discuss the topics of capital and interest, the notion of IGE is abandoned and replaced by the more traditional method based on the notion of stationary state, what the author calls a "permanent regime" (Allais, 1943, p. 666; 1947, p. 34).

The article is structured as follows: section II explores the *reasons* put forward by Allais to leave the IGE method behind. As we shall see, the most important of them turn round his misgivings about the sufficiently *correct foresight* entailed by that method, and which are none other than a symptom of its *inner contradiction*: the impossibility to establish, by means of the method based on the notion of IGE, a correspondence between observations and the variables determined by the theory. This will give me the opportunity to discuss in section III the features of an equilibrium notion that Allais believes to be essential to connect the results of the theory with actual phenomena. Sections IV and V argue that Allais' subsequent shift towards the notion of stationary equilibrium as a means to justify the plausibility of the neoclassical approach is untenable. For, I submit, a plausible notion of stationary equilibrium to connect the results of the theory with observations *obliges* the neoclassical approach to treat the endowment of capital as a single homogeneous factor, capable of changing form without changing in quantity, and hence measured in *value terms*, which is simply unacceptable. Section VI summarizes the argument and draws the main conclusions of the article.

II. ALLAIS' MISGIVINGS ABOUT THE IGE METHOD.

In the *Traité* Allais gives the earliest formalization of an intertemporal equilibrium in a finite-horizon economy with complete futures markets (see Allais, 1943, p. 514 and Annexe I H p. 845-846). His model is thus a

specifically deal with capital and interest; this chapter is missing however, and its content is the subject of *Économie*.

forerunner of the model later developed and popularized by Debreu (1959), and that serves as the basis on which neoclassical price-and-distribution theory stands at present. The specification of the economy is given by i) individuals' preferences; ii) the available technology; iii) the endowments of factors and of consumption goods and iv) the distribution of property-rights shares –which includes the distribution of fixed capital.[7]

Allais goes on to argue that the sequence of future prices can be interpreted in two alternative ways. If one assumes imperfect foresight, "the prices involved are expected prices. It is therefore of little importance whether these prices are actually realized or not". On the other hand, "to the extent that foresight can be considered as perfect, the representation thus made of the economy can be seen as a first approximation of the evolution of the real economy" (Allais, 1943, pp. 530-531).[8] We discuss below why Allais decides not to move in the direction of the temporary equilibrium framework popularized by Hicks (1939), and hence discards the interpretation of the future variables as expected variables that may generally be mistaken. For now it is sufficient to notice that, despite admitting the possibility to assume that the future prices will be actually realized, that is to say, despite assuming correct future foresight, Allais is clearly concerned with the strong limitations entailed by that assumption.

To grasp what according to Allais is a first expression of these limitations, one must first notice that Allais formalizes the hypothesis of correct future foresight by assuming *complete futures markets*. "This hypothesis [of correct foresight] itself", Allais indeed writes (1943, p. 534), "authorizes us to consider as many elementary markets as there are different future goods and services" (*ibid.*). However, he immediately continues,

> Such a representation is not consistent with reality; in fact, in the real economy there are not particular futures markets for each kind

[7] Allais does not provide however any formal proof on the existence of equilibrium and he limits himself to count the number of equations and unknowns to argue that the problem has a well-determined solution.

[8] The citations of Allais' 1943 and 1947 works here reproduced are translated into English by the author.

> of commodity. Only an abstract commodity is supplied and demanded, money capital, a monetary measure of material commodities. As a result, the actual aspect of phenomena is different from that of our Pure Economy (Allais, 1943, p. 534).

The passage is clear: complete futures markets *do not* exist; individuals allocate their resources over time through the capital market, by supplying and demanding an abstract commodity that Allais calls "money capital". The assumption of complete markets is indeed eventually relaxed and replaced by an "Economy in abstract goods" (*ibid.*, p. 492). In this alternative framework the missing markets are replaced by a loan market, i.e. the market for "abstract capital" (*ibid.*) and markets are open at the beginning of each period of the economy's life-span. At first sight, by apparently providing a truly sequential structure to the economy, this alternative framework seems to give a more accurate representation of the working of the actual economies. However, given that the assumption of correct future foresight is *still* maintained, the theoretical path so determined is a *very particular* path, since it is equivalent to the equilibrium path with complete futures markets.[9] It is clear therefore that the connection between this perfect-foresight path and the actual path is still in need of sufficient justification.

Allais' misgivings about the assumption of perfect foresight re-emerge behind a second shortcoming he detects in the IGE model: the *absence of fiat money*. He writes:

> The economy we will consider here is extremely abstract. Within this system, money is detached from every material representation. It is necessary to suppose a huge clearing-house system that at each instant of time is able to offset individuals' and firms' revenues and purchases in units of account. Thereby all payments are made by transfers and the entire economy is registered on accounting records. Such an economy is not actually conceivable other than in a world where information and foresight are perfect, because it ultimately rests on the existence of generalise credit and trust. (Allais, 1943, p. 536)

[9] For the equivalence between the perfect-foresight path and the intertemporal path c*f. e.g.* Rodano (1984, p. 42).

The passage seems to anticipate a discussion that, triggered by Hahn (1965), will later gain momentum among neoclassical economists: the difficulty to make room for fiat money within the method based on the notion of IGE. Indeed, Allais argues there that in the intertemporal equilibrium model he has formalized "it is necessary" to assume a "huge clearing house" that "at each instant of time" cancels out the mutually-agreed exchanges among individuals at market clearing prices, with the implication that no need arises for a medium of exchange. Moreover, considering, as Allais (1943, p. 60) argued before, that perfect foresight implies that "there is no error; risk is absent and every loan is completely reimbursed under the expected conditions", there seems to be no reason why individuals would be willing to hold positive amounts of money; they could instead buy capital goods, since they can be lent and provide a positive return with *certainty*. As Allais (1947, p. 235) later argues in *Économie*: "it should be shown why economic agents prefer to hold money rather than buying assets capable of yielding a net return". Money demand, he accordingly claims, may be justified on the grounds that it allows individuals to "equilibrate normal expenses and revenues over time" (*ibid*, p. 231); "to face unexpected needs" that normally emerge (*ibid*); "to face abnormal contingencies" (*ibid*, p. 232) and also to "profit" from investment opportunities that "emerge at each instant of time owing to the lack of correct foresight" (*ibid*, p. 233).[10] But all these reasons seem impossible to accommodate in the perfect-foresight model Allais formalizes. Little wonder that his discussion of monetary issues is postponed to *Économie*, that is outside the IGE framework and within the more traditional notion of stationary equilibrium. In fact, as we shall see below, within this notion the assumption of correct foresight in equilibrium need not exclude those elements Allais envisages to be relevant to justify the demand for fiat money.

Allais' final important misgiving about his IGE model is the assumption that the life-span of the economy is *finite*. He in fact openly recognizes that

[10] For instance, Allais explains, "if an economic agent expects a decrease in prices, he will have an incentive to postpone certain purchases; he will be induced to stock money." (*ibid.*).

"The conception of an economic life-span limited to a period $\bar{T}$, is naturally unreal" (*ibid.*, p. 535); he justifies the assumption owing to "purely mathematical" (*ibid.*) difficulties that would emerge in an infinite-horizon setting: when the horizon is infinite it is also infinite the number of variables and equations to be considered, an "extremely delicate" (*ibid, p.* 536) issue to deal with, so Allais claims. Upon careful reflection, in this case the problematic assumption of correct foresight comes out to be, not behind Allais' admission of the unreality of the finite-horizon assumption, but rather behind the way in which he tries to overcome the problem. Indeed, Allais argues in a footnote that "We will soon see how the problems relative to the infinite horizon can be solved in a rigorous way" (*ibid.*, fn. 3). And in order to show this he refers the reader to the discussion of stationary equilibria that takes place in section *F* of the *Traité* (pp. 665-669), a very brief introduction to the topics that will be much more developed later in the *Économie,* where the finite-horizon hypothesis of the *Traité* is in fact definitely left behind: Allais provides there the earliest formalization of an overlapping generations (OLG) model, but under stationary conditions. In other words, the *only way* indicated by Allais to surmount the finite-horizon assumption is the stationary economy; a framework that, conveniently enough, also allows him to avoid the implausibility of correct future foresight of *changing* relative prices over the infinite future.[11]

[11] The hypothesis of a finite-horizon economy entails a further difficulty related to perfect foresight. Although the issue is not discussed by Allais, the problem is worth mentioning anyway, for two different reasons: first, because it shows that when an assumption of the model *essentially* contradicts the working of actual economies, a truly conscious scholar worries *independently* of its possible contradictory manifestations, because what is being at stake is the correspondence between the theory and observations, and hence the *relevance* of the former. Second because, interestingly enough, the problem has been addressed by one of Allais' pupils, E. Malinvaud, precisely along the latter's discussion of Allais' overlapping generations (OLG) model, and as means to justify the usefulness of the stationary framework developed by Allais. The problem is the following: as long as it is admitted that the economy *does not* actually end in the final period of the economy under examination, say period $\bar{T}$, one must also admit that there will be some capital goods left *after* that period. However, unless one further assumes perfect foresight beyond $\bar{T}$, there are no bases to assume that the evolution of economic conditions after that period will be known by individuals when taken their investment and production decisions. The implication is that the composition of the capital stock left in that period *cannot but be arbitrary*, it would seem, since its usefulness in the production process can only be checked in the periods that follow $\bar{T}$, which by assumption are not considered by the model. As remarked by Malinvaud (1961, p. 150, emphasis added):

It still remains to be ascertained why, although Allais repeatedly insists that correct future foresight is a "highly abstract hypothesis" (see e.g. Allais, 1943, p. 61, 1947, 34), he comes to adopt it anyway. Only a partial answer is found in the *Traité*: it would be a necessary assumption "due to the need to eliminate the element of risk from the phenomenon of interest, since it is not susceptible of simple representation" (*ibid.*, p. 534). But in *Économie* he further explains that correct future foresight,

> by making each economic calculation concerning the future strictly accurate, by eliminating risk and thus arbitrary elements which are difficult to represent, will allow us to give a simple representation of the essential economic mechanisms that involve time; it will thus make it possible to provide a precise determination of the interest rate. (Allais, 1947, p. 33-34)

On the nature of these "arbitrary elements" Allais does not deepen. It might however be possible to get some insight about it if one considers that consumption, investment and production decisions are generally taken on the basis of *expectations* about future economic conditions, and hence that the avoidance of the assumption of correct foresight would force the theorist to model the *way* in which expected conditions influence the decisions taken by the different economic units, as it is e.g. the case in the temporary-equilibrium versions of the theory. But then one would have to model, e.g., how these decisions are affected by individuals' *subjective perceptions* about the

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"We cannot judge its [the capital stock, A.D.] usefulness since, by hypothesis, we *ignore* all economic activity that will take place after $\bar{T}$". Economic activity in a finite-horizon economy, Malinvaud (1986) further adds in his evaluation of Allais' (1947) model, "should lead to certain results at the terminal date, principally to leave a capital of which the volume and composition will have been specified in advance. To impose this last condition seems unsatisfactory, since only knowledge of the ensuing activity would determine what terminal capital would be suitable. That is why it was soon realized that the theory of capital should also consider alternative formulations in which time would be treated as unlimited and in which, consequently, the requirement for terminal conditions would disappear. The most convenient alternative consists in supposing that the environment remains the same through time and therefore in concentrating on stationary equilibria." (Malinvaud, 1986, p. 113). Notice indeed that the problem of the composition of capital left in the final period is surmounted if the economy under consideration is stationary: there is no last period in that framework –alternatively, the last period coincides with the first one- and hence the capital stock left in any period t is determined by the economic conditions that will prevail in $t+1$; conditions that, as we shall further argue below, can reasonably be assumed to be known owing to the stationary character of the economy.

possible *risks* involved in investing in the different industrial sectors; perceptions that may be precisely influenced by all sort of "arbitrary elements", and hence that, being "difficult to represent", may considerably weaken the level of generality of the results obtained regarding their effects on the variables under analysis, as for instance on the interest rate. "[I]n reality", Allais indeed openly admits,

> every time previsions are unstable, the evolution of the real economy is equally unstable. The issue of foresight constitutes one of the points where real phenomena can notably differ from the Pure Economy. (Allais, 1943, p. 485, fn. 6)

Allais thus seems to fully accept here that as long as forecasts about future economic conditions are not correctly envisaged by individuals -the normal case[12]-, the actual path may *considerably diverge* from the trends predicted by the intertemporal equilibrium path (the path of the "Pure Economy"), and in ways that do not seem to be susceptible of sufficiently general representation -as suggested by his writing "unstable" to characterize the evolution of the economy under imperfect foresight. It might thus be plausible to conjecture that, at least in part, it is to avoid *indefiniteness* in the results obtained under imperfect foresight that Allais advocates for the hypothesis of correct foresight, and despite his explicit reservations about such a hypothesis.[13]

III. A PLAUSIBLE JUSTIFICATION FOR CORRECT FORESIGHT.

At this juncture of the exposition it seems difficult to deny that the assumption of correct future foresight is still in need of robust justification.

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[12] As Allais also admits in this connection, "If, in fact, they [the equilibrium conditions, A.D.] are generally not verified, it is because of the *lack of foresight* that features the economic system and that usually makes firms to determine their activity and their productions on the basis of an incorrect evaluation of the future value of goods and products". (Allais, 1943, p. 540)

[13] This indefiniteness of the influence of expectations on prices and quantities seems to be further accepted when Allais writes: "it can perfectly be the case that this investment that is currently profitable under the present conditions of the market is not carried, if one expects an unfavourable evolution of prices. On the contrary, this other [investment] that is not profitable under the current conditions can be however realized, if the expected evolution of prices is favourable." (Allais, 1947, p. 117-118).

For if the method based on the notion of IGE rests on such "abstract hypothesis" (Allais, 1947, p. 34), the conclusion that the *relevance* of that method would be seriously impaired seems to be hardly avoidable. The question therefore emerges: can a plausible justification for the assumption of correct foresight be provided *within* the IGE method?

To examine the way in which Allais faces this problem, it will be convenient to pause for a moment and inspect first the notion of equilibrium that, in Allais' view, would make the correspondence between theory and observations possible. In this respect, it is Allais' opinion that economic theory is unable to determine the exact position of the economy at any single instant. So many are the causes that may possibly affect the prices that day by day are observed in the market, that their general investigation would be doomed to fail. As Allais (1943, p. 443) writes, "Economic movements are absolutely comparable to the movements of the sea". And he continues:

> The momentary price of a commodity, its current price on the market, will move like the waves of the sea because it is formed by the daily conditions of supply and demand, conditions that are subjected to quick and temporary changes. After an abundant fishing, the price of fish decreases, and it increases when the market is poorly supplied owing to a fortuitous circumstance, like a storm or a strike in the transport sector. But these accidentals shifts of supply and demand appear to be negligible when one searches for the laws of price formation over a period of time long enough that their influence disappears due to the action of averages. (Allais, 1943, pp. 443-444)

Notice that at the heart of Allais' argument it is possible to find, essentially, the same distinction, introduced for the first time by Adam Smith ([1776] 1979, p. 72-73), between the "market price" of a commodity, and its "natural price" (or as Marshall, 1920, calls it, its "normal price")[14], that is, the price that can be actually determined by the theory, and that has a clear empirical counterpart: it emerges over sufficient time, as Allais notes, as the

[14] The previous passage is in fact very similar to Marshall's (1920, p. 291), since he writes: "The actual value at any time, the market value as it is often called, is often more influenced by passing events and by causes whose action is fitful and short lived, than by those which work persistently. But in long periods these fitful and irregular causes in large measure efface one another's influence; so that in the long run persistent causes dominate value completely".

average of market prices. The former, on the other hand, is what Allais identifies as "the current price on the market", and is accordingly affected, like the waves of the sea, by any sort of "fortuitous circumstances", as for instance bad weather conditions or a strike, as Allais rightly notices; circumstances which doubt prevent market prices, and their respective quantities, from being susceptible of any theoretical determination -as also implied by Allais' referring to purely "accidental" shifts in supply and demand conditions. Now, it is clearly illegitimate to assume that there are situations in which those accidental forces stop exerting their influence on the variables under examination. However, as Allais' argument goes, these influences may be legitimately neglected at a theoretical level, since over relatively long periods their action will have sufficient time to correct or compensate each other, and the prices that are instead determined by the theory will emerge, as said, as a sort of *average* of the market prices. In other words, the possibility to ignore those short-lived accidental circumstances rests on the possibility to conceive the normal position of the system -or as Allais (1943, p 444) occasionally calls it, its "long-term equilibrium"- as a position that the economy tends to realize over sufficiently long periods.

Likewise, it is also clear that the causes that affect prices over long periods can be hardly assumed to be absolutely invariable. "In fact", so Allais argues,

> while the conditions that determine the average level of the sea remain immutable, this is not the case with the conditions that correspond to the long-term equilibrium. The conditions that define the structure of the economy suffer in fact from systematic modifications over time owing to legal, psychological, technical... developments of the economy. (Allais, 1943, p. 444)

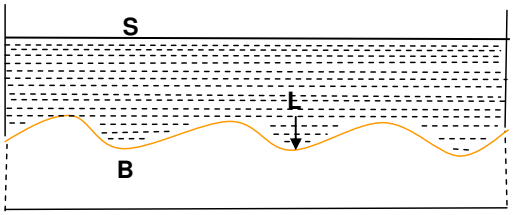
In sum, owing to the action of purely accidental forces, and also because the givens of the theory are continuously changing, the coincidence between the actual and the theoretical positions of the system cannot evidently be assumed. Confronted with this situation, the possibility to establish a relation between the former and the latter rests on the possibility to conceive the prices the theory is capable to determine as the *centre* around which the actual

prices are *constantly gravitating*. But even if stability can be assumed, to have the role of centres of gravity of market prices, the prices determined by the theory must be *sufficiently persistent,* that is to say, their speed of change must be of lower order of magnitude than the speed of gravitation of market prices towards the theoretical position. Allais is clearly aware of this, as he writes:

> In the real economy, when the market is not too imperfect and when forecasts are sufficiently accurate, in other terms, when the non-expected modifications in the structure of the economy per unit of time are sufficiently small relative to the speeds of adaptation, on can consider, as a first approximation, the economic parameters as provided by the equilibrium equations [...] [W]hen the conditions of structure vary so fast that the action of the adjustment mechanisms [towards equilibrium, A.D.] is constantly surpassed by modifications that are non-expected by the majority, the values of economic parameters will not be considered, even as a first approximation, as the solutions of the general equilibrium equations. (Allais, 1943, p. 548-549)[15]

We are now in position to resume our original question (can sufficiently correct foresight be justified within the IGE method?), since what we have

[15] To explain the notion of persistence, Allais (1943, pp. 545-546) relies on the following example. He considers a basin full of a viscous liquid (L) as in the figure below. Owing to its chemical characteristics, the liquid exercises a constant pressure (down-pointing arrow) over the irregular bottom of the basin (B) and hence the latter is continuously being "deformed" by the action of the former. Now, the question is, if we put a certain amount of liquid L in the basin, given that the bottom will be in fact constantly deformed by the action of L itself - and of course, assuming that the surface of the liquid (S) would have reached a position of rest had the liquid not deformed the bottom of the basin - is it possible to say that S will be in equilibrium?



What should be compared, Allais asserts, is the *speed of deformation* of the basin caused by the action of the liquid, and the *speed of adaptation* of the surface of the liquid towards its equilibrium level –i.e. the position the surface would have reached had the chemical features of the liquid been non-existent. And then, Allais argues, "one will be able to say that if the speed of deformation is sufficiently small relative to the speed of adaptation, in other words, if the equilibrium tends to be established at each instant of time faster than the speed with which the bottom is deformed, one will be able to consider as a first approximation the surface S as a flat surface and this approximation will be better the smaller the ratio between the speed of deformation and the speed of adaptation." (Allais, 1943, p. 545).

seen till now allows us to answer in the following way: the assumption that the prices determined by the theory will be correctly foreseen by the relevant economic units can be plausibly justified within the IGE method if, and only if, it is possible to justify the relation of gravitation within that method. Indeed, if the data of the theory change relatively slowly with respect to the speed of gravitation -and, again, if stability can be assumed- it can be reasonably argued that, through the repetitions of productions and transactions under essentially the *same* underlying conditions, expected prices will be the correct ones on average, since they will be eventually discovered by individuals under a trial-and-error process of experimentation. Correct foresight emerges in other terms not as an *unjustified* part of the definition of the equilibrium, but as an *outcome of the same adjustment* process of disequilibrium towards equilibrium magnitudes. This of course does not imply denying the influence that mistaken price expectations, unforeseen investment opportunities, etc. may exert on the evolution of the economy. However, this influence can be legitimately ignored at a first level of abstraction, when the determinants of prices and distribution are examined in their utmost generality. If necessary, at a second analytical stage, when for instance the causes of the trade cycle are examined, the influence of these factors might be in turn explored.

However, the relation of gravitation between observable and theoretical magnitudes *cannot* be reproduced within the IGE method. To see this, one must consider that intertemporal-equilibrium prices and quantities are determined on the basis of a given *set of physically heterogeneous capital goods*, whose quantities can be altered extremely fast, presumably with a speed of the same order of magnitude as the speed with which the demand for and the production of a consumption good tend to equality, a process that will generally involve trial and error and experimentation on the part of individuals, and hence sufficient time to assert itself, as Allais himself admits.[16] It would then be hardly acceptable to allow for the tendency

[16] This trial-and-error process is precisely how Allais justifies that the theoretical position emerges in the market. "In reality", he writes, "individuals and firms make trials and repeat

towards equilibrium between production of and demand for consumption goods without allowing, at the same time, the necessary modification in the relative composition of the existing capital goods.[17] Notice then that before the repetition of productions and transactions can correct or compensate the possible causes of disequilibria and the equilibrium position can emerge as a sort of average of the actual path, the initial composition of the capital stock - and hence the equilibrium path determined on its basis- will have changed considerably: IGE cannot therefore have the role of a centre of gravitation of actual variables.

It is now easier to understand why, despite in the *Traité* Allais (1943, p. 484) assumes that along the economy's life-span there is no technical change, that the rate of population growth is zero, that individuals' tastes remain the same and, finally, that individuals' property rights (presumably of land and of highly durable capital goods) do not change, he *still* feels the need to add the further qualification that "along it [the process of equilibration, A.D.] it will be convenient to assume that no effective transaction is realized" (*ibid.*). The reason for this, however, is not explained. But given that immediately before Allais has specifically referred to the *new firms* that will appear along the process of adjustment, the reason must be evidently that under the actual implementation of disequilibrium productions and transactions the initial endowments of capital goods (and of consumptions goods) will be modified considerably; hence the final position of the system, if reached, will strongly depend on the countless debris of disequilibrium, i.e. equilibrium is *path-dependent* because the effects of disequilibrium activities on the data are too relevant to be neglected. However, notice that if disequilibrium activities are only allowed to take a *virtual existence*, the path determined by the theory and

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them until they get their maximum levels of utility and profits compatible with the general conditions of the economic structure; equivalently, they solve, by successive approximations, the general system of equations of the equilibrium" (Allais, 1943 p. 531)

[17] On the same footing, consider that, when the real wage changes, this will likely impact on all sectors of the economy, and the possible effects will take considerable time to exert themselves fully. As Allais (1943, p. 669) observes when discusses the possible effects of a rise in the level of real wages: "these changes imply considerable modifications in individuals' activities, which under the most favourable hypothesis, cannot but occur slowly". Clearly, along this adjustment process to the new conditions it can hardly be denied that the composition of capital will change considerably.

the actual path of the economy would *exactly* match; that is to say, the theory in question would be able in principle to determine the position of the economy *at each instant of time*; but we have seen that this is impossible, as Allais has forcefully and rightly pointed out.[18]

A very important consequence of the previous discussion is that a notion of equilibrium that would be sufficiently persistent to make the correspondence between observations and theory possible cannot treat the endowments of capital goods as exogenously given magnitudes. This is indeed, as is well-known, the universally agreed vision among the founders of economic theory, both classically and neoclassically-oriented scholars; as for them the prices that would be capable of having the role of a centre of gravitation of market prices would exactly cover their minimum average costs, which include a uniform rate of return on the supply price of the capital goods. The reason is evidently that a situation characterized by different rates of return among sectors could not last, since the persistent action of competition would eventually cause movements of capital from the least profitable towards the more profitable industries. But the determination of cost-of-production relative prices as normal prices obliges the composition of the capital stock to be determined *endogenously*: there is in fact no reason why an arbitrarily given composition of capital would be able to satisfy forthcoming demand, a fact that will manifest in the existence of different rates of return in the different sectors. And while Allais does never seem to realize that this traditional view is utterly incompatible with the IGE model formalized in the *Traité*, he does seem to fully share it. As he writes first in the *Traité* and then in *Économie*:

[18] It could be argued at this juncture that Allais could have avoided these difficulties altogether had he directed his efforts towards the development of the notion of temporary equilibrium, since, after all, the assumption of correct foresight is not needed in those versions of the theory. This would be a big misunderstanding though: first, the temporary equilibrium is also built on the basis of a given vectorial endowment of capital goods. Second, as noticed by Petri (2004, p. 42) temporary equilibria are determined by including expectation functions among the data, which will also be generally affected by disequilibrium activities and hence are also deprived of the sufficient persistence.

> It is only in equilibrium that the law of cost of production is realized. This latter is therefore a law of tendency. (Allais, 1943, p. 267)

> In equilibrium, in fact, the value of every capital good is equal to its production costs. Naturally, this equality takes place only in equilibrium. It is therefore a law of tendency of the real economy. (Allais, 1947, p. 79)

IV. PERFECT FORESIGHT AND THE NOTION OF STATIONARY EQUILIBRIUM.

Our previous discussion has revealed, first, that sufficiently correct foresight may be justified if the relation of gravitation between observable and theoretical magnitudes can be plausibly argued; and second, that that relation of gravitation cannot be reproduced within the method based on the notion of IGE because some of its data are not sufficiently persistent. And indeed, as we now proceed to discuss, Allais will be forced to *abandon* the IGE method to determine a sufficiently persistent situation that allows connecting the results of the theory with observations; that is, to plausibly argue that "forecasts are sufficiently accurate [given that] the non-expected modifications in the structure of the economy per unit of time are sufficiently small relative to the speeds of adaptation" (Allais, 1943, p. 548). "[I]t is only to the extent that foresight is perfect", so Allais (*ibid*., p. 485) argues, that the equilibrium path "is equally valid for the actual economy" (*ibid*.). And he concludes:

> This is approximately the case when the real economy presents itself under the form of a fairly stationary regime. (Allais, 1943, p. 485)

We find essentially the same claim in *Économie*. Allais (1947, p. 117-118) asserts there that the adaptation of production to demand conditions requires "a lot of time", and hence that firms must *correctly foresee* expected demand when taking their production and investment decisions in order for those decisions to be *equilibrium* decisions. And he concludes that "accurate forecasts are possible only under economic and social conditions that are

relatively stable". Stationary regimes, Allais adds in this connection (both in the *Traité* and then in *Écomomie*),

> can be considered as a first approximation of the real economy, at least in periods of economic stability. In fact, in most economic sectors and for periods that may include several years, consumptions and productions remain essentially constant and of the same nature. (Allais, 1943, p 665; 1947, p. 35)

Allais' recourse to the notion of stationary equilibrium to connect the results of the theory with observations is hardly surprising, and the reason lies in the following consideration: these "fairly stationary" conditions are none other than the abovementioned normal position of the classical and early neoclassical scholars, and they reflect what Marshall ([1920] 1970, pp. 306-307) occasionally calls the stationary state of industry in a *statical sense*. This static notion of stationariness is, as Marshall writes, just a "fiction", since the *assumption* that net savings are zero is made in order to reflect the average situation of the economy along a particular stage in the process of capital accumulation; that is, a situation where the action of competition has come to rest, and hence prices exactly cover their minimum average costs. It is in other terms a position determined by sufficiently persistent factors to be capable of having the role of a centre of gravitation of actual variables, and for the reasons adduced in the previous section, also capable of providing a plausible justification for the claim that price expectations will be correct in equilibrium, at least on average.

We shall see in the next section that the formalization of these relatively stationary conditions by means of neoclassical data -preferences, technology and factor endowments- faces an insuperable difficulty in the treatment of the factor capital. For the time being, let us further notice that given the slowness of capital accumulation relative to the speed of adaptation of market prices towards their minimum average costs, the assumption that the

economy is stationary, although not strictly necessary[19], seems fully justified when the aim is to isolate those persistent causes that, over sufficient time, may explain relative prices and income distribution.[20] As Allais specifically writes with respect to the determinants of the rate of interest in capitalist economies:

> [T]he existence of a nominal rate of interest that is every day positive constitutes a constancy whose explanation must be sought, not from essentially contingent probabilities, but from given data which are equally permanent. (Allais, 1947, p. 479) [21]

We thus see that, in order to justify the assumption of sufficiently correct foresight, Allais is *forced* to abandon the notion of IGE he presents and develops in the *Traité* and to resume, instead, the more traditional notion of stationary state embraced by the founders of the neoclassical approach such as Marshall or Wicksell. This is in fact the path Allais takes in his *Économie*[22], and in this direction we shall also move. [23]

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[19] See e.g. Petri (2004, chapter 4) for a formalization of a normal position within neoclassical theory where the economy is not stationary.

[20] On the same footing, it is the relative persistence of these causes what authorizes the theorist to neglect the possible changes that the relative prices may experience over time in the definition of the theoretical position, as if the prices so determined were actually stationary. This abstraction ceases to be legitimate within the IGE or the TGE versions of the theory, since the rapid changes that the given composition of the capital stock will experience in the not-very-far future, a fact that cannot be ignored by individuals when taking their decisions, obliges the theorist to incorporate price changes in the definition of the equilibrium. This is done either by assuming that complete markets exist (IGE) or, alternatively, by introducing expectations about future prices changes (TGE).

[21] We find essentially the same idea, e.g., in Wicksell ([1901] 1934, pp. 154-155), who argued that "The real theoretical difficulty is [...] to explain how, under stationary conditions, the possession of capital can remain a permanent source of income. The application to non-stationary conditions offers no difficulty in principle. [...] Both logically and for purposes of exposition it would seem right to begin by examining the effects of a given supply of capital already accumulated, and *then* to inquire the causes which influence, and eventually alter, this supply.". *Cf.* Kurz (2000) for a detailed analysis of Wicksell' position with respect to the notion of stationary state.

[22] As noticed above, in the *Traité* the notion of stationary equilibrium is only very briefly discussed in section *F*.

[23] One may notice at this juncture that with his shift to the notion of stationary equilibrium as a persistent situation reached only after sufficient time, Allais can plausibly solve the problem relative to the difficulty of making room for fiat money in the IGE method, since the reasons he adduces to justify money demand can be plausibly accommodated within the notion of equilibrium he is now adopting. As is well known, within the neoclassical approach only 'real forces' matter for determining the persistent causes of prices and distribution. However, since a centre-of-gravitation notion of equilibrium need not assume that all economic activities are simultaneously carried out, agents need on average a certain amount

V. THE TREATMENT OF THE CAPITAL ENDOWMENT UNDER STATIONARY CONDITIONS.

As argued in the previous section, in the *Économie* Allais definitely abandons the IGE method and adopts, instead, the more traditional method based on the notion of stationary state to explain the determinants of income distribution and relative prices. However, an insurmountable difficulty in the treatment of the factor capital arises for neoclassical theory in the attempt to formalize a situation in which those "fairly stationary" conditions prevail: for if capital must be included, as is distinctive of the neoclassical approach, among the givens of the theory, the determination of cost-of-production relative prices obliges the approach to treat capital as a single factor, *capable of changing form* so as to allow an endogenous determination of the composition of the capital stock in equilibrium, but *without changing in quantity*. And the only general way in which capital can be so treated is by measuring it in value terms. While unlike the vectorial specification, the value specification of the factor capital does satisfy the requisite of persistence since the total quantity of capital of the economy is only *very slowly modified* by individuals' net savings[24], the fact is that this treatment of capital is unacceptable. If for no other reason, because this magnitude depends on the relative prices that the theory should seek to determine. This is the most evident expression of the

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of money to perform their normal economic activities, and money therefore emerges as a universally accepted means of exchange. It is also admitted, given that transactions and productions are generally performed at market (disequilibrium) prices, that money is necessary to face the possible contingences that are bound to occur, and also to profit from possible unforeseen investment opportunities in the stock market. But given that only 'real forces' determine the centre of gravitation of the economy, it was also legitimate to face the specific problems posed by money at a second analytical stage, in particular when the problem of economic fluctuations around the centre of gravitation was considered. Indeed, according to the approach, the existence of money and the banking sector, by decreasing the connection between investment demand and the rate of interest, temporally create mismatches between saving and investment decisions. This is precisely the vision of Allais in *Économie* (*cf.* chapter 9). "A deep study", he e.g. writes, "allows showing that only [...] the possibility of money emission by the banking sector and the hoarding of circulating money are at the origin of the short-run fluctuations that constitute the economic cycles". (Allais, 1947, p. 417 fn. 5).

[24] So for instance Knight (1931, p. 208-209) writes that the "total supply of capital is 'very large' in comparison with possible variations in it, and that the opportunity for further investment is on a similar scale with the total [...] Manifestly no possible variation in the amount saved in a year could make enough of a variation in the total supply [...] The cumulative result would be detectable after a considerable number of years."

illegitimacy of treating the factor capital as a single-valued quantity. We shall see below that Allais is indeed aware of this shortcoming; a problem that, following Petri (2004, p. 32-33), we may label as the problem of capital from its "supply side"; we shall also assess Allais' alleged solution to it.

We know however since the contributions of Sraffa (1960) and Garegnani (1960) that the problem *cannot be* solved; and, in fact, Allais *does not* solve it. A first proof of this is that, to justify the uniqueness and stability of the theoretical position, Allais *unquestionably* shares the view of the founders of the neoclassical approach that the demand for value capital is a decreasing function of the interest rate-real wage ratio. He is in other terms clearly unaware of another expression of the problem derived from the treatment of capital as a single-valued factor, and which we may identify as the problem of capital from its "demand side": namely, that contrary to what is assumed by the theory, the demand for the given endowment of value capital by the profit-maximizing firms *may not* be a negative function of the interest rate, with the implication that, even if the quantity of capital could be somehow measured independently of distribution, the theoretical position may be neither unique nor stable. A fact, no doubt, that puts at serious risk the role of the former as a centre of gravitation of the actually observed prices and quantities on the market. But as said, these problems are ignored in *Économie,* since the demand for value-capital is assumed to be a well-behaved function of the rate of interest[25]; and this why Allais is able to determine a unique and stable equilibrium.[26]

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[25] Allais in fact closely follows the Austrian school in that book and argues, by means of the notion of *average period of production* (*cf.* Allais, 1947, p. 119-120), that a decrease in the rate of interest will induce firms to adopt techniques that employ more value capital per unit of output. Is not the aim of this article to discuss the strong limitations of this argument. The reader is referred to Garegnani (1960, p. 130-134) and Petri (2004, p. 2004, appendix 3A).

[26] While a thorough discussion of Allais' analysis of the demand for value capital in the *Économie* escapes the aim of this article, it may be useful to consider some passages of that book, since they will allow us to detect an additional aspect of the notion of capital as a single factor within neoclassical theory that is not often noticed. Allais writes that a decrease in the rate of interest will "constitute a determinant element in the *renewal* of the capital equipment according to more indirect and more productive processes." (Allais, 1947, p. 151, emphasis added). Notice how Allais accepts here that, when the additional savings push the real interest rate downwards, the new, more-capitalistic technique will be initially implemented in *new plants* only. The reason is that, given that at each moment of time the aggregate endowment of capital will have taken a *specific* physical form, when conditions change the

Let us now turn to the supply-side problem. As said, in *Économie* Allais shows sufficient awareness that, being a value magnitude, the quantity of capital of the economy cannot be known before distribution and hence relative prices are known. "One of the most important difficulties in the theory of interest", he writes,

> comes from the fact that the quantity 'capital' is not a physical magnitude like a quantity of flour or of wine, but rather is an economic magnitude whose value directly depends on the rate of interest. This difficulty is not generally clearly perceived. The classical authors were used to say that the rate of interest is determined by the intersection of the supply and demand curves for capital, which is true, but by omitting to state that these curves themselves depend on the rate of interest, their theory presented a singular lacuna. (Allais, 1947, p. 517)

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new capital-labour ratio, which will generally require the employment of capital goods of a *different* kind, can be implemented only gradually, as the old capital goods are scrapped and replaced, and the value they embody -plus the additional savings- can meet the labour freed by the gradual closure of old plants. The implication, only implicit in the founders of the neoclassical approach and brought back to general attention by Garegnani (1978) and further developed by Petri (2004, p. 127-128) is the following: because initially only new investments adopt the more-capital-intensive methods, while the capital goods in the existing plants still keep their original forms, the rate of interest *must* necessarily be determined in the market for capital in *free form*; the market where, as Petri (*ibid.*) argues, "the demand for (the flow of) 'free' capital met its supply (also a flow)". In other words, in the savings-investment market. And in fact, Allais (1947, p. 144) discusses this point explicitly: he argues that, given that the total demand for capital as a stock includes "old capital goods" (*ibid.*) which cannot be turned back to consumption in the period because they "correspond to immobilized goods" (*ibid.*), the relevant demand schedule to be considered when explaining the forces that determine the interest rate is the demand for capital that corresponds to the *new investments*, that is, the demand for capital as a flow. "Instead of considering the total demand for capital", so Allais (1947, p. 144) writes, "one can only include that demand reduced by the value of old capital goods, that is to say, the value of new investments". Similar considerations are provided for the total supply of capital, i.e. only the supply of free capital (as a flow) must be taken into account in the explanation of the interest rate. Allais (*ibid.*) thus concludes: "The point of intersection between the two curves thus obtained corresponds, naturally, to the same value of the interest rate". That is, the rate of interest determined by the intersection between the supply of and the demand for capital considered as flows must be the same as the interest rate that results from the intersection between the supply and demand for capital as stocks. This last observation is correct –of course, taking for granted that the premises of neoclassical theory are sound- as long as the analysis is restricted to conditions of stationariness, because in stationary equilibrium the demand for capital as a flow is a reduced-scale representation of the demand for capital as a stock. Outside stationary equilibrium however, the rate of interest that is determined in the savings-investment market will *not* necessarily reflect the equilibrium conditions between the *total* demand for and supply of capital because the capital sock that is still invested in the old plants has not yet adjusted to the conditions of the new equilibrium.

The problem described in this passage is enough to demolish the neoclassical explanation of relative prices and distribution since, once it is admitted that the quantity of capital cannot be determined before the rate of interest is known, the general equilibrium equations that attempt to represent those average conditions of the economy over sufficient time, are simply *undetermined*.

Why, however, Allais does not seem to envisage this issue as a seriously disturbing problem for the neoclassical approach to prices and distribution? I wish to suggest that the reason lies in the following consideration: in *Économie*, when Allais comes to formalize the OLG model under stationary conditions (Annexe II, pp. 641-771) to allegedly represent those "fairly stable" conditions that prevail in actual industrial sectors over sufficient time, Allais *does not* include the quantity of value capital among the givens of the equilibrium, as the static neoclassical notion of stationariness dictates; as I show in the APPENDIX, in the model Allais determines the quantity of capital *endogenously* by dropping the traditional neoclassical assumption that net savings are zero[27]; he instead envisages the supply of savings as a function of prices, incomes and distribution derived from consumer intertemporal choices, and equilibrium is defined as a price-quantity configuration such that individuals are *induced* to make zero net savings. Allais does not in other terms determine a static, but rather a *secular* stationary state.

While Allais does avoid the illegitimacy of including a value magnitude among the givens, his solution cannot be accepted. It will be convenient to discuss the reasons in some detail, considering that the difference between the notions of secular and static stationariness has been largely lost sight of.[28] First, even if one were to defend the secular equilibrium determined in Allais' model as a plausible equilibrium notion to study the very-long-run trends in

[27] Let us recall that this *assumption* of zero net accumulation is justified due to the relative persistence of the factors that determine cost-of-production relative prices within the neoclassical approach. In particular, owing to the slowness of capital accumulation relative to the speed of gravitation of market prices towards their equilibrium values. Moreover, it should be noticed that the given amounts of factors (among which capital the value single factor) do not in the least prevent the analysis from asking about the effects of slow, or once-and-for-all, effects of changes in those amounts.

[28] The distinction is clearly made by Lionel Robbins (1930).

income distribution and relative prices along neoclassical lines[29], in actual economies *positive* rates of accumulation are typically observed; if, therefore, Allais' claim that "fairly stable" conditions prevail in most sectors actually meant that in real economies accumulation is close to coming to a halt, the claim loses plausibility and is liable to objection.

Second, the secular equilibrium determined in Allais' model is based on contradictory hypotheses: while he assumes that capital accumulation has stopped, the data relative to preferences, population and technical knowledge are the same as those determining a static equilibrium. In a truly secular equilibrium, for instance, the quantity of labour should be endogenously determined too. And the assumption of *given* preferences and technical knowledge must also be removed. However, it seems impossible to predict those preferences or technical knowledge ruling in the very-far future, i.e. when capital accumulation and population growth would have come to a halt. And even if this could be somehow ascertained, the connection between secular equilibrium conditions and actual economic conditions would be completely lost.

To conclude I would like to point out a further aspect of the problem that has been little noticed: even assuming that the forces of supply and demand do tend to establish a secular equilibrium, this position *is not persistent* in the sense discussed in section III above, that is, it is not characterized by a speed of change of the data of lower order of magnitude than the speed of convergence towards equilibrium. The reason is that the speed of adaptation of the endogenous variables towards a stationary position in a secular sense is so slow that it is, probably, of lower order of magnitude (or at most of the same order of magnitude) than the speed of change of the determinants of the secular equilibrium (preferences, technical knowledge, rate of population growth, etc.). One seems then authorized to conclude that Allais' solution to the problem of capital from its supply side must be rejected because, even

[29] Yet, a noticed by Petri (2004, p. 122, fn.6), this role of the secular equilibrium would still require the traditional notion of equilibrium -and hence the notion of capital as single factor- to plausibly justify the tendency of the actual path, not continuously in equilibrium, to the secular equilibrium state.

granting stability, his suggested notion of equilibrium violates the requisite of persistence, a concept that Allais himself carefully discusses and considers essential for an equilibrium notion to be capable of having the role of a centre of gravitation of actually observed magnitudes.[30]

VI. CONCLUDING REMARKS.

The whole argument can be briefly summarized in the following way: while in the *Traité* Allais provides the earliest formalization of an IGE in a finite horizon setting, he puts forward harsh critiques on this notion as a means of ascertaining a correspondence between theory and observations, and ends up by abandoning it and resuming instead the more traditional notion of stationary equilibrium when, in the *Économie*, he fully develops his theory of capital and interest. While I have subsequently argued that Allais' formalization of the stationary equilibrium cannot be accepted, I do share his criticisms to IGE theory; criticisms whose tone, incidentally, will increase with the passage of time, in particular against the version of IGE developed and popularized by Debreu (1959) (*cf.* Allais, 1971; 1989, chapter III).[31]

I only wish to add that the impossibility to establish a correspondence between the neoclassical approach and observations by means of the IGE method is only a *symptom* of the illness of the former. For as we have seen,

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[30] This lack of persistence of the data that determine a secular equilibrium is what seems to have prompted Knight (1930, p. 198) to wonder "whether accumulation is to be treated as an equilibrating process" or not, and to answer that the tendency towards a secular equilibrium "is indefinitely remote in time, giving 'other things' indefinite scope for action (*ibid.*, p. 200); where the "other things" are the "given conditions" (*ibid.*) -or data- of the secular equilibrium. Knight accordingly concludes with the following warning: "For very small changes it is admissible to assume that while any element or condition changes, the others in the same group remain fixed. But in discussing the trends over any considerable period of time this must not be done. The greatest caution needs to be exercised in determining and specifying the systems of constants or long-periods processes and of variables adjusting to them (and to each other), if the notion of tendency towards equilibrium is to yield sound results." (*ibid.*, 1930, p. 200)

[31] Not only in those contributions will Allais harshly criticise the assumption that no economic activity takes place before equilibrium is reached (1971, p. 161; 1989, p. 342). Moreover, he will argue that Debreu's (1959) model is highly misleading owing to i) the treatment of uncertainty by means of the hypothesis of contingent markets, since it is a "serious distortion of the true nature of reality" (1971, p. 149); ii) the assumption of a given number of firms (1971, p. 152); iii) the assumption of general convexity of consumption and production sets (1971, p. 151-153; 1989, p. 343) since it neglects, e.g., the empirical fact of increasing returns to scale.

this correspondence can be *only* argued by *illegitimately* treating capital as a single factor of production measured in value terms. From this difficulty the approach cannot escape, it would seem; and the conclusion that a satisfactory explanation of prices and distribution needs the approach to be abandoned, and replaced by a different theory, seems to be unavoidable. That Allais has never pushed his critiques to this bitter end should not in the least obscure the fact that these criticisms are no doubt an important step in the direction of showing that something is clearly wrong with the neoclassical approach. And while some scholars, notably Profs. Garegnani and Petri, have been insisting on the issues here discussed for a long time, the fact that these critiques also come from one of the 'fathers', so to speak, of the method based on the notion of IGE, may contribute to strengthen their importance. I close then by quoting Paul Samuelson (1983, as quoted in Grandmont, 1989, p. 27), who argued that "Had Allais' earliest writings been in English, a generation of economic theory would have taken a different course".

APPENDIX: THE QUANTITY OF CAPITAL IN ALLAIS' (1947) OLG MODEL.

This appendix aims to show that the equations of Allais' OLG model (*Économie*, Annexe II) determine a *secular* stationary state.

A1. General setting.

- In each period, there are *2n* consumers (*n* young; *n* old); Each consumer lives for two periods. The aggregate endowment of labour is equal to X. Each young consumer is endowed with $\frac{X}{n}$ units of labour that are supplied inelastically at the wage rate x. Old people do not work. Stationary conditions prevail.

A2. Consumption side.

- All consumers have identical preferences, represented by a logarithmic additive utility function, which depends on the consumption of good A in the first period (A_0) and in the second period (A_1).
- Consumer's demands for A_0 and A_1 are derived from the following maximization problem: $\max: \mathrm{U}(\mathrm{A}_0; \mathrm{A}_1) = \ln(\mathrm{A}_0) + \alpha\ln(\mathrm{A}_1)\ \mathrm{s.t.}: R^i = \bar{\mathrm{a}}\mathrm{A}_0 + \frac{\bar{\mathrm{a}}}{1+i}\mathrm{A}_1$, where $\bar{a}$ stands for the price of good A, i for the rate of interest, $R^i \stackrel{\text{def}}{=} x\frac{X}{n}$ for the income of the single individual and $\alpha \in (0,1)$ represents individuals' preference for present consumption.

A3. Production side.

- There are two productive sectors. Sector 1 produces the consumption good (A) and sector 2 an "indirect good" (H), i.e. a circulating capital good.
- Good A is produced by labour and the capital good under a twice differentiable production function, homogeneous of degree one: $\bar{A} = f(X_A, H)$, where $\bar{A}$ stands for the supply of consumption good, and X_A and H respectively represent the quantity of labour and of the capital good used in the production of A.[32]
- The capital good is produced with the aid of labour only, under a twice-differentiable homogeneous of degree one production function $\bar{H} = g(X_H)$, where $\bar{H}$ stands for the supply of new capital goods and X_H is the quantity of labour used in the production of the capital good.

A4. Equilibrium equations.

[32] In some variants of the OLG model, Allais includes land as a second, non-produced factor of production. And Allais considers different possibilities for the distribution of land property rights, e.g. a case where land is equally distributed to the old generation, and then sold to the young, or a case in which land is publicly owned, and land rents are equally distributed to the young or, alternatively, to the old. Given the limited purposes of this appendix, all this is omitted here since the inclusion of land will not alter the main results, i.e. that Allais's equations determine a secular equilibrium. Allais also considers a variant of the model where there is one more variable to be considered, the amount of debt issued by the Government, which I also neglect.

The equilibrium conditions of the problem are given by the following equations:

$$A_0 = \frac{R}{\bar{a}(1+\alpha)} \quad \text{(A.1)}$$

$$A_1 = \frac{R}{\bar{a}(1+\alpha)}\alpha(1+i) \quad \text{(A.2)}$$

$$A_0 + A_1 = A = \frac{R}{\bar{a}(1+\alpha)}[1+\alpha(1+i)] \quad \text{(A.3)}$$

$$\bar{A} = f(X_A, H) \quad \text{(A.4)}$$

$$\bar{a}\bar{A}'_x = x \quad \text{(A.5)}$$

$$\bar{a}\bar{A}'_H = h \quad \text{(A.6)}$$

$$\bar{H} = g(X_H) \quad \text{(A.7)}$$

$$\bar{h}\bar{H}'_X = x \quad \text{(A.8)}$$

$$1+i = \frac{h}{\bar{h}} \quad \text{(A.9)}$$

$$X_A + X_H = X \quad \text{(A.10)}$$

$$H = \bar{H} \quad \text{(A.11)}$$

$$A = \bar{A} \quad \text{(A.12)}$$

Equations (A.1)-(A.2) stand for the aggregate demand for the consumption good of the young (A_0) and the old (A_1) respectively, and $R = \sum_{i=1}^{n} R^i = xX$ stands for the aggregate income of the young; equation (A.3) is the total demand for the consumption good A. Equations (A.4) and (A.7) respectively represent the supplies of the consumption good ($\bar{A}$) and of the new capital goods produced ($\bar{H}$); equations (A.5) and (A.6) respectively equalize, in the consumption-good sector, the wage rate (x) with the value marginal product of labour ($\bar{a}\bar{A}'_x$)($\bar{a}$ being the selling price of good A), the rental price of the capital good (h) with its value marginal product ($\bar{a}\bar{A}'_H$); equation (A.8) equalizes, in the capital good industry, the wage rate with the marginal product of labour used in the capital-good industry ($\bar{h}\bar{H}'_x$); equation (A.9) establishes that, in equilibrium, the rate of interest (i) must be equal to the ratio between the rental price of the circulating capital goods (h) and their selling price ($\bar{h}$); equations (A.10)-(A.12) establish the market clearing conditions in the labour market, in the capital-good market and in the consumption-good market respectively. Equations (A.1)-(A.12) thus define a system in 12 equations[33] to determine 13 unknowns:

$$\begin{cases} prices: \bar{a}, h, \bar{h}, x, i \\ quantities: A_o, A_1, A, H, X_A, X_H, \bar{A}, \bar{H} \end{cases}$$

[33] Of them only 11 are independent, since (A3) is equal to (A1)+(A2). We thus have 11 independent equations in 12 unknowns since once A_0 and A_1 are determined A is determined as well.

We thus determine relative prices by setting $\bar{a} = 1$. In Allais's model, the equations of equilibrium are numbered by (4)-(5)-(9)-(21)-(22)-(23), 2 equations (24), equations (25)-(28) and 2 equations (31), which respectively correspond to my equations (A.4)-(A.7)-(A.10)-(A.1)-(A.2)-(A.3)-(A.5)-(A.6)-(A.8)-(A.9)- (A.11)-(A.12).

Notice that the assumption of homogeneity of degree 1 of the production functions *f(.)* and *g(.)* allows us to derive the selling price=cost conditions. For instance, we know that $\bar{A} = \overline{A'}_{X_A} X_A + \overline{A'}_H H$; hence, by multiplying equation (A.5) by X_A, equation (A.6) by H and adding, we obtain:

$$\bar{a}\bar{A} = xX_A + hH \tag{A.13}$$

Analogously, given that $\overline{H} = \overline{H'}_{X_H} X_H$, by multiplying equation (A.8) by X_H we obtain,

$$h\overline{H} = xX_H \tag{A.14}$$

A5. The stationary assumption and the quantity of capital.

Note first that conditions (A.4) and (A.11) already show that the economy is stationary because the demand for the endowments of capital goods (H) that are used as inputs in the consumption good sector, and hence already produced in the previous period, is equal to the new capital goods that are being currently produced, $\overline{H}$. In order to shed some further light on this issue we could replace equation (A.11) by three equations (A.11') and two additional unknowns, H^E and H^I, that would explicitly represent, respectively, the endowment of existing capital goods and the investment demand for new capital goods.

$$\begin{cases} H^E = H \\ H^I = H^E \\ H^I = \overline{H} \end{cases} \tag{A.11'}$$

The first of equations (A.11') equalizes the endowment of existing capital goods with their forthcoming demand in the consumption-good sector; the second equation explicitly establishes stationariness, since the demand for new capital goods (H^I) must be equal to the existing endowments; finally, the last equation establishes the market clearing condition for the production of *new* capital goods. Of course, the set of equations (A.11') can be reduced to (A.11); their usefulness is that they make explicit the stationary character of the economy. These conditions however, are also compatible with the traditional notion of *static* stationary state.

To show that in Allais' OLG model the endowment of capital in value terms is not included among the givens but is rather a *resultant* of the adjustment process, i.e. it determines a *secular* equilibrium state, I closely follow Fratini (2007: section V). But first there is a further aspect needing for discussion. Because only one capital good is produced in the model, it is still possible to determine a static stationary equilibrium –where, *trivially*, the return on the supply prices of the new capital goods will be the same- without having recourse to a value specification of the factor capital: the physical endowment (H^E) of the unique capital good could be taken as given and specified in its *own* technical units, so it is not immediately explicit why Allais does not have recourse to the static stationary equilibrium.[34] In order to avoid this pathological case, which only emerges owing to the very special assumptions of Allais' model, one should only modify the original model by assuming, e.g., that the consumption good is produced by the aid of labour and *two* capital goods, in turn produced by unassisted labour. Then to show that the equilibrium determined in this modified model is a secular equilibrium, and to make the relevant modifications to transform it into a static equilibrium, one must follow the same procedure as the one we will develop below.

[34] Suppose in fact that in equations (A.11') H^E is no longer an unknown but is given. We would lose one variable. However, once the rental price (h) and the selling price of the capital goods ($\bar{h}$) are given, condition (A9) will be satisfied, and since there is only one *kind* of capital goods, then all existing capital goods will trivially yield the same return on their supply prices without any need to specify the capital endowment in value terms (we may drop then equation A.20' below)

Note first that the supply of savings is only implicitly determined because the budget constraint of the individual is written in its intertemporal form. Aggregate savings (*S*) are thus determined by the difference between aggregate income in the first period (R) and the aggregate value of the first period consumption (A_0). That is, $S = R - \bar{a}A_0$. Therefore:

$$S = R\frac{\alpha}{1+\alpha} \tag{A.15}$$

Let us now introduce equation (A.9) into (A.13) and add equation (A.14). Recall that equilibrium implies: $X_A + X_H = X$ and that $H = \bar{H}$. We thus have:

$$\bar{a}\bar{A} + \bar{h}\bar{H} = xX + (1+i)\bar{h}\bar{H} \tag{A.16}$$

Equation (A.16) says that in equilibrium, gross production, GP (left hand side), is equal to the full-employment level of gross income, GI (right hand side). Now, from the individuals' budget constraint of each period, we known that the full-employment level of gross income is devoted to consumption demand –both of the young and the old- and gross savings.

$$GI = \bar{a}A_0 + \bar{a}A_1 + \mathrm{S} \tag{A.17}$$

This means that,

$$\bar{a}\bar{A} + \bar{h}\bar{H} = \bar{a}A_0 + \bar{a}A_1 + S \tag{A.18}$$

Therefore, recalling that aggregate demand, A, is equal to $A_0 + A_1$, we have that

$$\bar{a}(\bar{A} - A) = S - \bar{h}\bar{H} \tag{A.19}$$

Equation (A.19) implies that, in equilibrium, full-employment gross savings are equal to the gross production of capital goods, which is equal to investment demand. Moreover, because of (A.19), equation (A.12) is satisfied *if and only if* the condition

$$S = \bar{h}\bar{H} \qquad \text{(A.20)}$$

is satisfied. We can thus substitute condition (A.20) for condition (A.12) *without* altering the significance of the system. Notice that the equilibrium determined is a s*ecular stationary equilibrium* because the quantity of capital in value terms is *endogenously* determined by condition (A.20), which implies zero net accumulation in equilibrium. To determine instead a *static* stationary equilibrium we can replace the value of gross savings by an *exogenously given quantity of capital*. We change equation (A.20) by

$$K^* = \bar{h}\bar{H} \qquad \text{(A.20')}$$

where K^* stands for the given endowment of capital in value terms.[35] Under this new specification, net savings are *assumed* to be zero (whatever prices and distribution)[36]; let me insist, a legitimate approximation due to the slowness of capital accumulation.

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[35] Although we are replacing a stock (K^*) for a flow, gross savings, given that all capital is circulating capital both magnitudes coincide.

[36] As to the distribution of the capital endowment, we may for instance assume that each young individual is endowed with a fraction K^*/n.

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